Falkner-Skan equation with Heat Transfer: A new Optimal approach

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Abstract

In this paper we used the newly developed optimal approach namely Optimal Auxilary Function Method (OAFM) for the solution of the Falkner-Skan equation with heat transfer. The constitutive equations obtained from Navier-Stokes equations are converted into a set of nonlinear Ordinary Differential Equations (ODEs) with the help of a similarity transformation. For the solution of the obtained ODEs with boundary conditions is solved by OAFM. The OAFM along with convergence is studied in detail. The effects of the physical parameters are discussed with the help of tabular data and graphs. The reliability and effectiveness of the method is achieved by comparing the results available in the literature.

Keywords: OAFM, Falkner-Skan equations, Navier Stokes Equations, Heat transfer, Non-linear differential equations.

1. Introduction

The boundary layer flow (BLF) of an incompressible fluid over a stretching sheet (SS) is common in many engineering and industrial processes. The field has attracted researchers in the last few decades. BLF flow has major applications in industries such as aerodynamic extraction of polymer paper from debris, thermal wrapping, cooling plate with no cooling tuber, and boundary layer next to liquid film in condensation phase, glass-fiber development [1-3]. By immersing them in quiescent liquids, many metal processes need to cool continuous such as fibers. The mechanical features of the final product depend only on the drawing costs and the process temperature. Sakiadis [4-5] has experimented with new work in this area and many researchers in the field have investigated the flow of the boundary layer into the ongoing SS at an increasing speed.

The third order nonlinear two point boundary value problem with no exact solution known as Falkner-Skan equation (FSE). Due to the importance of the boundary theory the (FSE) is considered widely in the last forty years. Due to the nonlinear nature of the (FSE) having no exact solutions available in the literature, the scientists have tried the analytical and numerical approaches. Hartree [6] obtain a solution of the (FSE) numerically. Smith and Cebeci [7-8] solved (FSE) by shooting method. Maksyn [9] solved (FSE) by analytic approximation. Aasaithambi [10-12] found (FSE) solution by finite differences, S.J. Liao [13] applied homotopy analysis method (HAM) to solved (FSE) and Vera [14] applied Fourier series for the solution of FSE. The important special case of (FSE) is known as Blasius equation (BE). The BE solved by Rosales and Valencia [15] with Fourier series. Boyd [16] establish the result by numerical method.

The analytical and numerical methods studied have some advantages as well as some limitations. The numerical methods required linearization, discretization which may affects the accuracy. The analytical methods are used by many researchers such as Adomian Decomposition Method (ADM) [17], Variational Iteration Method (VIM) [18], Differential Transform Method (DTM) [19], Radial basis function [19], and Homotopy Perturbation Method (HPM) [20-22] , Artificial Parameters Method [23], Homotopy Analysis Method (HAM) [24] are used for the solutions of nonlinear equations. All these techniques either required the assumption of small parameter like HPM or an initial guess. Again it’s improper selection affect the accuracy. Herisanu et.al [25-26], currently introduced an optimal approach (OAFM). (OAFM) do not required the assumption of small parameter and initial guess. In this paper we propose the (OAFM) for the FSE with heat transfer.

In the succeeding section, the basic idea of OAFM is formulated. Section 3 is dedicated to results and discussion while in section 4 the conclusion is presented.

2. Basic concept of OAFM

Let us see the OAFM to nonlinear ODE

$$\mathcal{L}\left(f\left(\xi\right)\right)+s\left(\xi\right)+\mathcal{N}\left(f\left(\xi\right)\right)=0, \tag{1}$$

where $\mathcal{L},\mathcal{N}$ are the linear, nonlinear operator, s source function, $f\left(\xi\right)$ is an unknown function at this stage.

The initial/boundary conditions are

$$\mathcal{B}\left(f\left(\xi\right),\frac{df\left(\xi\right)}{d\xi}\right)=0. \tag{2}$$

Since the exact solution of strongly nonlinear equations is very hard to find. The proposed approximate solution is given as

$$\widehat{f}\left(\xi,E_k\right)=f_0\left(\xi\right)+f_1\left(\xi,E_k\right),\qquad k=1,2,...,s. \tag{3}$$

Using Eq. (3) in Eq. (1), we obtain

$$\mathcal{L}\left(f_0\left(\xi\right)\right)+\mathcal{L}\left(f_1\left(\xi,E_k\right)\right)+s\left(\xi\right)+\mathcal{N}\left(f_0\left(\xi\right)+f_1\left(\xi,E_k\right)\right)=0, \tag{4}$$

$E_k, k=1,2,...,s$ are control convergence parameters, to be determined.

The initial approximation is determined as

$$\mathcal{L}\left(f_0(\xi)\right)+s(\xi)=0, \quad \mathcal{B}\left(f_0(\xi), \frac{f_0(\xi)}{d\xi}\right)=0. \tag{5}$$

The first approximation is obtained as

$$\mathcal{L}\left(f_1\left(\xi,E_k\right)\right)+\mathcal{N}\left(f_0\left(\xi\right)+f_1\left(\xi,E_k\right)\right)=0, \quad \mathcal{B}\left(f_1(\xi), \frac{f_1(\xi)}{d\xi}\right)=0. \tag{6}$$

The nonlinear term is expressed as

$$\mathcal{N}\left(f_0\left(\xi\right)+f_1\left(\xi,E_k\right)\right)=\mathcal{N}\left(f_0\left(\xi\right)\right)+\sum_{l=1}^{\infty}u^{l}{}_{1}\left(t,E_k\right)\mathcal{N}^{\ell}\left(f_0\left(\xi\right)\right). \tag{7}$$

The last term in Eq. (7) seems difficult to solve, so to avoid this difficulty and to fast the convergence of the solution. Eq. (6) can be written as

$$\begin{gathered}\mathcal{L}\left(f_1\left(\xi,E_k\right)\right)+D_1\left(\left(f_0\left(\xi\right),E_m\right)F\left(\mathcal{N}\left(f_0\left(\xi\right)\right)\right)\right)+D_2\left(f_0\left(\xi\right),E_n\right)=0,\\ \mathcal{B}\left(f_1(\xi,E_k), \frac{df_1(\xi,E_k)}{d\xi}\right)=0,\ \ n=1,2,...,q, m=q+1,q+2,....s\end{gathered} \tag{8}$$

where D_1, D_2 are optimal auxiliary functions depends on $f_0\left(\xi\right)$ and E_n, E_m and $F\left(\mathcal{N}\left(f_0\left(\xi\right)\right)\right)$are functions which depends on the expression appearing in within in the nonlinear term of in $\mathcal{N}\left(f_0\left(\xi\right)\right)$. The optimal auxiliary functions D_1, D_2 should be expressed in the sum form of $f_0\left(\xi\right)$ such as if $f_0\left(\xi\right)$are polynomial, exponential and trigonometric then D_1, D_2 would be the sum of polynomial, exponential and trigonometric respectively. Also $f_0\left(\xi\right)$ would be the exact solution of the original problem if $\mathcal{N}\left(f_0\left(\xi\right)\right)=0.$ The Optimal auxiliary functions can be obtained from Method of least square, collocation method, Galerkin method and Ritz Method.

Convergence of the Method: In order to obtain the convergent solution, we calculate the optimal constants also known as control convergence constant by Method of Least Squares: These optimal constants are re-submitted into original equation to get the series solution

$$J\left(E_1,E_2,...E_s\right)=\int_I R^2\left(\xi,E_1,E_2,...E_s\right)d\xi, \tag{9}$$

where I is equation domain.

The unknown constants are establish as

$$\partial_{E_1}J=0,\ \partial_{E_2}J=0,\ ....\partial_{E_s}J=0.. \tag{10}$$

3. Problem formulation and solution procedure

We consider two dimensional laminar viscous flows over a semi-infinite flat plate under the boundary layer approximation. The governing equations

$$\frac{\partial \tilde{u}}{\partial x}+\frac{\partial \tilde{v}}{\partial y}=0 \tag{10}$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial x}+v\frac{\partial \tilde{v}}{\partial y}=U_s\frac{dU_s}{dx}+v\frac{\partial^2 \tilde{u}}{\partial^2 y}\quad , \tag{11}$$

$$\tilde{u}\frac{\partial \tilde{T}}{\partial x}+\tilde{v}\frac{\partial \tilde{T}}{\partial y}=a\frac{\partial^2 \tilde{T}}{\partial y^2} \tag{12}$$

where $U_s\left(x\right)$ is the free stream velocity, $\tilde{u}$ and $\tilde{v}$ are velocity components in x and y directions, $"a"$is the thermal diffusivity , υ is the kinematic viscosity.

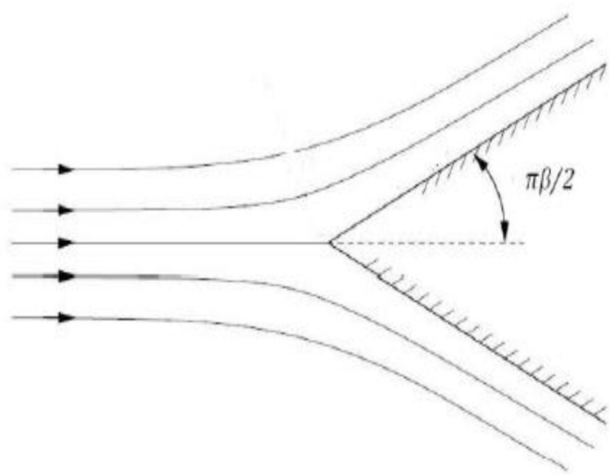

Fig. 1. Geometry of the problem

The incompressible boundary layer flow over a wedge of angle $\beta\pi$ as shown in Fig. 1, when $U_e\left(x\right)=cx^r$. Using the similarity transformation

$$\begin{gathered}\tilde{u}\left(x,y\right)=U_s f^{'}\left(\xi\right),\psi=\sqrt{\frac{\left(r+1\right)}{2}}\sqrt{\nu xU}\,f\left(\xi\right),\xi=\sqrt{\frac{\left(r+1\right)}{2}}\sqrt{\frac{\nu U}{x}}y \\ \tilde{v}=-\sqrt{\frac{\left(r+1\right)}{2}}\sqrt{\frac{\nu U}{x}}\,f\left(\xi\right),\;\theta\left(\xi\right)=\frac{T-T_\infty}{T_w-T_\infty}.\end{gathered} \tag{13}$$

Using Eq. (13) into Eq. (10)-(12), we obtained the Falkner-Skan equation

$$f'''(\xi)+f(\xi)f''(\xi)+\beta\left(1-\left(f'(\xi)\right)^2\right)=0,$$
$$\theta''(\xi)+\Pr f(\xi)\theta'(\xi)=0, \tag{14}$$

with boundary conditions

$$f(0)=0,\ f'(0)=0,\ f'(\infty)=1,$$
$$\theta(0)=1,\theta(\infty)=0. \tag{15}$$

where Pr is the prandl number.

The linear and nonlinear operators are given as

$$\mathcal{L}\left(f(\xi)\right)=f'''(\xi)$$
$$\mathcal{L}\left(\theta(\xi)\right)=\theta''(\xi), \tag{16}$$

$$\mathcal{N}\left(f(\xi)\right)=f(\xi)f''(\xi)+\beta\left(1-\left(f'(\xi)\right)^2\right),$$
$$\mathcal{N}\left(\theta(\xi)\right)=Pr\theta(\xi)\theta'(\xi). \tag{17}$$

From Eq. (5), we have

$$f_0'''(\xi)=0,\qquad f_0(0)=0,\ f_0'(0)=0,\ f_0'(\infty)=1,$$
$$\theta_0''(\xi)=0,\qquad \theta_0(0)=1,\theta_0(\infty)=0. \tag{18}$$

has solution

$$f_0(\xi)=\xi-\left(1-e^{-\xi}\right),$$
$$\theta_0(\xi)=e^{-\xi}. \tag{19}$$

Based on Eq. (19), we get

$$\mathcal{N}\left(f_0(\xi)\right)=-1+\xi+e^{-2\xi}\left(\alpha+e^{\xi}\left(2\beta+\xi\right)\right),$$
$$\mathcal{N}\left(\theta_0(\xi)\right)=\Pr\left(\left(1-\xi\right)e^{-\xi}-e^{-2\xi}\right). \tag{20}$$

where $\alpha=1-\beta.$

The first approximation on based of Eq. (8), (16) and (20), we have

$$f_1'''(\xi)+D_1\left(e^{-\xi},e^{-2\xi},E_m\right)\left(-1+\xi+e^{-2\xi}\left(\alpha+e^{\xi}\left(2\beta+\xi\right)\right)\right)+D_2\left(e^{-\xi},e^{-2\xi},E_n\right)=0,$$
$$\theta_1''(\xi)+D_3\left(e^{-\xi},e^{-2\xi},E_p\right)\Pr\left(\left(1-\xi\right)e^{-\xi}-e^{-2\xi}\right)+D_4\left(e^{-\xi},e^{-2\xi},E_r\right)=0, \tag{21}$$

With BCS

$$f_1(0)=0,\ f_1'(0)=0,\ f_1'(\infty)=0,$$
$$\theta_1(0)=0,\theta_1(\infty)=0. \tag{22}$$

The OAF can be chosen freely as

$$
\begin{aligned}
&D_1\left(f_0(\xi), E_m\right) = -\left(E_1 + E_2\xi\right),\\
&D_2\left(f_0(\xi), E_n\right) = -\left(E_3 + E_4\xi\right)e^{-\xi} - \left(E_5 + E_6\xi + E_7\xi^2\right)e^{-2\xi},\\
&D_3\left(f_0(\xi), E_p\right) = 0\\
&D_4\left(f_0(\xi), E_r\right) = -\left(E_8 + E_9\xi\right)e^{-\xi} - \left(E_{10} + E_{11}\xi + E_{12}\xi^2\right)e^{-2\xi},
\end{aligned}
\tag{23}
$$

We obtained the first approximate solution

$$
\begin{aligned}
&f_1'''(\xi) + D_1\left(e^{-\xi}, e^{-2\xi}, E_m\right)\left(-1 + \xi + e^{-2\xi}\left(\alpha + e^{\xi}\left(2\beta + \xi\right)\right)\right) + D_2\left(e^{-\xi}, e^{-2\xi}, E_m\right) = 0,\\
&\theta_1''(\xi) + D_3\left(e^{-\xi}, e^{-2\xi}, E_p\right)\Pr\left(\left(1-\xi\right)e^{-\xi} - e^{-2\xi}\right) + D_4\left(e^{-\xi}, e^{-2\xi}, E_r\right) = 0,
\end{aligned}
\tag{24}
$$

and its solution is given as

$$
\begin{aligned}
&f_1(\xi) = \frac{1}{72}e^{-3\xi}\begin{pmatrix} -3 + 89\, e^{2\xi} - 3\,\beta\,(-1 + e^{\xi})^2\,(-1 + 6\,e^{\xi}) \\ -\,2\,e^{\xi}\,(5 + 6\,\xi) + e^{3\xi}\,(-76 + 72\,\xi) \end{pmatrix},\\
&\theta_1(\xi) = \frac{1}{72}e^{-3\xi}\begin{pmatrix} -9\Pr + e^{2\xi}\,(48 + 17\Pr) \\ -\,8\,e^{\xi}\,(-3 + \Pr + 3\Pr\,\xi) \end{pmatrix},
\end{aligned}
\tag{25}
$$

Table 1. Presenting the values of the optimal constants obtained from method of least square given in Eq. (9)-(10)

E_1	-0.001196478535	E_7	0.0000124357867
E_2	0.0001458765872	E_8	0.0000004578932
E_3	0.0000457821458	E_9	0.00004786531245
E_4	0.0000045789534	E_{10}	0.00000045721423
E_5	0.0000478515785	E_{11}	-0.0000014523678
E_6	0.0000781425637	E_{12}	0.00000022214587

Table 2. Initial slope $f''(0)$ obtained by OAFM for different values of β.

β	Hartree [6]	Asaithambi [11]	Asaithambi [12]	Salama [27]	Zhang et al [28]	Vera [29]	OAFM
2	1.687	1.687222	1.687218	1.687218	1.687218	1.687218	1.687218
1	1.233	1.23589	1.232588	1.232588	1.232587	1.232587	1.232587
0.5	0.927	0.927682	0.927680	0.927680	0.927680	0.927680	0.927680
-0.1	0.319	0.319270	0.319270	0.319270	0.319270	0.319270	0.319270

Table 3 Absolute errors (OAFM and Numerical methods (RK 4 Method)) for $\beta = 1$

ξ	OAFM	Numerical results (RKM- 4)	Absolute errors
0.0	$-1.304512053934559\times10-15$	$-1.304512053934559\times10-15$	0.0000000000000000
0.5	0.7081810142900181	0.7081810142900187	$1.2457896356\times10-15$
1.0	2.1288696163158923	2.1288696163158923	$2.4587963125\times10-15$
1.5	3.6831058803801161	3.6831058803801161	$1.8456789632\times10-15$
2.0	5.1401507626328793	5.1401507626328792	$3.2154587665\times10-15$
2.5	6.4256229119798556	6.4256229119798556	$2.4578965835\times10-15$
3.0	7.5338423942076735	7.5338423942076735	$3.2142045768\times10-15$
3.5	8.4867626915090735	8.4867626915090734	$2.0124578963\times10-15$
4.0	9.3145070754938912	9.3145070754938912	$2.1457862353\times10-15$
4.5	10.046470822358745	10.046470822358744	$2.1457856396\times10-15$
5.0	10.707761095553249	10.707761095553248	$2.1457896523\times10-15$
5.5	11.318298386998947	11.318298386998948	$2.1457896523\times10-14$
6.0	11.893148651390864	11.893148651390865	$8.4578596321\times10-14$

.

Fig. 2. Velocity distribution $f'(\xi)$ for different values of β

Fig. 3. Temperature profile $\theta(\xi)$ for different values of Pr .

Fig. 4. Velocity distribution for Blasius equation $f'(\xi)$ for $\beta = 0$.

In case of $\beta = 0$ in we obtain the famous Blasius equation.

Fig. 5. Comparison of Velocity distribution for different values of β (OAFM, RK-4 Method)

Fig. 6. Comparison of Temperature profile $\theta(\xi)$ for different values of Pr (OAFM, RK-4 Method)

4. Results and Discussions

The detail OAFM presented in section 2, deliver a high accurate and fastly convergent solution for the BLF model. For the computational analysis we have used Mathematica 11. The Table 1 presented the values of the optimal constants obtained by the method of least square and being used for obtaining the first order solution of OAFM. The Table 2 demonstrates the comparison of the proposed method results with the results already available in the literature and we have found an excellent agreement even at first order approximation of the OAFM. The graphical comparisons of the velocity profile for $f'(\xi)$ with respect to ξ and temperature profile for $\theta(\xi)$ with respect to ξ are elaborated in Figs. 2-6. Fig. 1 presents the geometry of the flow problem. From Figs. 5-6, excellent agreements of the OAFM results with Numerical method results (Runge Kutta method of order 4) are obtained. The absolute errors demonstrated in Table 3 reveal the accuracy of the OAFM. Also the physical parameters effects are given in figs. 2-3 for velocity and temperature distributions. From these figures it is observed that by increasing the values of β and Pr the boundary layer thickness reduces for velocity and temperature distributions. Fig. 4 shows the well BE with strong agreement with the results in literature [30].

5. Conclusion

In this study a new analytical method is suggested for the solution of the boundary layer flow model (FSE). We obtain the first order series solution for the governing equations of the FSE model and achieved the first order solution with high accuracy. For the accuracy and validity of our method, we compared the OAFM results with the results available in the literature and the numerical results obtained by using the RK Method of order, from the comparison it is concluded that the suggested method is very accurate and good agreement of our results with the numerical results proves the validity of our method. OAFM is applicable is very easy in applicable to high nonlinear initial and boundary value problems even if the nonlinear initial/ boundary value problem does not contain the small parameter. In comparison with other analytical method, OAFM is very easy in applicability and provide us good results of more complex nonlinear initial/boundary value problems. OAFM contain the optimal auxiliary constants through which we can control the convergence as OAFM contain the auxiliary functions D_1, D_2, D_3, E_4 in which the optimal constants E_m, E_n, E_r, E_p the control convergence parameters exist to play an important role to get the convergent solution which are obtained rigorously. The computational work in OAFM is less when compared to other methods and even a low specification computer can do the computational work easily. Upto now there is no limitation of this method which enable us to implement this efficient and fast convergent method in our future work for more complex models arising from real world problems.

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