

# Supplemental Figures for *On the comparative utility of entropic learning versus deep learning for long-range ENSO prediction*

Michael Groom,<sup>\*</sup> Davide Bassetti,<sup>†</sup> Illia Horenko<sup>‡</sup>  
and Terence J. O’Kane<sup>§</sup>

## 1 Additional Plots

This section contains additional plots referenced throughout the paper.

### 1.1 ACCESS-OM2 composites

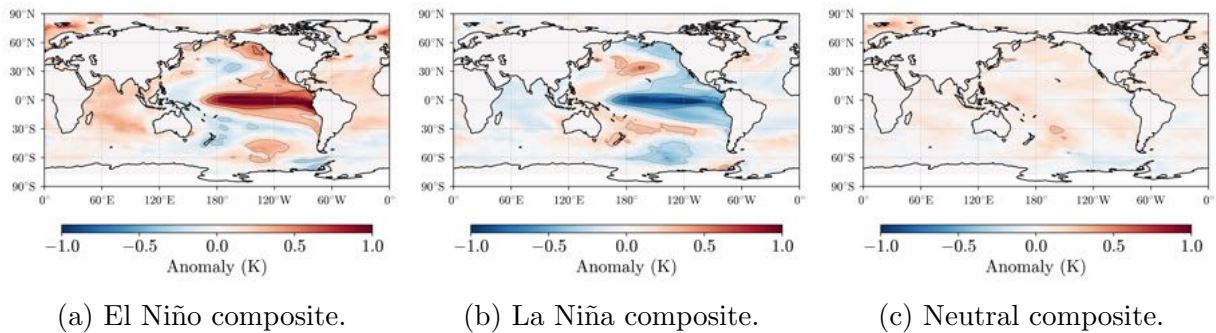


Figure 1: ACCESS-OM2 SST composite images. Black solid/dashed contour lines enclose an anomaly of  $\pm 0.5^\circ$  and grey solid/dashed lines enclose an anomaly of  $\pm 0.25^\circ$ .

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<sup>\*</sup>CSIRO Environment, Eveleigh, NSW 2015, Australia. Email: michael.groom@csiro.au (Corresponding Author)

<sup>†</sup>Faculty of Mathematics, Rheinland-Pfälzische Technische Universität Kaiserslautern Landau, Kaiserslautern 67663, Germany

<sup>‡</sup>Chair for Mathematics of AI, Rheinland-Pfälzische Technische Universität Kaiserslautern Landau, Kaiserslautern 67663, Germany

<sup>§</sup>CSIRO Environment, Battery Point, TAS 7004, Australia

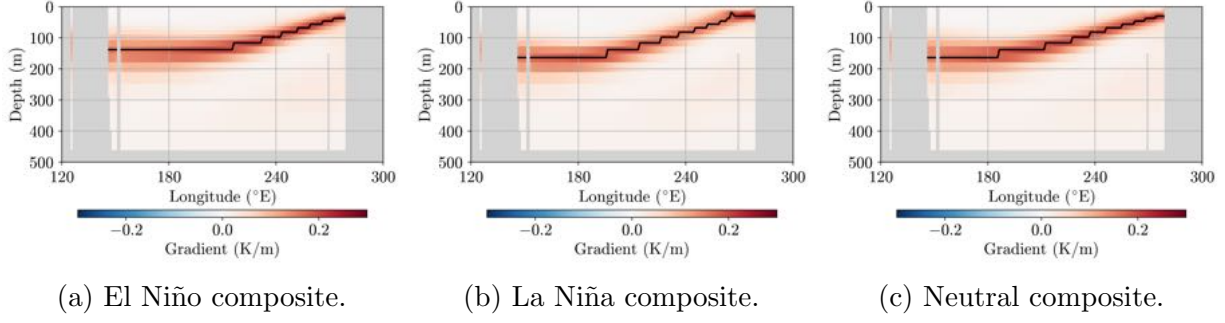


Figure 2: ACCESS-OM2  $dT/dz$  composite images. The black line indicates the depth of maximum  $dT/dz$  at each longitude.

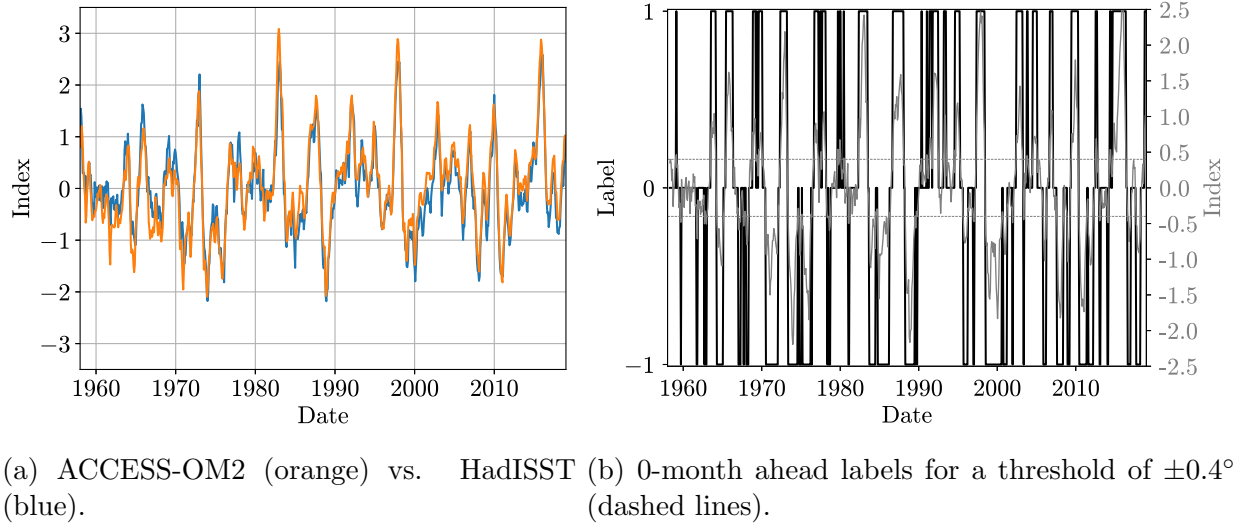
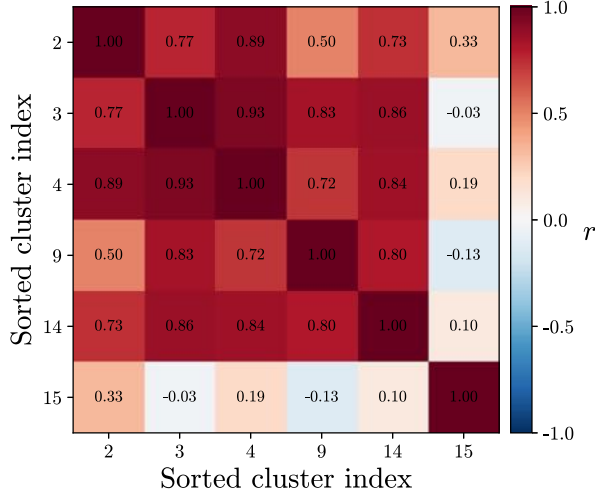
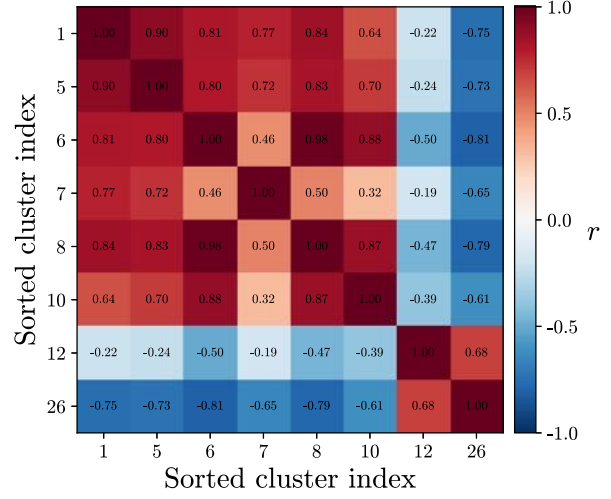


Figure 3: Niño 3.4 indices and corresponding labels.

## 1.2 Pattern Correlations

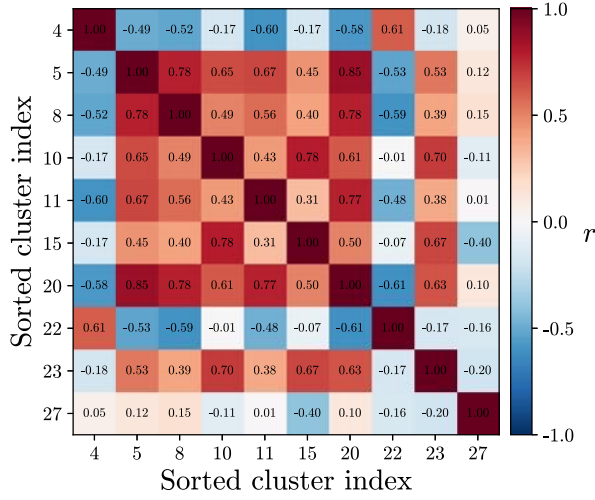


(a) El Niño pattern correlations.

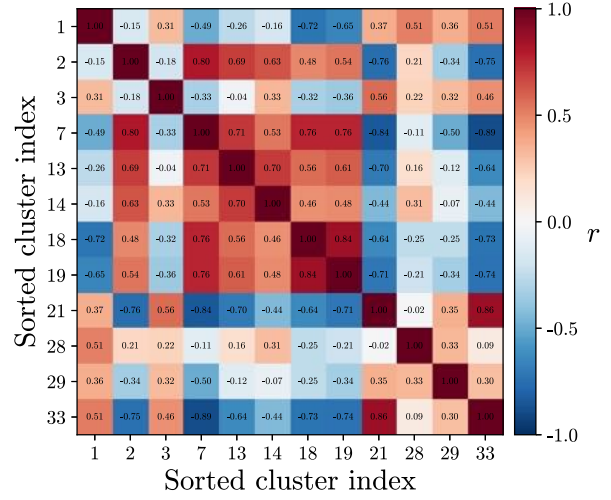


(b) La Niña pattern correlations.

Figure 4: Pattern correlations for 3 months lead time.

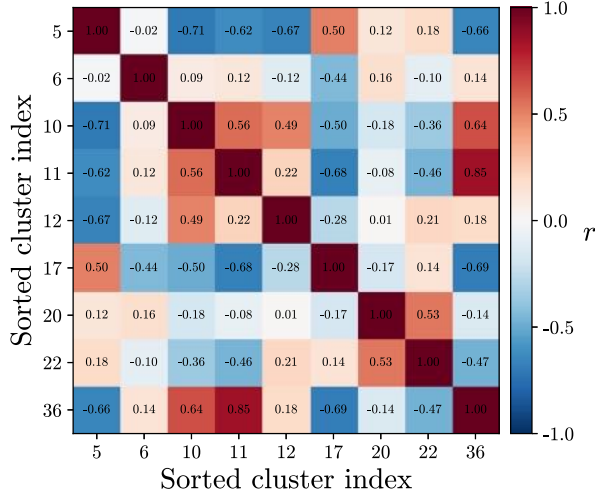


(a) El Niño pattern correlations.

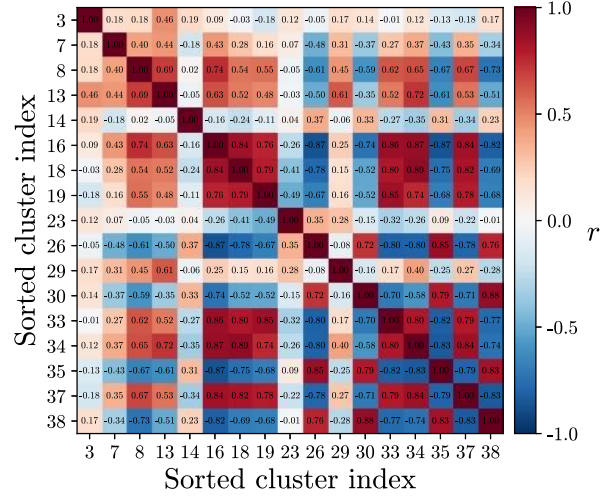


(b) La Niña pattern correlations.

Figure 5: Pattern correlations for 6 months lead time.

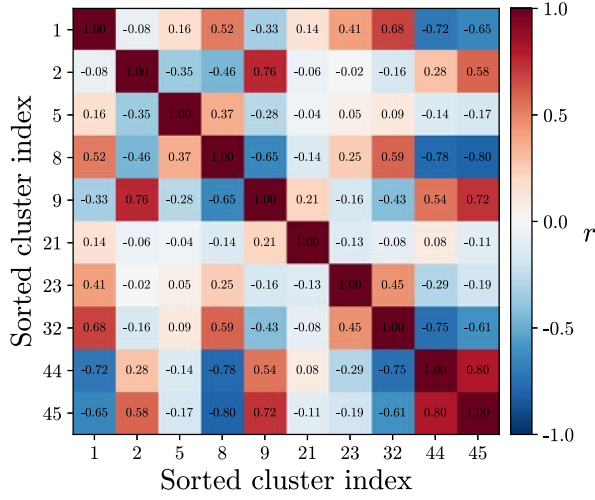


(a) El Niño pattern correlations.

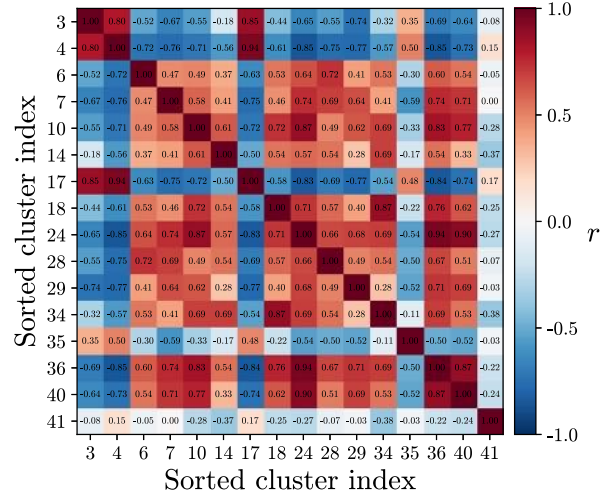


(b) La Niña pattern correlations.

Figure 6: Pattern correlations for 9 months lead time.



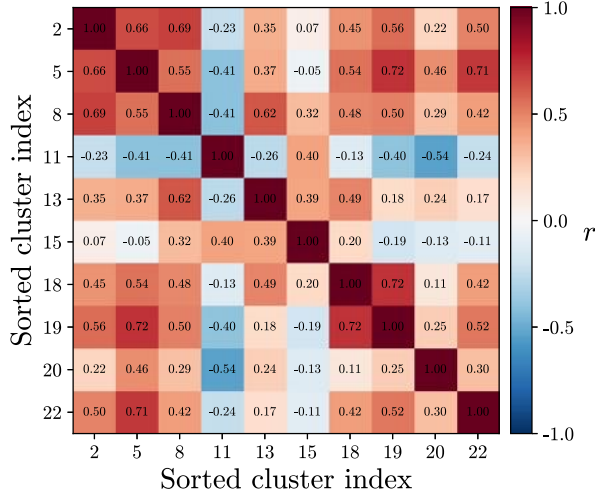
(a) El Niño pattern correlations.



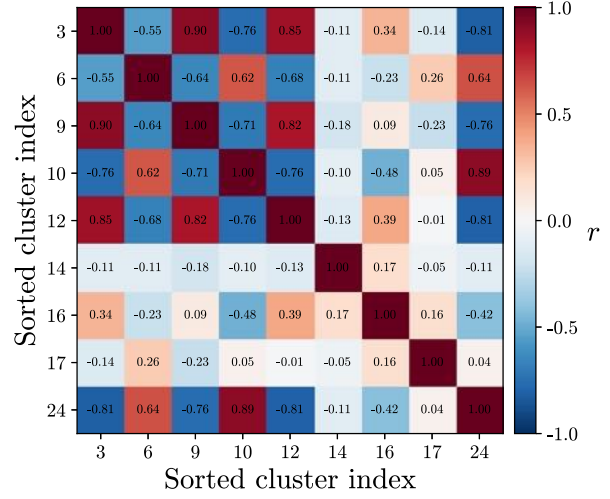
(b) La Niña pattern correlations.

Figure 7: Pattern correlations for 12 months lead time.



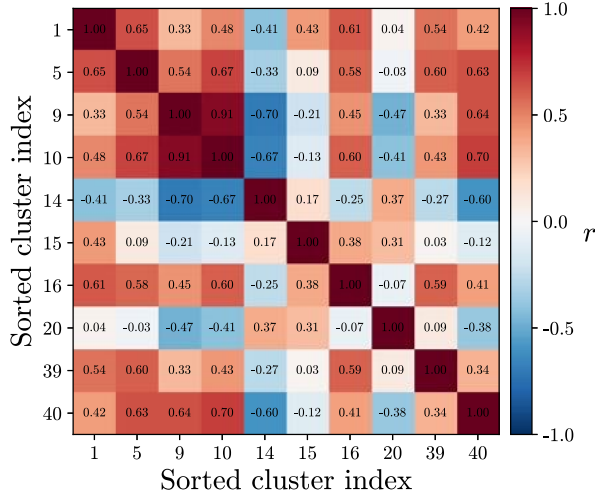


(a) El Niño pattern correlations.

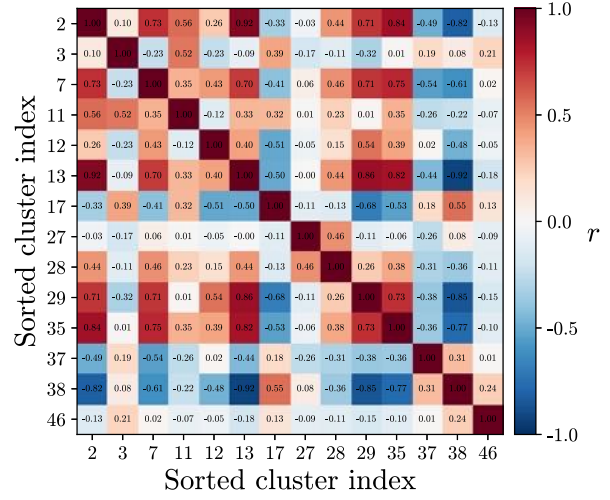


(b) La Niña pattern correlations.

Figure 8: Pattern correlations for 18 months lead time.



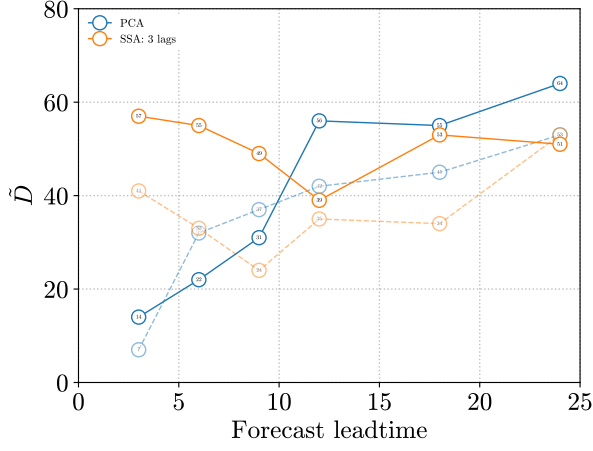
(a) El Niño pattern correlations.



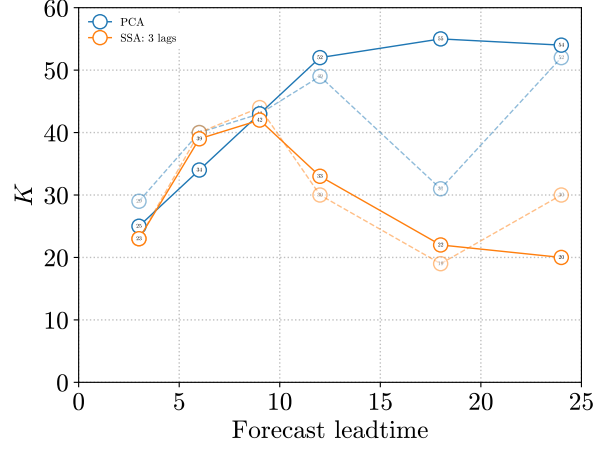
(b) La Niña pattern correlations.

Figure 9: Pattern correlations for 24 months lead time.

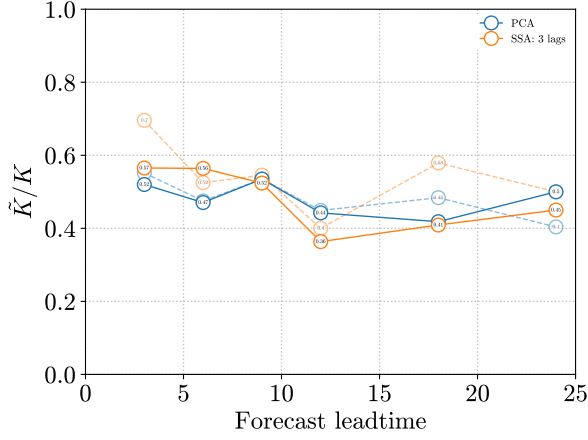
### 1.3 Parsimony Plots



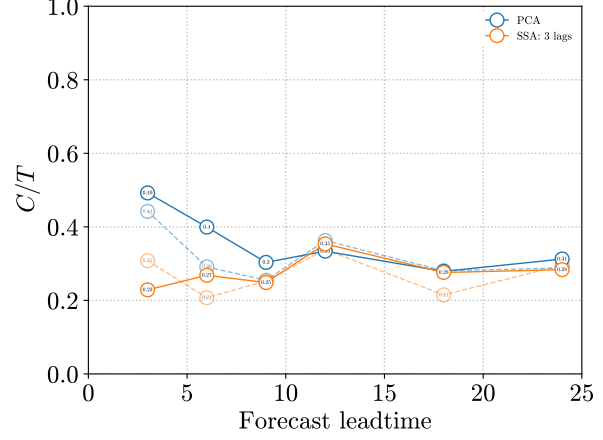
(a) Number of important features.



(b) Number of clusters.

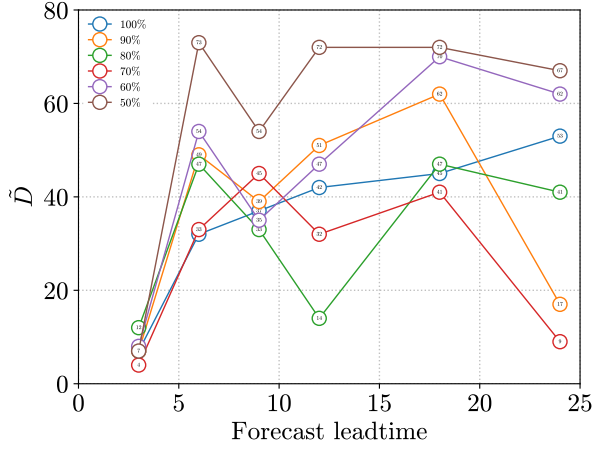


(c) Ratio of significant clusters.

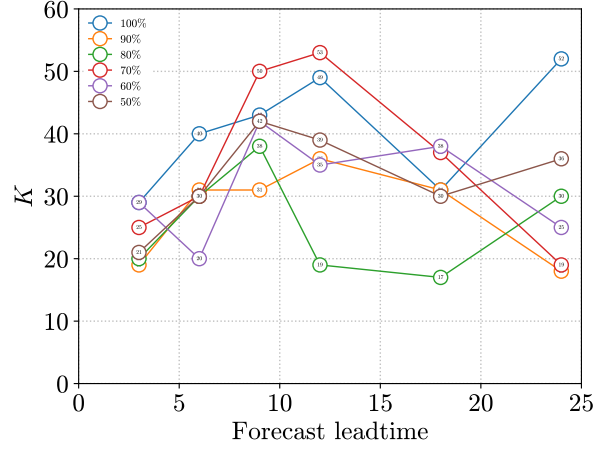


(d) Number of switches between clusters.

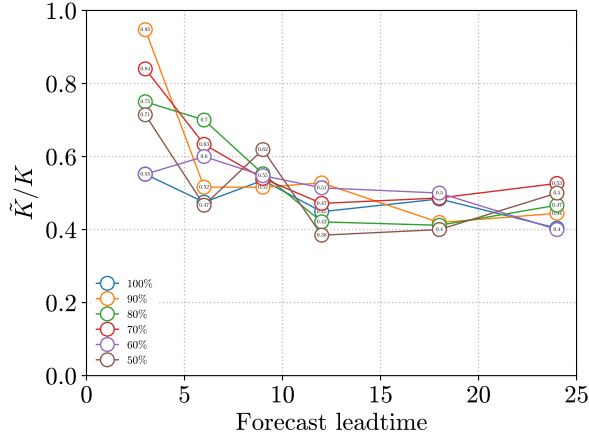
Figure 10: Effects of filtered labels on measures of parsimony. Results for unfiltered labels are given by dashed lines.



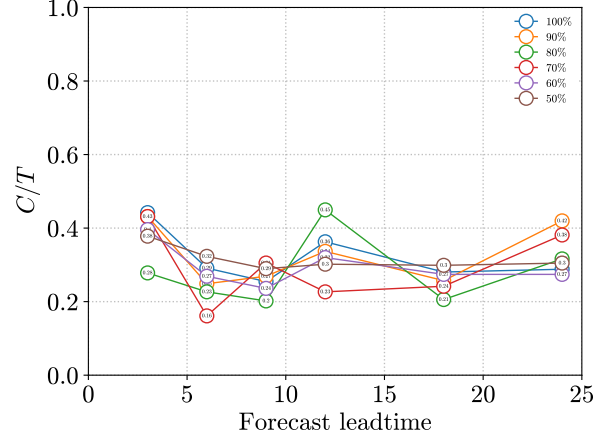
(a) Number of important features.



(b) Number of clusters.

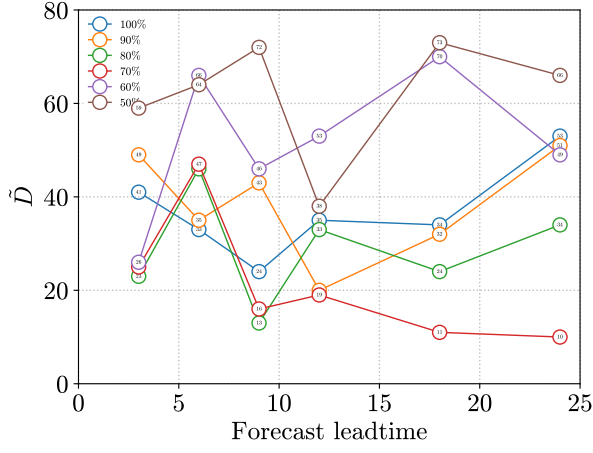


(c) Ratio of significant clusters.

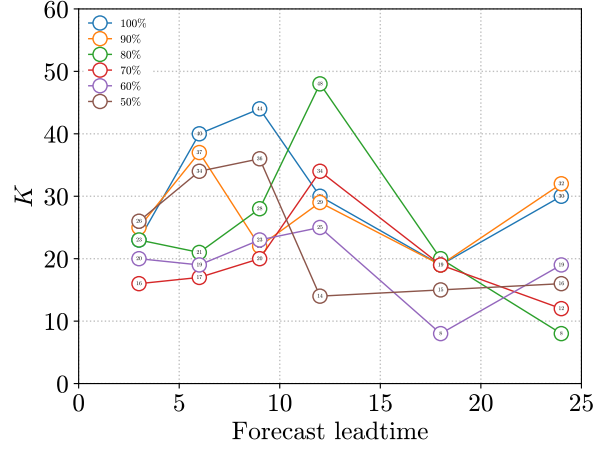


(d) Number of switches between clusters.

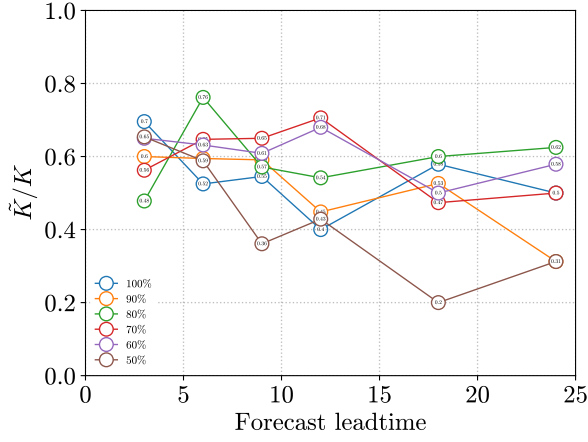
Figure 11: Effects of amount of training data on measures of parsimony for PCA. The legend refers to the amount of data retained relative to the baseline dataset.



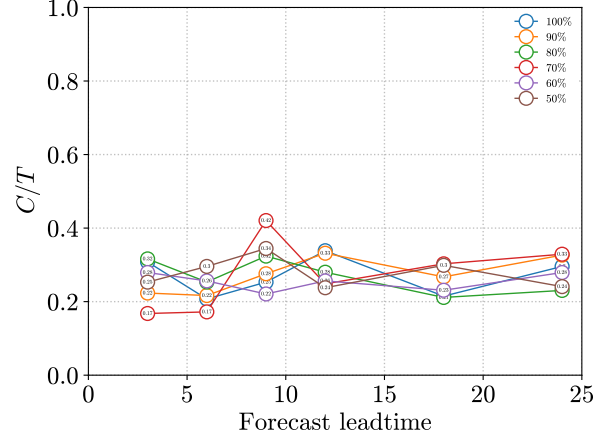
(a) Number of important features.



(b) Number of clusters.



(c) Ratio of significant clusters.



(d) Number of switches between clusters.

Figure 12: Effects of amount of training data on measures of parsimony for SSA: 3 lags. The legend refers to the amount of data retained relative to the baseline dataset.

## 2 ACCESS-OM2 PCs and EOFs

### 2.1 PCA

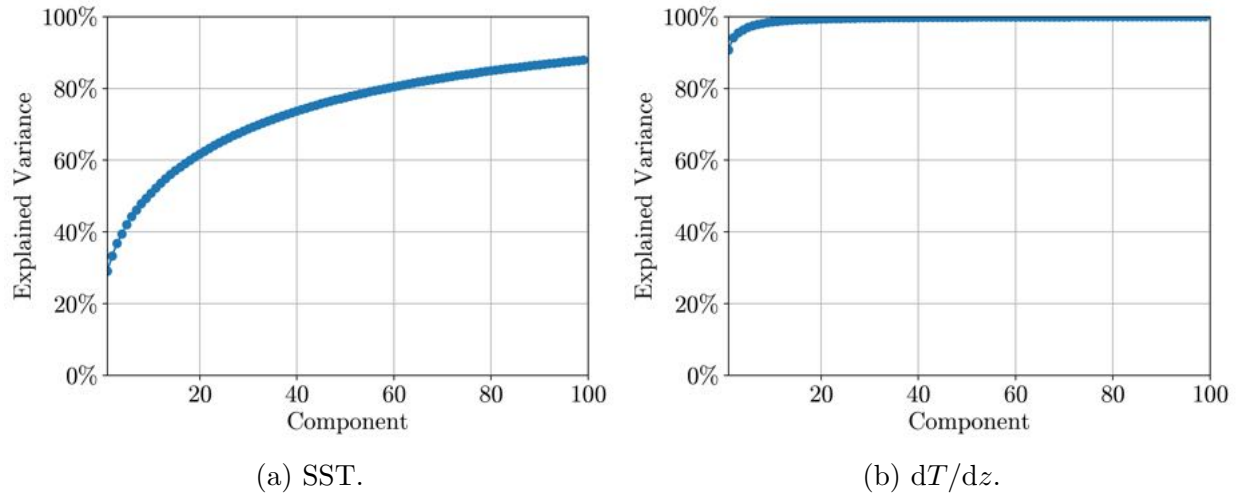


Figure 13: Cumulative explained variance of EOFs from PCA.

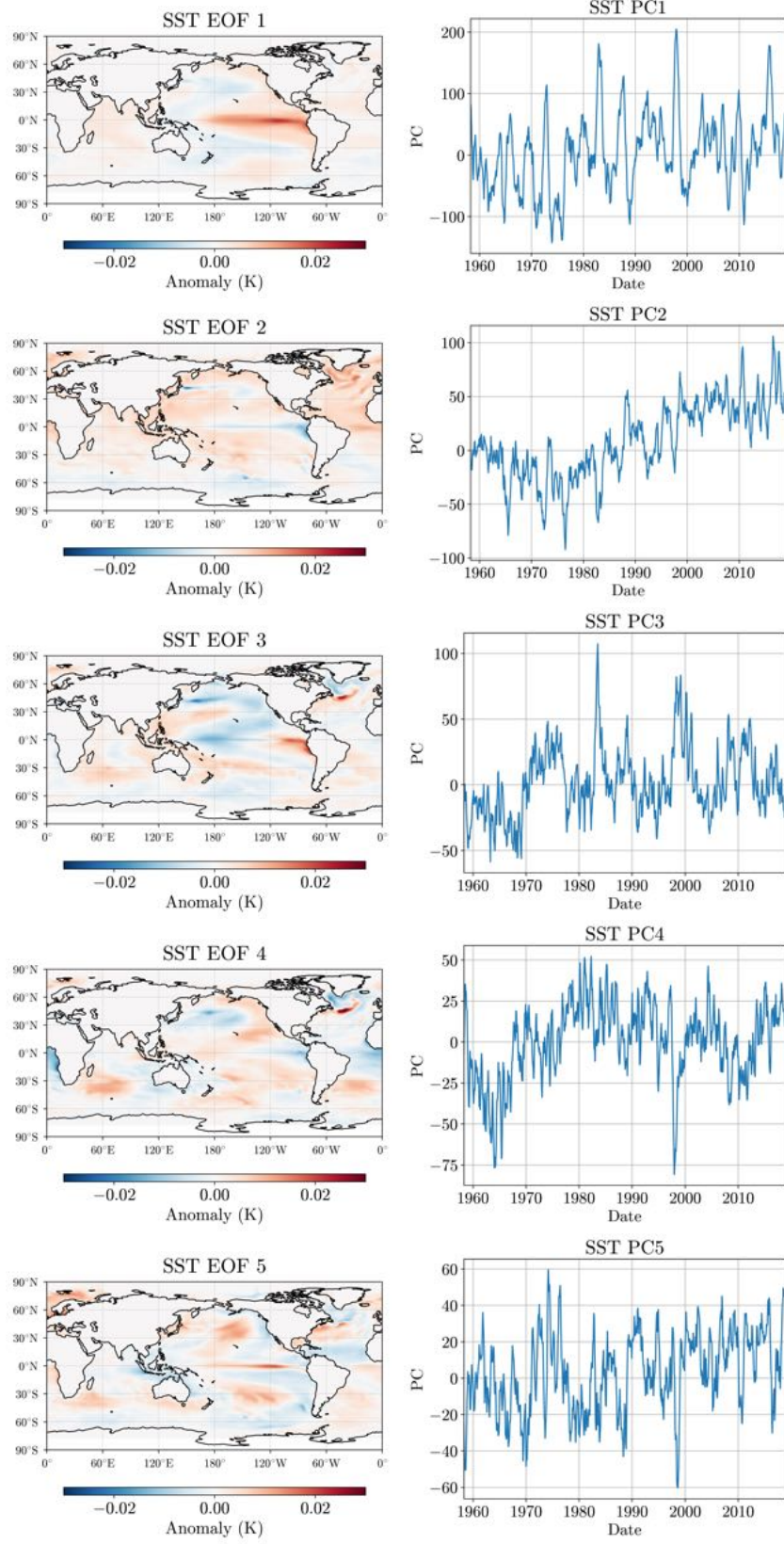


Figure 14: ACCESS-OM2 SST EOFs 1-5 and corresponding PCs for PCA.



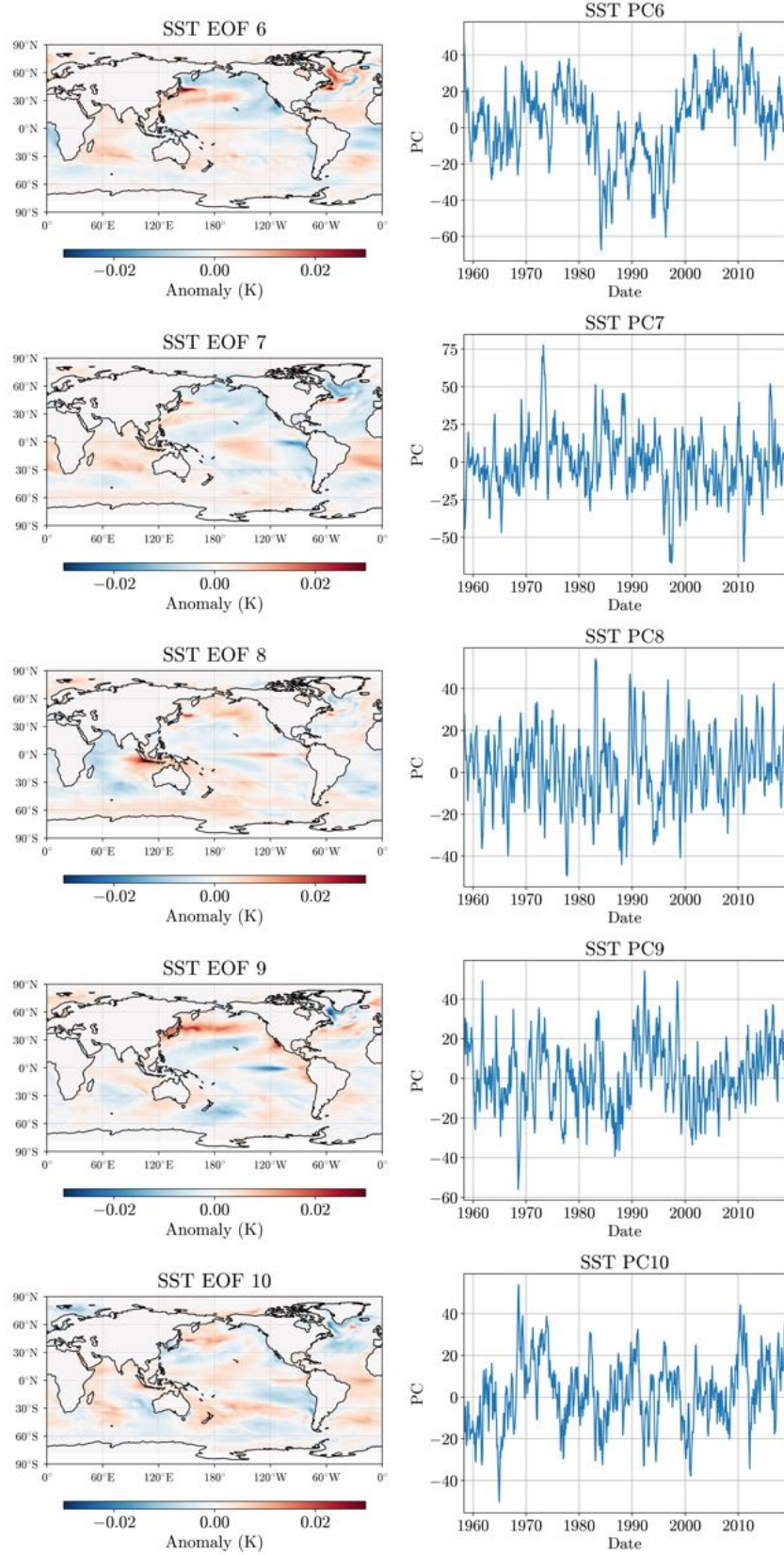


Figure 15: ACCESS-OM2 SST EOFs 6-10 and corresponding PCs for PCA.

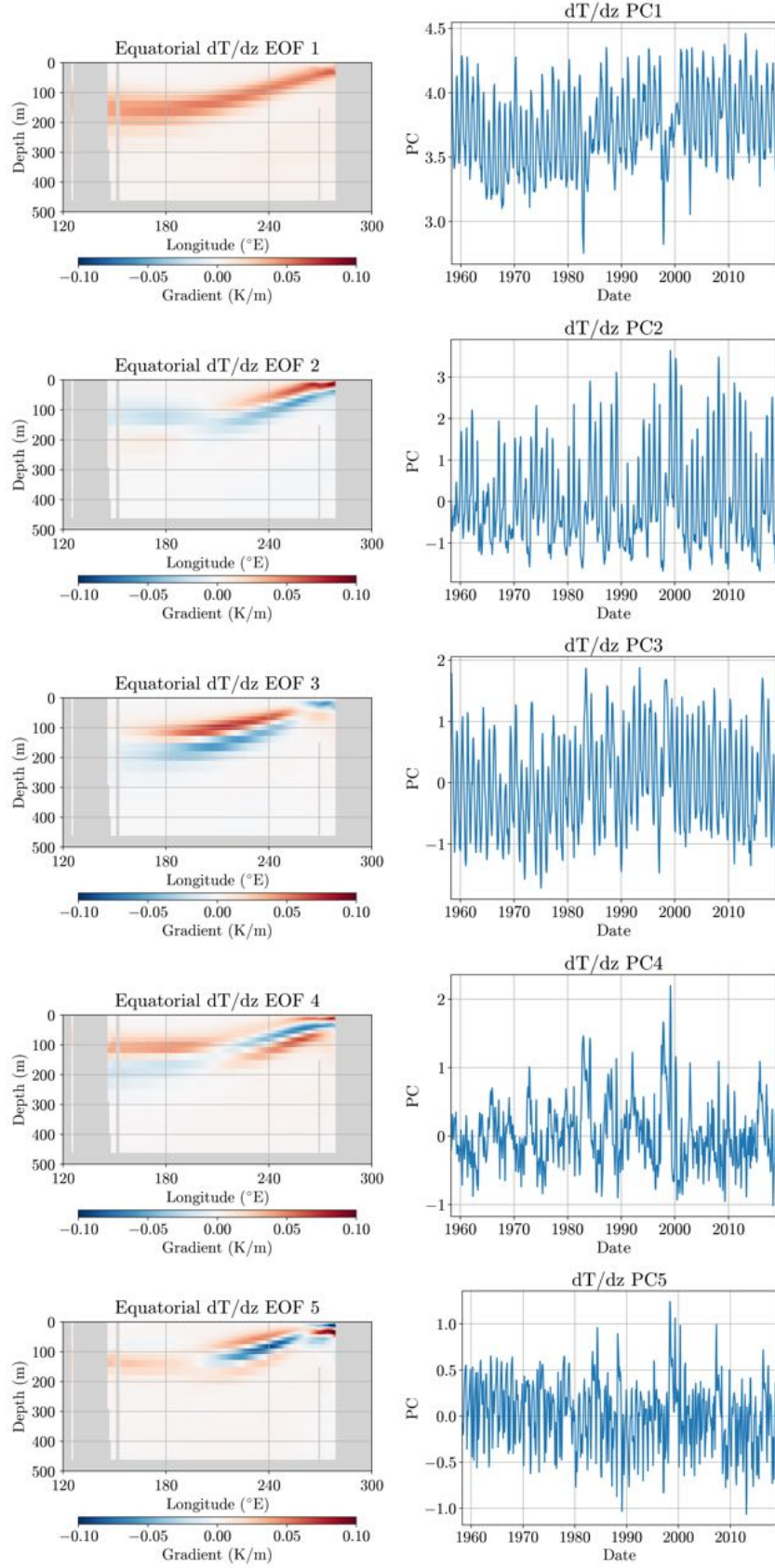


Figure 16: ACCESS-OM2  $dT/dz$  EOFs 1-5 and corresponding PCs for PCA.

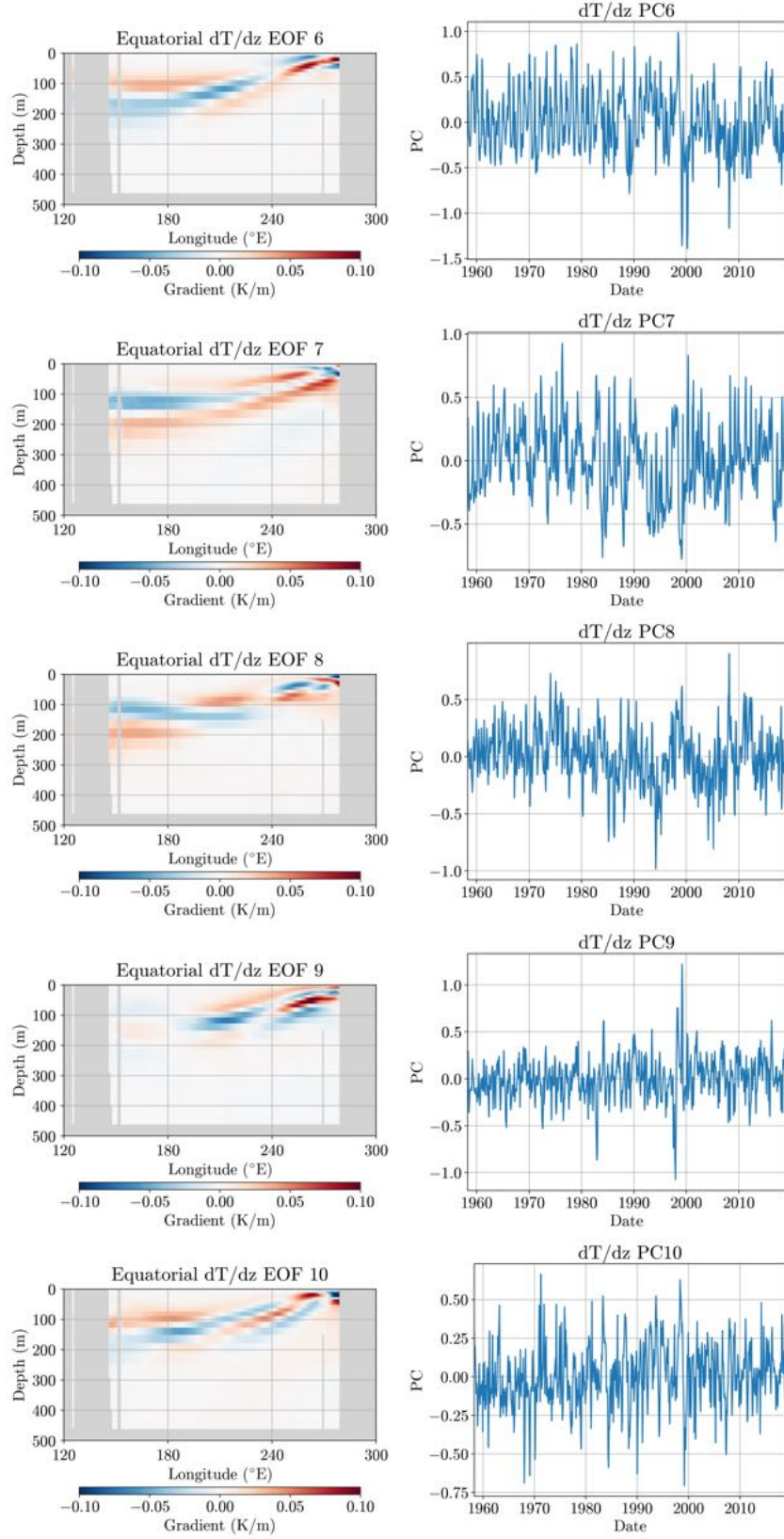
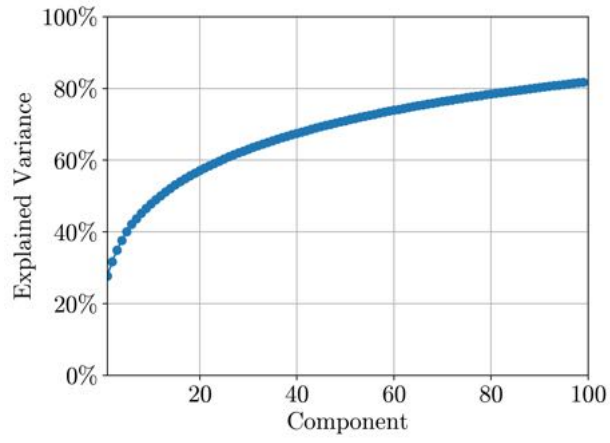
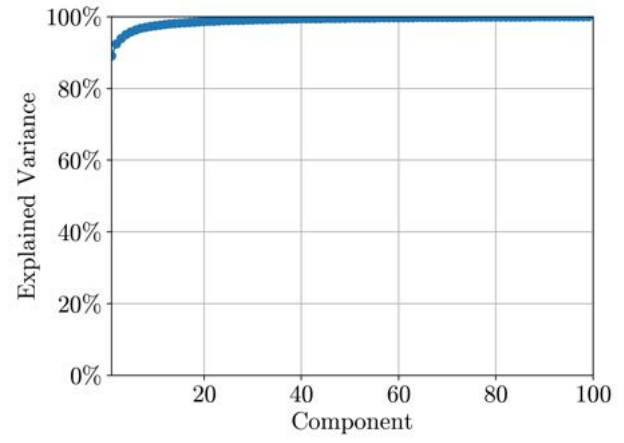


Figure 17: ACCESS-OM2  $dT/dz$  EOFs 6-10 and corresponding PCs for PCA.

## 2.2 SSA



(a) SST.



(b)  $dT/dz$ .

Figure 18: Cumulative explained variance of EOFs from SSA.



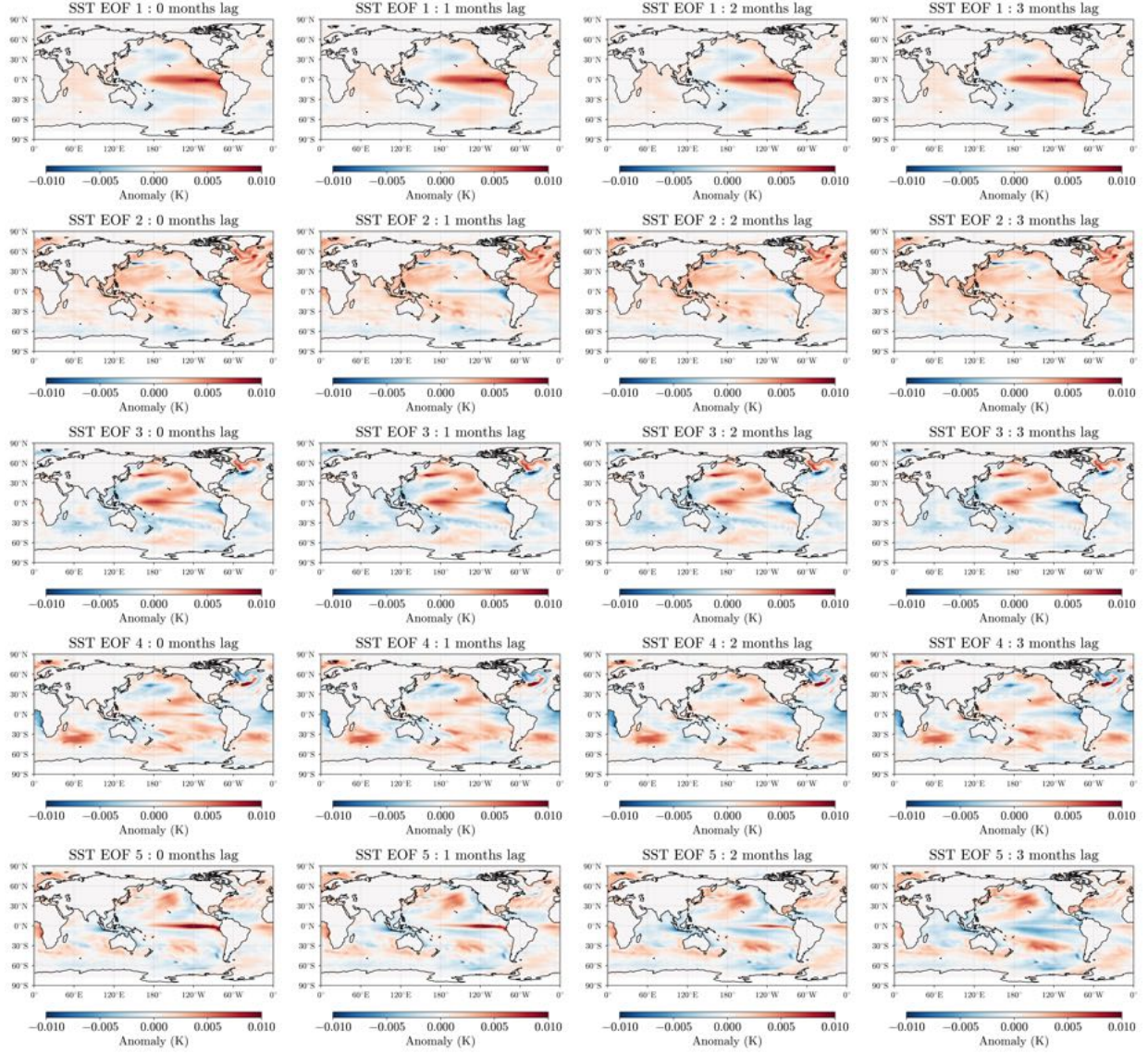


Figure 19: ACCESS-OM2 SST EOFs 1-5 for SSA.

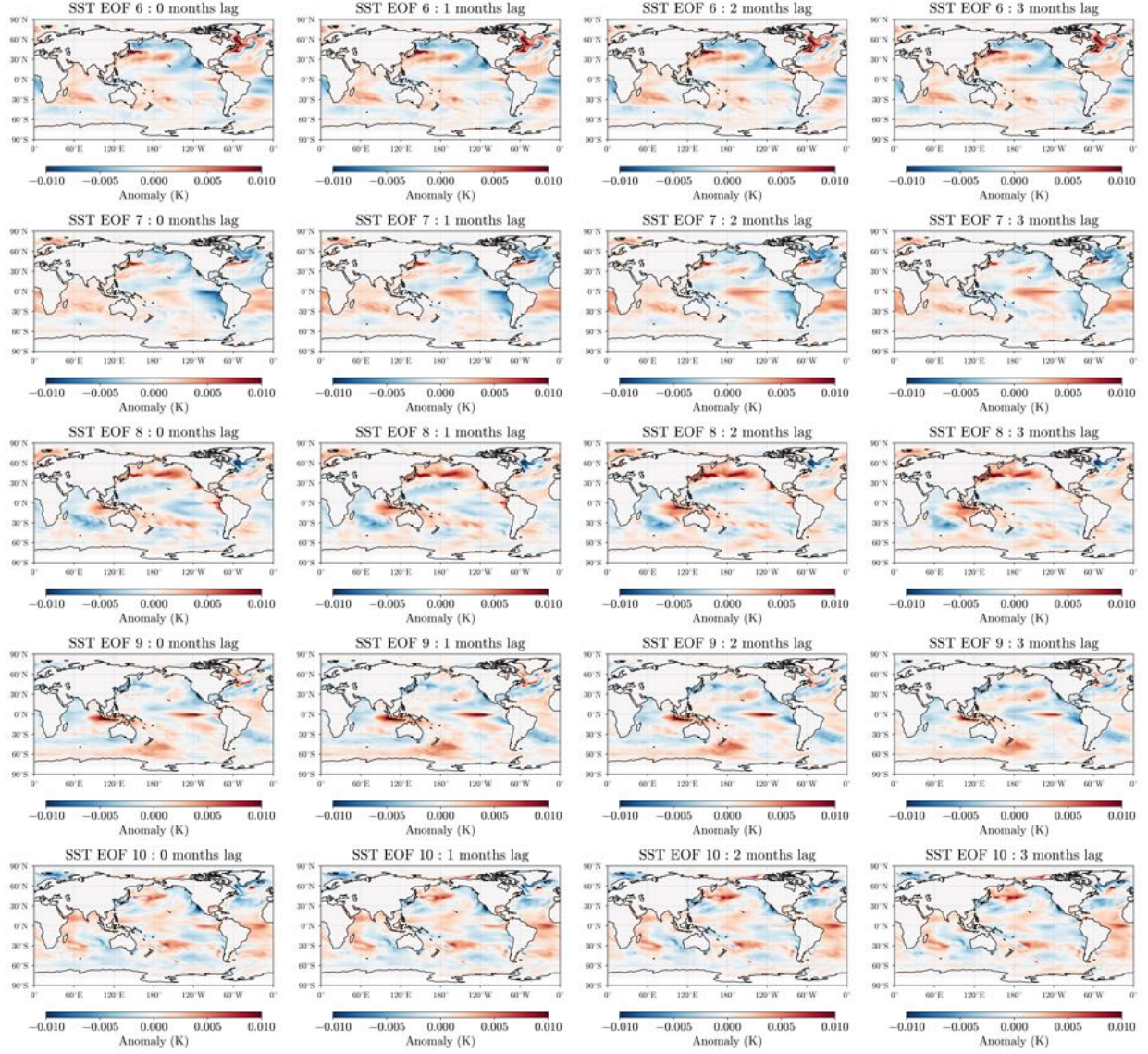


Figure 20: ACCESS-OM2 SST EOFs 6-10 for SSA.



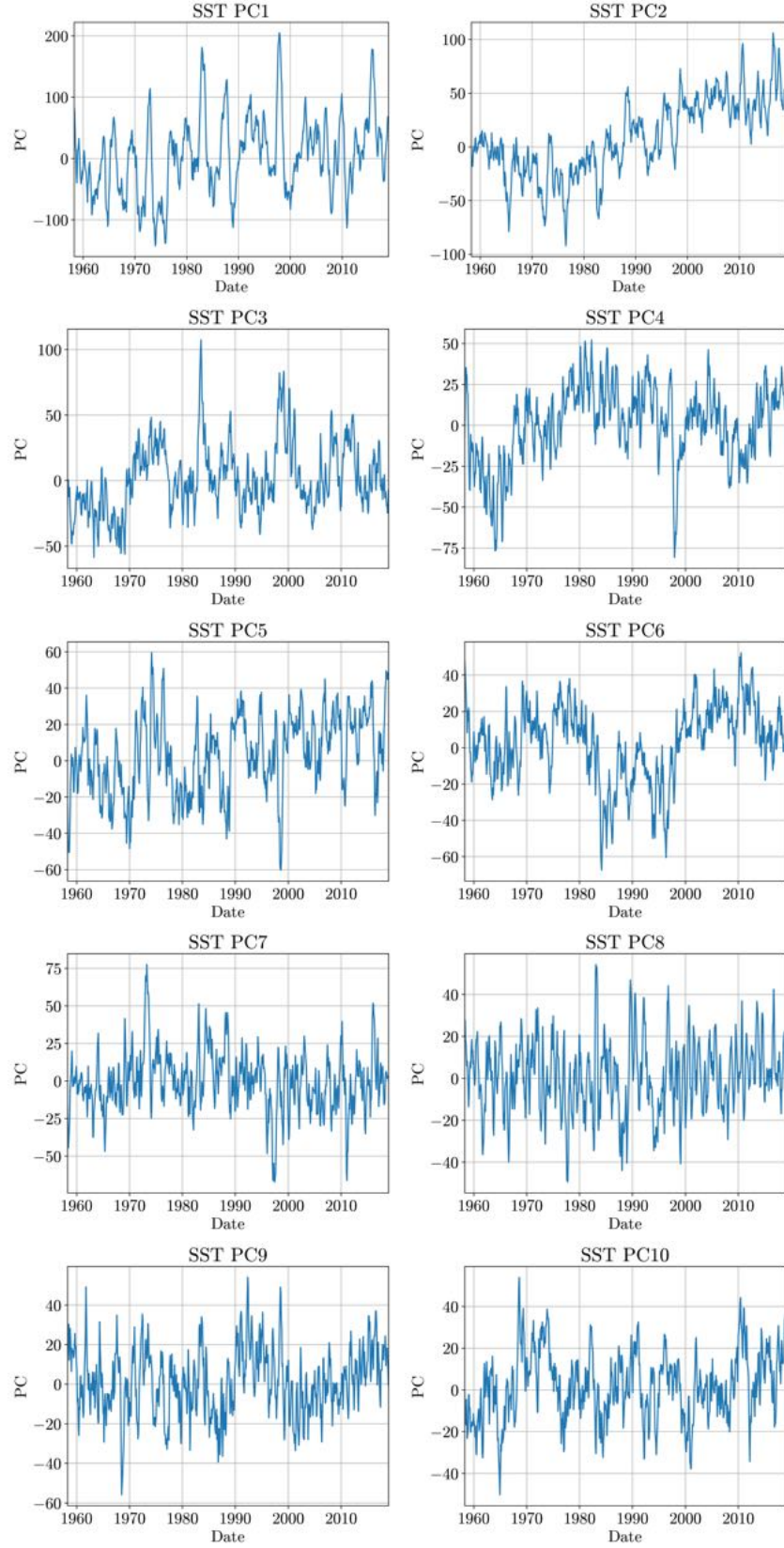


Figure 21: ACCESS-OM2 SST PCs 1-10 for SSA.

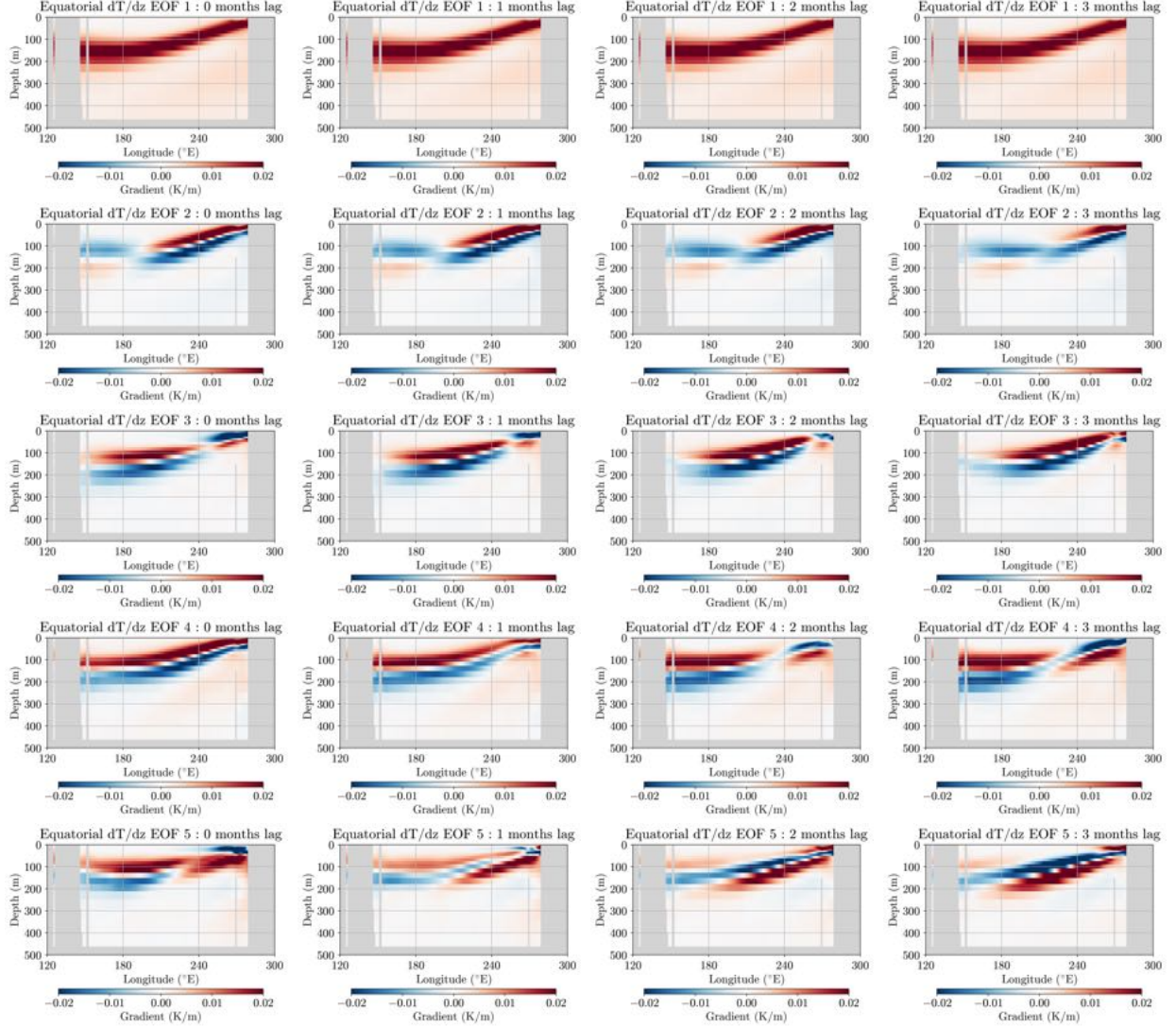


Figure 22: ACCESS-OM2  $dT/dz$  EOFs 1-5 for SSA.

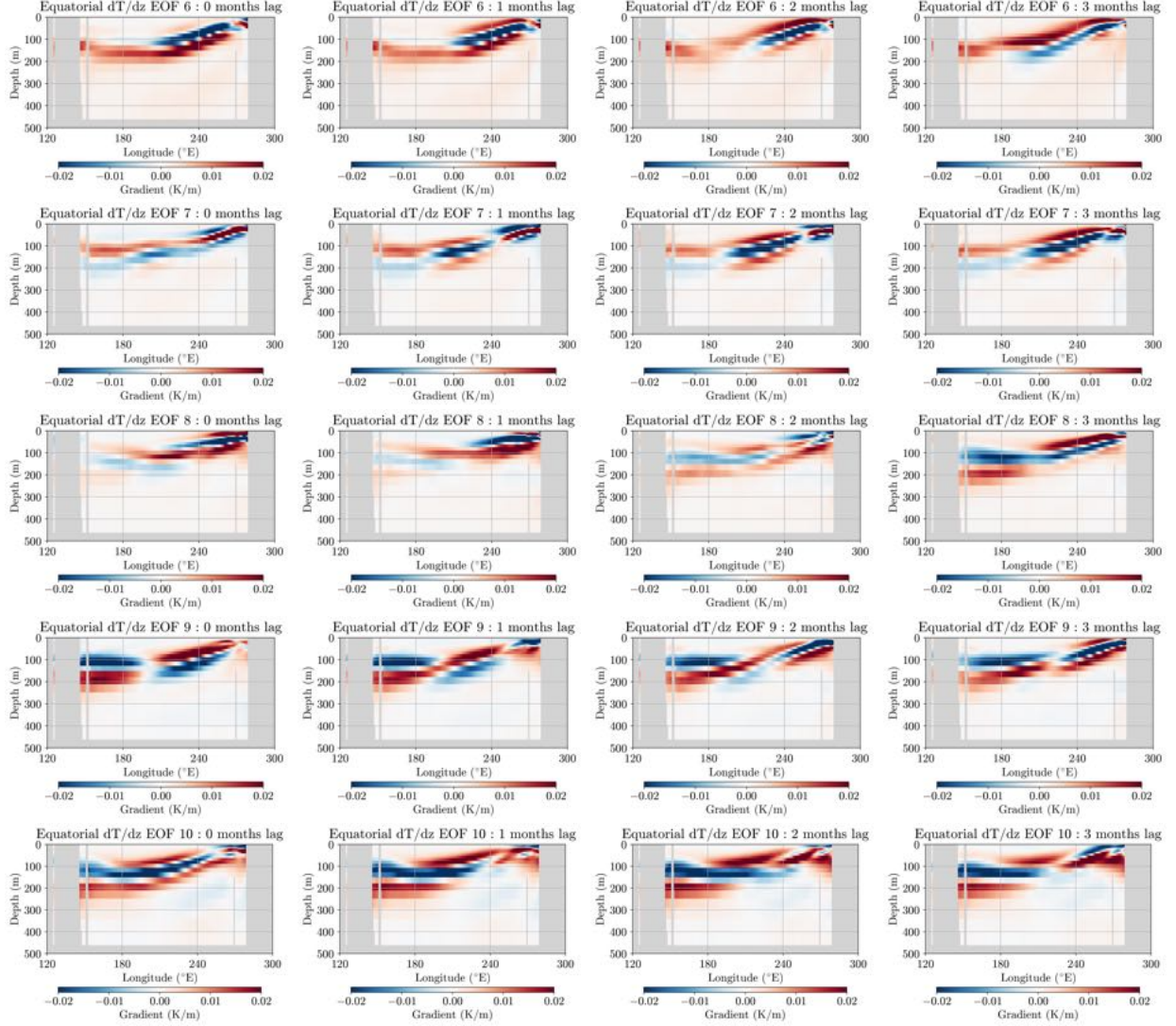


Figure 23: ACCESS-OM2  $dT/dz$  EOFs 6-10 for SSA.

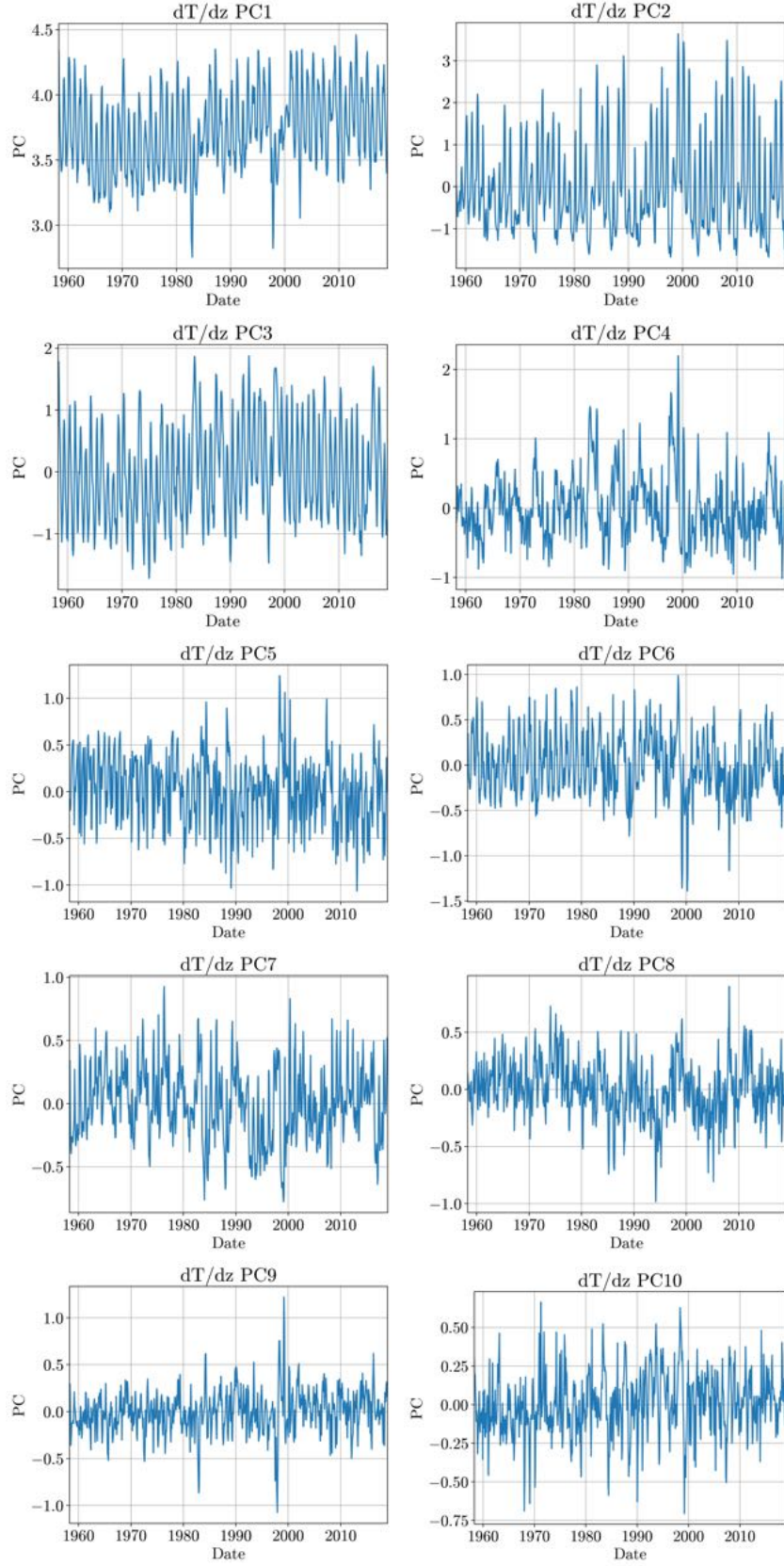


Figure 24: ACCESS-OM2  $dT/dz$  PCs 1-10 for SSA.

### 3 Test set predictions

Red circles indicate El Niño events, blue circles indicate La Niña events and white circles indicate neutral events. The Niño3.4 index (grey lines) and corresponding labels (black lines) for the target date are also shown.

#### 3.1 PCA: 100%

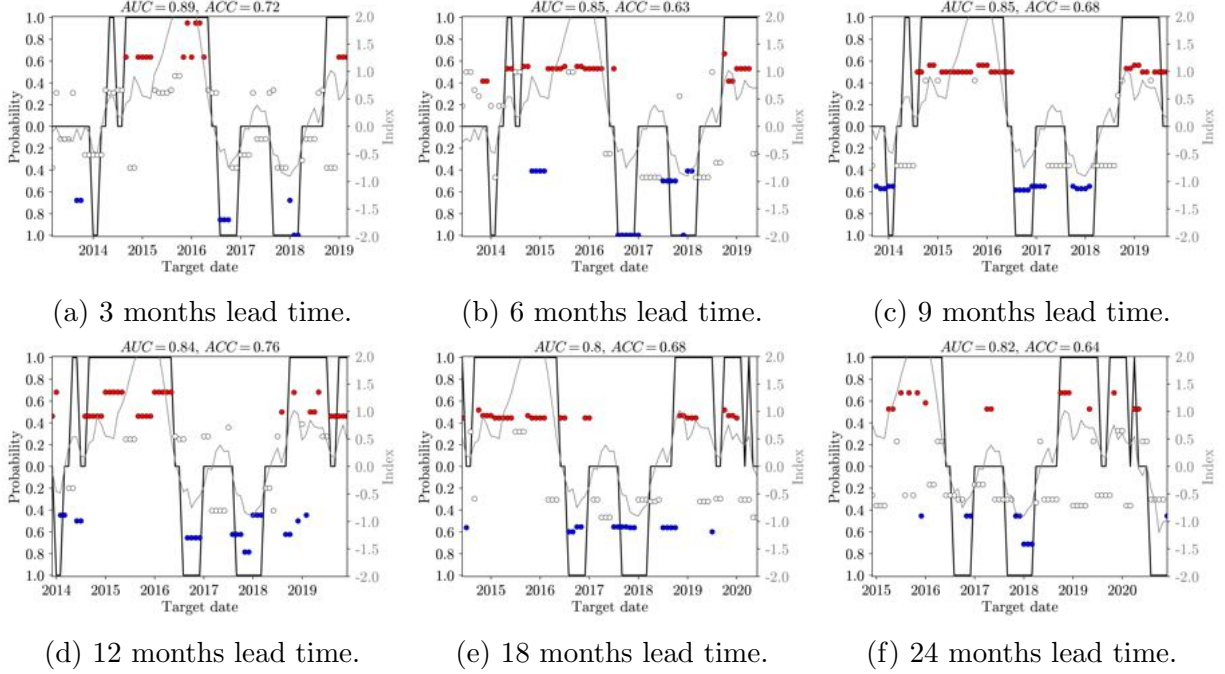


Figure 25: eSPA test set predictions using PCA features and 100% of the dataset.



### 3.2 PCA: 90%

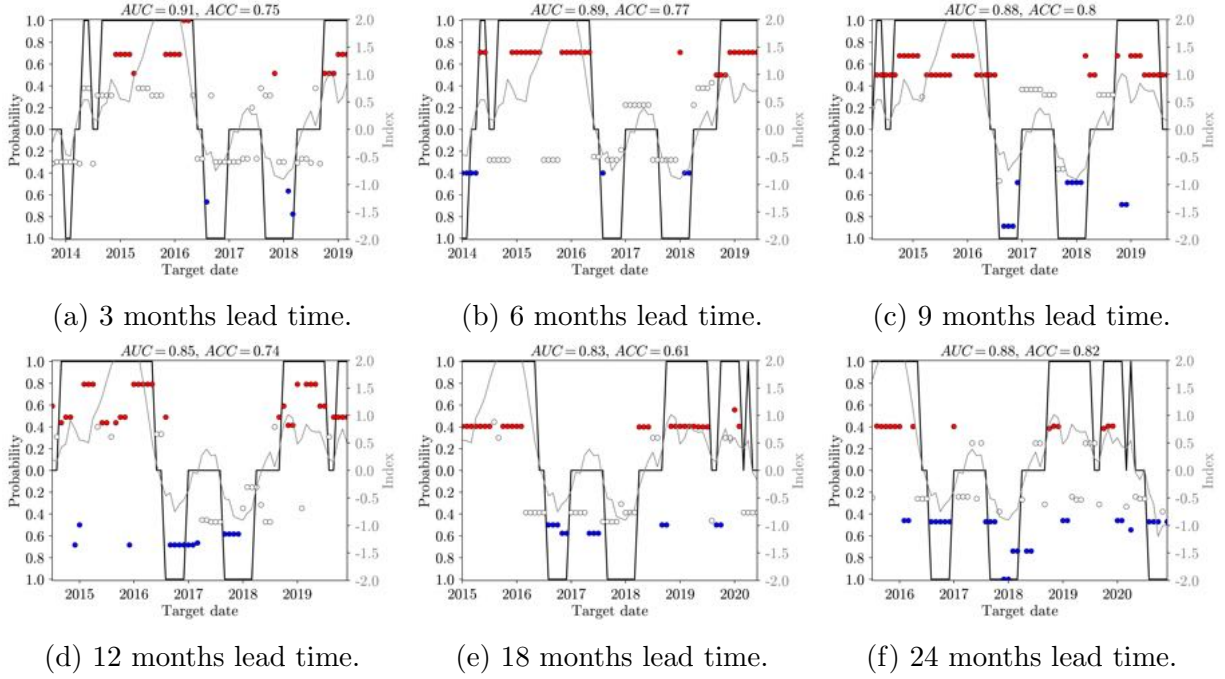


Figure 26: eSPA test set predictions using PCA features and 90% of the dataset.

### 3.3 PCA: 80%

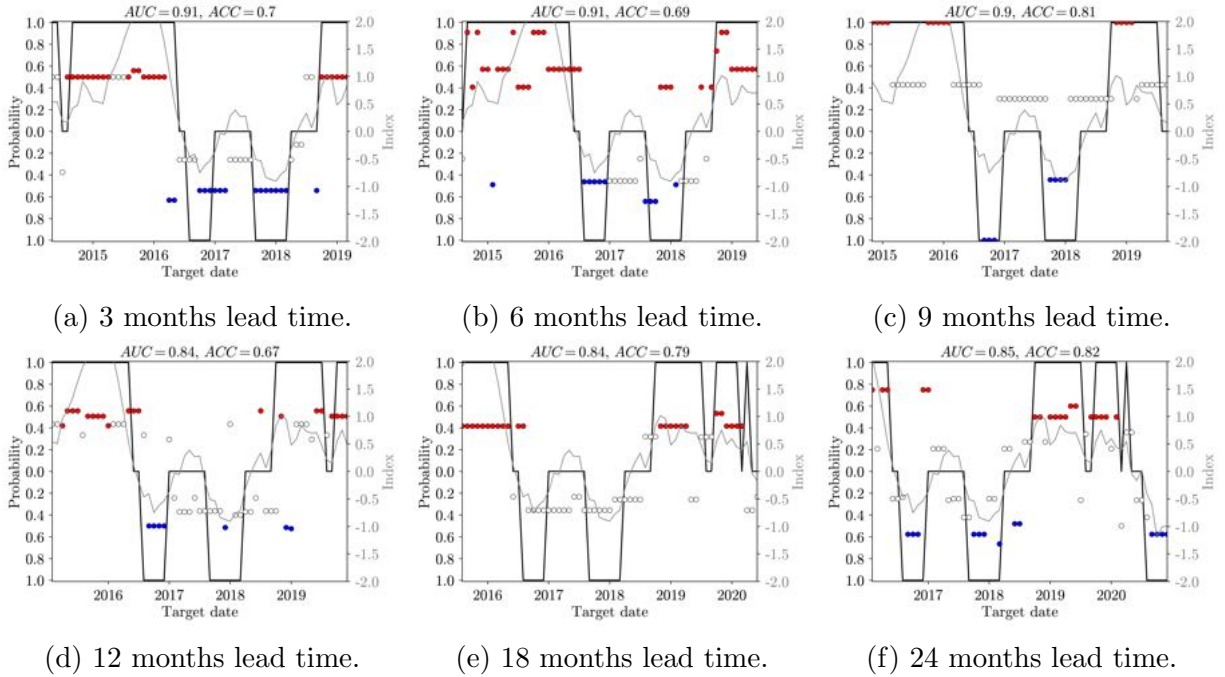


Figure 27: eSPA test set predictions using PCA features and 80% of the dataset.



### 3.4 PCA: 70%

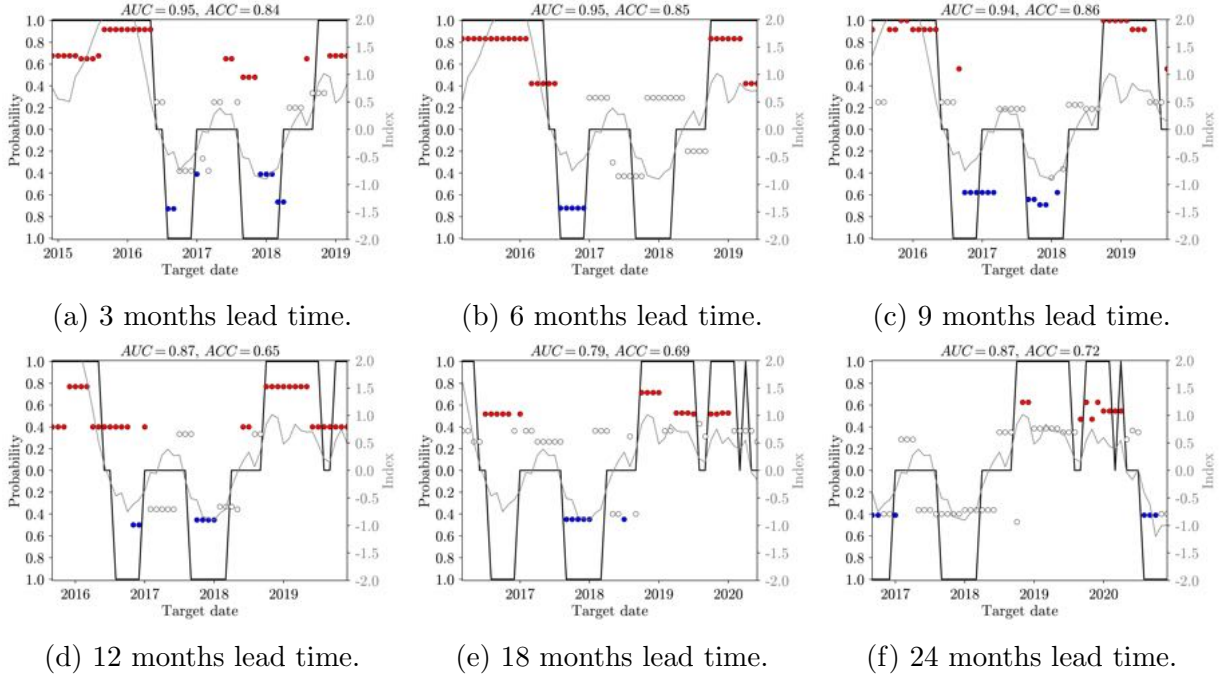


Figure 28: eSPA test set predictions using PCA features and 70% of the dataset.

### 3.5 PCA: 60%

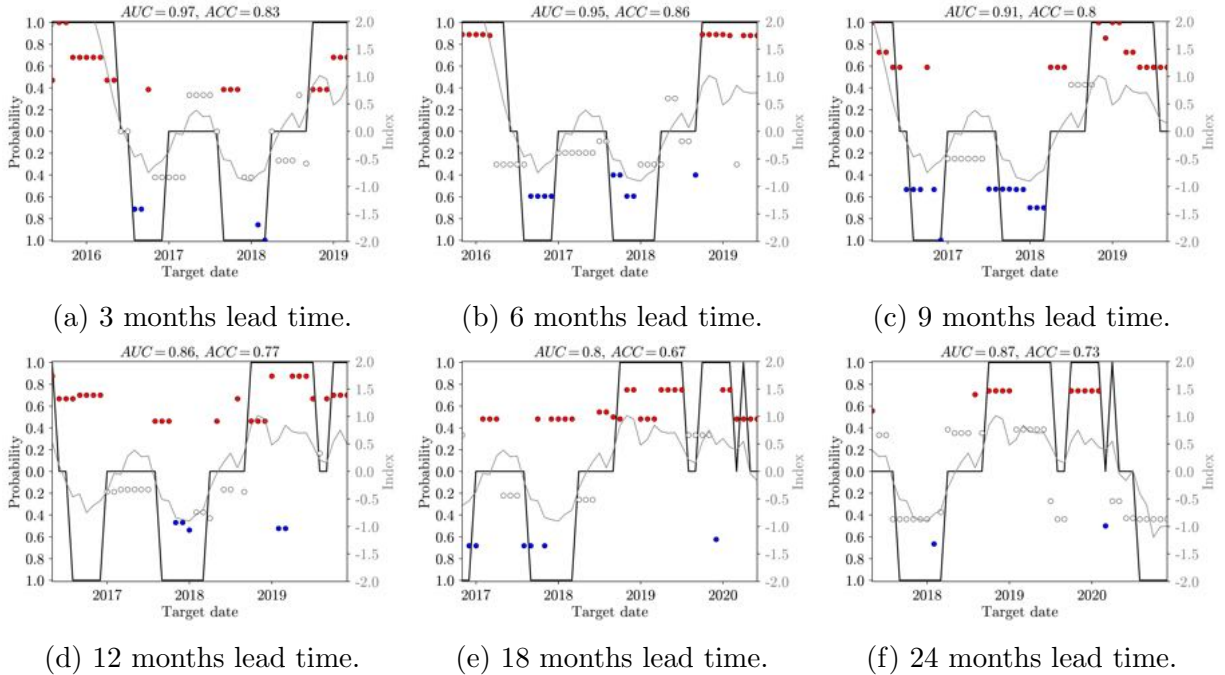


Figure 29: eSPA test set predictions using PCA features and 60% of the dataset.

### 3.6 PCA: 50%

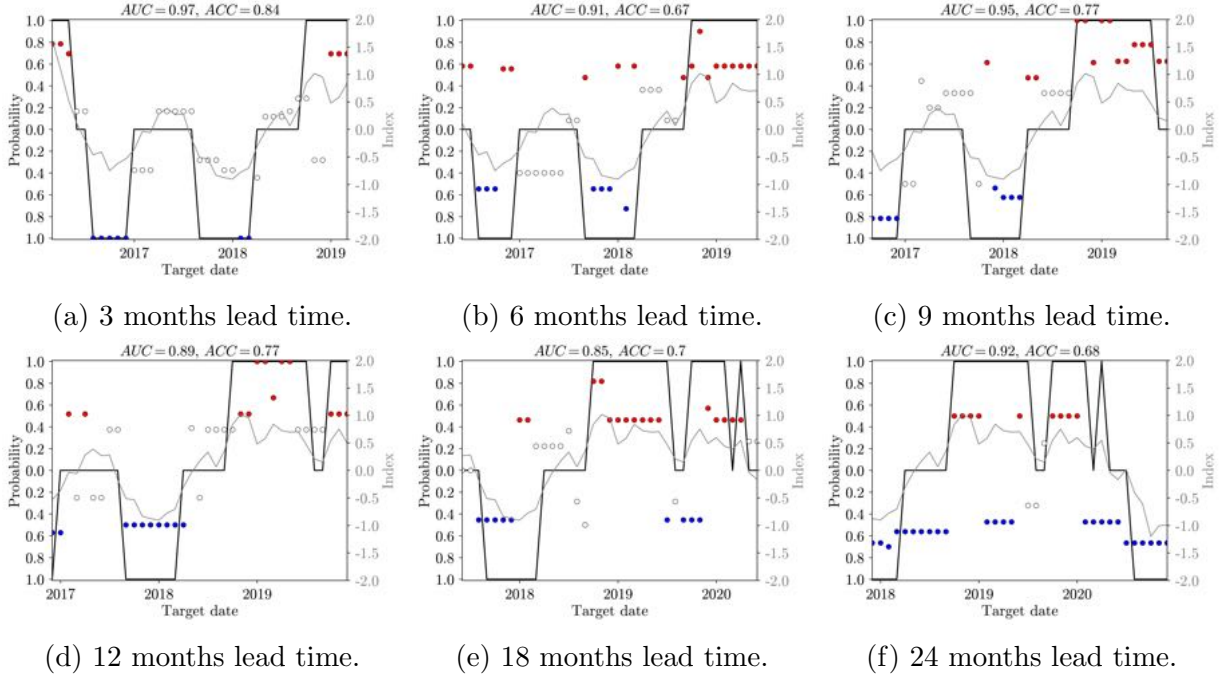


Figure 30: eSPA test set predictions using PCA features and 50% of the dataset.

### 3.7 SSA: 100%

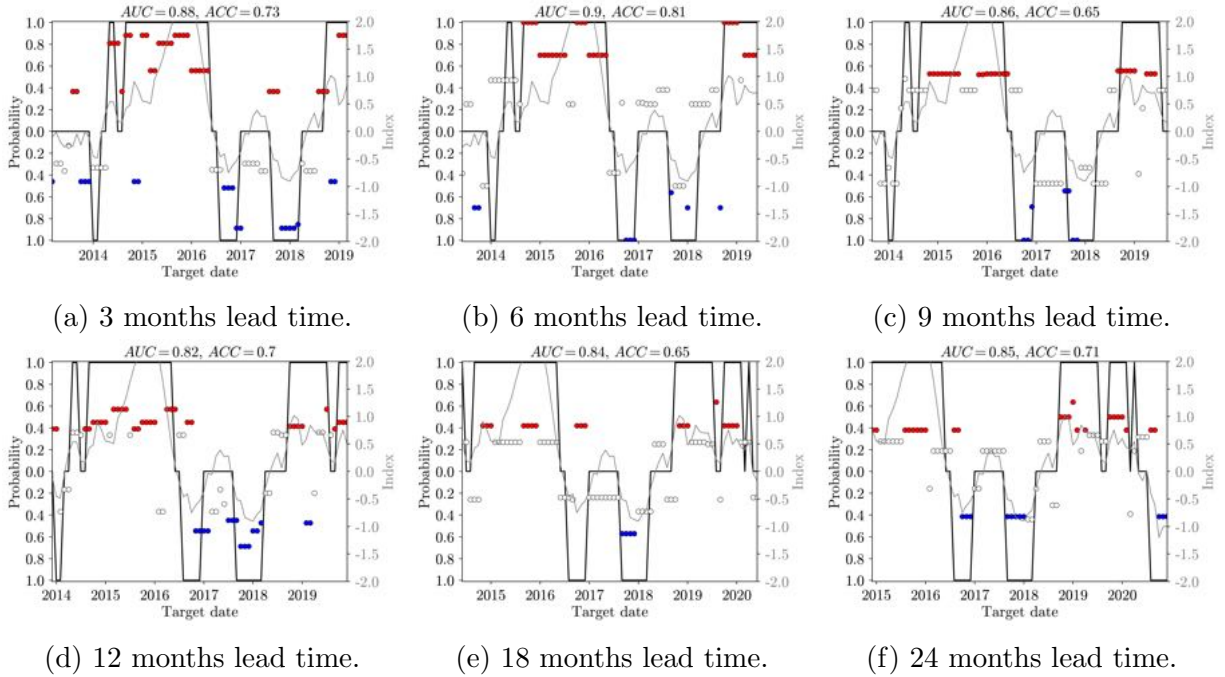


Figure 31: eSPA test set predictions using SSA features and 100% of the dataset.

### 3.8 SSA: 90%

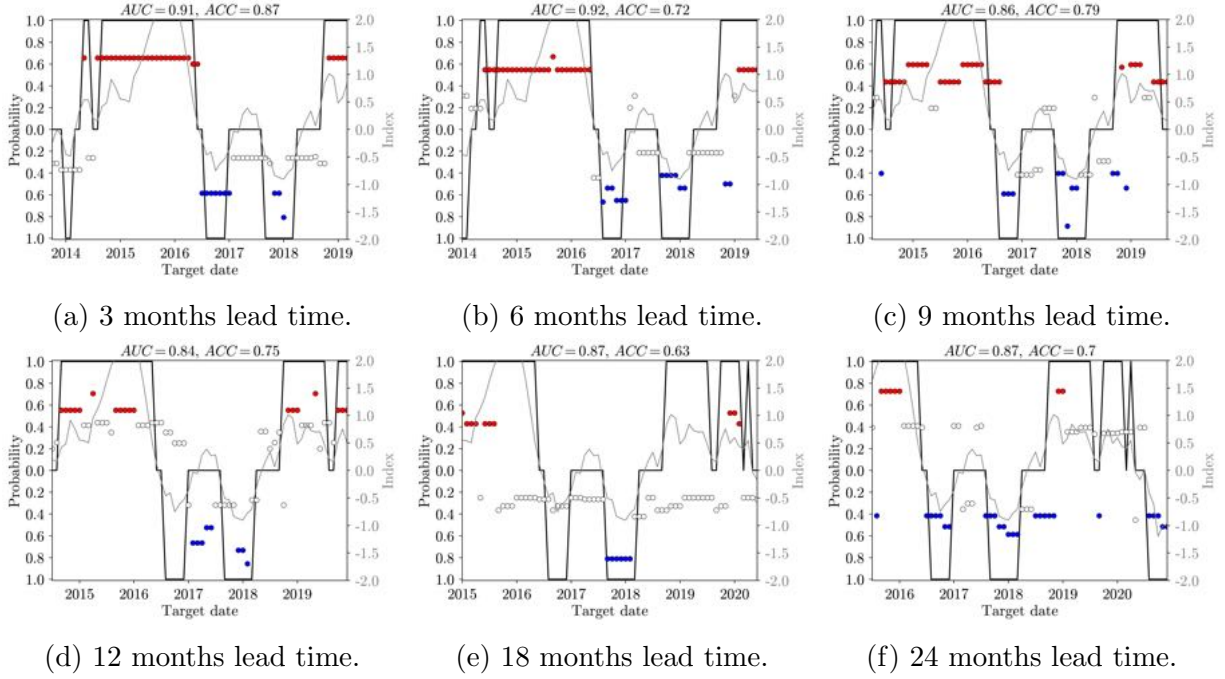


Figure 32: eSPA test set predictions using SSA features and 90% of the dataset.

### 3.9 SSA: 80%

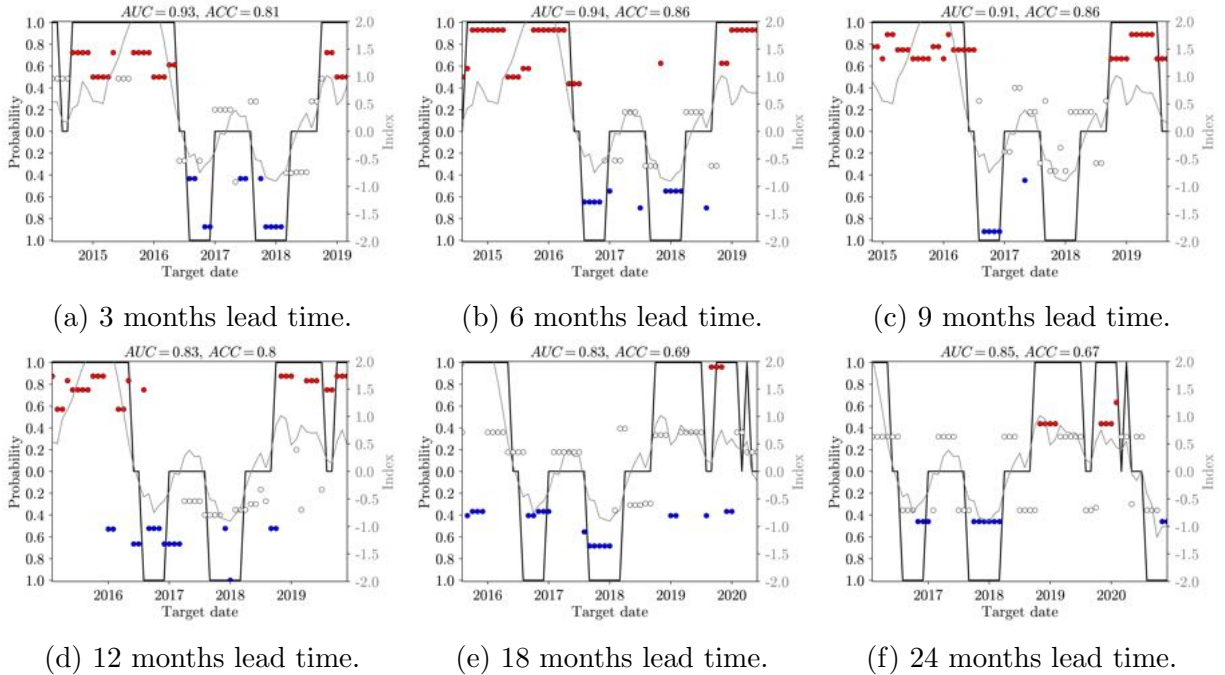


Figure 33: eSPA test set predictions using SSA features and 80% of the dataset.

### 3.10 SSA: 70%

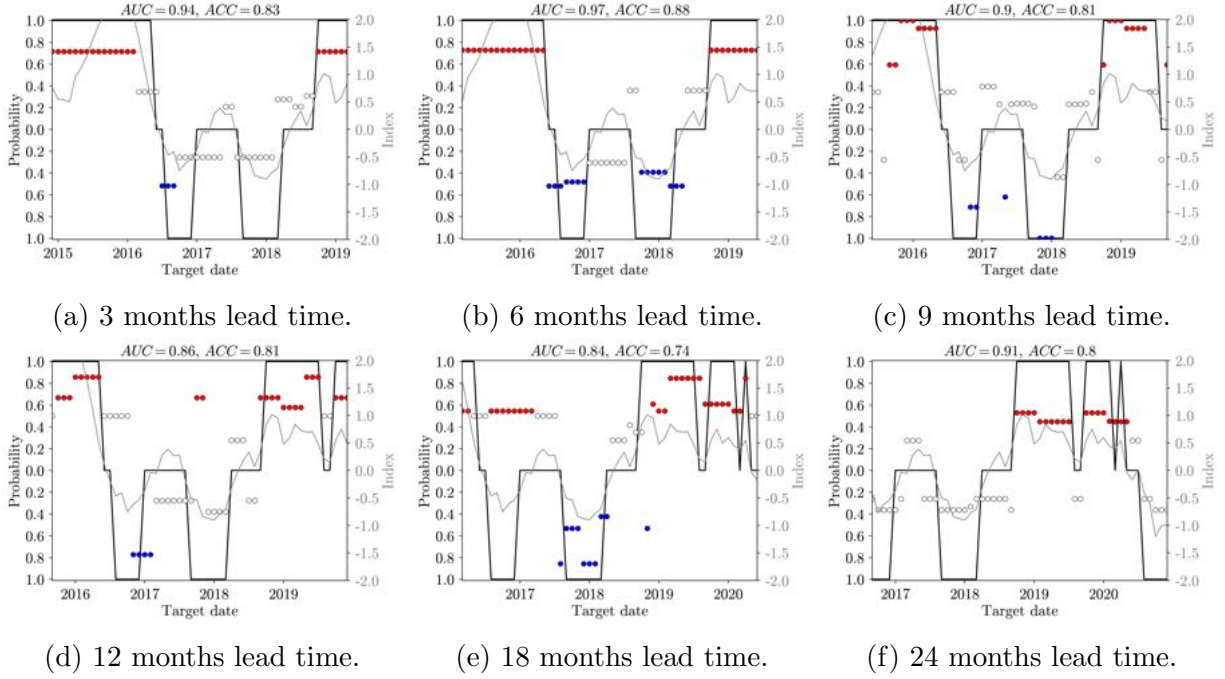


Figure 34: eSPA test set predictions using SSA features and 70% of the dataset.

### 3.11 SSA: 60%

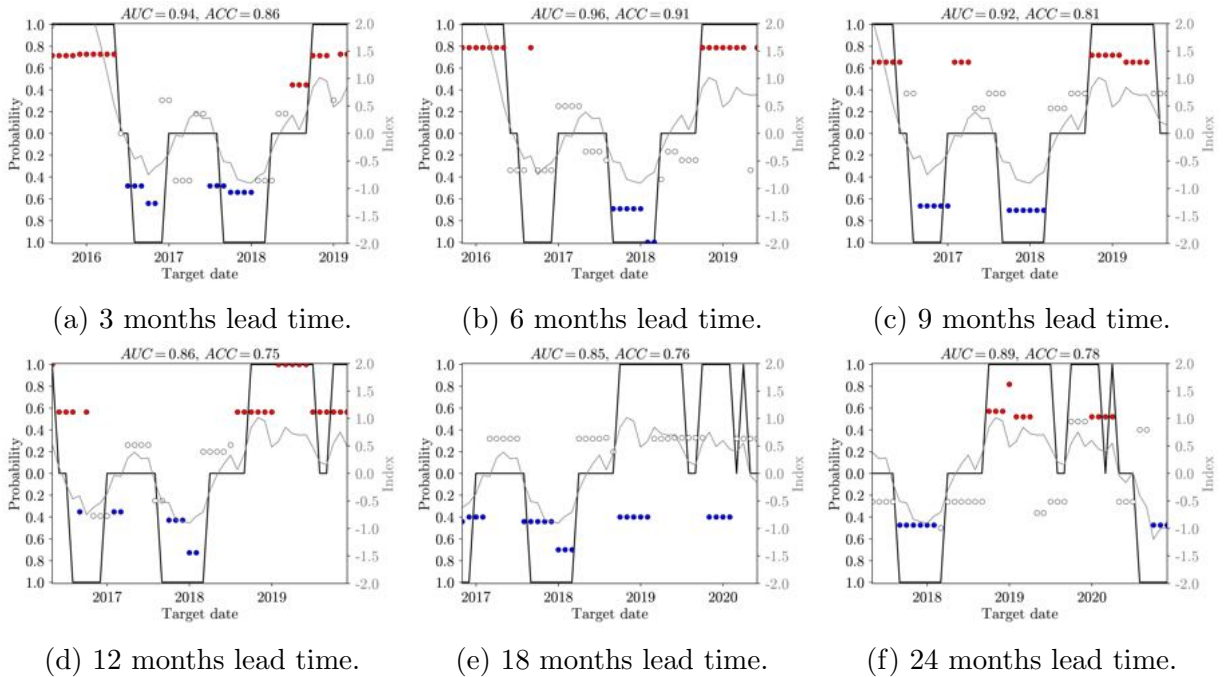


Figure 35: eSPA test set predictions using SSA features and 60% of the dataset.

### 3.12 SSA: 50%

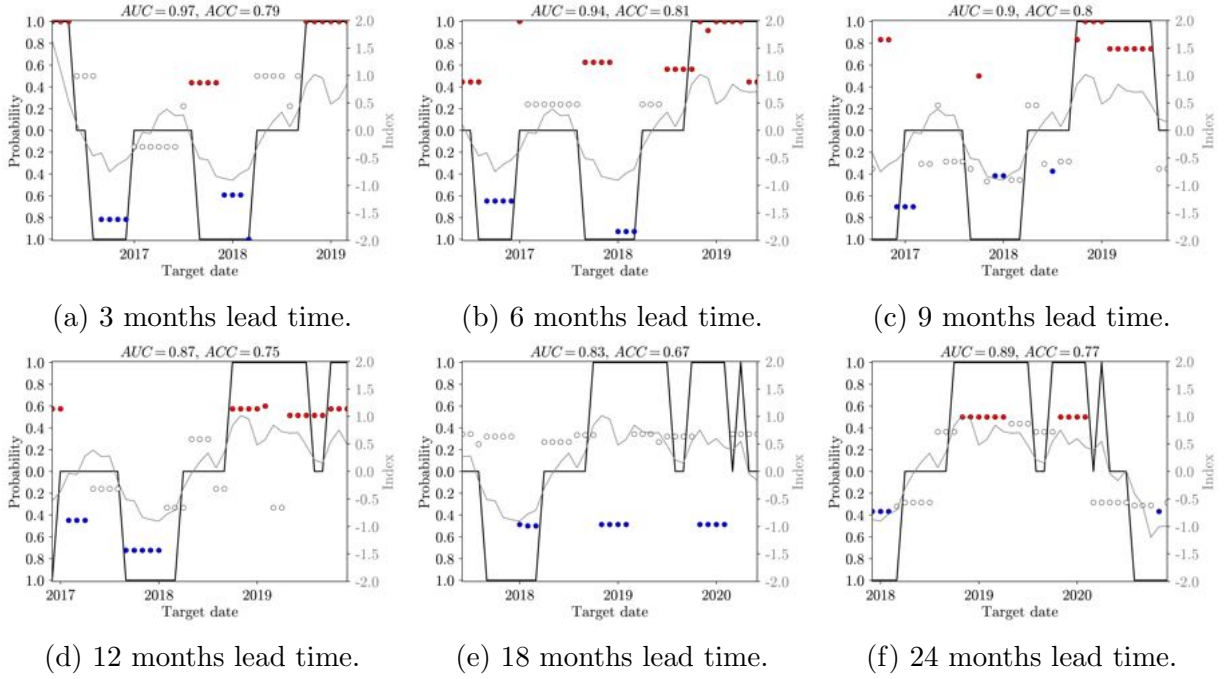


Figure 36: eSPA test set predictions using SSA features and 50% of the dataset.

## 4 Feature importance plots

The number of features with probability greater than  $1/D$  is given in the title of each plot.

### 4.1 PCA: 100%

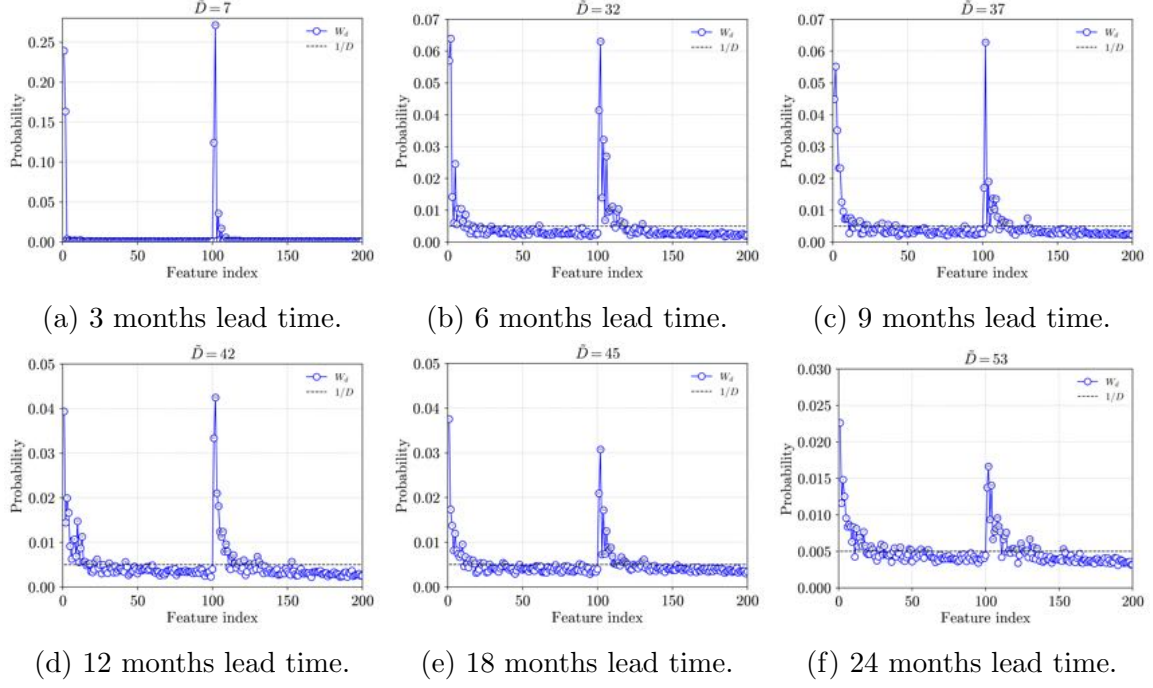


Figure 37: Feature importance plots using PCA features and 100% of the dataset.



## 4.2 PCA: 90%

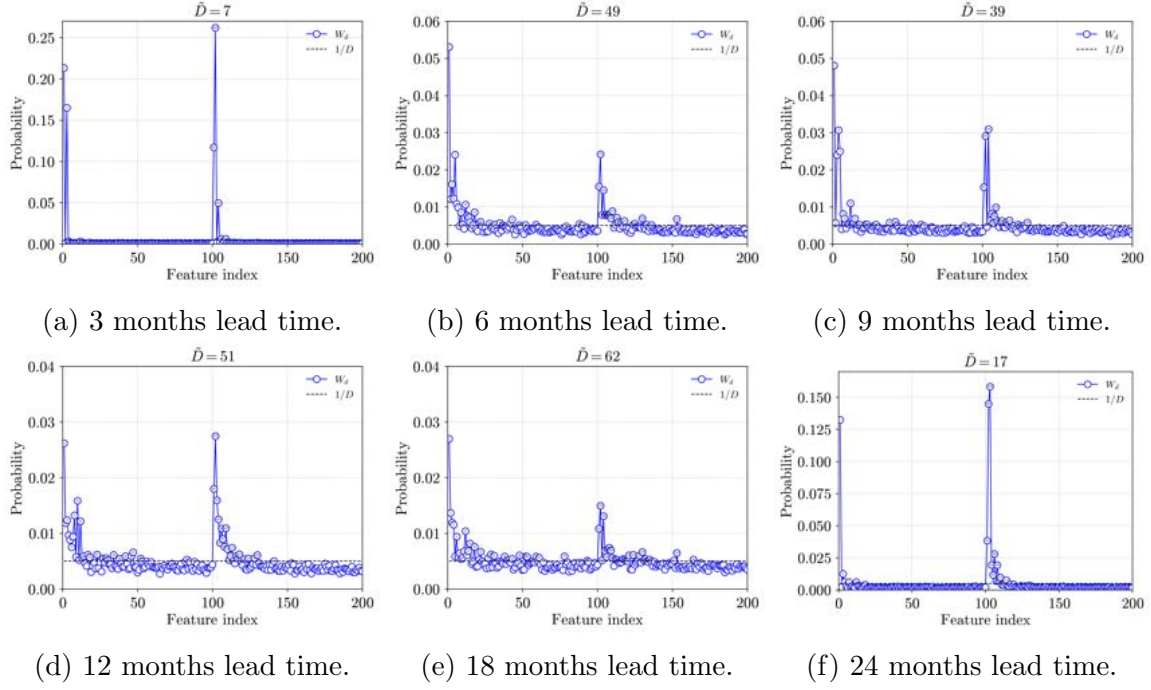


Figure 38: Feature importance plots using PCA features and 90% of the dataset.

## 4.3 PCA: 80%

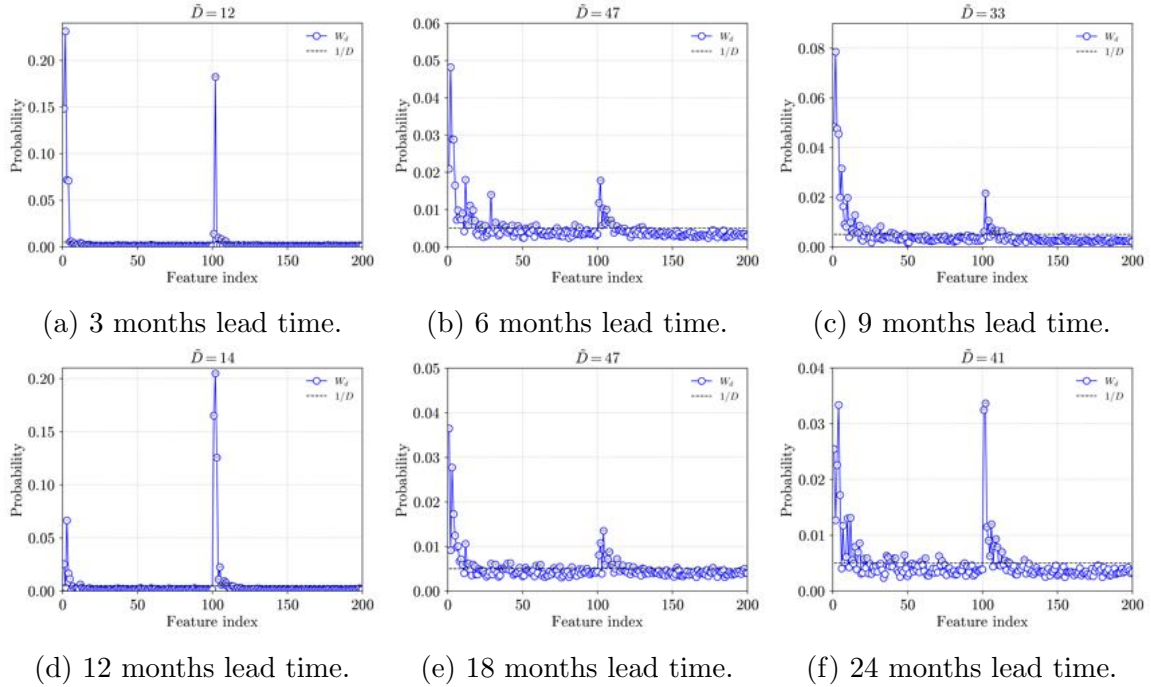


Figure 39: Feature importance plots using PCA features and 80% of the dataset.

#### 4.4 PCA: 70%

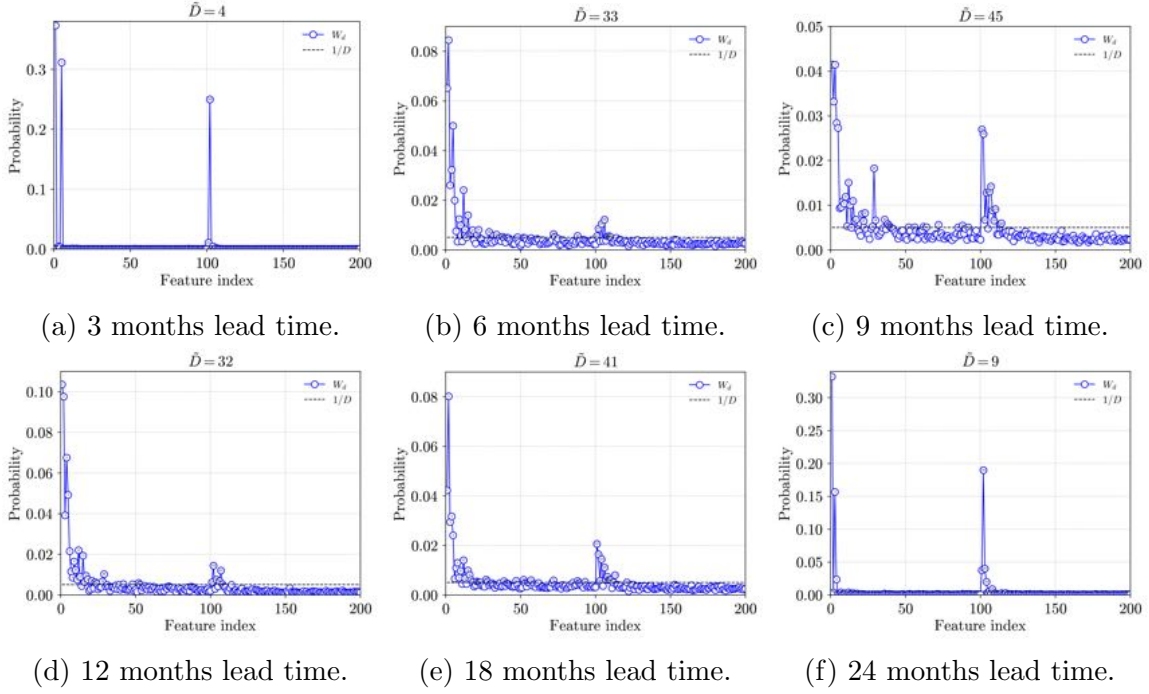


Figure 40: Feature importance plots using PCA features and 70% of the dataset.

#### 4.5 PCA: 60%

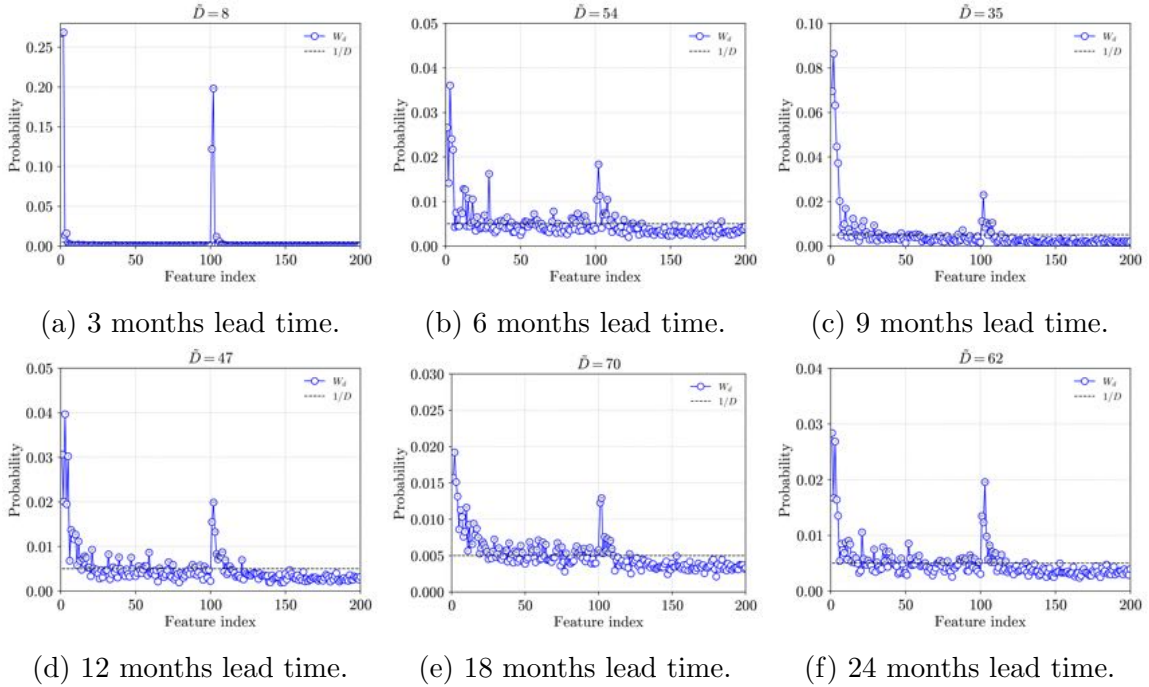


Figure 41: Feature importance plots using PCA features and 60% of the dataset.

## 4.6 PCA: 50%

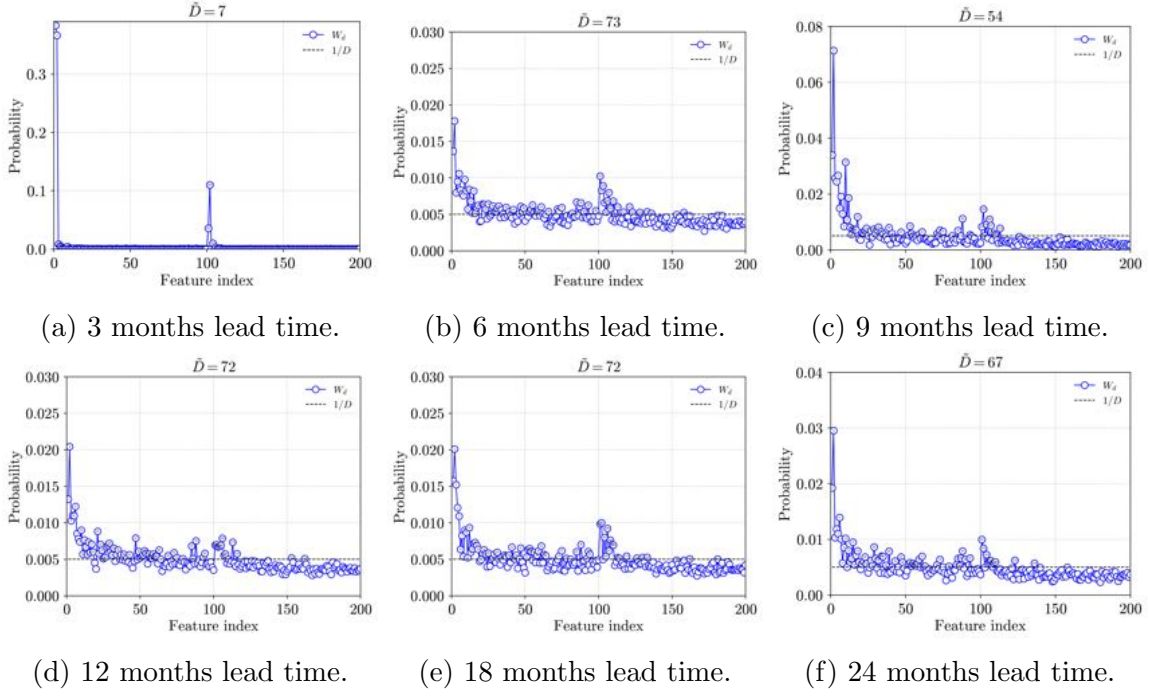


Figure 42: Feature importance plots using PCA features and 50% of the dataset.

## 4.7 SSA: 100%

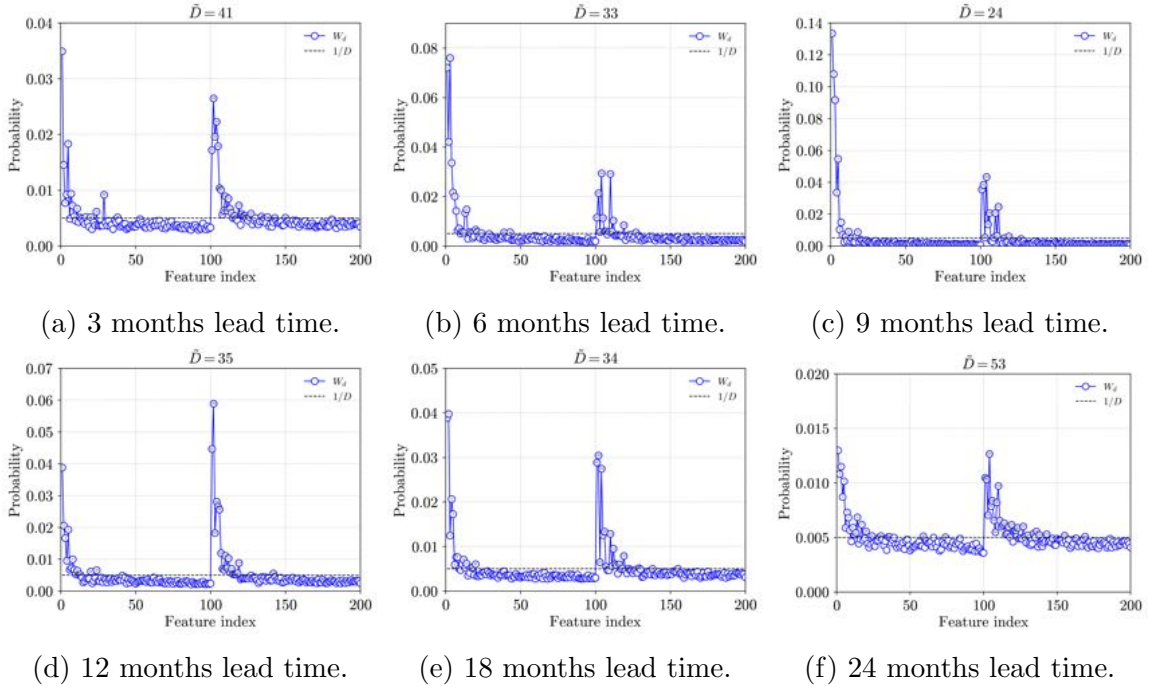


Figure 43: Feature importance plots using SSA features and 100% of the dataset.

## 4.8 SSA: 90%

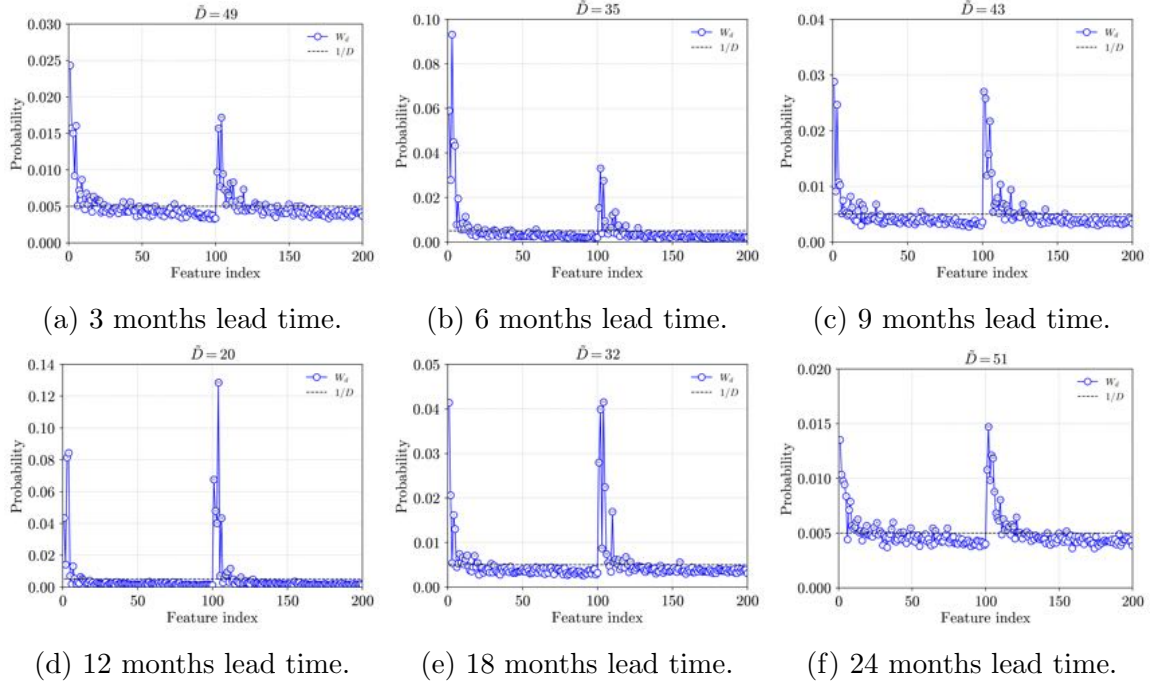


Figure 44: Feature importance plots using SSA features and 90% of the dataset.

## 4.9 SSA: 80%

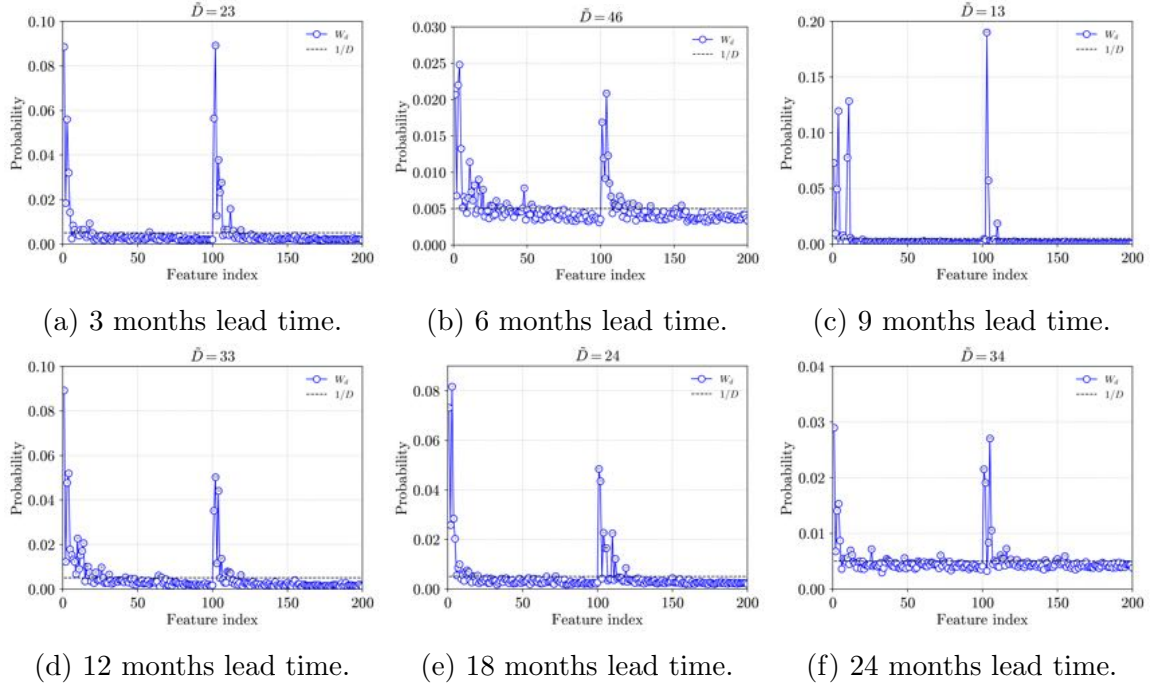


Figure 45: Feature importance plots using SSA features and 80% of the dataset.

#### 4.10 SSA: 70%

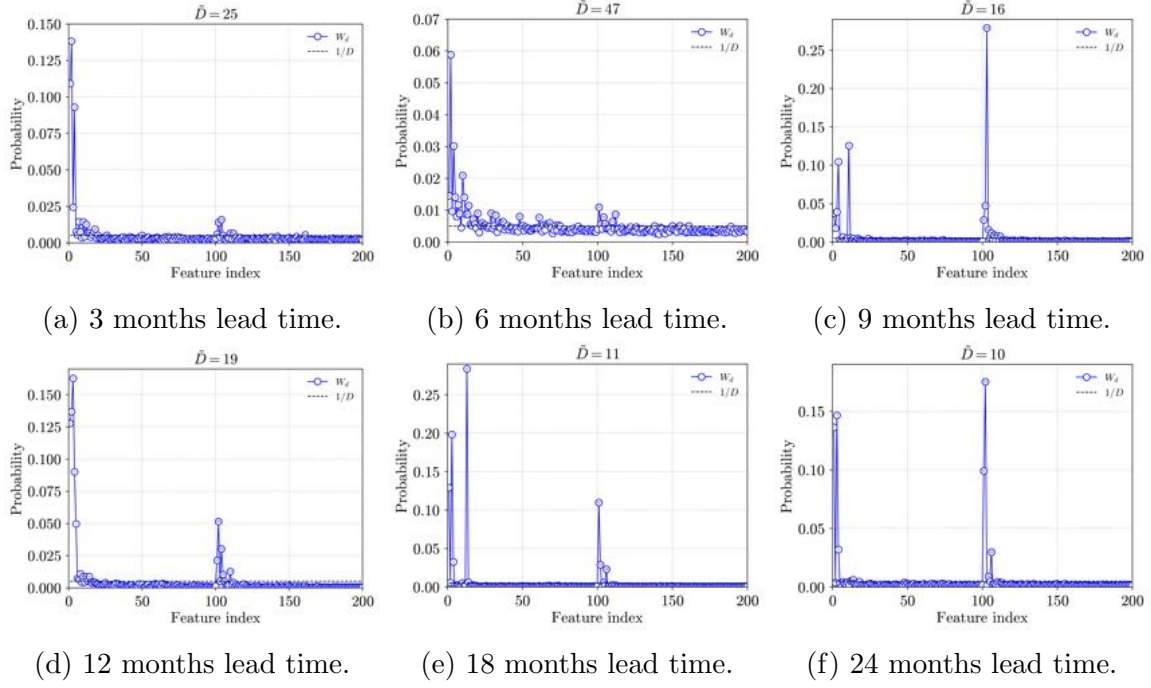


Figure 46: Feature importance plots using SSA features and 70% of the dataset.

#### 4.11 SSA: 60%

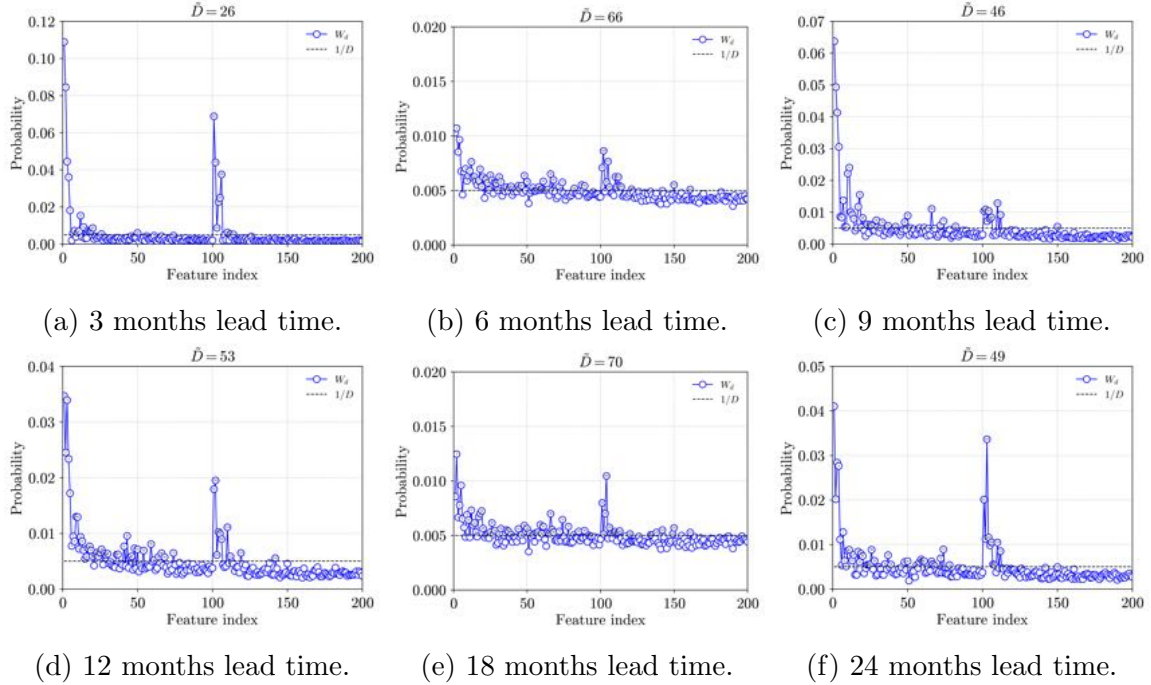


Figure 47: Feature importance plots using SSA features and 60% of the dataset.



## 4.12 SSA: 50%

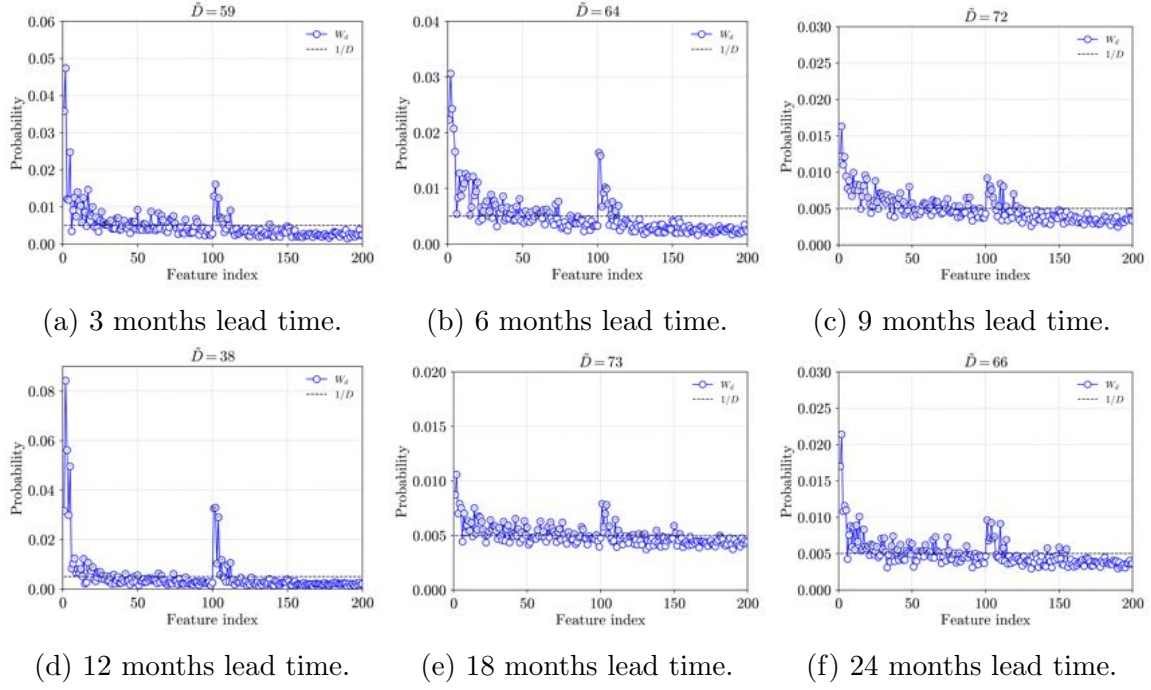


Figure 48: Feature importance plots using SSA features and 50% of the dataset.

## 5 Cluster occupation plots

El Niño clusters are coloured red, La Niña clusters are coloured blue and neutral clusters are coloured grey, while the dashed black line demarcates clusters whose  $p$ -value is less than 0.05.

### 5.1 PCA: 100%

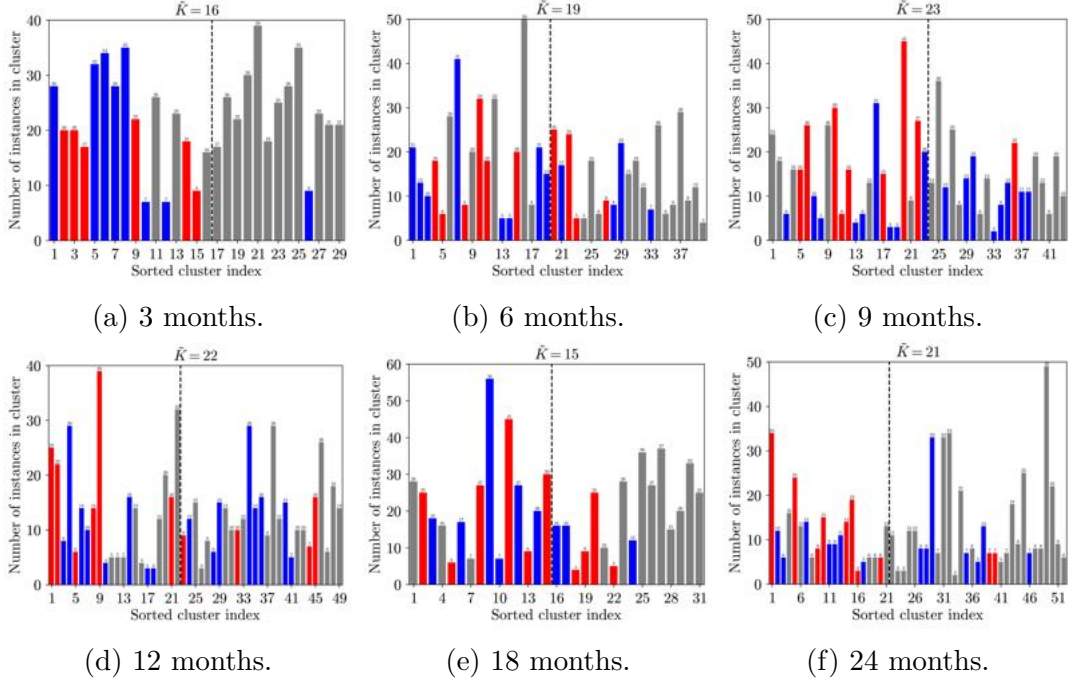


Figure 49: Training set cluster occupation plots using PCA features and 100% of the dataset.

## 5.2 PCA: 90%

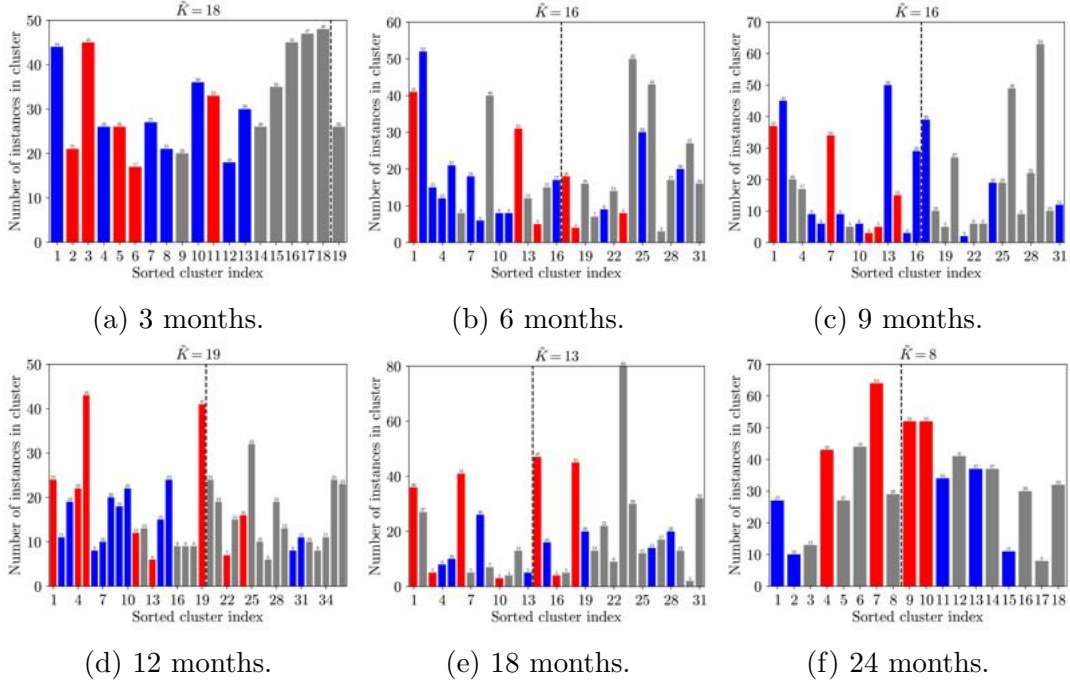


Figure 50: Training set cluster occupation plots using PCA features and 90% of the dataset.

## 5.3 PCA: 80%

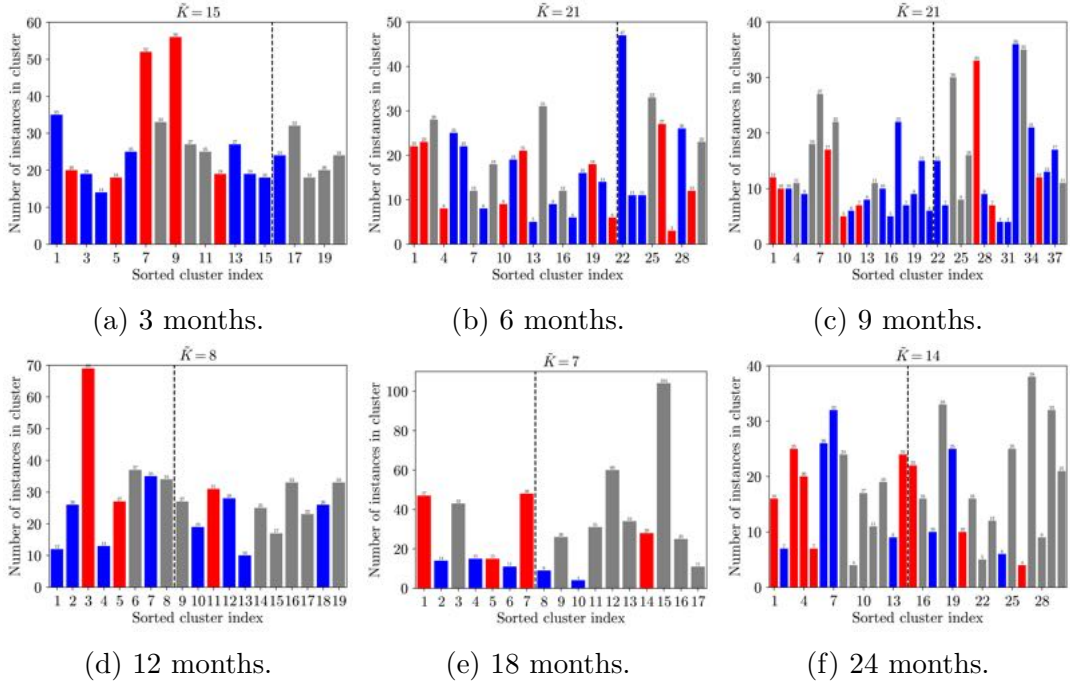


Figure 51: Training set cluster occupation plots using PCA features and 80% of the dataset.

## 5.4 PCA: 70%

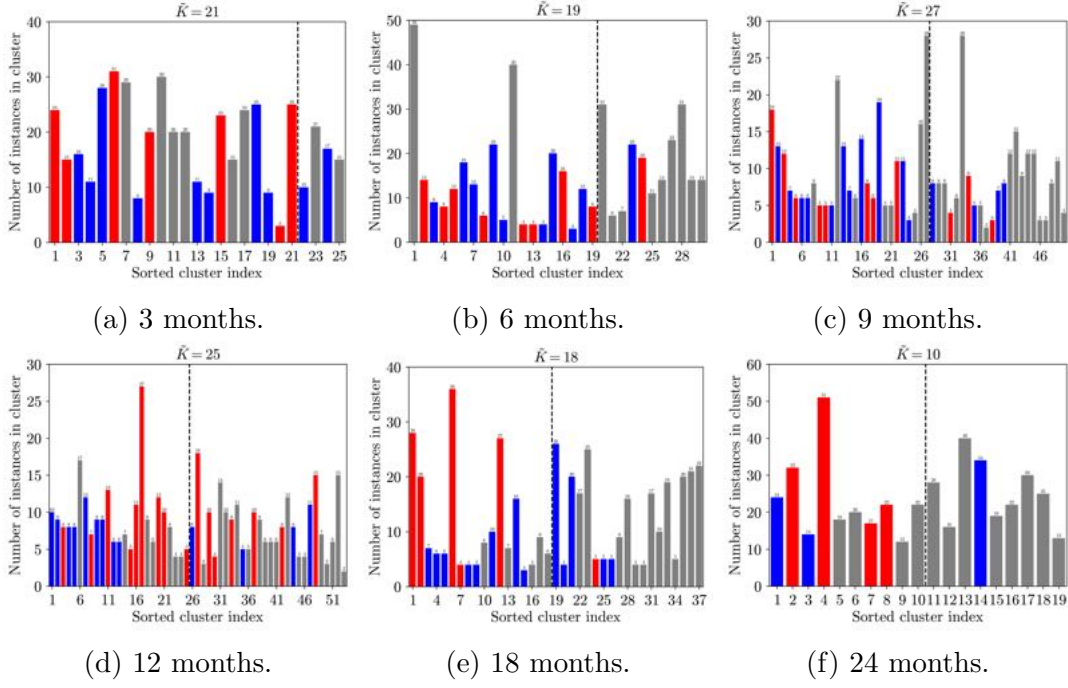


Figure 52: Training set cluster occupation plots using PCA features and 70% of the dataset.

## 5.5 PCA: 60%

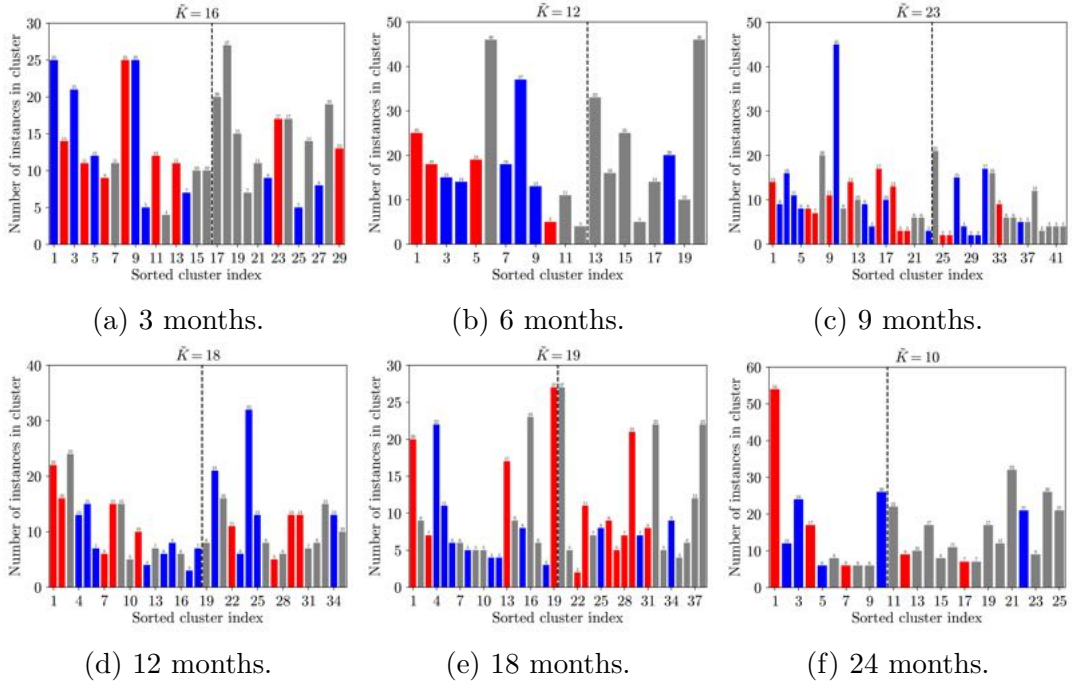


Figure 53: Training set cluster occupation plots using PCA features and 60% of the dataset.

## 5.6 PCA: 50%

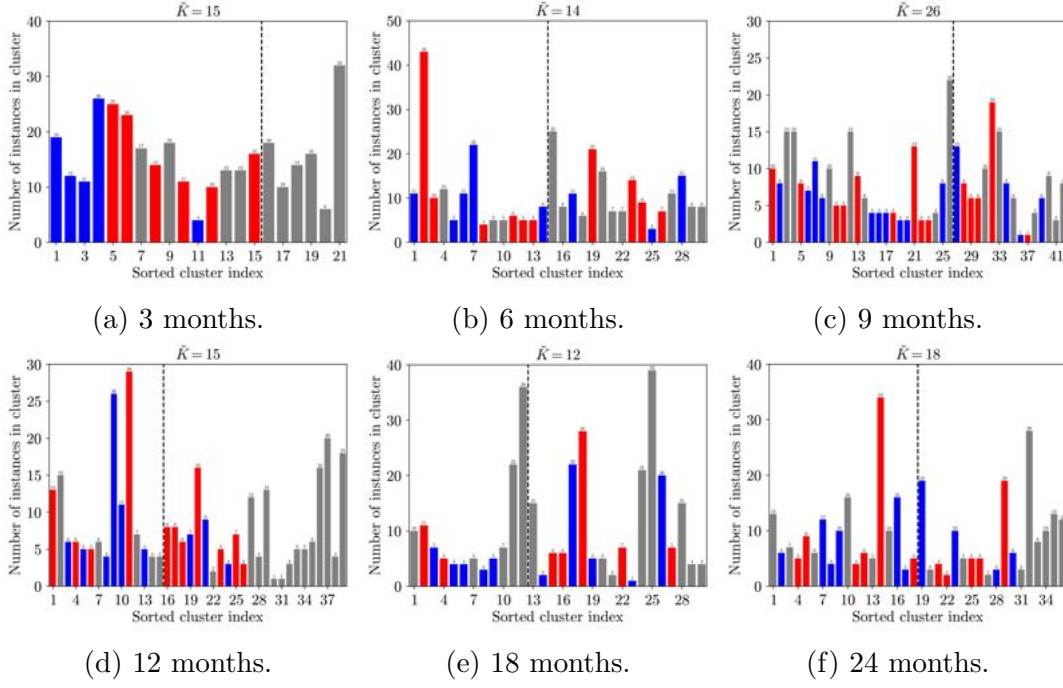


Figure 54: Training set cluster occupation plots using PCA features and 50% of the dataset.

## 5.7 SSA: 100%

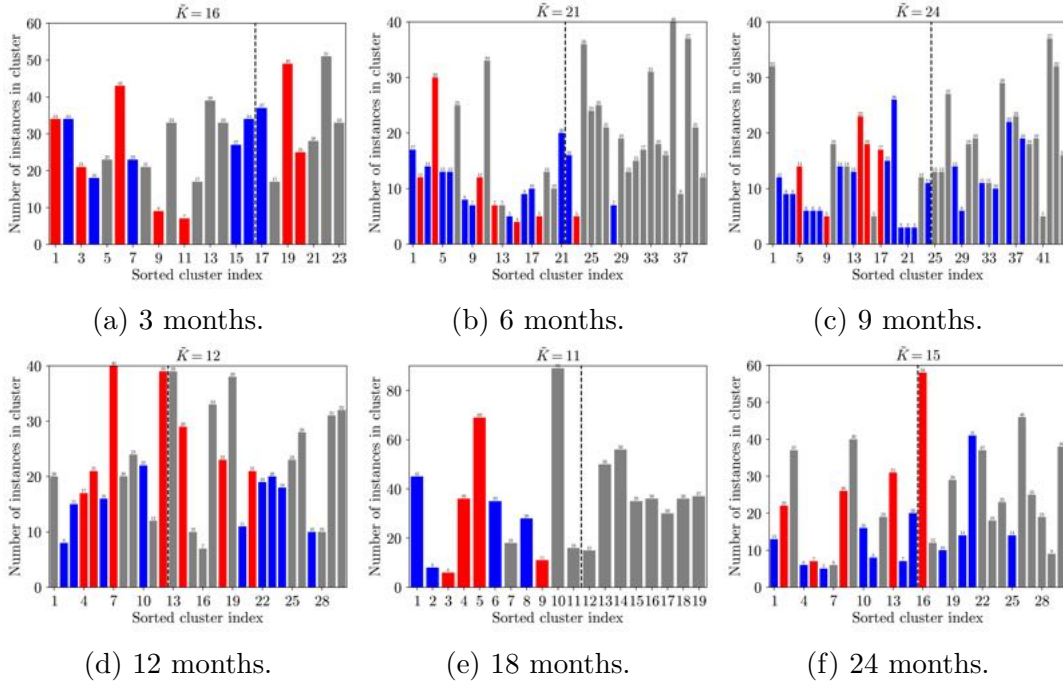


Figure 55: Training set cluster occupation plots using SSA features and 100% of the dataset.



## 5.8 SSA: 90%

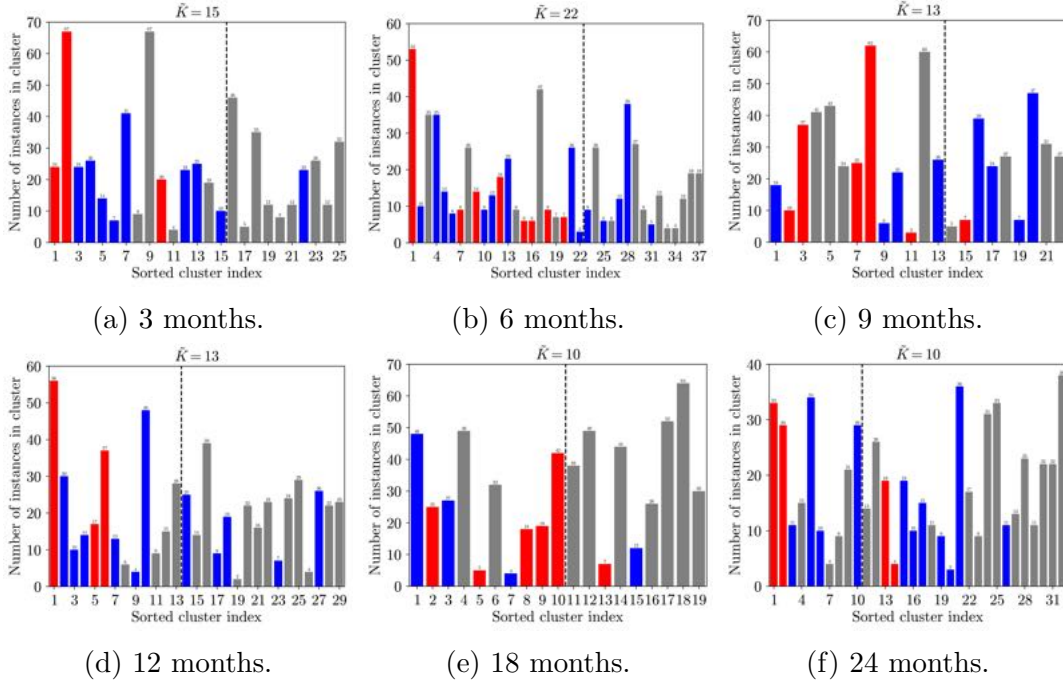


Figure 56: Training set cluster occupation plots using SSA features and 90% of the dataset.

## 5.9 SSA: 80%

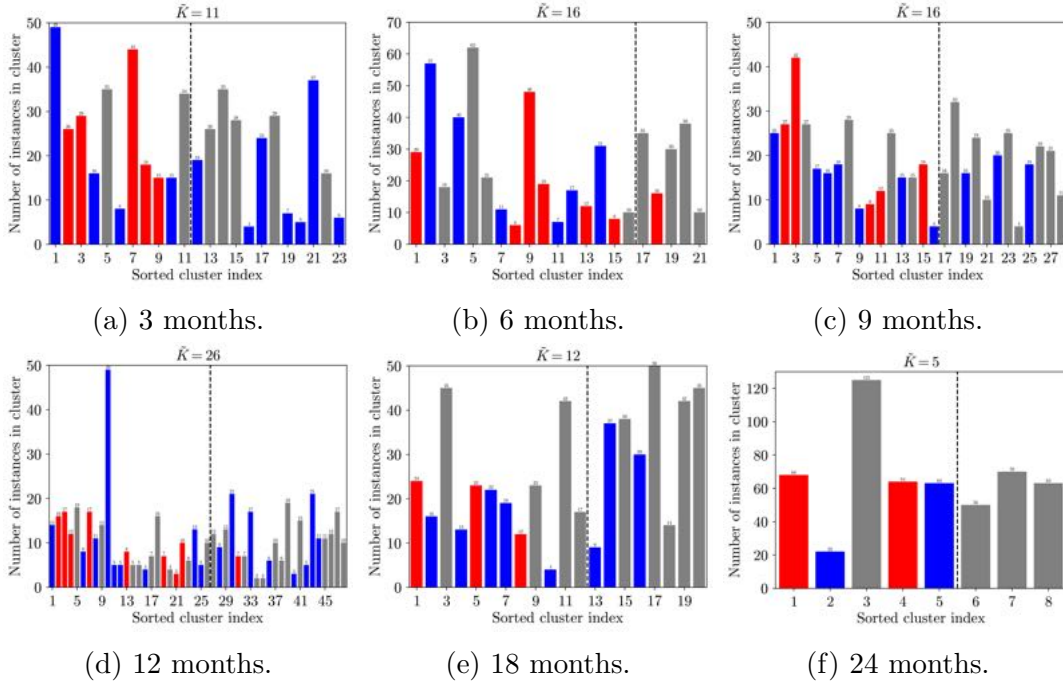


Figure 57: Training set cluster occupation plots using SSA features and 80% of the dataset.

## 5.10 SSA: 70%

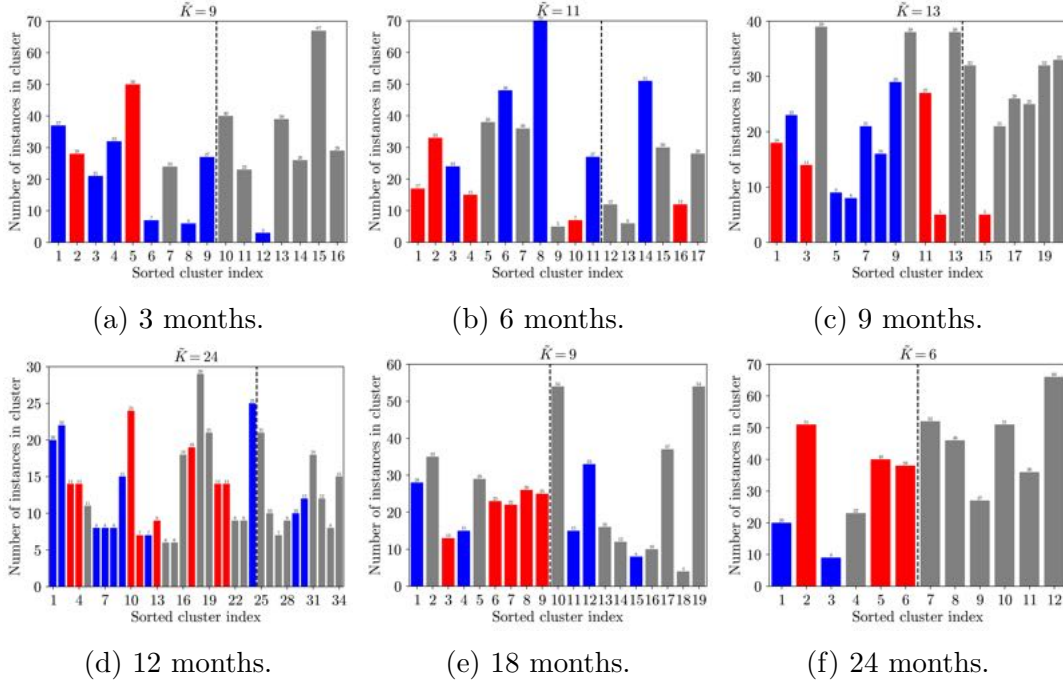


Figure 58: Training set cluster occupation plots using SSA features and 70% of the dataset.

## 5.11 SSA: 60%

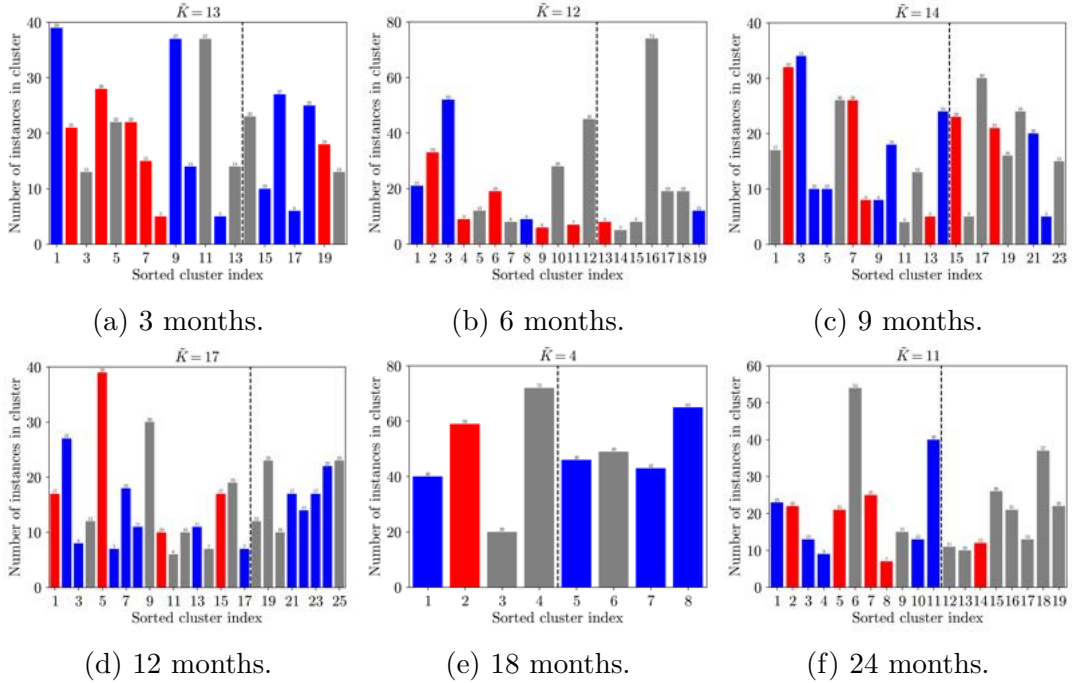


Figure 59: Training set cluster occupation plots using SSA features and 60% of the dataset.

## 5.12 SSA: 50%

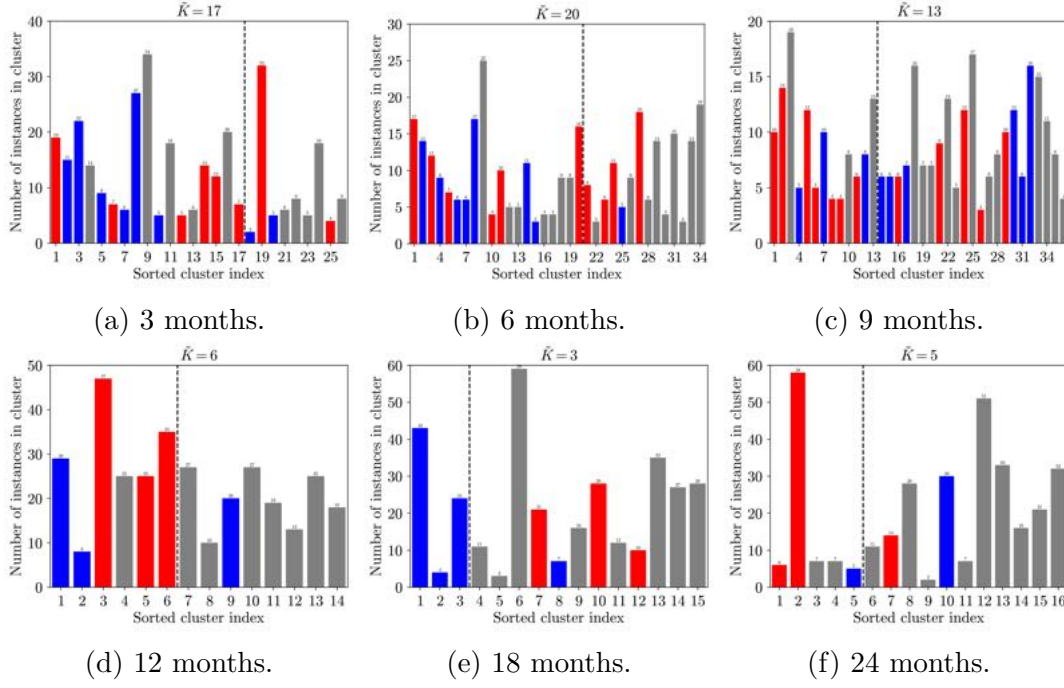
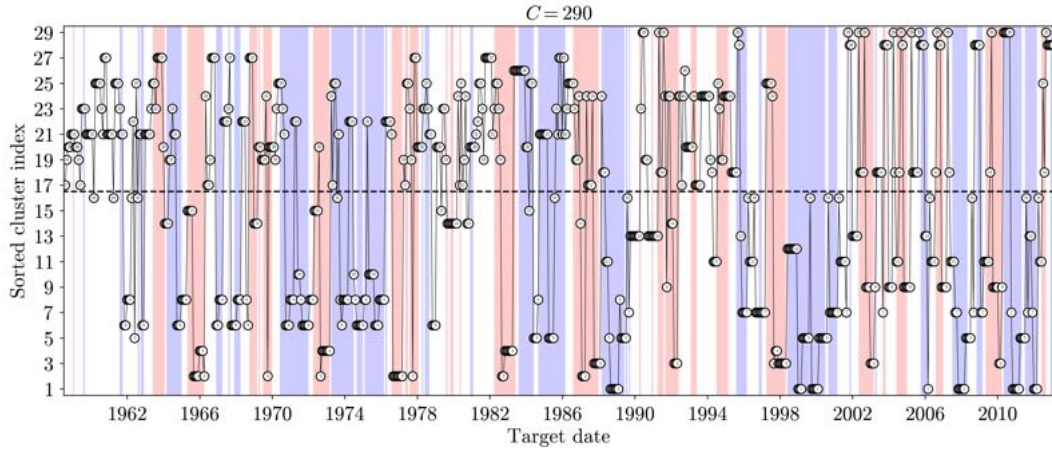


Figure 60: Training set cluster occupation plots using SSA features and 50% of the dataset.

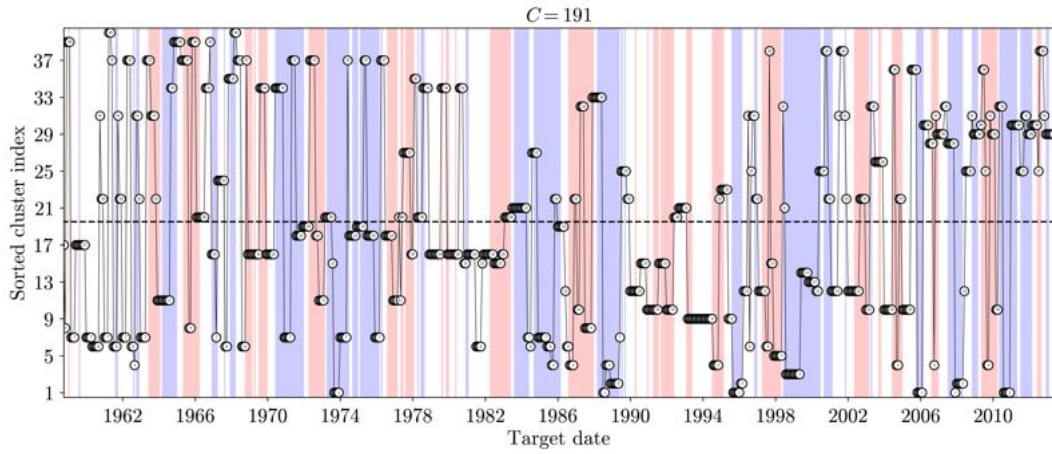
## 6 Cluster affiliation plots: training set

The red, blue and white background shading indicates an El Niño, La Niña or neutral event occurring on that target date, while the number of switches between clusters is given in the title of each plot.

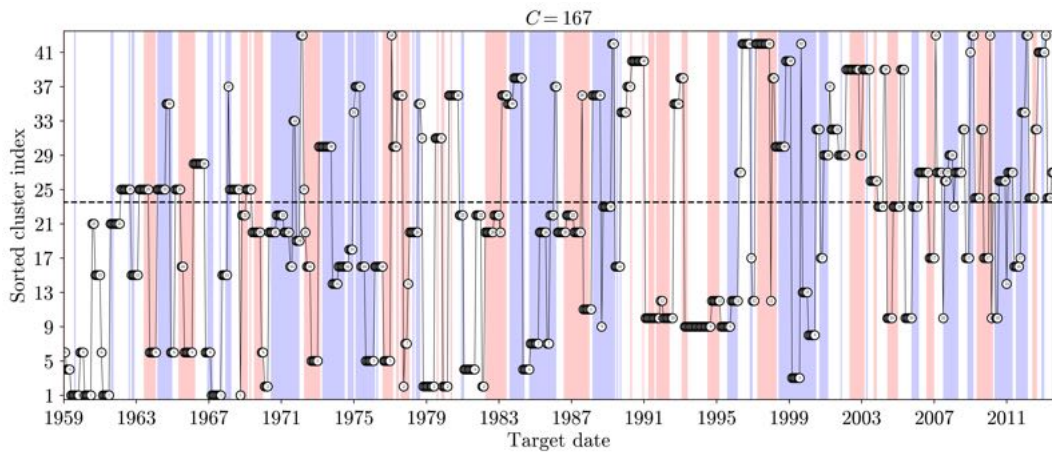
## 6.1 PCA: 100%



(a) 3 months.



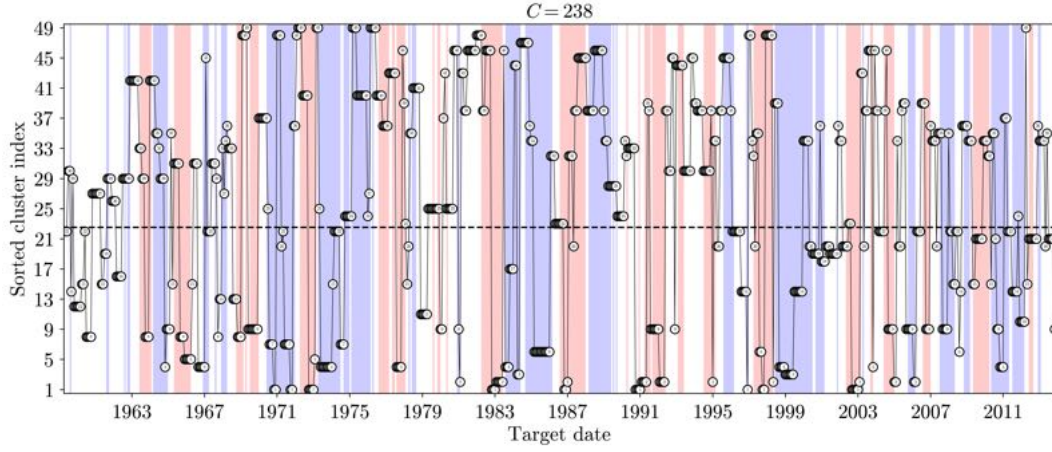
(b) 6 months.



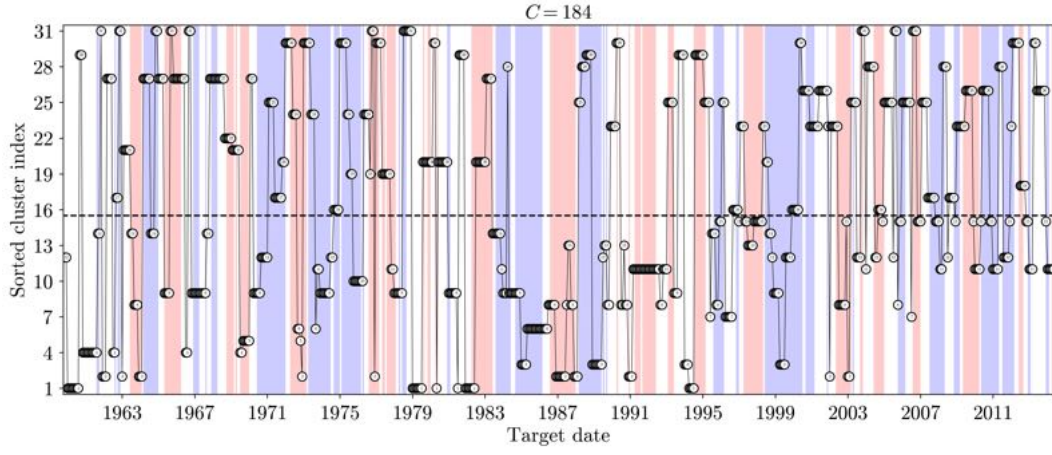
(c) 9 months.

Figure 61: Training set cluster affiliation plots using PCA features and 100% of the dataset: 3-9 months

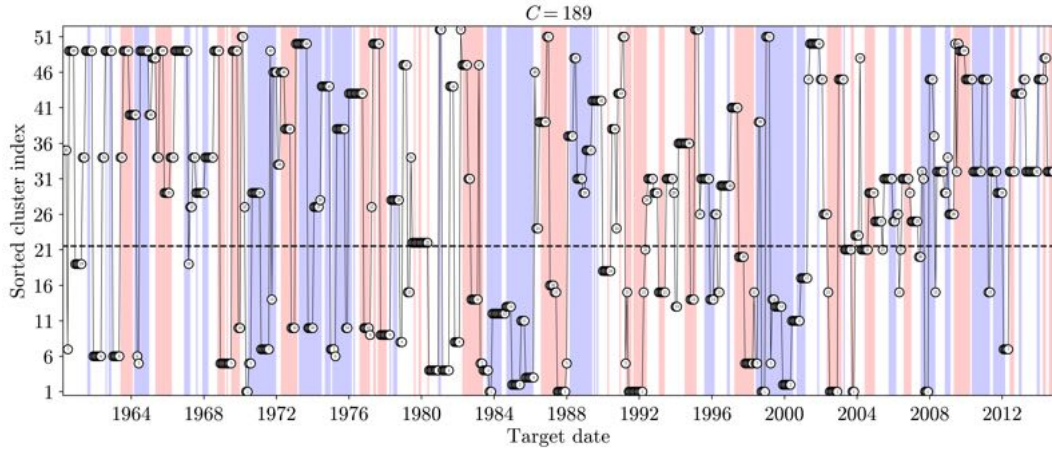




(a) 12 months.



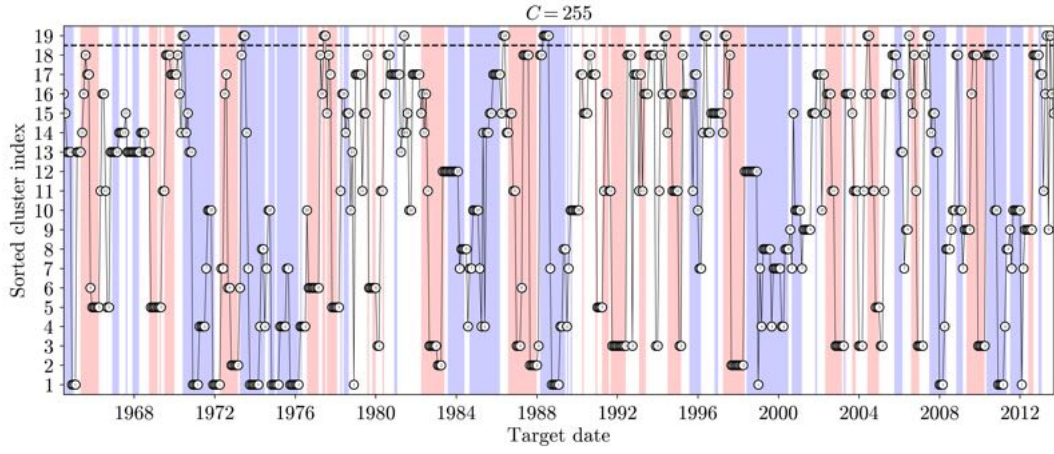
(b) 18 months.



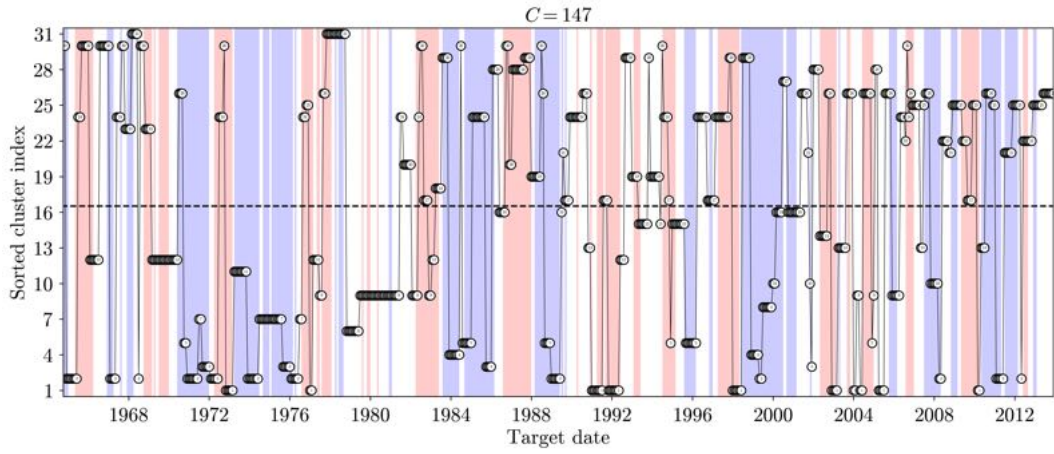
(c) 24 months.

Figure 62: Training set cluster affiliation plots using PCA features and 100% of the dataset: 12-24 months

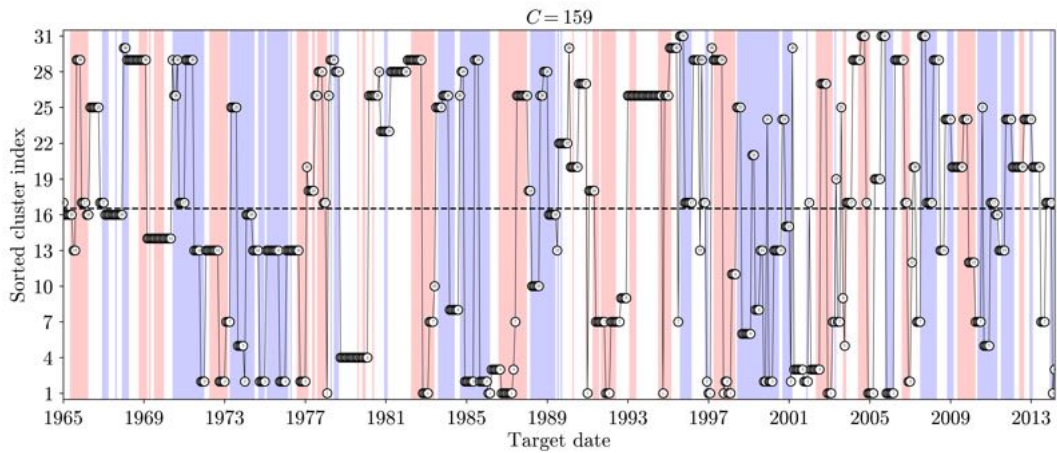
## 6.2 PCA: 90%



(a) 3 months.

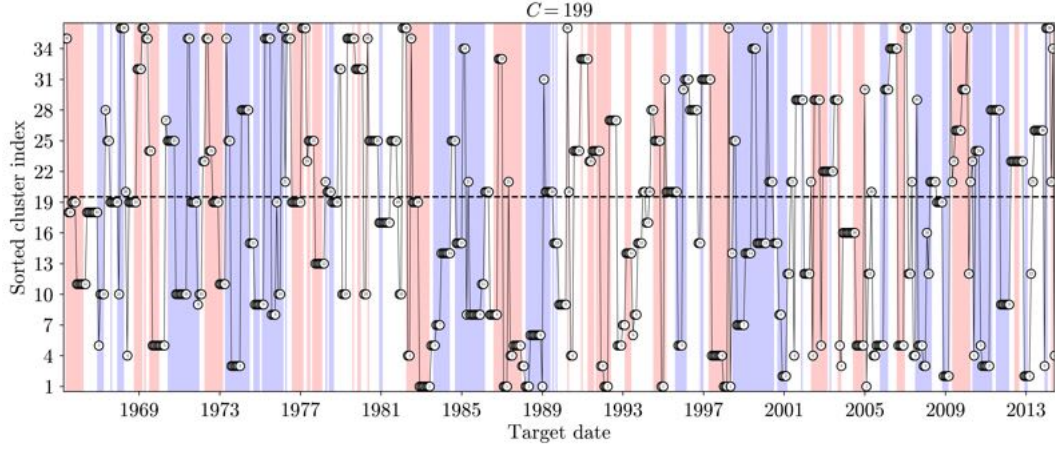


(b) 6 months.

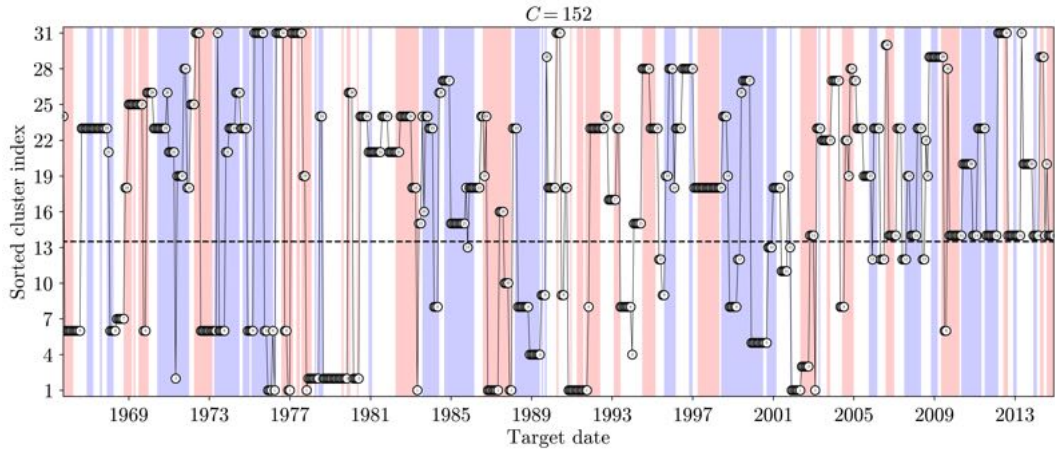


(c) 9 months.

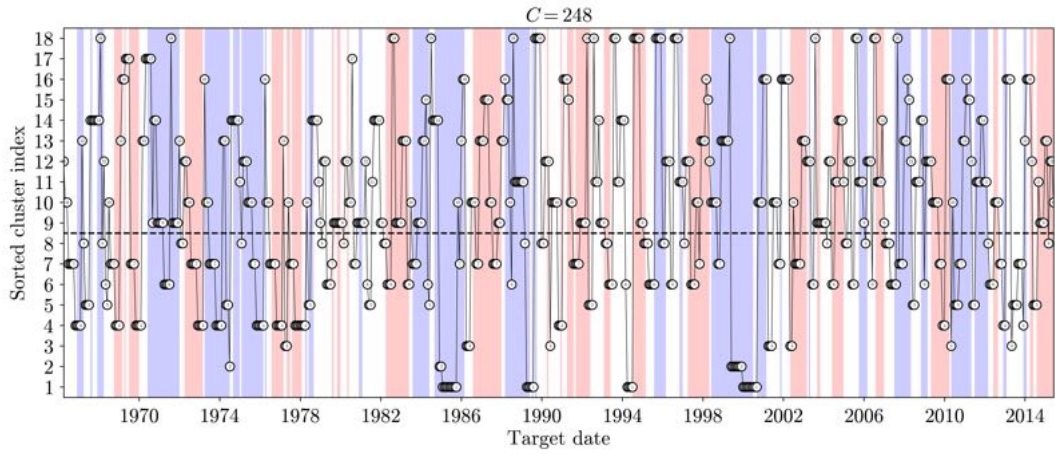
Figure 63: Training set cluster affiliation plots using PCA features and 90% of the dataset: 3-9 months



(a) 12 months.



(b) 18 months.

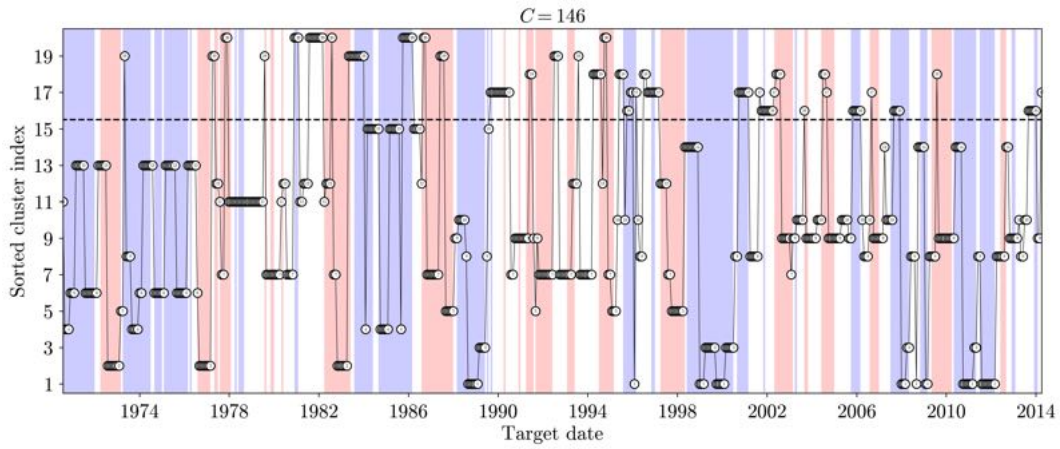


(c) 24 months.

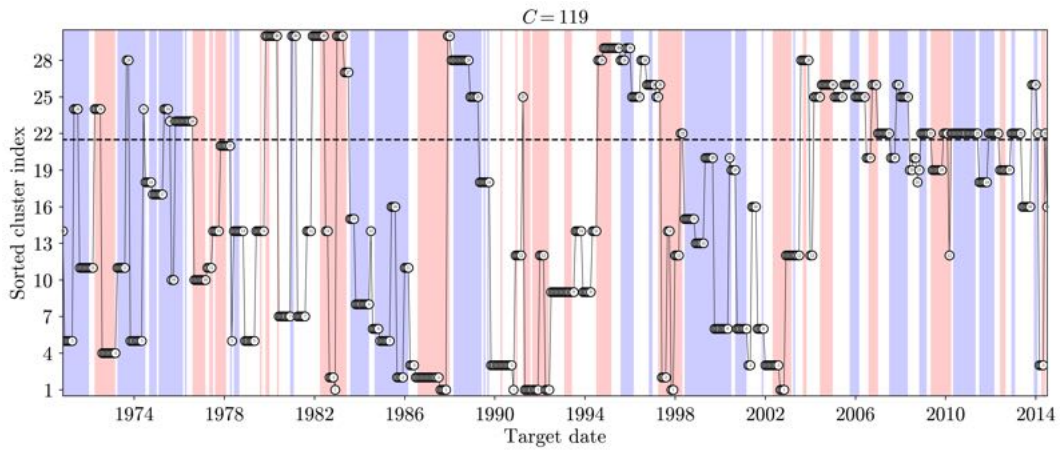
Figure 64: Training set cluster affiliation plots using PCA features and 90% of the dataset: 12-24 months



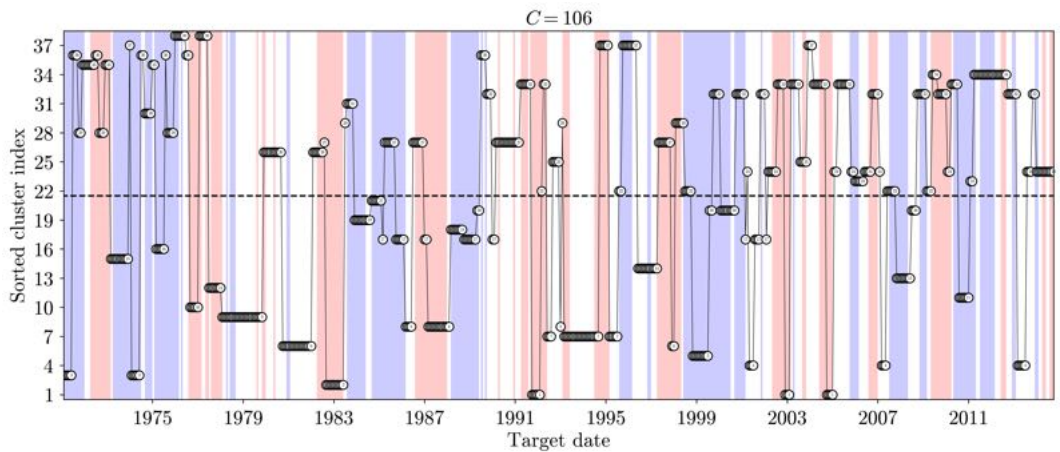
### 6.3 PCA: 80%



(a) 3 months.

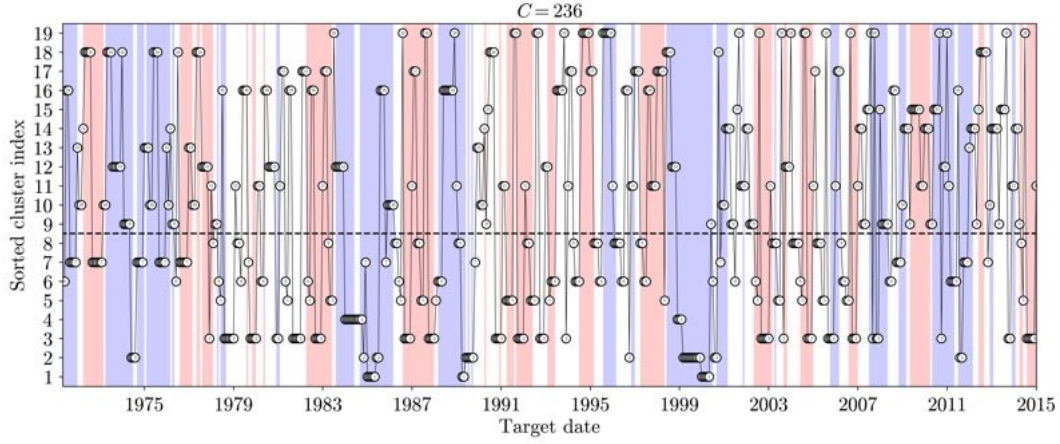


(b) 6 months.

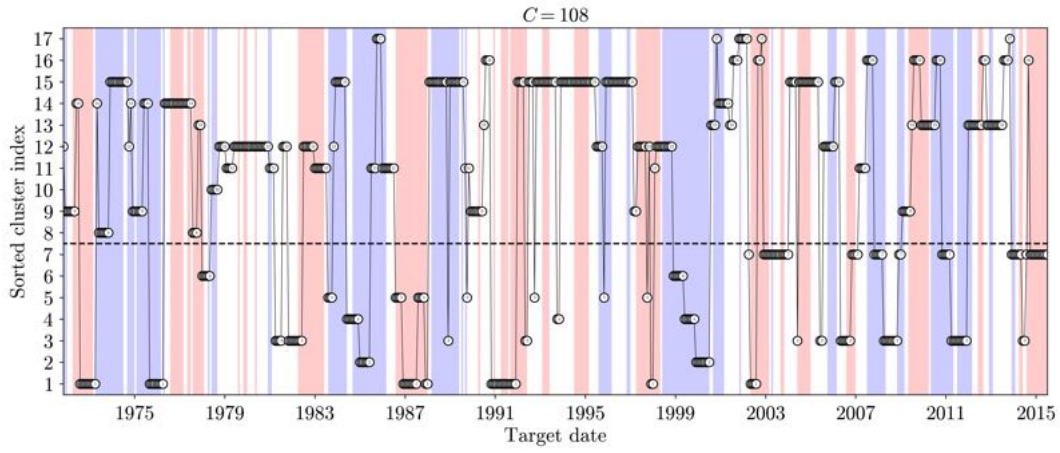


(c) 9 months.

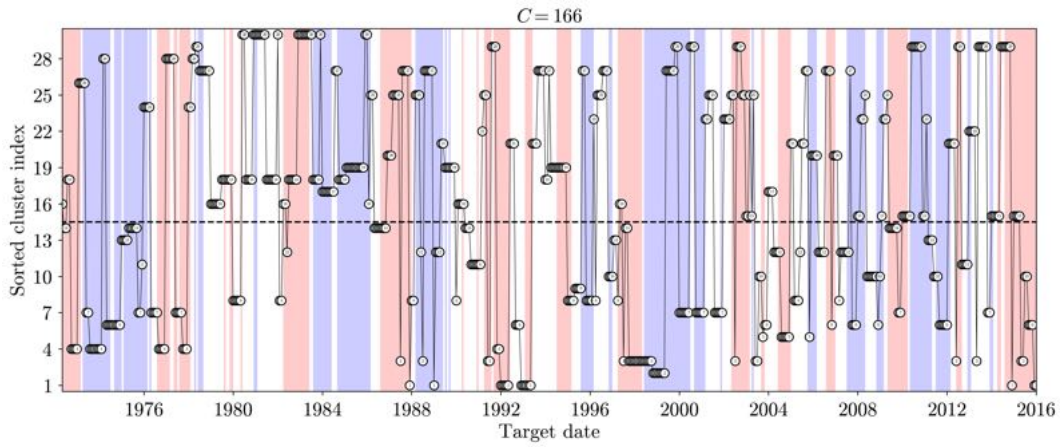
Figure 65: Training set cluster affiliation plots using PCA features and 80% of the dataset: 3-9 months



(a) 12 months.



(b) 18 months.

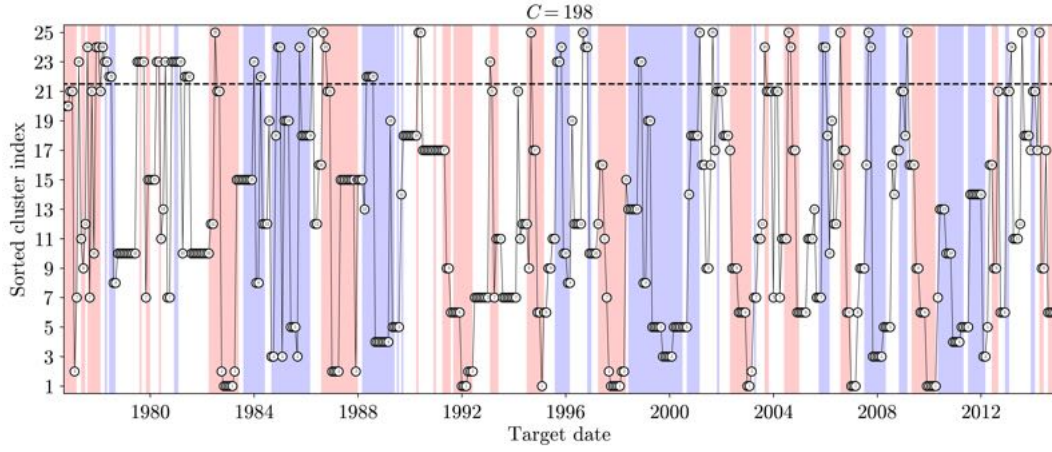


(c) 24 months.

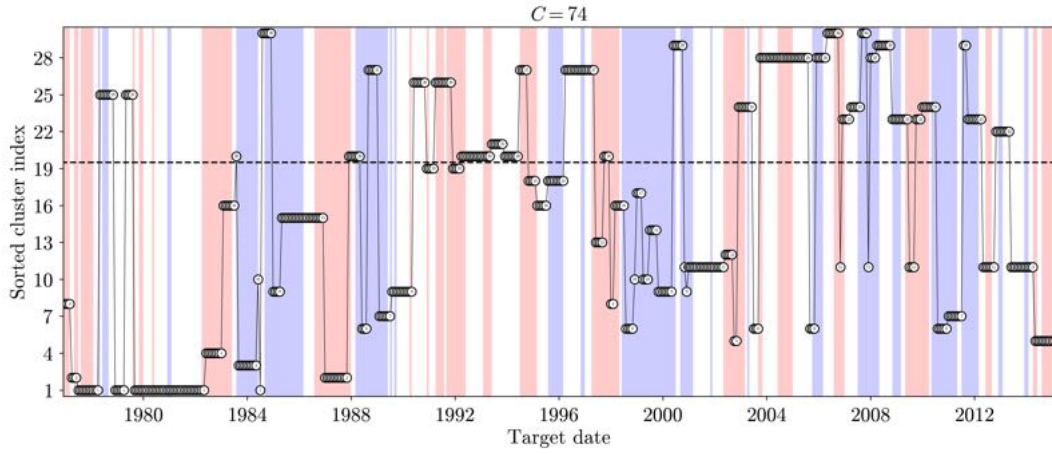
Figure 66: Training set cluster affiliation plots using PCA features and 80% of the dataset: 12-24 months



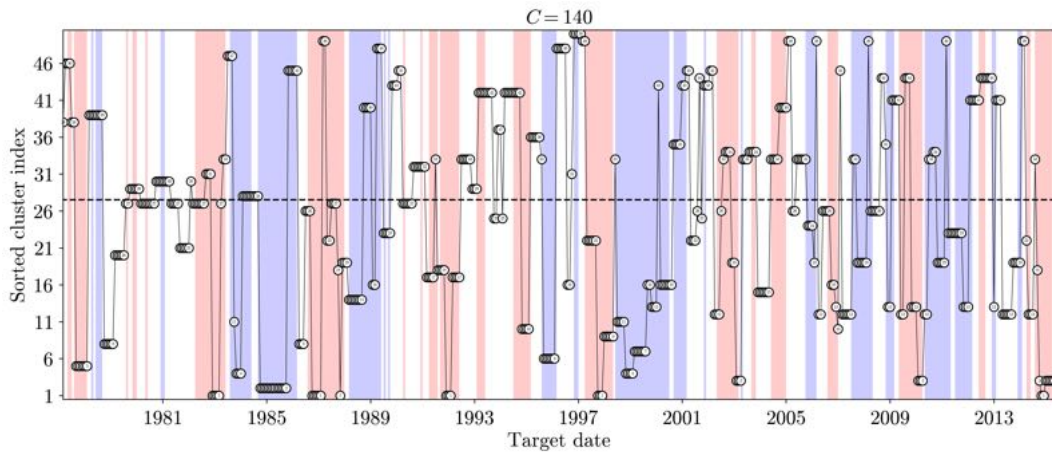
## 6.4 PCA: 70%



(a) 3 months.

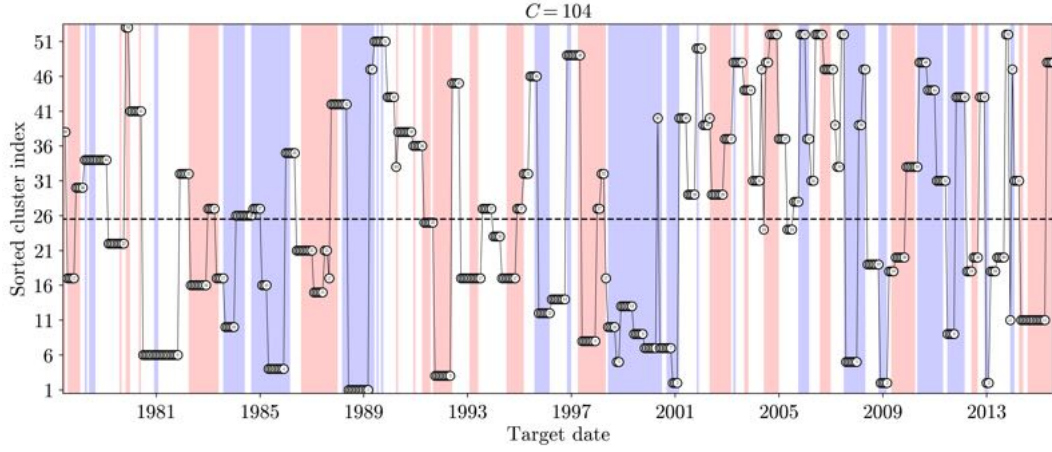


(b) 6 months.

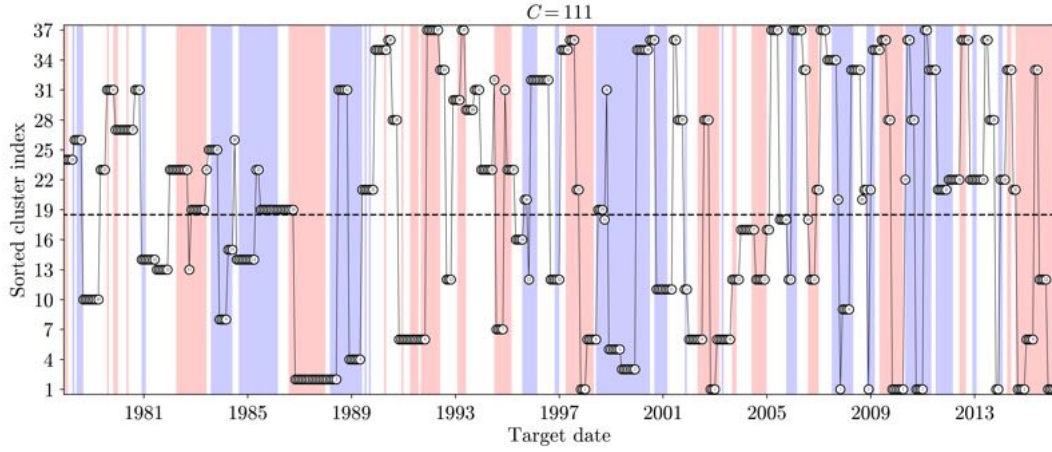


(c) 9 months.

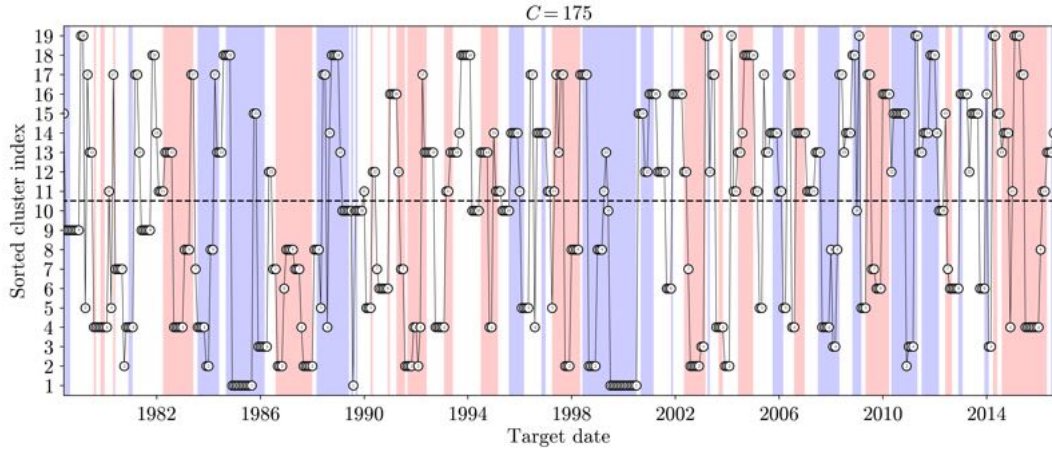
Figure 67: Training set cluster affiliation plots using PCA features and 70% of the dataset: 3-9 months



(a) 12 months.



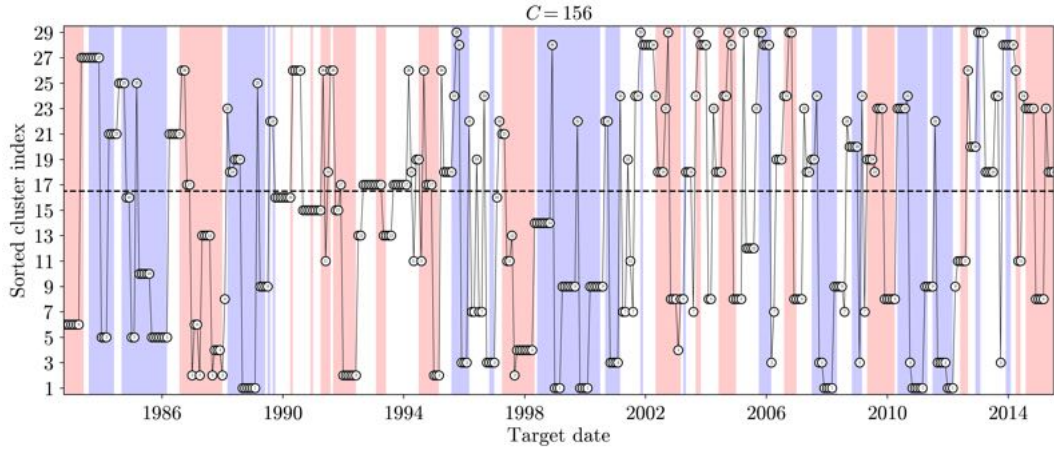
(b) 18 months.



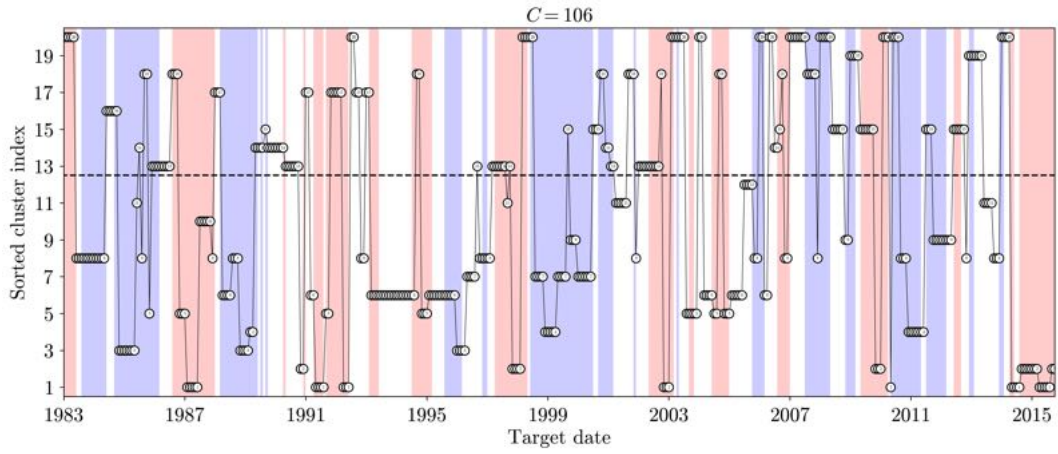
(c) 24 months.

Figure 68: Training set cluster affiliation plots using PCA features and 70% of the dataset: 12-24 months

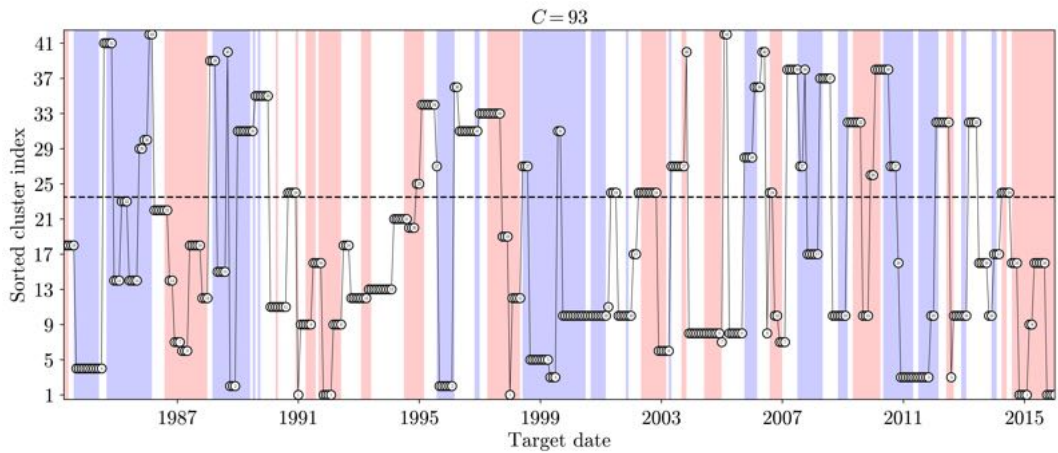
## 6.5 PCA: 60%



(a) 3 months.



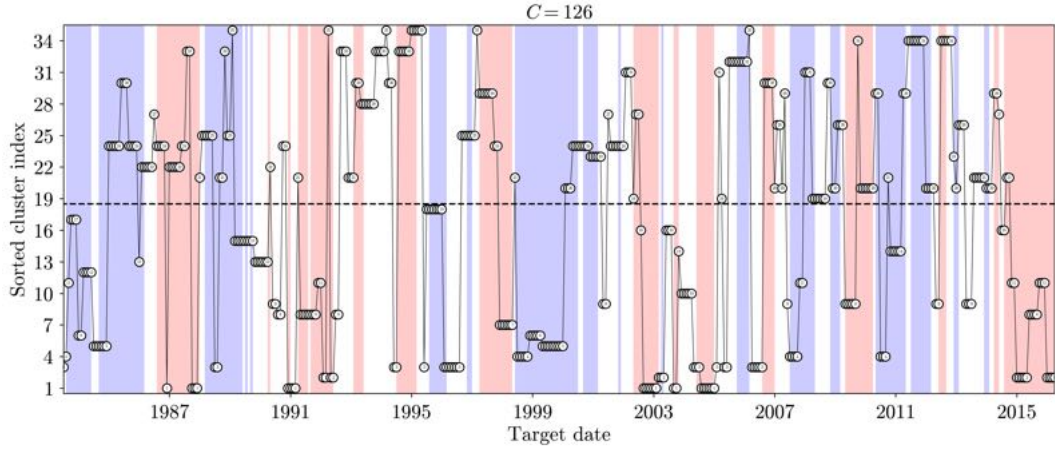
(b) 6 months.



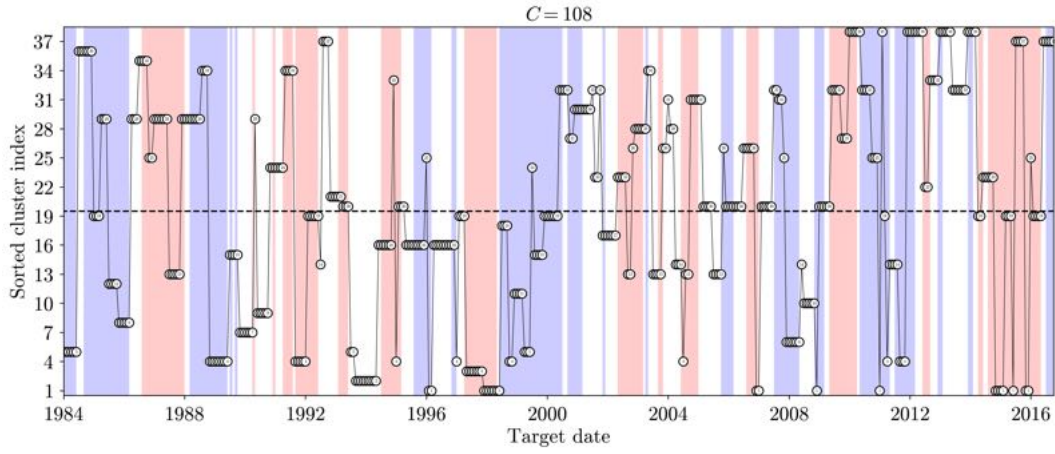
(c) 9 months.

Figure 69: Training set cluster affiliation plots using PCA features and 60% of the dataset: 3-9 months

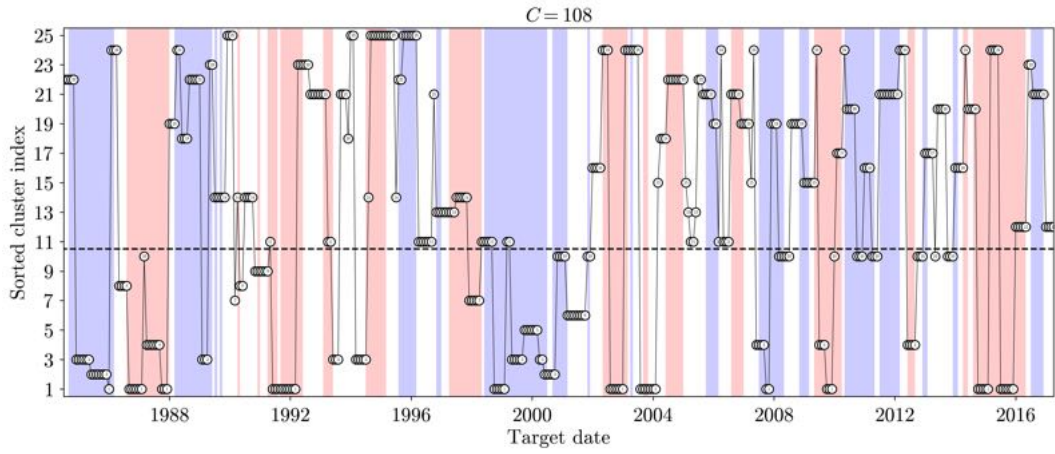




(a) 12 months.



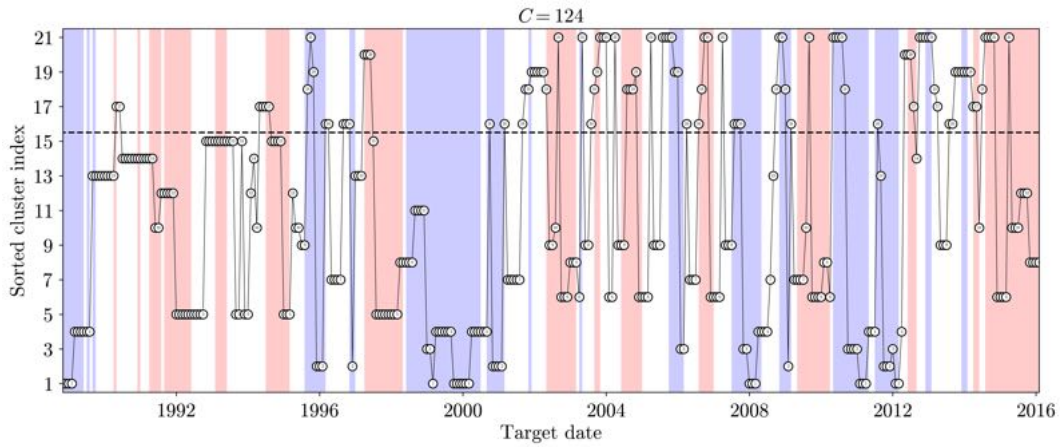
(b) 18 months.



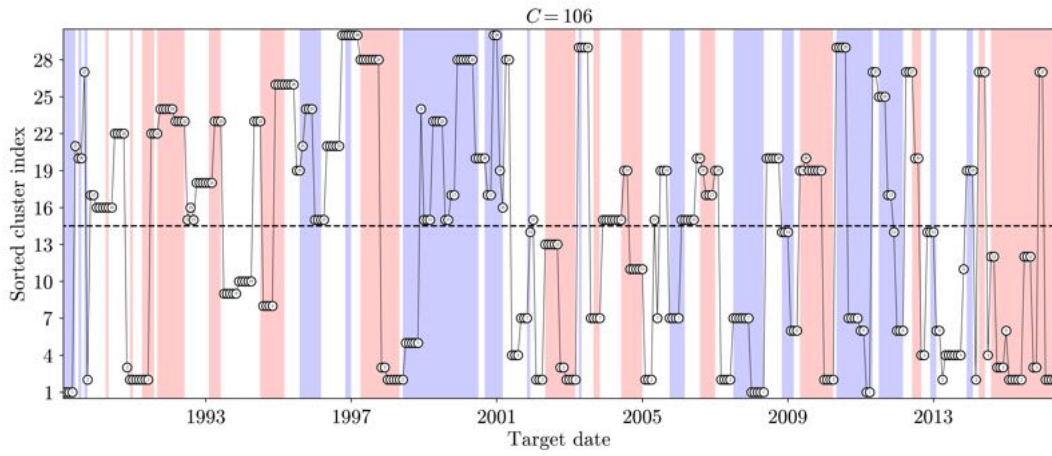
(c) 24 months.

Figure 70: Training set cluster affiliation plots using PCA features and 60% of the dataset: 12-24 months

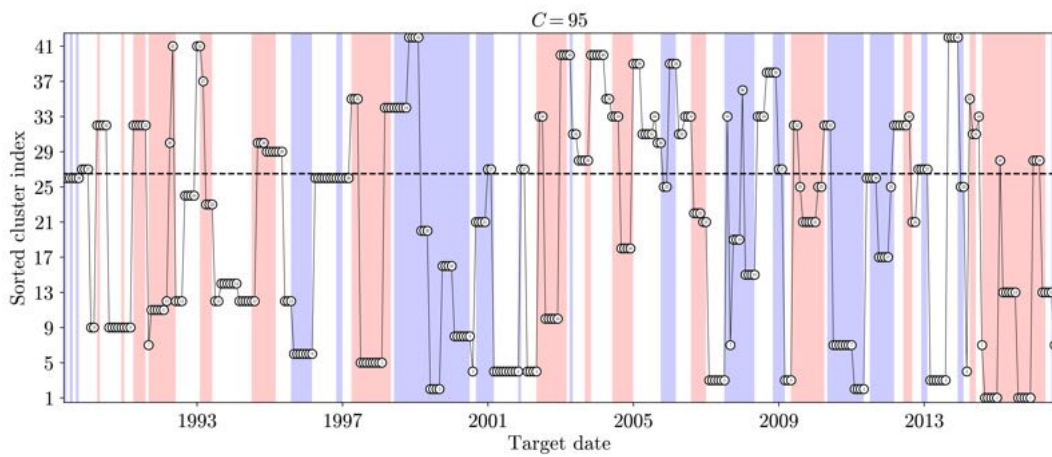
## 6.6 PCA: 50%



(a) 3 months.



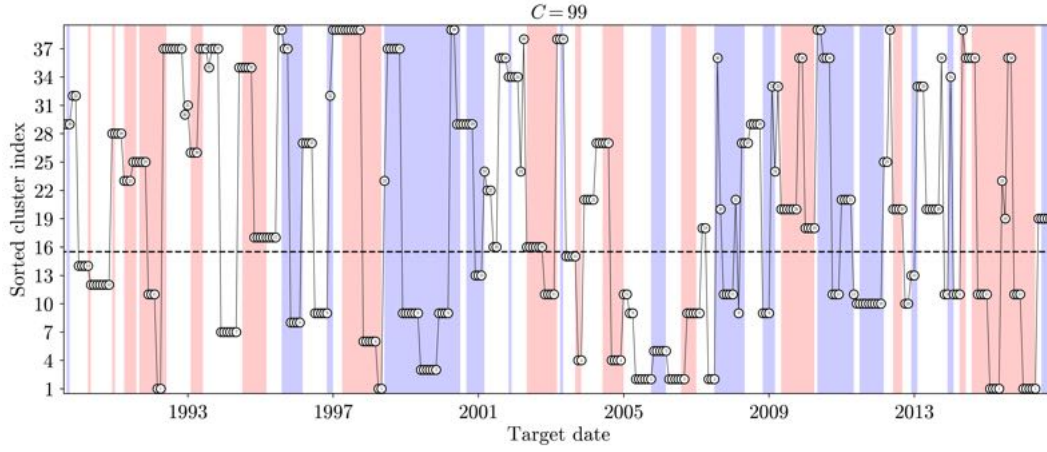
(b) 6 months.



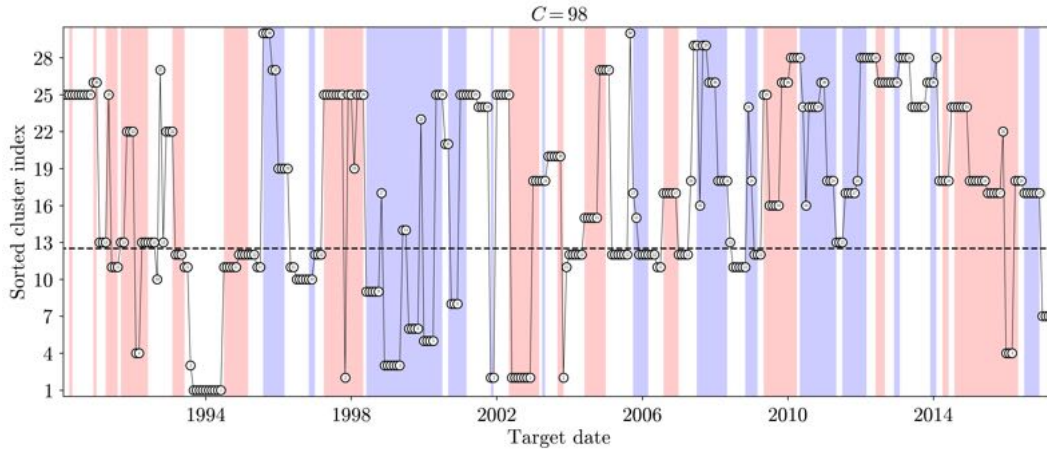
(c) 9 months.

Figure 71: Training set cluster affiliation plots using PCA features and 50% of the dataset: 3-9 months

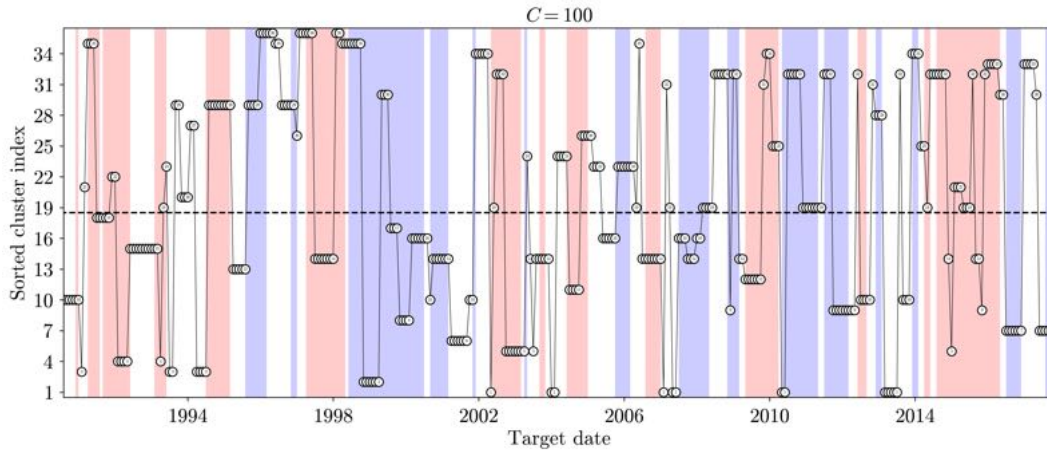




(a) 12 months.



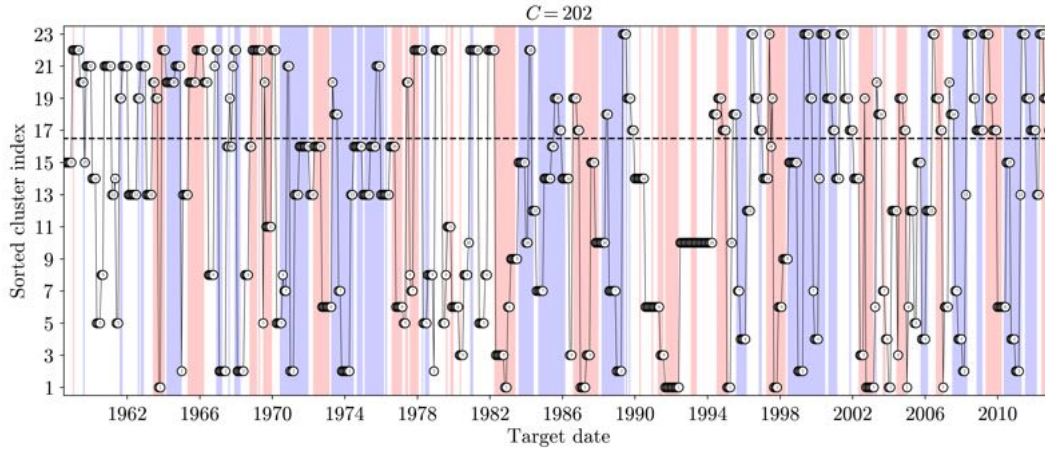
(b) 18 months.



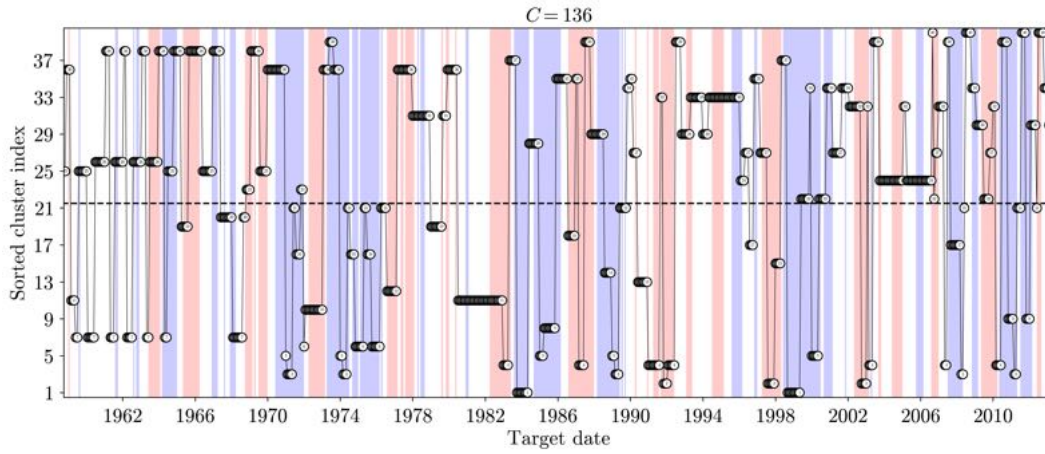
(c) 24 months.

Figure 72: Training set cluster affiliation plots using PCA features and 50% of the dataset: 12-24 months

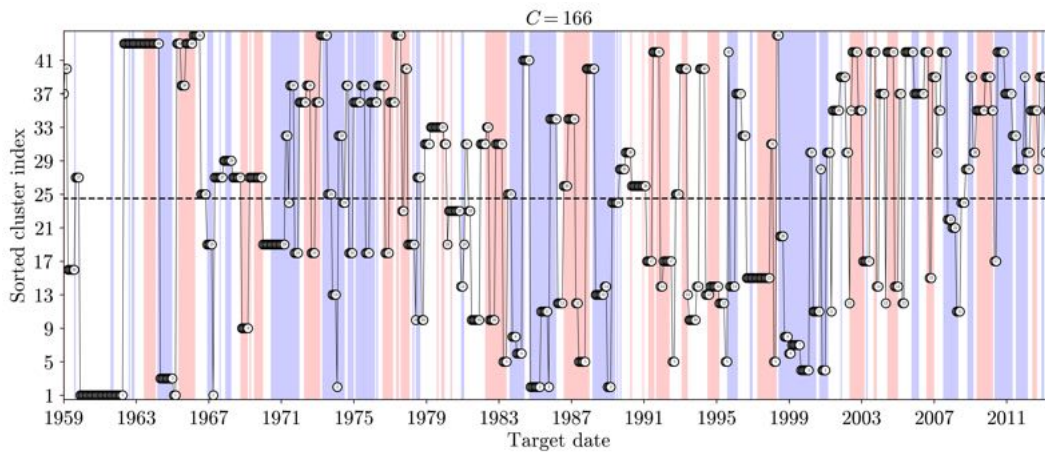
## 6.7 SSA: 100%



(a) 3 months.

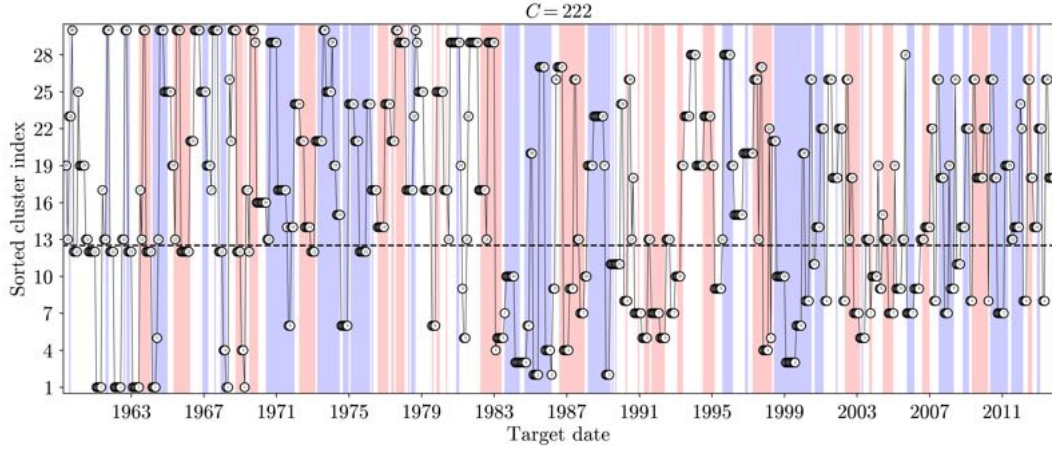


(b) 6 months.

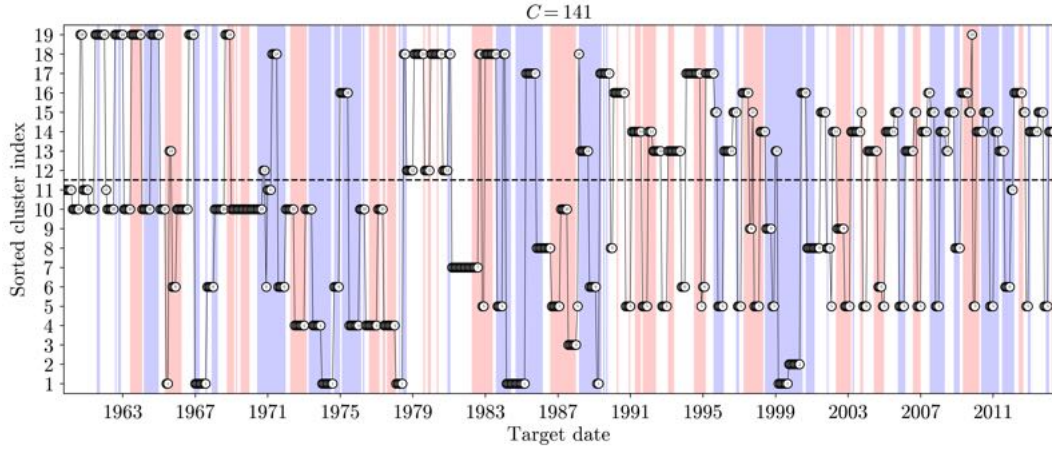


(c) 9 months.

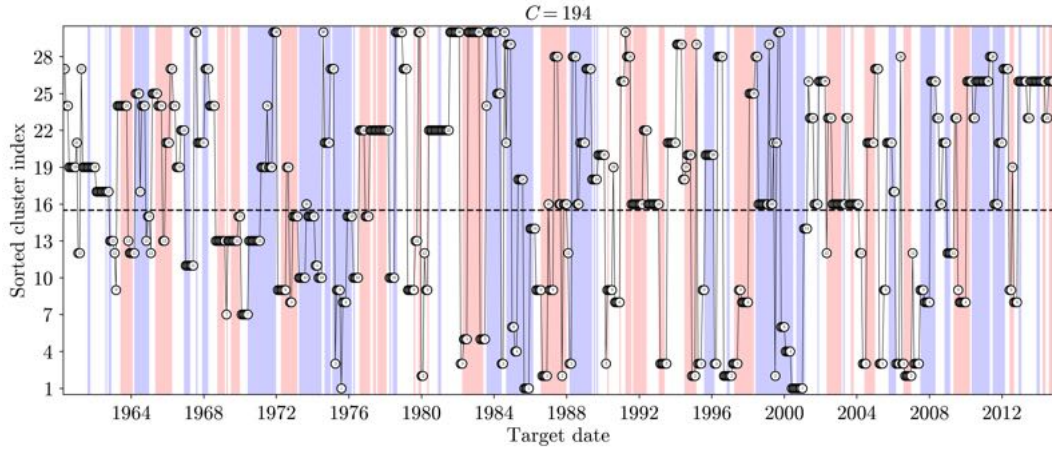
Figure 73: Training set cluster affiliation plots using SSA features and 100% of the dataset: 3-9 months



(a) 12 months.



(b) 18 months.

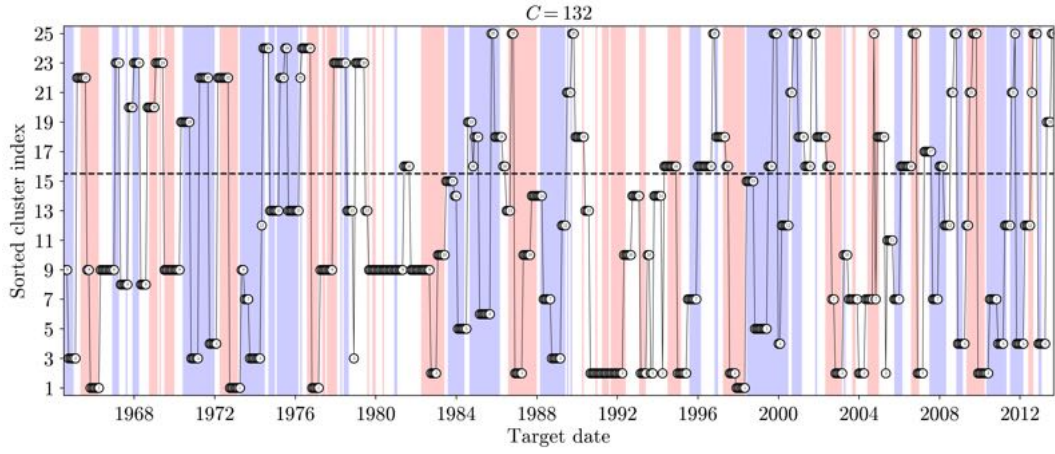


(c) 24 months.

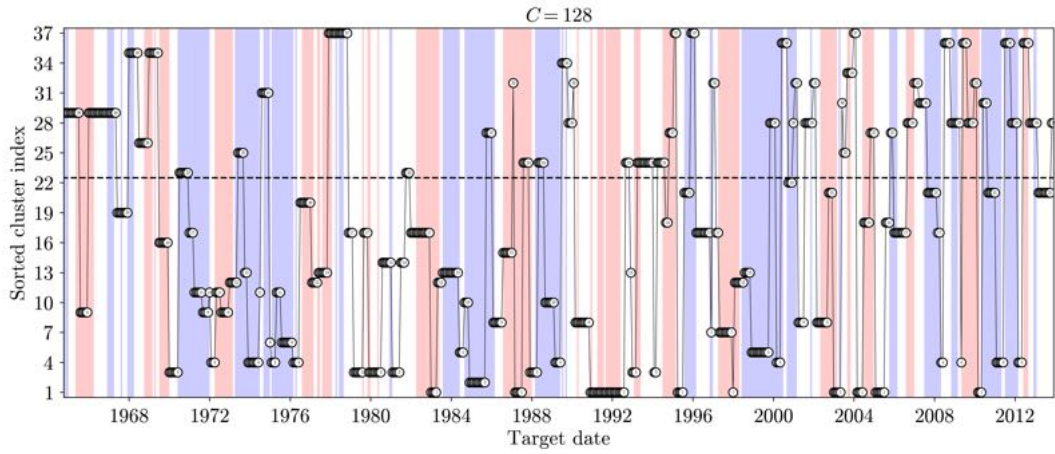
Figure 74: Training set cluster affiliation plots using SSA features and 100% of the dataset: 12-24 months



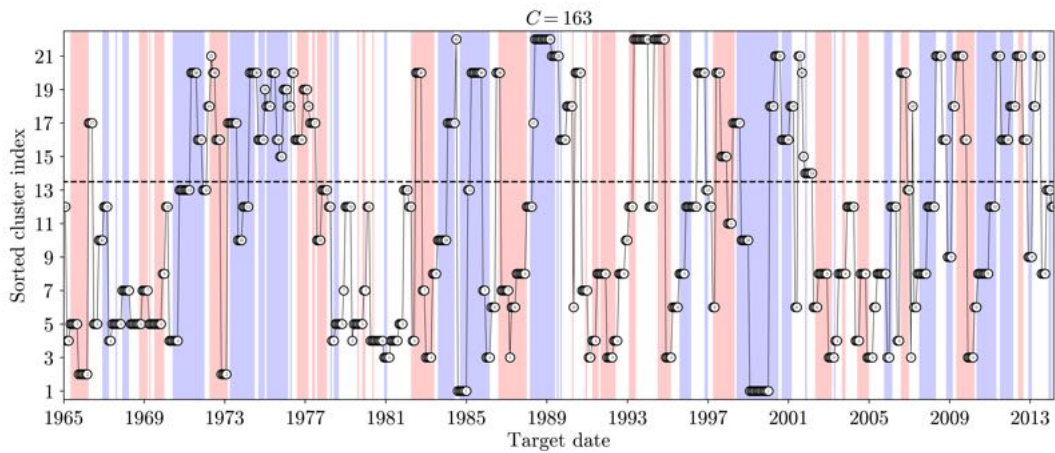
## 6.8 SSA: 90%



(a) 3 months.

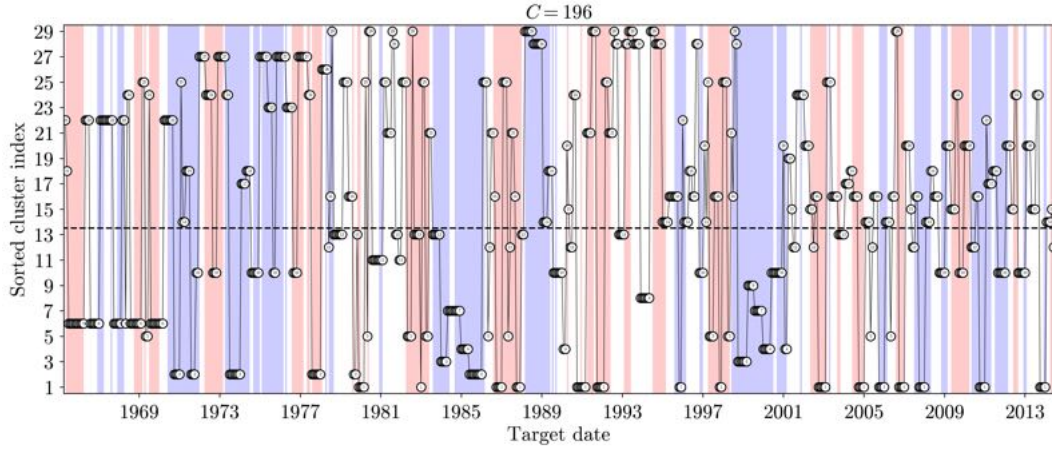


(b) 6 months.

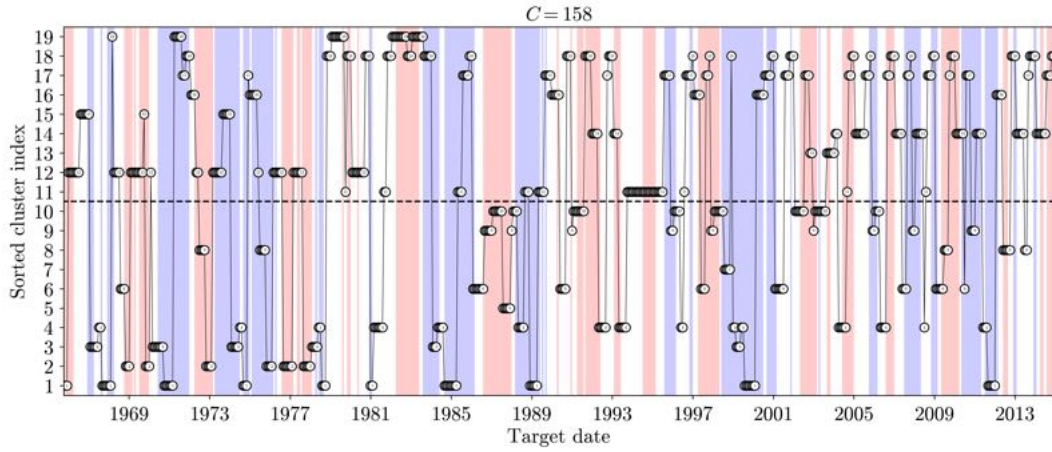


(c) 9 months.

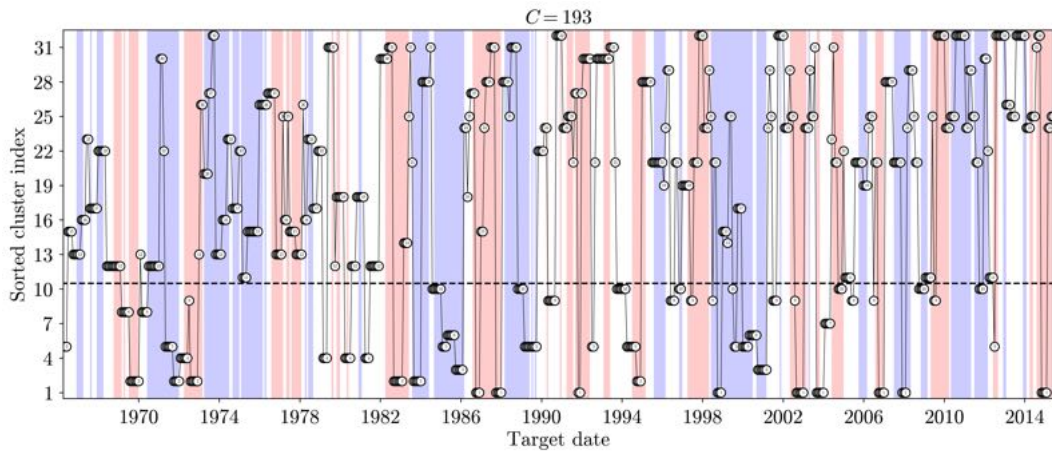
Figure 75: Training set cluster affiliation plots using SSA features and 90% of the dataset: 3-9 months



(a) 12 months.



(b) 18 months.

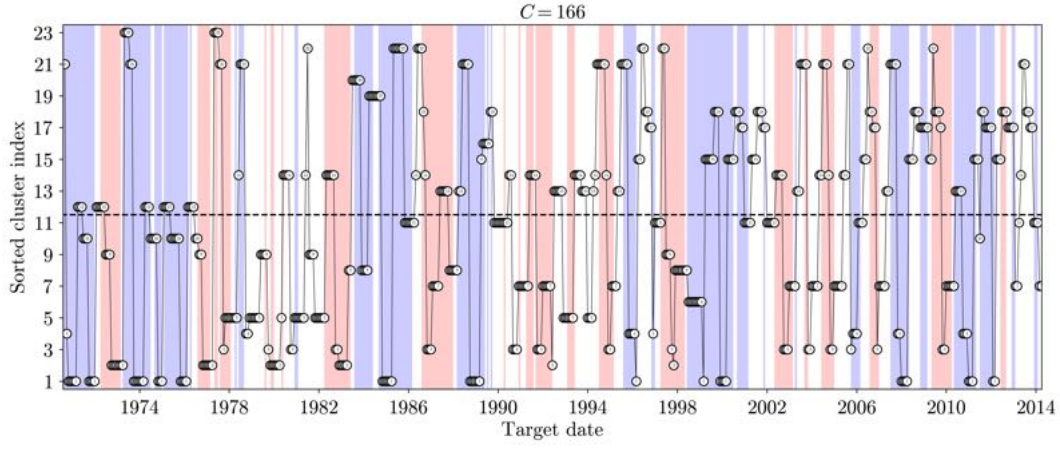


(c) 24 months.

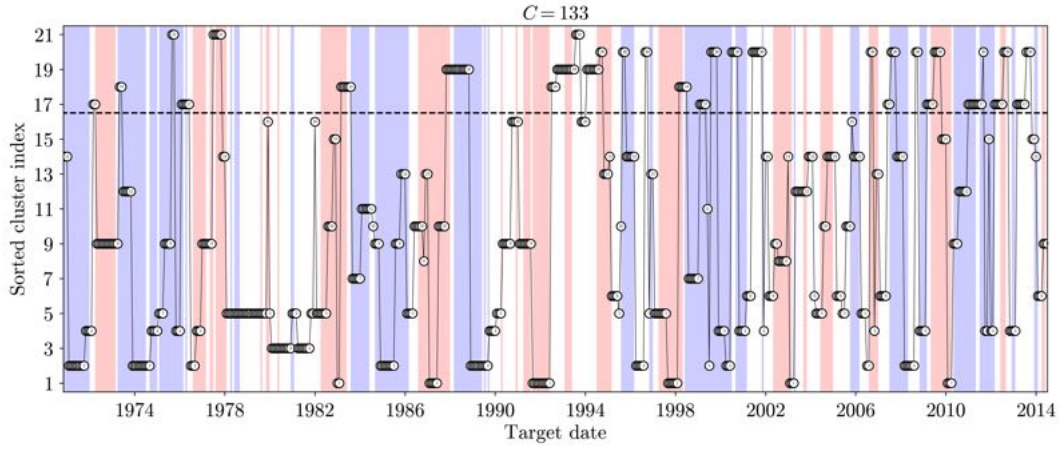
Figure 76: Training set cluster affiliation plots using SSA features and 90% of the dataset: 12-24 months



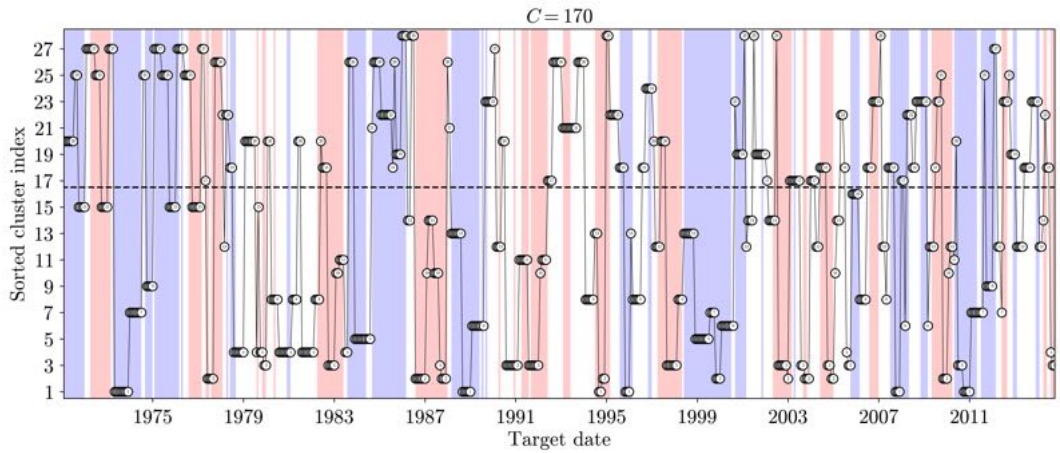
## 6.9 SSA: 80%



(a) 3 months.

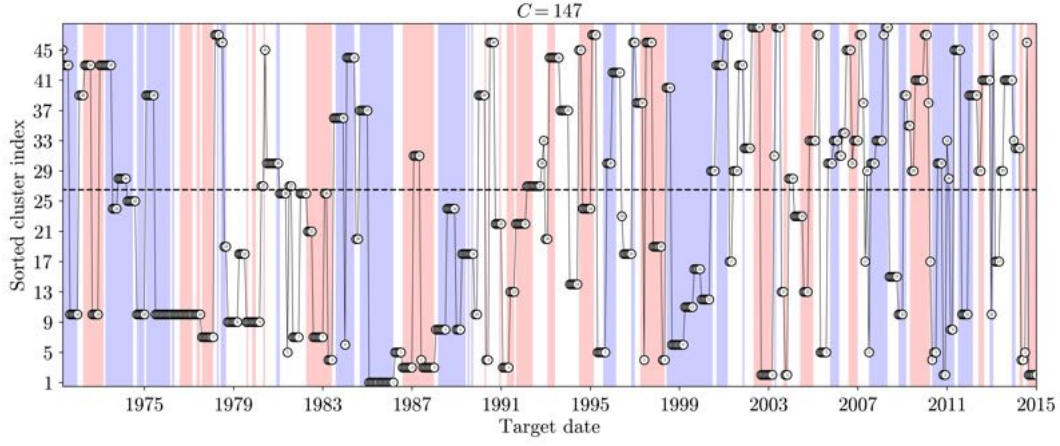


(b) 6 months.

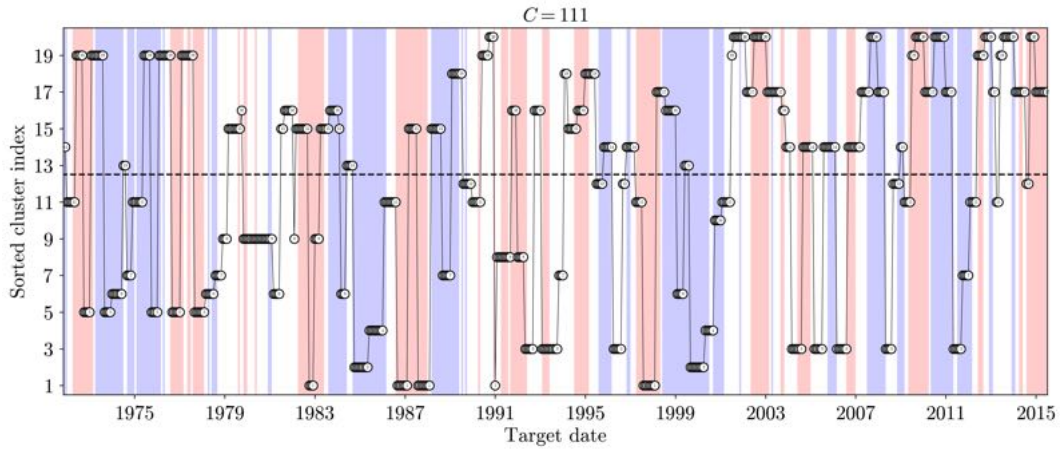


(c) 9 months.

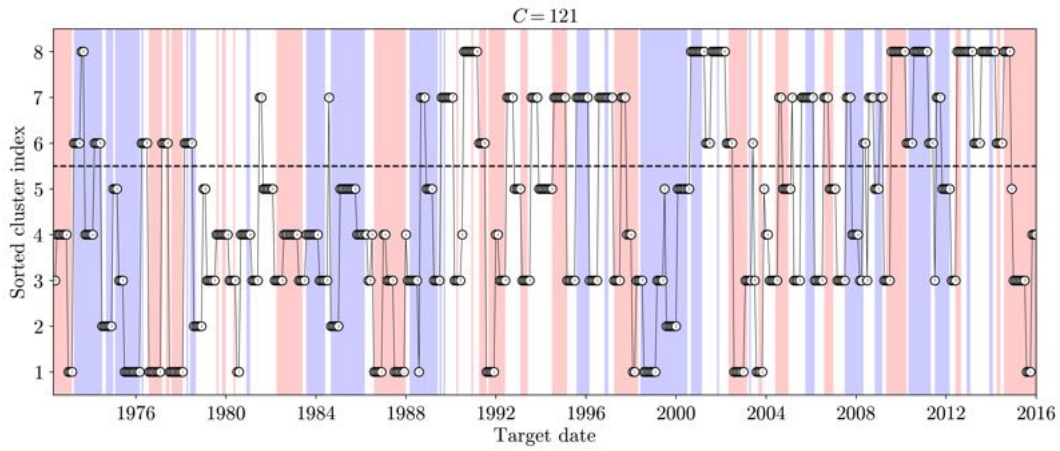
Figure 77: Training set cluster affiliation plots using SSA features and 80% of the dataset: 3-9 months



(a) 12 months.



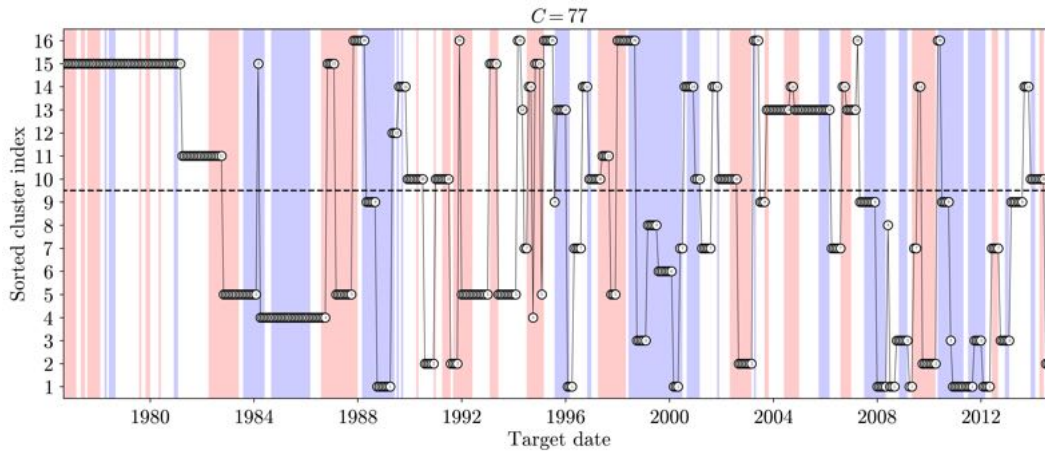
(b) 18 months.



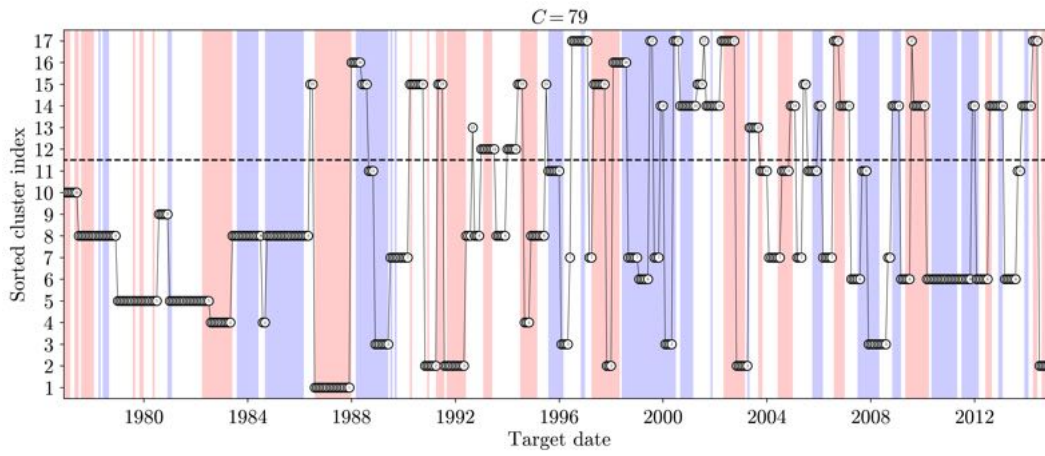
(c) 24 months.

Figure 78: Training set cluster affiliation plots using SSA features and 80% of the dataset: 12-24 months

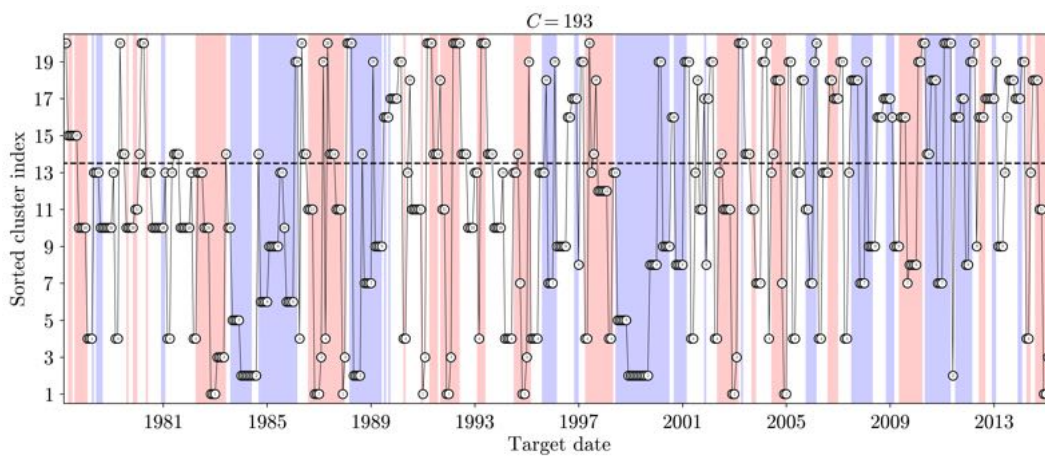
## 6.10 SSA: 70%



(a) 3 months.



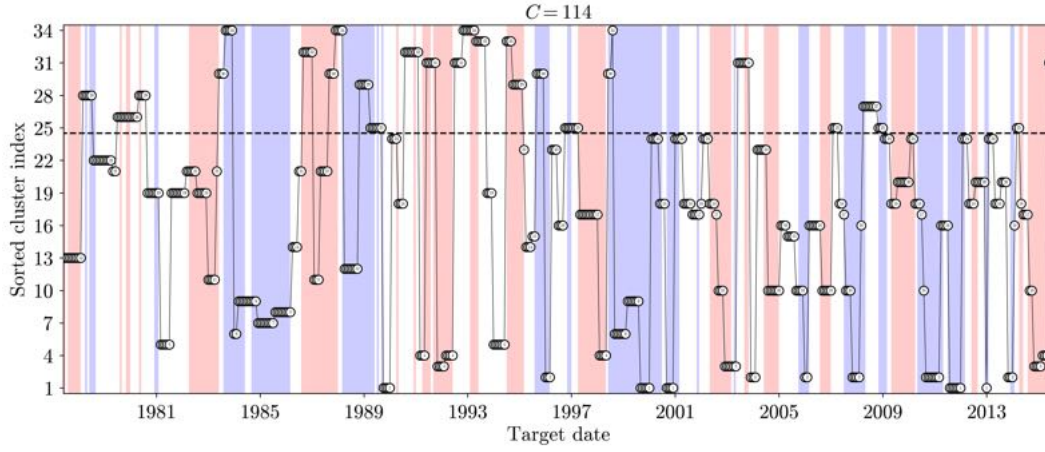
(b) 6 months.



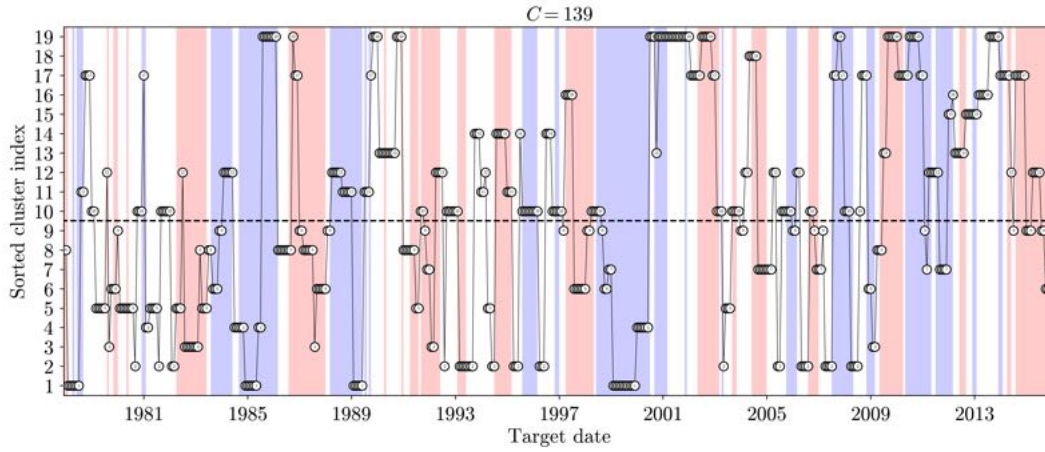
(c) 9 months.

Figure 79: Training set cluster affiliation plots using SSA features and 70% of the dataset: 3-9 months

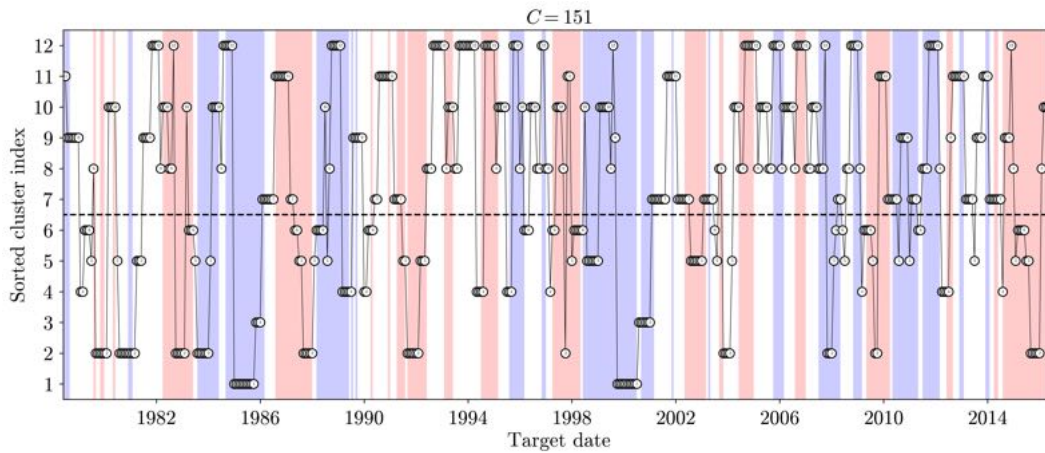




(a) 12 months.



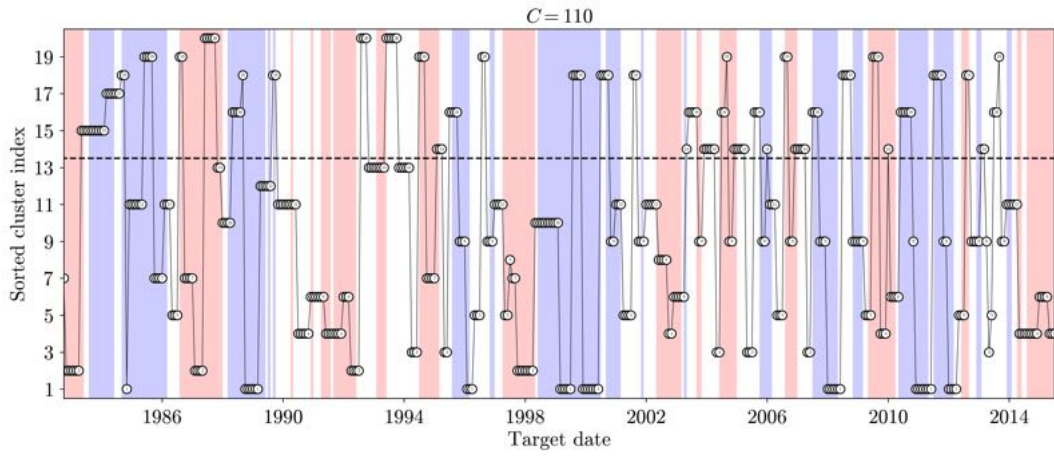
(b) 18 months.



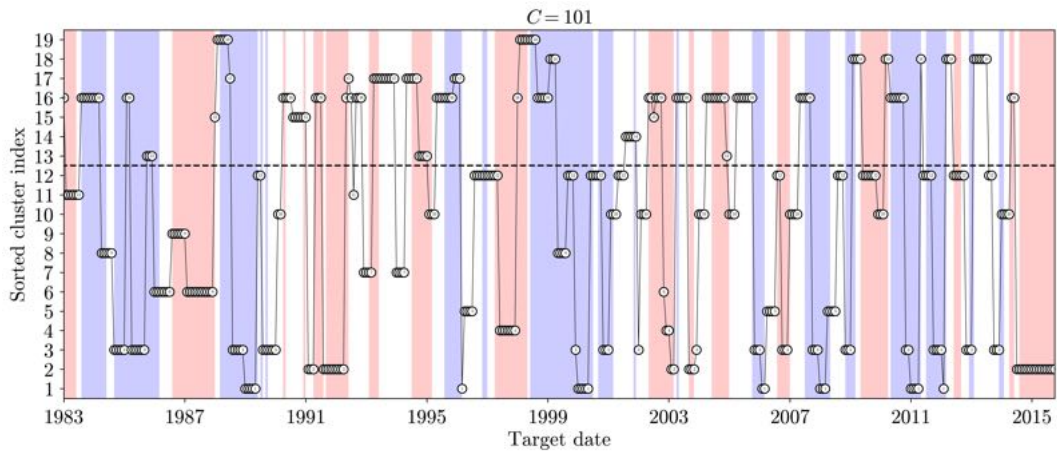
(c) 24 months.

Figure 80: Training set cluster affiliation plots using SSA features and 70% of the dataset: 12-24 months

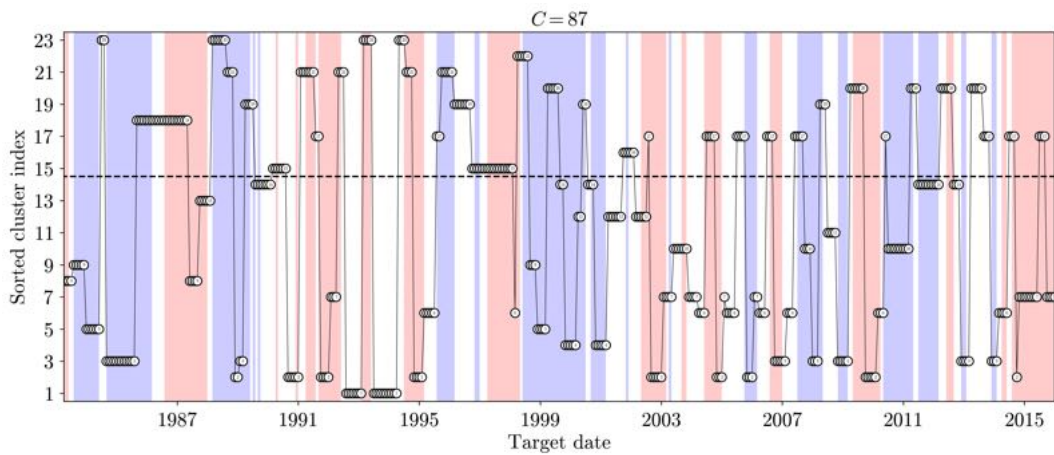
## 6.11 SSA: 60%



(a) 3 months.



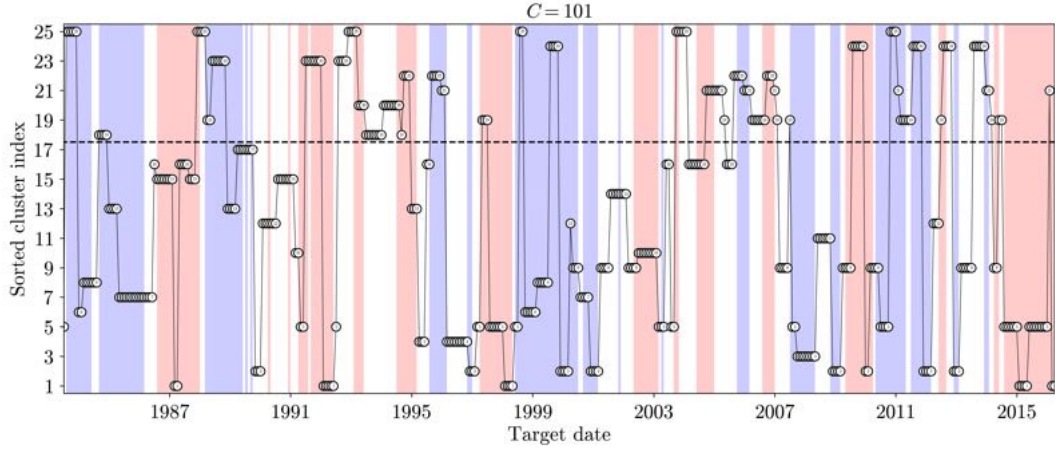
(b) 6 months.



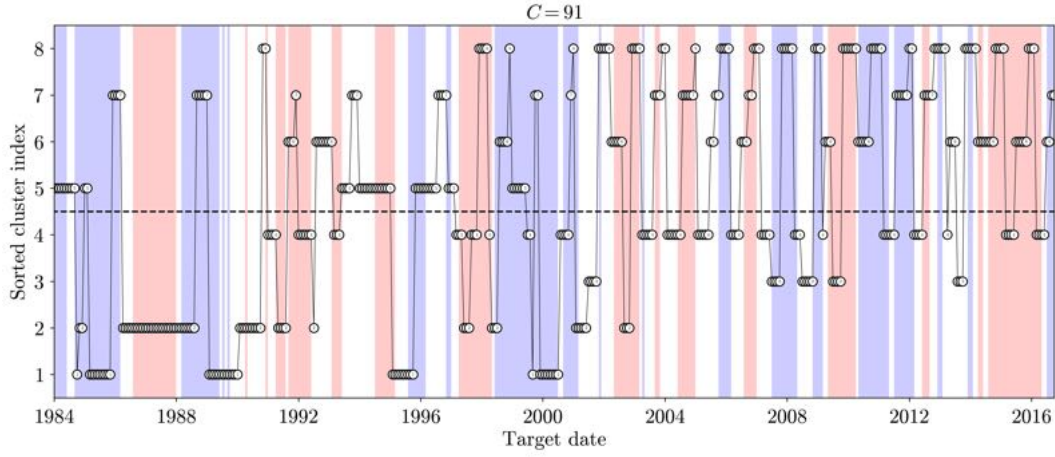
(c) 9 months.

Figure 81: Training set cluster affiliation plots using SSA features and 60% of the dataset: 3-9 months

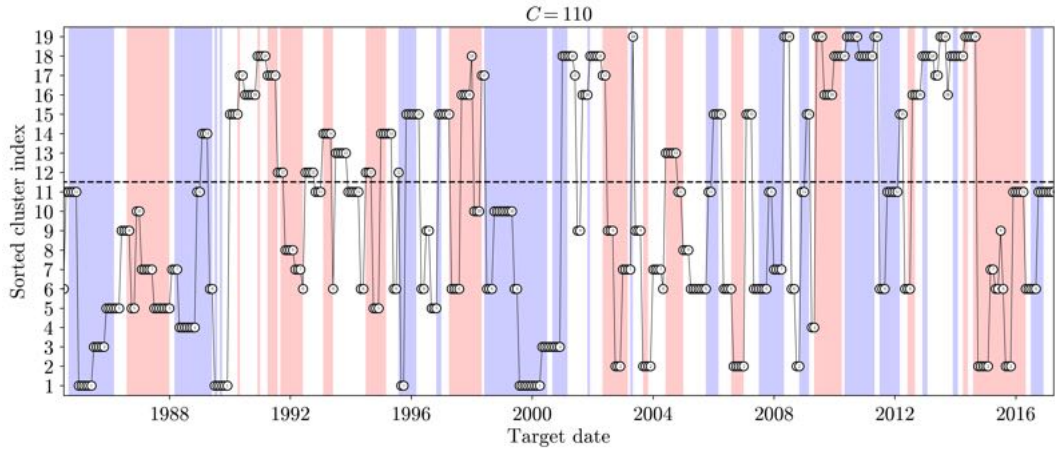




(a) 12 months.



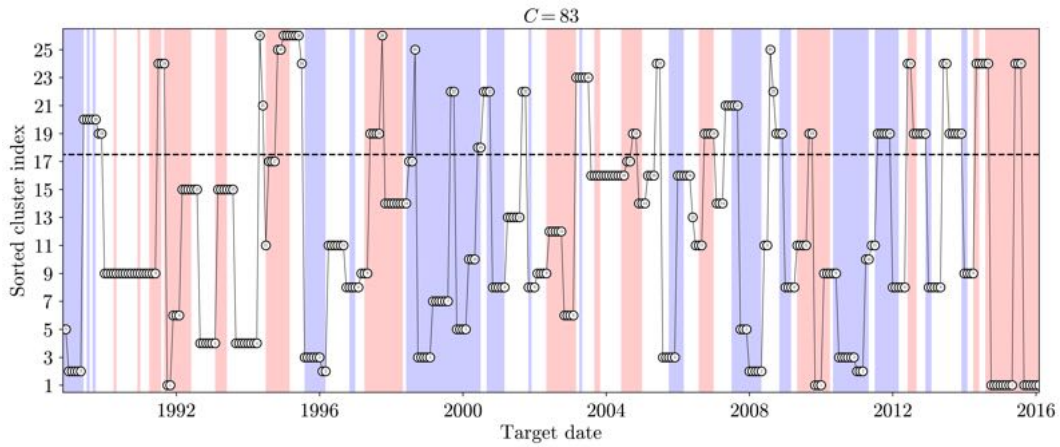
(b) 18 months.



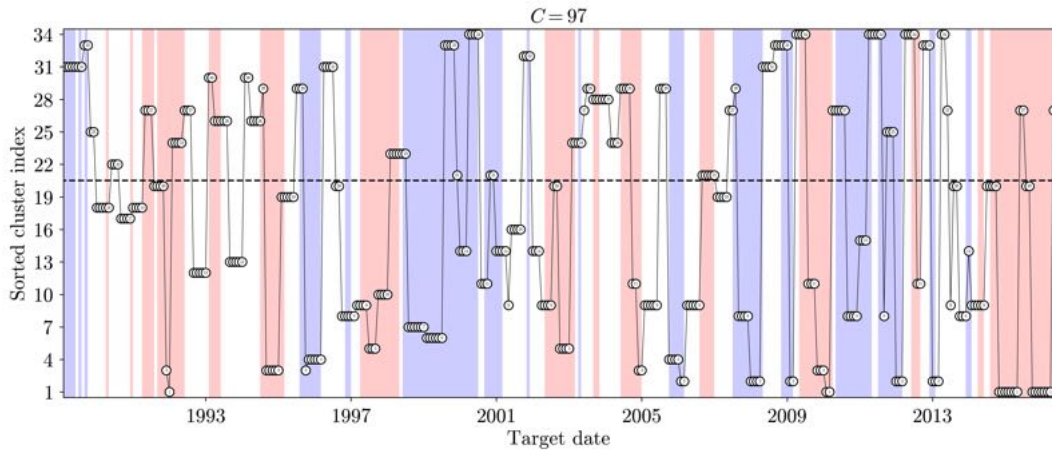
(c) 24 months.

Figure 82: Training set cluster affiliation plots using SSA features and 60% of the dataset: 12-24 months

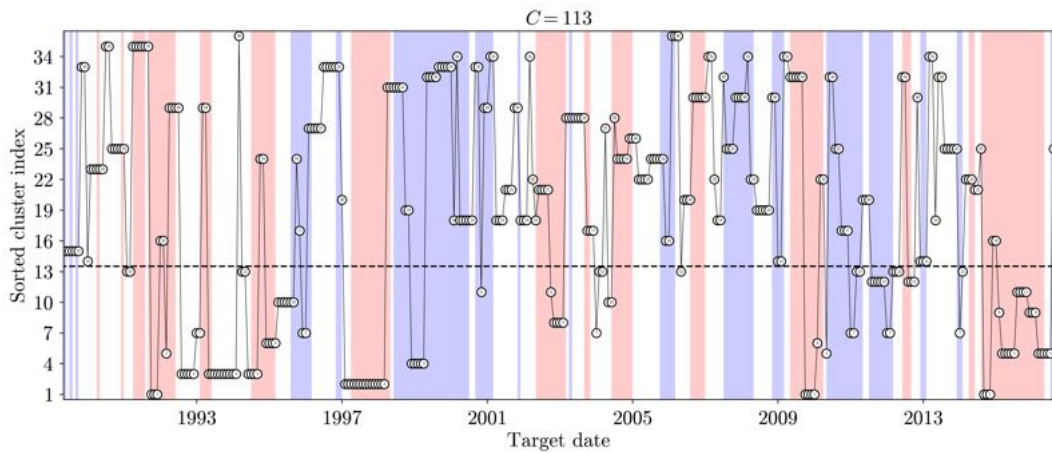
## 6.12 SSA: 50%



(a) 3 months.

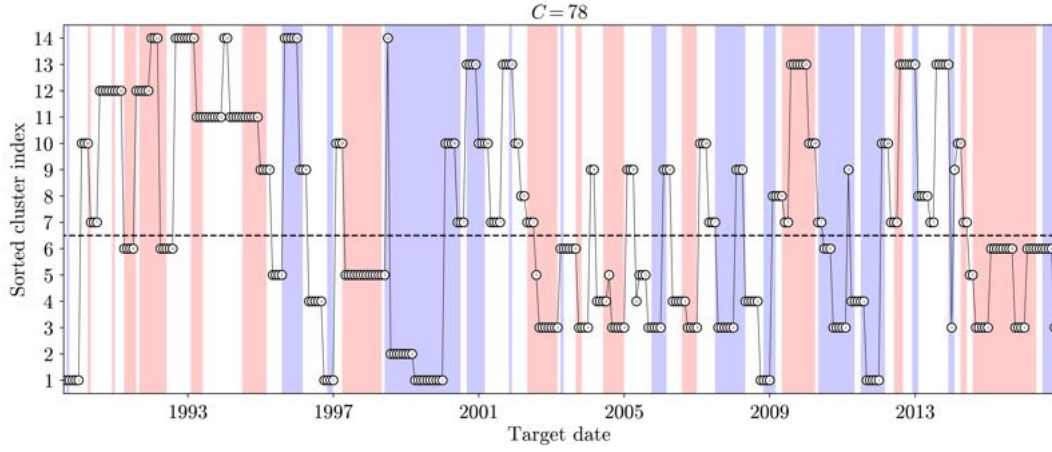


(b) 6 months.

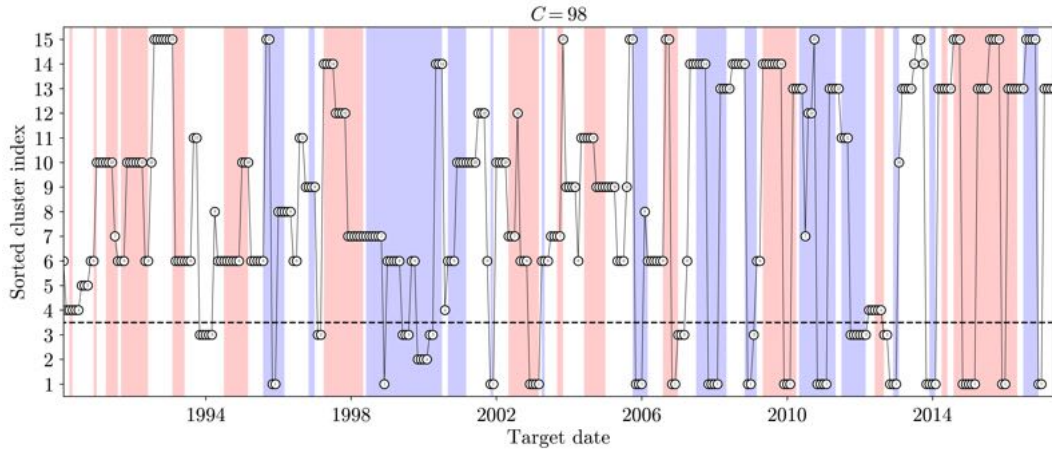


(c) 9 months.

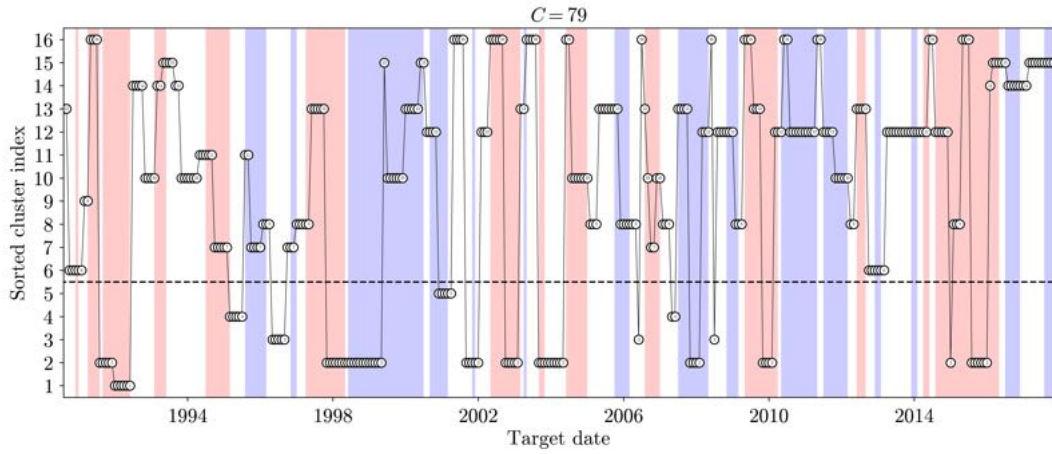
Figure 83: Training set cluster affiliation plots using SSA features and 50% of the dataset: 3-9 months



(a) 12 months.



(b) 18 months.



(c) 24 months.

Figure 84: Training set cluster affiliation plots using SSA features and 50% of the dataset: 12-24 months

## 7 Cluster affiliation plots: test set

The red, blue and white background shading indicates an El Niño, La Niña or neutral event occurring on that target date, while the number of switches between clusters is given in the title of each plot.

### 7.1 PCA: 100%

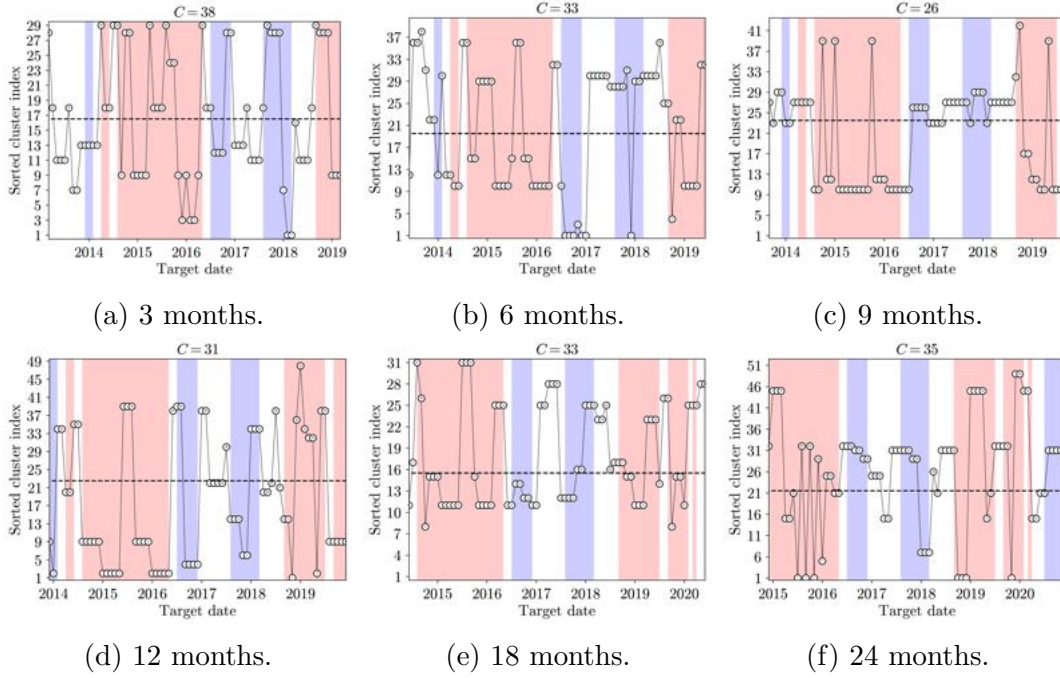


Figure 85: Test set cluster affiliation plots using PCA features and 100% of the dataset.



## 7.2 PCA: 90%

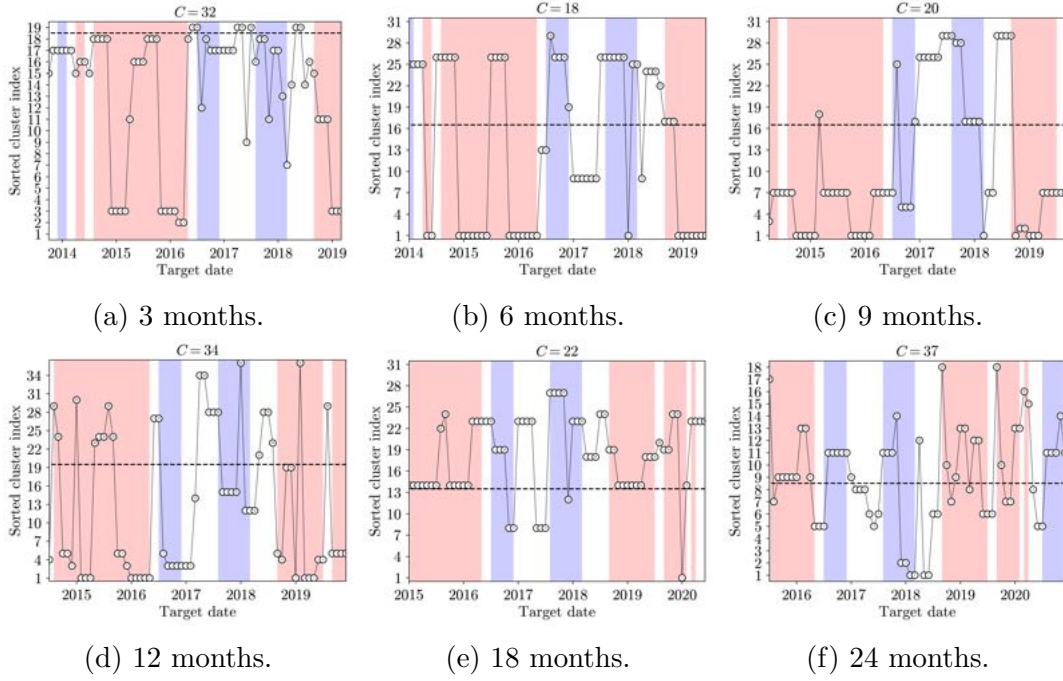


Figure 86: Test set cluster affiliation plots using PCA features and 90% of the dataset.

## 7.3 PCA: 80%

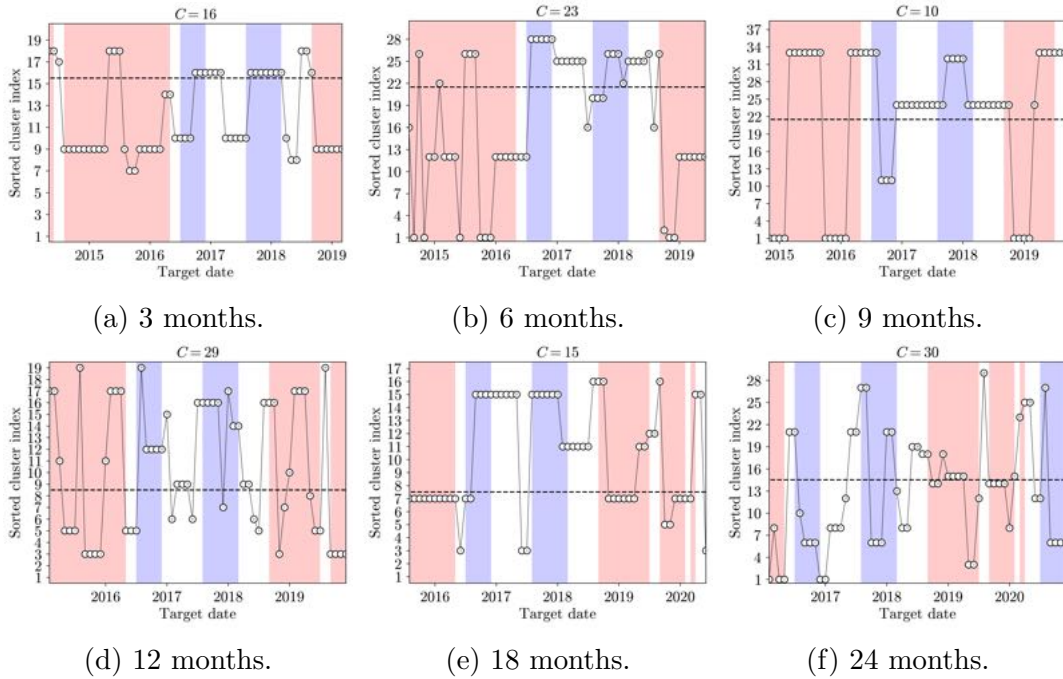


Figure 87: Test set cluster affiliation plots using PCA features and 80% of the dataset.



## 7.4 PCA: 70%

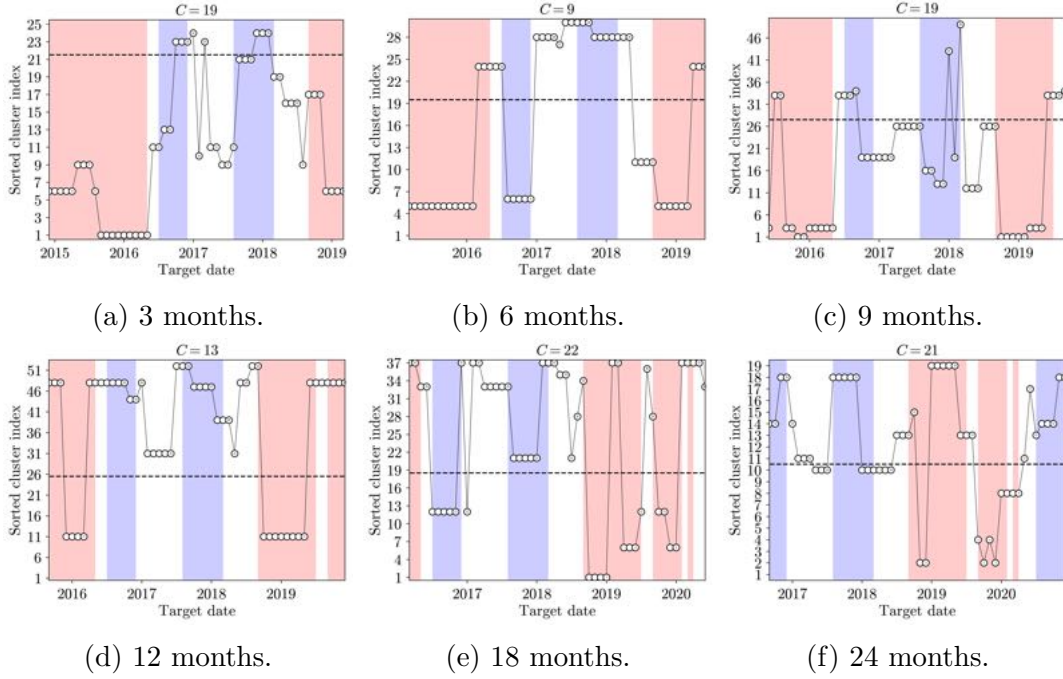


Figure 88: Test set cluster affiliation plots using PCA features and 70% of the dataset.

## 7.5 PCA: 60%

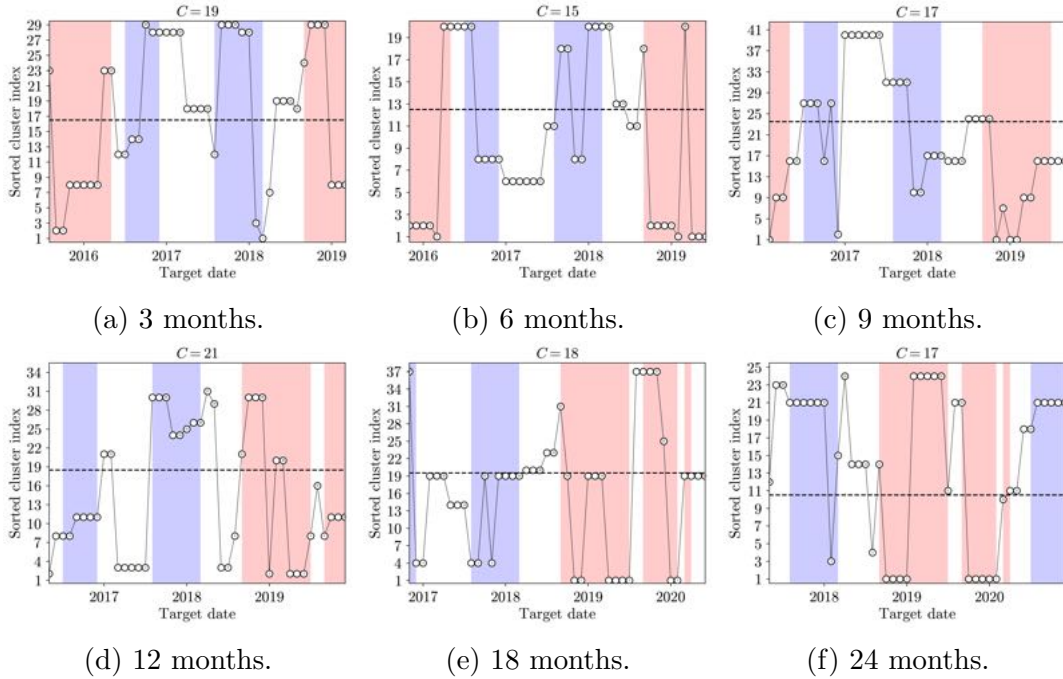


Figure 89: Test set cluster affiliation plots using PCA features and 60% of the dataset.

## 7.6 PCA: 50%

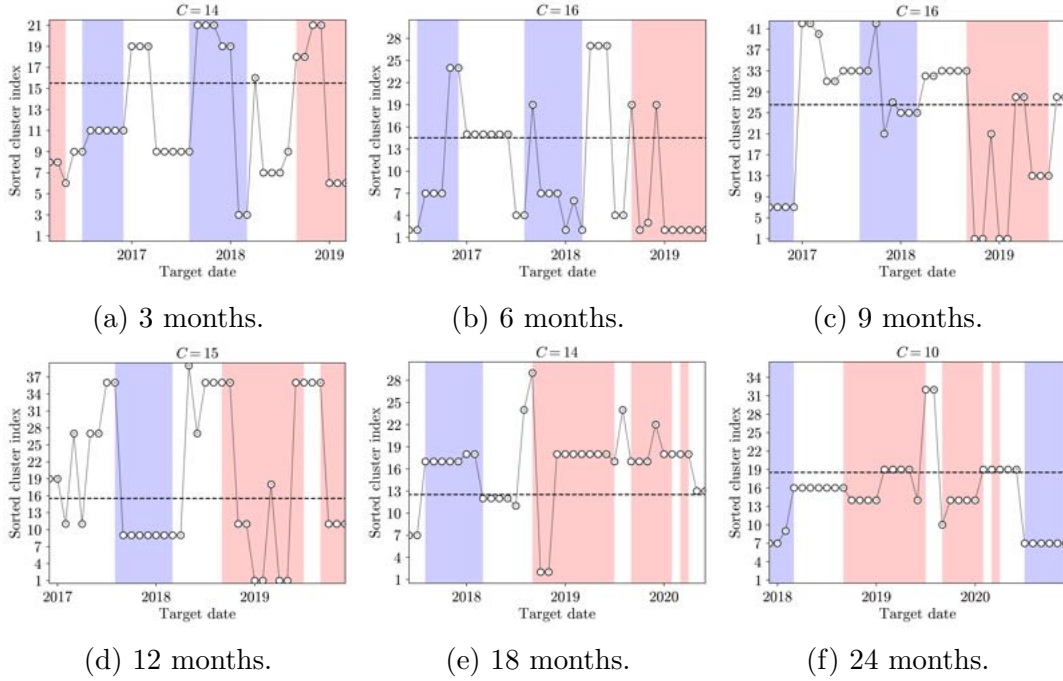


Figure 90: Test set cluster affiliation plots using PCA features and 50% of the dataset.

## 7.7 SSA: 100%

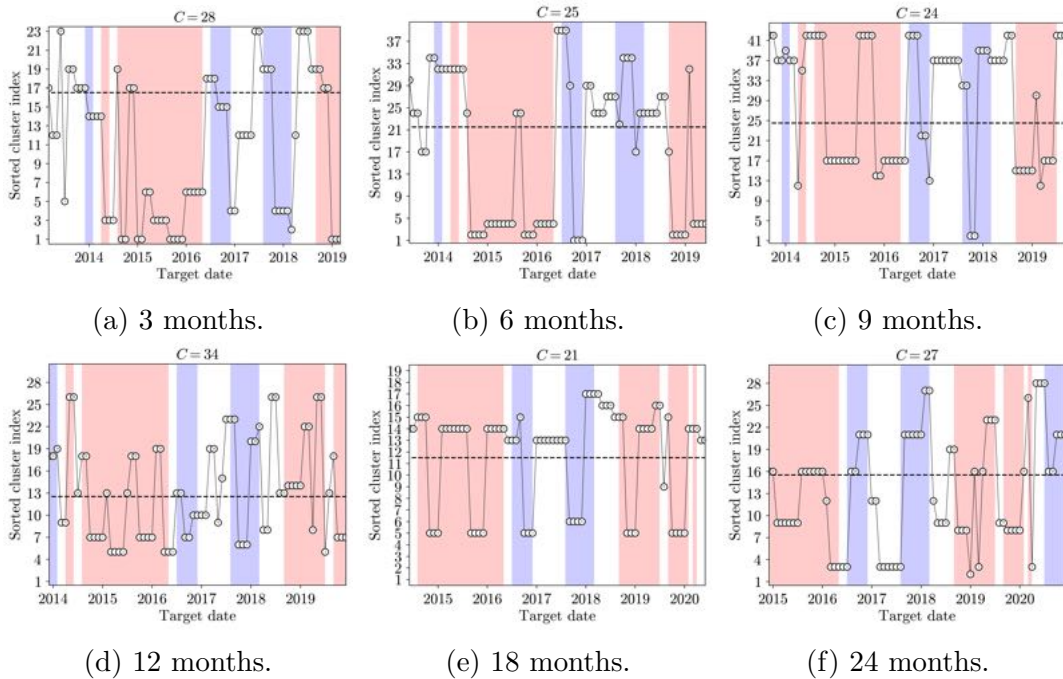


Figure 91: Test set cluster affiliation plots using SSA features and 100% of the dataset.

## 7.8 SSA: 90%

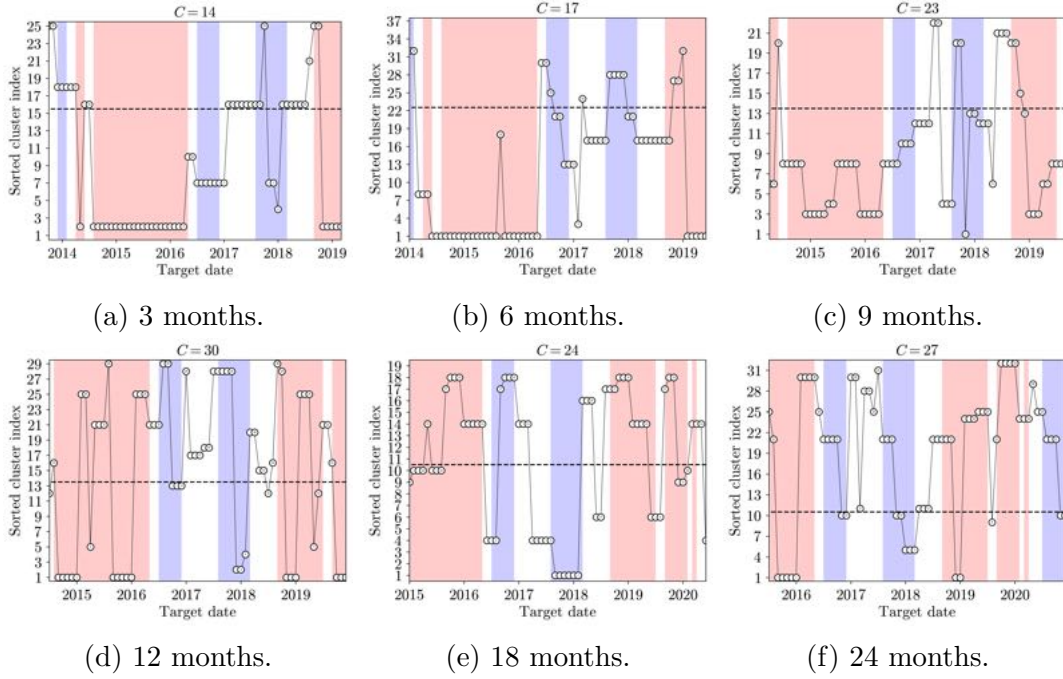


Figure 92: Test set cluster affiliation plots using SSA features and 90% of the dataset.

## 7.9 SSA: 80%

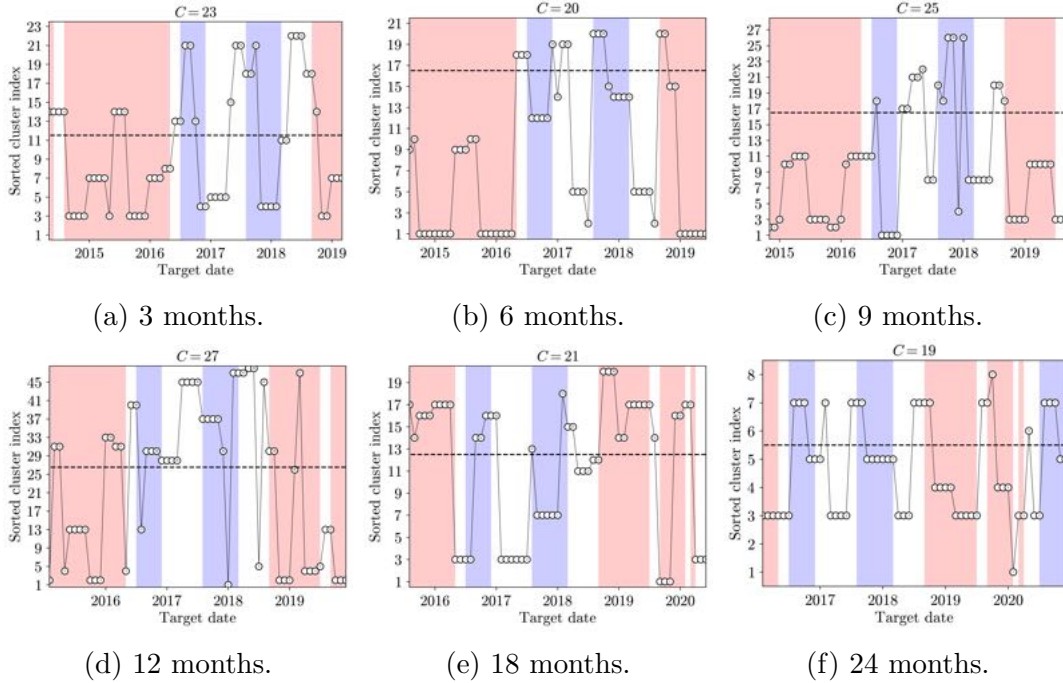


Figure 93: Test set cluster affiliation plots using SSA features and 80% of the dataset.

## 7.10 SSA: 70%

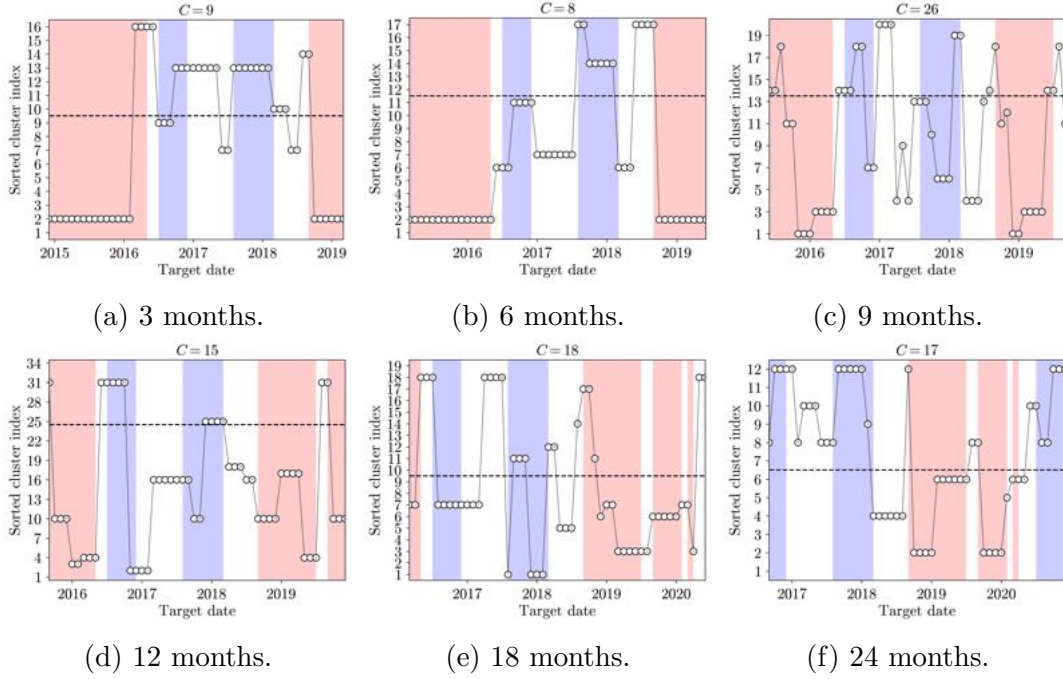


Figure 94: Test set cluster affiliation plots using SSA features and 70% of the dataset.

## 7.11 SSA: 60%

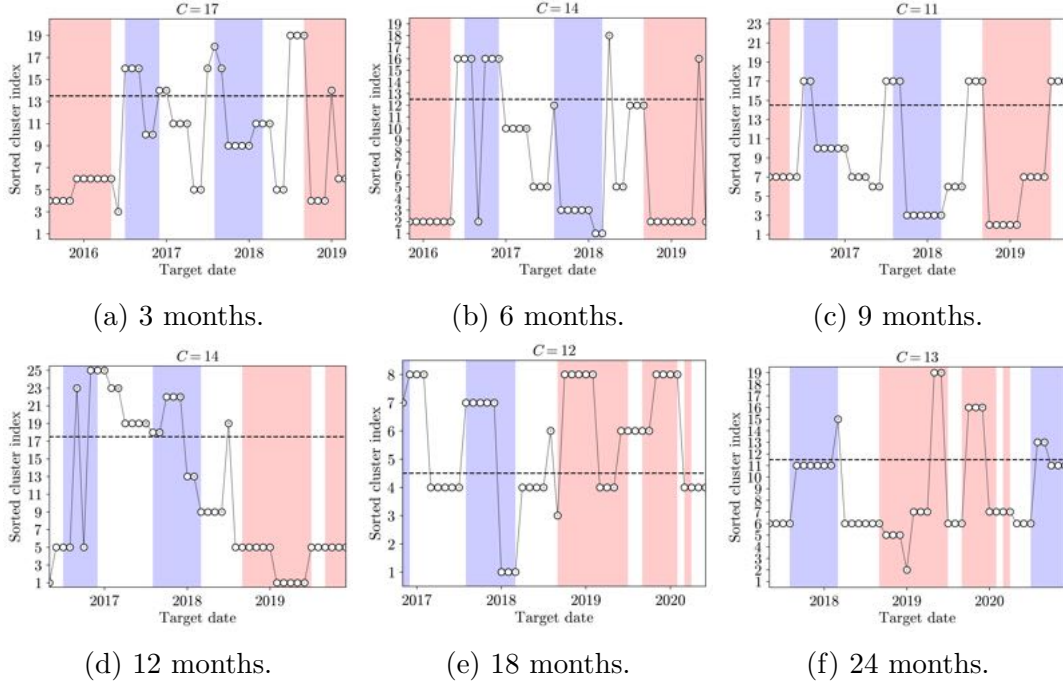


Figure 95: Test set cluster affiliation plots using SSA features and 60% of the dataset.



## 7.12 SSA: 50%

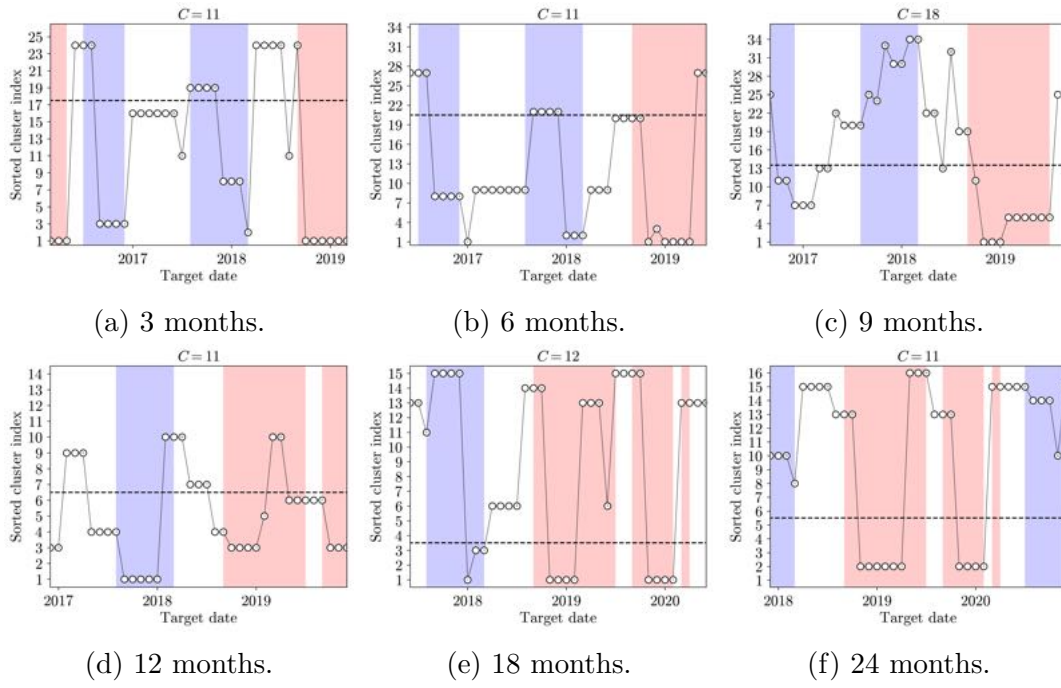


Figure 96: Test set cluster affiliation plots using SSA features and 50% of the dataset.



## 8 Cluster composites

Available online at: <https://zenodo.org/records/10582420>