

1 **Multigrid spatially constrained dispersion curve**  
 2 **inversion: towards distributed acoustic sensing**  
 3 **surface wave imaging**

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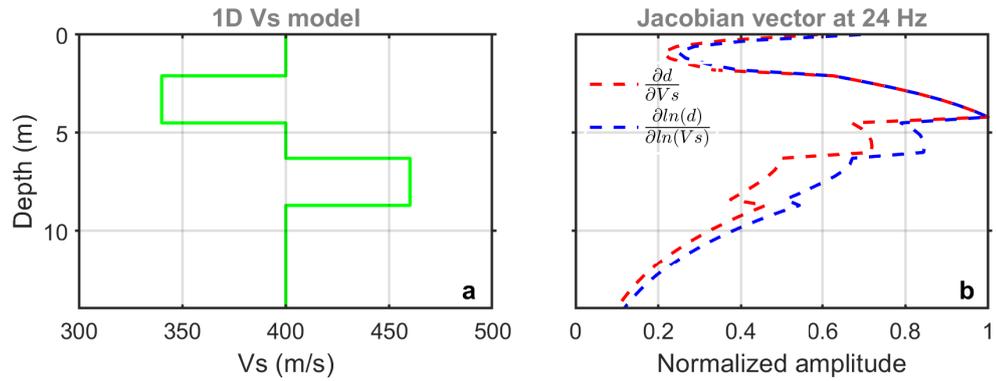
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28 5. Figure S5: The established initial  $V_s$  model in field application.

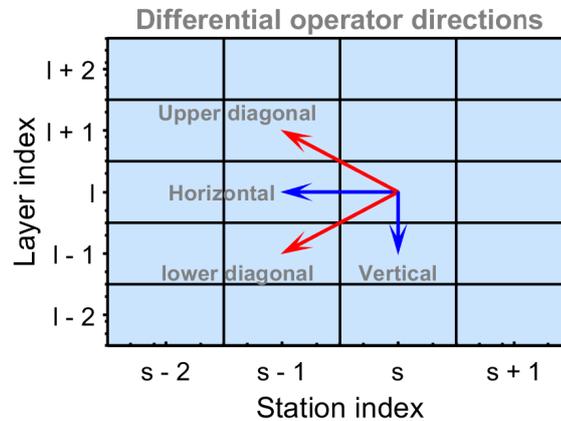
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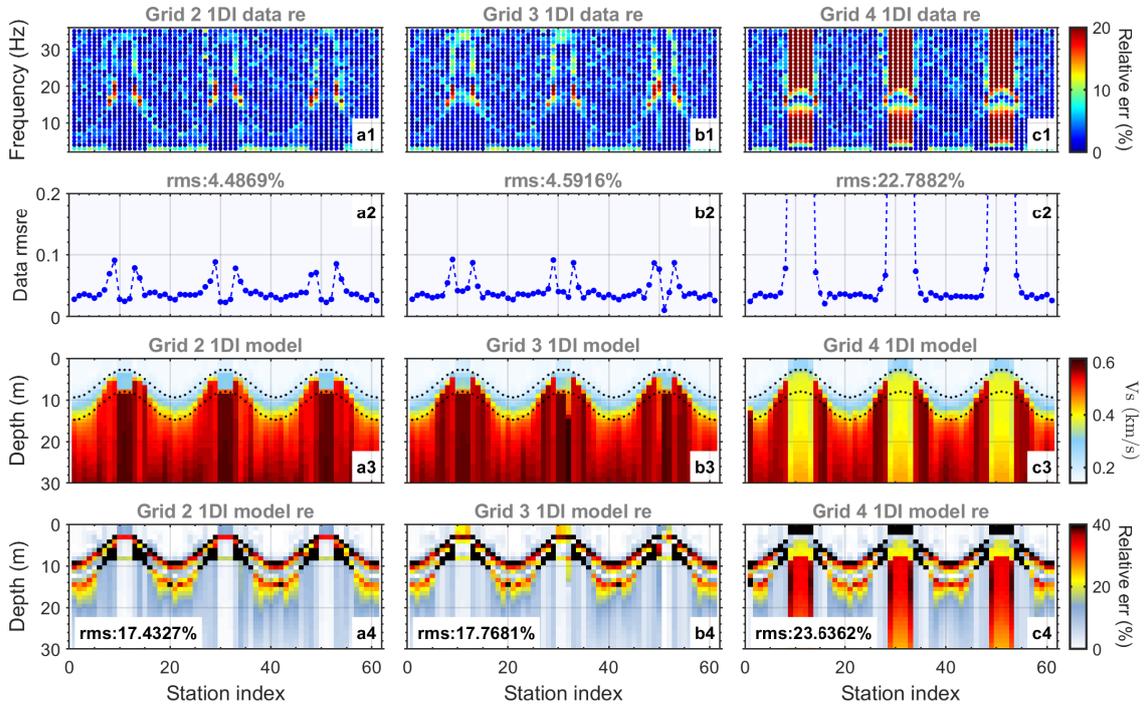
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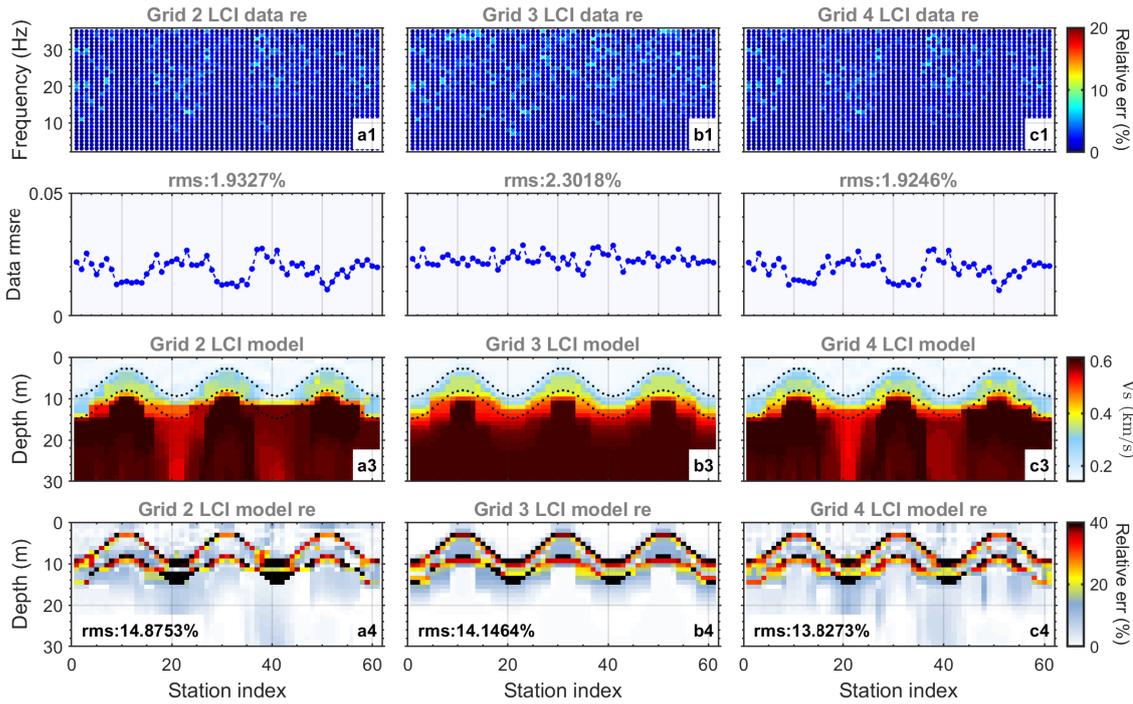
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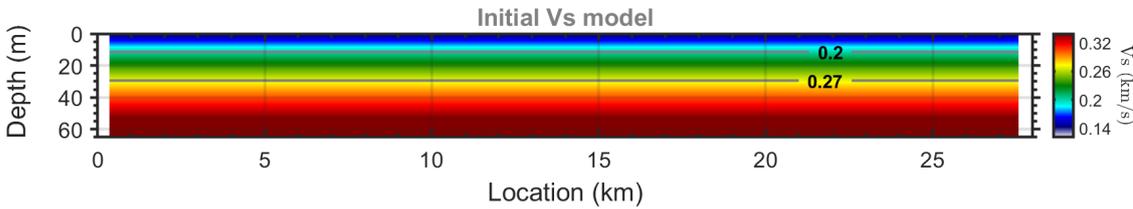
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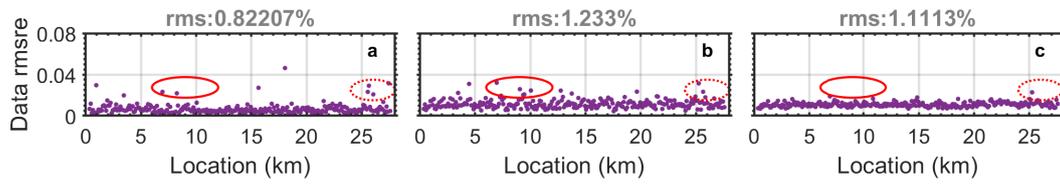
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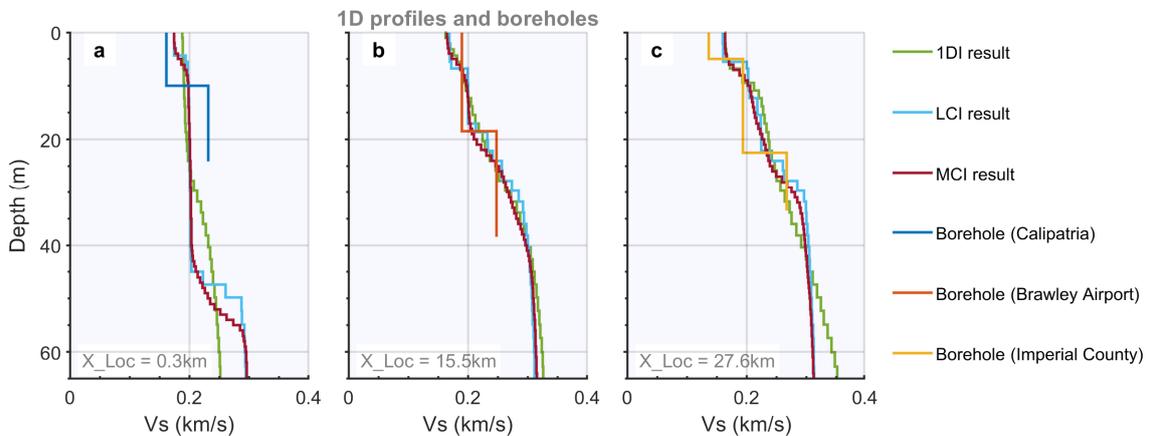
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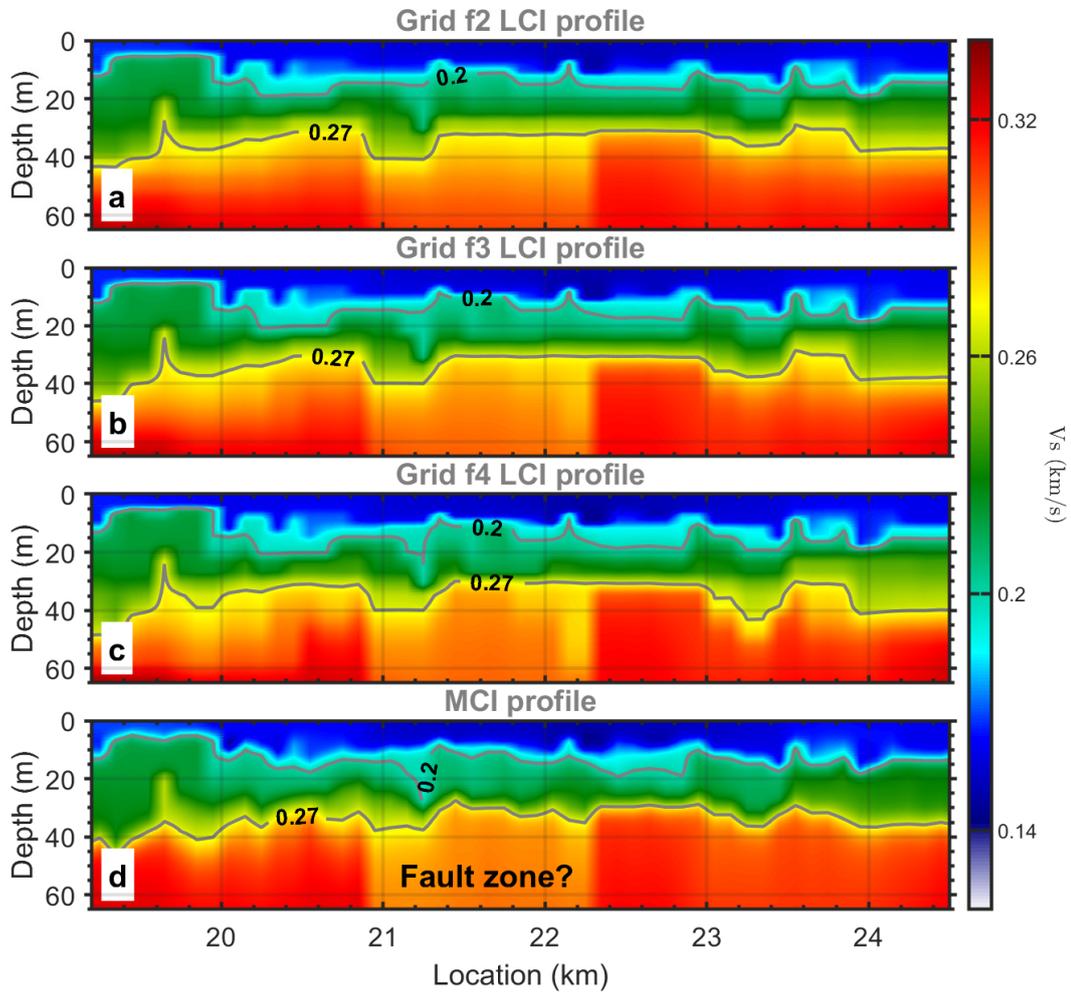
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