

1 **Effective Characterization of Fractured Media with PEDL: A Deep Learning-Based**
2 **Data Assimilation Approach**

3 Tongchao Nan^{1,2}, Jiangjiang Zhang^{1,2*}, Yifan Xie³, Chenglong Cao^{1,2}, Jichun Wu⁴, and Chunhui
4 Lu^{1,2,3,5}

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6 ¹Yangtze Institute for Conservation and Development, Hohai University, Nanjing, China,

7 ²The National Key Laboratory of Water Disaster Prevention, Hohai University, Nanjing, China,

8 ³College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing, China,

9 ⁴Key Laboratory of Surficial Geochemistry of Ministry of Education, School of Earth Sciences and
10 Engineering, Nanjing University, Nanjing, China,

11 ⁵College of Hydrology and Water Resources, Hohai University, Nanjing, China.

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13 **Corresponding Author:** Jiangjiang Zhang (zhangjiangjiang@hhu.edu.cn)

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15 **Key Points**

- 16 • A deep learning-based data assimilation method, named PEDL, is proposed to
17 characterize fractured media with highly non-Gaussian features.
- 18 • PEDL outperforms three popular data assimilation methods based on deep learning or the
19 Kalman formula.
- 20 • Sensitivity analyses confirm PEDL's validity and adaptability across various ensemble
21 sizes and DL model architectures.

22 **Abstract**

23 In various research fields such as hydrogeology, environmental science and energy engineering,
24 geological formations with fractures are frequently encountered. Accurately characterizing these
25 fractured media is of paramount importance when it comes to tasks that demand precise
26 predictions of liquid flow and the transport of solute and energy within them. Since directly
27 measuring fractured media poses inherent challenges, data assimilation (DA) techniques are
28 typically employed to derive inverse estimates of media properties using observed state variables
29 like hydraulic head, concentration, and temperature. Nonetheless, the considerable difficulties
30 arising from the strong heterogeneity and non-Gaussian nature of fractured media have
31 diminished the effectiveness of existing DA methods. In this study, we formulate a novel DA
32 approach known as PEDL (parameter estimator with deep learning) that harnesses the
33 capabilities of DL to capture nonlinear relationships and extract non-Gaussian features. To
34 evaluate PEDL's performance, we conduct two numerical case studies with increasing
35 complexity. Our results unequivocally demonstrate that PEDL outperforms three popular DA
36 methods: ensemble smoother with multiple DA (ESMDA), iterative local updating ES (ILUES),
37 and ES with DL-based update (ESDL). Sensitivity analyses confirm PEDL's validity and
38 adaptability across various ensemble sizes and DL model architectures. Moreover, even in
39 scenarios where structural difference exists between the accurate reference model and the
40 simplified forecast model, PEDL adeptly identifies the primary characteristics of fracture
41 networks.

42 **1 Introduction**

43 Fractures are narrow openings or mechanical discontinuities in geological formations, typically
44 found in rocks, soils, and aquifers. The significance of fractures lies in their pivotal role across
45 numerous processes: they offer preferential pathways for the rapid transport of liquids, and act as
46 reservoirs for fluid storage and release (Viswanathan et al., 2022). Despite their relatively minor
47 presence within the subsurface media, fractures hold importance in diverse fields, including
48 Karst aquifer management (S Li et al., 2020), geothermal energy production (Guo et al., 2022),
49 hydrocarbon extraction (F Zhang & Emami-Meybodi, 2022), geological sequestration of CO₂
50 (Luo et al., 2022), and nuclear waste disposal (Saceanu et al., 2022). Nonetheless, the complexity

51 of fractured media is deeply ingrained in the coexistence of fractures with varying sizes,
52 orientations, and apertures. This complexity engenders significant heterogeneity, anisotropy, and
53 discontinuity in the hydraulic characteristics of fractured media, ultimately giving rise to
54 elevated levels of uncertainty in flow and transport predictions (Neuman, 2005). For achieving
55 trustworthy process comprehension and simulation outcomes, it is imperative to effectively
56 characterize the attributes of fractured media and reasonably quantify the associated uncertainty
57 (Hyman, 2020; Klepikova et al., 2020).

58 Given the inherently opaque and complex nature of fractured media, the direct observation of
59 fractures poses a formidable challenge. Although X-ray computed tomography can be employed
60 to study the structure of fracture in specific scenarios, it is subject to limitations related to
61 method resolution and sample size (C Jiang et al., 2019; Hao Wu et al., 2019). An alternative
62 approach to characterizing fractured media involves the use of numerical models that explicitly
63 represent fracture networks, particularly the major ones (Berre et al., 2019). These models, often
64 referred to as discrete fracture models (DFMs), encompass a range of methodologies, including
65 channel networks (Hyman, 2020), discrete fracture networks (Cacas et al., 1990), discrete
66 fracture-matrix models (Koohbor et al., 2020), and embedded discrete fracture-matrix (EDFM)
67 models (J Jiang & Younis, 2017). To characterize the inherent structural complexity and scale
68 disparities within fracture networks, it is often imperative to employ fine spatial resolutions.
69 Nevertheless, this approach can make simulations prone to gridding failures, result in high
70 computational expenses, and, as a consequence, prove infeasible for extensive, large-scale
71 problems (Viswanathan et al., 2022). A simpler yet less precise approach for representing
72 fractures involves utilizing continuum models, where effective parameter values of the matrix
73 are employed. These models include the stochastic continuum (SC) model (Tsang et al., 1996)
74 and the dual-permeability model (Presho et al., 2011). While these models may not provide the
75 same level of accuracy as DFMs, they are known for their ease of implementation and cost-
76 effectiveness in practical applications (National Academies of Sciences, 2020).

77 In the aforementioned models, a substantial number of parameters are indispensable for
78 representing complex fractures. However, these parameters remain predominantly unknown,
79 introducing considerable uncertainty when simulating flow and transport processes happening
80 within the fractured media. To confront this challenge, researchers have underscored the growing
81 importance of incorporating diverse data types, encompassing hydraulic, geophysical, and hydro-

82 chemical measurement data, through the implementation of data assimilation (DA) techniques
83 (Elahi & Jafarpour, 2018; Y Li et al., 2016; Miskimins, 2009; Ping & Zhang, 2013; Hui Wu et al.,
84 2021). DA is devoted to seamlessly integrating theoretical knowledge, often represented as
85 numerical models, with observational data to achieve optimized estimations of system states,
86 parameters, and initial/boundary conditions. It has found extensive applications across various
87 research domains of geosciences (Carrassi et al., 2018; Fletcher, 2022), and can serve as a
88 valuable tool for enhancing our understanding of fractures and their properties.

89 When characterizing fractured media, the prevailing DA approach in use is ensemble Kalman
90 filter (EnKF) (Evensen, 2009) and its iterative variations, such as ESMDA (Emerick & Reynolds,
91 2013) and EnRML (Gu & Oliver, 2007). For instance, Vogt et al. (2012) employed EnKF to
92 assimilate tracer data, aiming to estimate equivalent permeability field of discrete fracture
93 networks at an enhanced geothermal system reservoir. However, their predictions display a
94 noticeable degree of uncertainty, which could be attributed to the inherent constraints of EnKF
95 and its variants in handling non-Gaussian distributions. To tackle this challenge, a viable strategy
96 involves transforming non-Gaussian distributed variables into Gaussian ones using various
97 techniques. For example, Hui Wu et al. (2021) applied principal component analysis, Chen et al.
98 (2023) used deep generative model, Ping & Zhang (2013) utilized level set function, and Lu &
99 Zhang (2015) employed Hough transformation, respectively, to reparameterize complex
100 fractured media with Gaussian random variables. These Gaussian distributed variables can be
101 effectively updated using EnKF or its variants, and then transformed back to non-Gaussian
102 distributed fracture parameters to make predictions with dynamic models. Nevertheless, it's
103 important to note that the reparameterization and updating processes may lead to the loss of
104 some basic features and continuity of fractures (Yao et al., 2018). Another strategy for
105 addressing the non-Gaussianity issue entails adopting more theoretically robust DA methods,
106 such as Markov chain Monte Carlo (MCMC) (Vrugt, 2016) and particle filter (PF) (Djuric et al.,
107 2003). For example, Blaheta et al. (2020) and Xue et al. (2020) respectively employed MCMC
108 and PF for the characterization of fractured media in subsurface applications. Nevertheless, the
109 computational costs associated with MCMC and PF can become prohibitive when dealing with
110 high-dimensional inverse problems. Even with the incorporation of lower-fidelity or surrogate
111 models to improve efficiency in the two studies, the practical application of MCMC and PF
112 techniques remains a formidable challenge. In addition to the challenge posed by non-

113 Gaussianity, another prevalent and complex issue in the realm of DA for subsurface
114 characterization is equifinality, which refers to the scenario where various model structures and
115 parameter combinations can yield acceptable reproductions of observed system behaviors (Beven
116 & Freer, 2001). From the Bayesian perspective, equifinality means that the parameter
117 distribution is multi-modal in ensemble sense. Several studies have tried to handle multi-modal
118 distributions in DA by conceptualizing them as a mixture of Gaussian distributions, with each of
119 these distributions being updated individually (Elsheikh et al., 2013; Sun et al., 2009; J Zhang et
120 al., 2018). Among them, the iterative local updating ensemble smoother (ILUES) proposed by J
121 Zhang et al. (2018) has been used as the basic DA method in various subsurface characterization
122 problems.

123 Over the past decade, deep learning (DL) has gained substantial attention within the field of
124 hydrology and water resources (Shen, 2018). DL's prowess in unveiling complex nonlinear
125 relationships and intricate patterns from data has endowed it with formidable analytical
126 capabilities. Moreover, the strong alignment between DL and DA in terms of statistical
127 principles and methodologies has been well investigated and recognized (Abarbanel et al., 2018;
128 Berry & Harlim, 2017). In an effort to tackle the challenges posed by high-dimensionality and
129 non-Gaussianity in DA problems, J Zhang et al. (2020) put forth an innovative approach. They
130 proposed the utilization of DL to establish a nonlinear updating scheme that supplants the
131 conventional Kalman updating in EnKF and its variants. Their findings showcased the efficacy
132 of this DL-based DA method, known as ESDL, in effectively characterizing high-dimensional,
133 non-Gaussian parameter fields. Due to its capability to address complex DA problems, ESDL
134 and its variants have found applications in numerous subsurface characterization scenarios
135 (Godoy et al., 2022; Man et al., 2022; Wang & Yan, 2022; Xiao et al., 2023; J Zhang et al.,
136 2023).

137 In the present study, we adopt the ESDL method for the first time to characterize fractured media.
138 While ESDL exhibits improved performance compared to Kalman-based DA methods like
139 ESM DA and ILUES, it is not without its limitations, as will be demonstrated later in this work.
140 The fracture networks inferred by ESDL exhibit some irregularities, possibly due to the updating
141 process from the innovation vector (i.e., the difference between observations and model
142 predictions) to the update vector (i.e., the difference between posterior and prior parameters).
143 Based on this finding, we improve ESDL by introducing a new updating rule. This new DA

144 method, named parameter estimator with DL (PEDL), establishes a direct update from the multi-
 145 sourced observation vector to the posterior parameter vector. To validate PEDL’s effectiveness,
 146 we compare its performance with several existing DA methods, including ESM DA, ILUES, and
 147 ESDL, in a groundwater flow problem within fractured media. As will be demonstrated later,
 148 PEDL outperforms the other three DA methods significantly in characterizing the fractured
 149 media.

150 The rest of this paper is structured as follows. In Section 2, we present the implementation details
 151 of PEDL, along with three well-known DA methods for comparative analysis, namely ESM DA,
 152 ILUES, and ESDL. Section 3 features two illustrative cases aimed at evaluating the performance
 153 of PEDL in the context of DA within fractured media. Here, we consider two scenarios with and
 154 without considering the model structural error, respectively. Finally, in Section 4, we draw
 155 conclusions and engage in a discussion about the applicability of PEDL in a broader perspective.

156 **2 Methodology**

157 Here, we assume that the process of concern (e.g., groundwater flow within a fractured aquifer)
 158 is simulated by a numerical model $\mathcal{F}(\mathbf{m})$. The true parameters \mathbf{m}^* describing the media
 159 properties (e.g., permeability) are unknown and can be inferred from noisy measurement data

$$\tilde{\mathbf{d}} = \mathcal{F}(\mathbf{m}^*) + \boldsymbol{\varepsilon}, \quad (1)$$

160 where $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_D)$ represent the error terms. Based on the Bayesian theory, our updated
 161 understanding about the parameters can be expressed as the posterior distribution

$$p(\mathbf{m}|\tilde{\mathbf{d}}) = \frac{p(\mathbf{m})p(\tilde{\mathbf{d}}|\mathbf{m})}{p(\tilde{\mathbf{d}})}, \quad (2)$$

162 where $p(\mathbf{m})$ is the prior distribution of \mathbf{m} , $p(\tilde{\mathbf{d}}|\mathbf{m})$ demotes the likelihood function, and $p(\tilde{\mathbf{d}}) =$
 163 $\int p(\tilde{\mathbf{d}}|\mathbf{m})p(\mathbf{m})d\mathbf{m}$ signifies the evidence, a normalization constant.

164 For complex problems, analytical form of $p(\mathbf{m}|\tilde{\mathbf{d}})$ is not available, and Monte Carlo methods
 165 can be employed to approximate the posterior. According to the background knowledge of \mathbf{m} , N_e
 166 random samples can be drawn to form the prior ensemble, $\mathbf{M}^0 = \{\mathbf{m}_i^0 | i = 1, \dots, N_e\}$. With the
 167 numerical model, the corresponding model outputs can be calculated, i.e., $\mathbf{D}^0 = \{\mathbf{d}_i^0 =$

168 $\mathcal{F}(\mathbf{m}_i^0) | i = 1, \dots, N_e\}$. Below we will introduce how to update \mathbf{M}^0 from $\tilde{\mathbf{d}}$ with various DA
 169 methods, including ESMDA, ILUES, ESDL and PEDL.

170 **2.1 Ensemble Smoother with Multiple Data Assimilation: ESMDA**

171 Using the Kalman formula, we can update each sample in \mathbf{M}^0 as follows:

$$\mathbf{m}_i^1 = \mathbf{m}_i^0 + \mathbf{C}_{\text{MD}}^0 (\mathbf{C}_{\text{DD}}^0 + \mathbf{C}_{\text{D}})^{-1} (\tilde{\mathbf{d}} + \boldsymbol{\varepsilon}_i - \mathbf{d}_i^0), \quad (3)$$

172 where $\mathbf{M}^1 = \{\mathbf{m}_i^1 | i = 1, \dots, N_e\}$ is the updated ensemble representing our posterior
 173 understanding about the model parameters, \mathbf{C}_{MD}^0 is the cross-covariance between \mathbf{M}^0 and \mathbf{D}^0 ,
 174 \mathbf{C}_{DD}^0 is the auto-covariance of \mathbf{D}^0 , and $\boldsymbol{\varepsilon}_i$ is a random realization of measurement errors,
 175 respectively. For highly nonlinear problems, iterative application of the Kalman updating of
 176 equation (3) can be adopted:

$$\mathbf{m}_i^t = \mathbf{m}_i^{t-1} + \mathbf{C}_{\text{MD}}^{t-1} (\mathbf{C}_{\text{DD}}^{t-1} + \alpha^t \mathbf{C}_{\text{D}})^{-1} (\tilde{\mathbf{d}} + \sqrt{\alpha^t} \boldsymbol{\varepsilon}_i - \mathbf{d}_i^{t-1}), \quad (4)$$

177 where $t = 1, \dots, N_{\text{iter}}$, α^t is the inflation factor that satisfies $\sum_{t=1}^{N_{\text{iter}}} 1/\alpha^t = 1$, and N_{iter} is the
 178 iteration number. Finally, we use $\mathbf{M}^{N_{\text{iter}}} = \{\mathbf{m}_i^{N_{\text{iter}}} | i = 1, \dots, N_e\}$ to approximate the posterior
 179 distribution of \mathbf{m} . For the theory and implementation details of ESMDA, one can refer to
 180 (Emerick & Reynolds, 2013).

181 **2.2 Iterative Local Updating Ensemble Smoother: ILUES**

182 When the posterior distribution of parameters \mathbf{m} exhibits multiple modes, signifying that distinct
 183 parameter sets can equally reproduce the measurement data, the straightforward application of
 184 ESMDA may lead to biased results. This phenomenon, known as the equifinality issue, is a
 185 common challenge encountered in DA for complex systems, particularly when the available
 186 information contained in the measurement data is insufficient. To reasonably represent the multi-
 187 modal posterior distribution, J Zhang et al. (2018) proposed to identify and update the local
 188 ensemble of each sample \mathbf{m}_i^{t-1} in \mathbf{M}^{t-1} independently at iteration $t = 1, \dots, N_{\text{iter}}$. The local
 189 ensemble of \mathbf{m}_i^{t-1} is based on the following measure:

$$\mathcal{J}(\mathbf{m}) = \mathcal{J}_1(\mathbf{m})/\mathcal{J}_1^{\max} + \mathcal{J}_2(\mathbf{m})/\mathcal{J}_2^{\max}, \quad (5)$$

190 where $J_1(\mathbf{m}) = (\mathbf{m} - \mathbf{m}_i^{t-1})\mathbf{C}_{\text{MM}}^{-1}(\mathbf{m} - \mathbf{m}_i^{t-1})^T$, $J_2(\mathbf{m}) = (\mathcal{F}(\mathbf{m}) - \tilde{\mathbf{d}})\mathbf{C}_{\text{D}}^{-1}(\mathcal{F}(\mathbf{m}) - \tilde{\mathbf{d}})^T$, \mathbf{C}_{MM}
 191 is the auto-covariance of model parameters, J_1^{max} is the maximum value of J_1 , and J_2^{max} is the
 192 maximum value of J_2 , respectively. The local ensemble of \mathbf{m}_i^{t-1} is the $N_L = \beta N_e$ ($0 < \beta \leq 1$)
 193 samples in \mathbf{M}^{t-1} with the smallest J values, i.e., $\mathbf{M}_i^{t-1} = \{\mathbf{m}_{i,j}^{t-1} | j = 1, \dots, N_L\}$. Each sample in
 194 \mathbf{M}_i^{t-1} can be updated as:

$$\mathbf{m}_{i,j}^t = \mathbf{m}_{i,j}^{t-1} + \mathbf{C}_{i,\text{MD}}^{t-1}(\mathbf{C}_{i,\text{DD}}^{t-1} + \alpha^t \mathbf{C}_{\text{D}})^{-1} (\tilde{\mathbf{d}} + \sqrt{\alpha^t} \boldsymbol{\varepsilon}_j - \mathbf{d}_{i,j}^{t-1}), \quad (6)$$

195 where $\mathbf{C}_{i,\text{MD}}^{t-1}$ is the cross-covariance between \mathbf{M}_i^{t-1} and $\mathbf{D}_i^{t-1} = \{\mathbf{d}_{i,j}^{t-1} = \mathcal{F}(\mathbf{m}_{i,j}^{t-1}) | j = 1, \dots, N_L\}$,
 196 $\mathbf{C}_{i,\text{DD}}^{t-1}$ is the auto-covariance of \mathbf{D}_i^{t-1} , respectively. The updated sample of \mathbf{m}_i^{t-1} , i.e., \mathbf{m}_i^t , can be
 197 randomly drawn from the updated local ensemble, $\mathbf{M}_i^t = \{\mathbf{m}_{i,j}^t | j = 1, \dots, N_L\}$. Finally, we use
 198 $\mathbf{M}^{N_{\text{iter}}} = \{\mathbf{m}_i^{N_{\text{iter}}} | i = 1, \dots, N_e\}$ to approximate the posterior distribution of \mathbf{m} with possible
 199 multi-modes. More details about ILUES can be found in (J Zhang et al., 2018).

200 **2.3 Ensemble Smoother with Deep Learning-based Update: ESDL**

201 Essentially, the Kalman update used in ESM DA and ILUES builds a linear relationship between
 202 the innovation vector, $\Delta \mathbf{d}_i^t = \tilde{\mathbf{d}} + \sqrt{\alpha^t} \boldsymbol{\varepsilon}_i - \mathbf{d}_i^{t-1}$, and the update vector, $\Delta \mathbf{m}_i^t = \mathbf{m}_i^t - \mathbf{m}_i^{t-1}$:

$$\Delta \mathbf{m}_i^t = \mathbf{K}^t \Delta \mathbf{d}_i^t, \quad (7)$$

203 where $\mathbf{K}^t = \mathbf{C}_{\text{MD}}^{t-1}(\mathbf{C}_{\text{DD}}^{t-1} + \alpha^t \mathbf{C}_{\text{D}})^{-1}$ is the so-called Kalman gain matrix. As the above calculation
 204 is only based on the mean and covariance, the Kalman-based DA is subjected to the Gaussian
 205 assumption. Inspired by the universal approximation and pattern recognition abilities of DL, J
 206 Zhang et al. (2020) proposed to supplant the Kalman-based update in EnKF and its variants with
 207 a DL-based counterpart:

$$\Delta \mathbf{m}_i^t = \mathcal{G}_{\text{DL}}^t(\Delta \mathbf{d}_i^t), \quad (8)$$

208 where $\mathcal{G}_{\text{DL}}^t(\cdot)$ is a nonlinear mapping from the innovation vector to the update vector based on
 209 DL. The effective training of $\mathcal{G}_{\text{DL}}^t(\cdot)$ can be facilitated by the vast amount of data directly
 210 generated from \mathbf{M}^{t-1} and \mathbf{D}^{t-1} , i.e., $\mathbf{X}^t = \{\mathbf{x}_{\text{input}} = \mathbf{d}_i^{t-1} + \sqrt{\alpha^t} \boldsymbol{\varepsilon}_{ij} - \mathbf{d}_j^{t-1}, \mathbf{x}_{\text{target}} = \mathbf{m}_i^{t-1} -$
 211 $\mathbf{m}_j^{t-1} | i = 1, \dots, N_e - 1, i < j \leq N_e\}$. Here, we treat $\{\mathbf{m}_i^{t-1}, \mathbf{d}_i^{t-1}\}$ as the synthetic truth and can

212 produce $N_e - 1$ pairs of innovation and update vectors from the rest ensemble members. In total,
 213 there are $C_{N_e}^2 = N_e(N_e - 1)/2$ training samples in \mathbf{X}^t . Results shown that ESDL can better solve
 214 DA problems involving high-dimensional and non-Gaussian distributed parameters than its
 215 Kalman-based counterpart (J Zhang et al., 2020).

216 **2.4 Parameter Estimator with Deep Learning: PEDL**

217 In ESDL, what $\mathcal{G}_{\text{DL}}^t(\cdot)$ tries to capture is the relationship between the increment in parameters,
 218 i.e., $\Delta\mathbf{m}_i^t$, and the displacement in model responses, i.e., $\Delta\mathbf{d}_i^t$, and the updated parameters are
 219 obtained as:

$$\mathbf{m}_i^t = \mathbf{m}_i^{t-1} + \Delta\mathbf{m}_i^t. \quad (9)$$

220 In the context of fractured media, \mathbf{m} describe the spatial distribution of hydraulic properties,
 221 encompassing distinct parameter values (high values for the fractures, and low values for the
 222 matrix). During the training of $\mathcal{G}_{\text{DL}}^t(\cdot)$, the information about the starting point of $\Delta\mathbf{m}_i^t$, i.e., \mathbf{m}_i^{t-1} ,
 223 is not considered. Consequently, there is a potential mismatch between the regions with high and
 224 low values in \mathbf{m}_i^{t-1} and $\Delta\mathbf{m}_i^t$, as the inferences made by $\mathcal{G}_{\text{DL}}^t(\cdot)$ are not flawless. When \mathbf{m}_i^{t-1} and
 225 $\Delta\mathbf{m}_i^t$ are added up, this imperfection can lead to irregularities in the fracture networks inferred by
 226 ESDL, as illustrated in Figure 5 (Section 3.1.2), highlighting this limitation.

227 To tackle this concern, here we modify ESDL by shifting the target of DL from the update vector
 228 $\Delta\mathbf{m}_i^t$ to the target parameter set \mathbf{m}_i^t directly:

$$\mathbf{m}_i^t = \mathcal{G}_{\text{DL}}^t(\mathbf{m}_i^{t-1}, \tilde{\mathbf{d}} + \sqrt{\alpha^t}\boldsymbol{\varepsilon}_i, \mathbf{d}_i^{t-1}). \quad (10)$$

229 To distinguish it from ESDL, we have named the new method PEDL (parameter estimator with
 230 DL). PEDL avoids the use of the difference vectors in ESMDA, ILUES and ESDL. Thus, the
 231 mismatches between \mathbf{m}_i^{t-1} and $\Delta\mathbf{m}_i^t$ in ESDL will not affect the results anymore. However, the
 232 implementation of PEDL as described in equation (10) is ambiguous. After comprehensive
 233 testing, we suggest to adopt a simplified form of equation (10), i.e.,

$$\mathbf{m}_i^t = \mathcal{G}_{\text{DL}}^t(\tilde{\mathbf{d}} + \sqrt{\alpha^t}\boldsymbol{\varepsilon}_i). \quad (11)$$

234 Here, the training data for the DL model are $\{\mathbf{x}_{\text{input}} = \mathbf{d}_i^{t-1} + \sqrt{\alpha^t} \boldsymbol{\epsilon}_i, \mathbf{x}_{\text{target}} = \mathbf{m}_i^{t-1} | i =$
 235 $1, \dots, N_e\}$. As will be demonstrated in latter part of this work, this new method is very easy to
 236 implement, and can obtain more reliable estimations of fractured media than the other three DA
 237 methods, i.e., ESMDA, ILUES, and ESDL.

238 **3 Illustrative Case Studies**

239 **3.1 Case 1: Data Assimilation without Considering Model Structural Error**

240 3.1.1 Model Settings

241 In this section, we set up a case study involving transient groundwater flow in a two-dimensional
 242 (2-D) fractured aquifer to evaluate the performance of PEDL. We employ the SC model (Tsang
 243 et al., 1996) as both the reference model providing the ground truth and the forecast model used
 244 in DA. In this context, we can disregard the error stemming from model structures, allowing us
 245 to directly compare the performance of PEDL with ESMDA, ILUES, and ESDL.

246 The flow domain is $205[\text{L}] \times 205[\text{L}]$ (in unit of length) and is discretized into 41×41 grids with a
 247 uniform spacing of $5[\text{L}]$ in the numerical model. All four lateral boundaries are impermeable.
 248 The domain predominantly consists of a matrix with permeability of $k_m = 10^{-13}[\text{L}^2]$, but there
 249 exist regions with considerably higher permeability values due to the presence of fractures.
 250 Figure 1 illustrates the reference distribution of the fractured zones, with isotropic permeability
 251 of $k_f = 90k_m$. The simulation period spans $T_s = 86400[\text{T}]$ (in unit of time) and is evenly
 252 divided into 20 timesteps. The initial pressure across the entire domain is $p_0 = 10^7[\text{ML}^{-1}\text{T}^{-2}]$,
 253 where $[\text{M}]$ represents any consistent unit of mass. An injection well (I) at the center of the
 254 domain conducts water injection at a constant rate of $Q_0 = 0.1[\text{L}^2\text{T}^{-1}]$, and eight pumping wells
 255 (P1-P8) initiate pumping at a constant pressure of $0.5p_0$. The pressure dynamics are described by
 256 the following equation:

$$\phi c_{\text{rw}} \frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{\mathbf{k}}{\mu} \nabla p \right) + q, \quad (12)$$

257 where ϕ is the porosity of medium (-), c_{rw} is the total compressibility of rock and water
 258 $[\text{M}^{-1}\text{LT}^2]$, p is the hydraulic pressure $[\text{ML}^{-1}\text{T}^{-2}]$, t is the time $[\text{T}]$, \mathbf{k} is the permeability tensor
 259 $[\text{L}^2]$, μ is the water viscosity $[\text{ML}^{-1}\text{T}^{-1}]$, and q is the source/sink term $[\text{T}^{-1}]$, respectively. Here,

260 $\phi = 0.3$, $c_{rw} = 6 \times 10^{-10} [\text{M}^{-1} \text{L} \text{T}^2]$, and $\mu = 10^{-3} [\text{ML}^{-1} \text{T}^{-1}]$. In this scenario, the tensor \mathbf{k}
 261 are simplified into a location-dependent scalar function, $k(x, y)$, where the values within the
 262 1681 cells constitute the sole unknown parameters that need calibration. The governing equation
 263 is numerically solved with the MATLAB Reservoir Simulation Toolbox (Lie & Møyner, 2021).

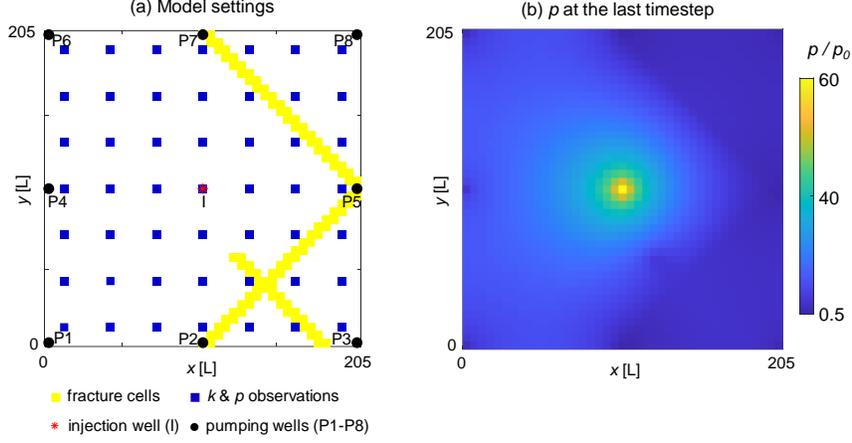


Figure 1. (a) Schematic overview of the model settings in Case 1. All four boundaries are impermeable. An injection well is positioned at the domain’s center, and eight pumping wells (P1-P8) are situated near the boundaries. The blue cells indicate locations where measurements of k and p are acquired. The yellow cells depict the reference distribution of fractures. (b) Distribution of pressure (p) at the last timestep, highlighting the influence of fractured cells.

264 The observational dataset employed for inferring the distribution of $k(x, y)$ includes 49
 265 permeability (k) values at the blue nodes (as shown in Figure 1a), 49 pressure (p) values at the
 266 same nodes at 21 timesteps (49×21 values in total), and 9 flowrates (Q) values at the single
 267 injection well and eight pumping wells at the 21 timesteps (9×21 values in total). The
 268 observation errors associated with k , p , and Q are modeled with Gaussian distributions, i.e., $\varepsilon_k \sim$
 269 $\mathcal{N}(0, \sigma_k^2)$, $\varepsilon_p \sim \mathcal{N}(0, \sigma_p^2)$, and $\varepsilon_Q \sim \mathcal{N}(0, \sigma_Q^2)$, where $\sigma_k = 5k_m$, $\sigma_p = 0.05p_0$, and $\sigma_Q =$
 270 $0.005Q_0$, respectively. In the reference model, there are 106 fractured cells, comprising
 271 approximately 6.3% of the total cells. These high-permeability fracture zones are indicated by
 272 the yellow cells in Figure 1(a). Figure 1(b) illustrates the pressure distribution in the reference
 273 model at the last timestep, clearly demonstrating the impact of these high-permeability fractured
 274 zones.

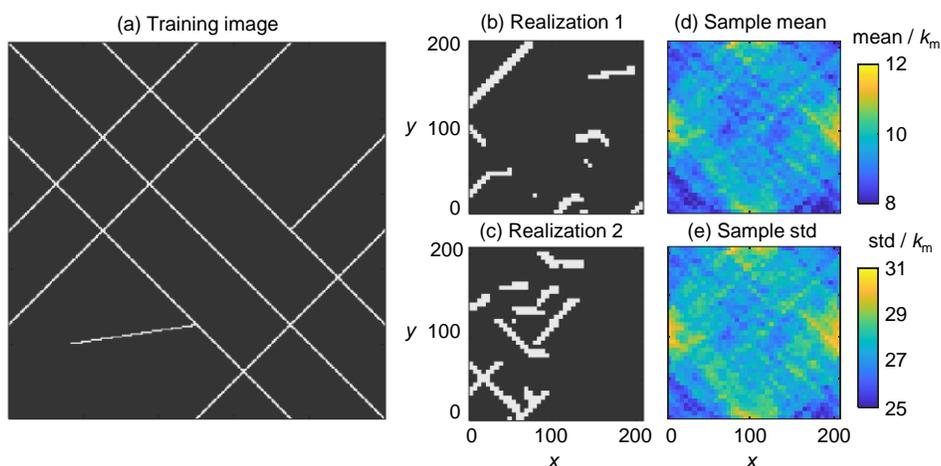


Figure 2. (a) The training image used to generate random realizations of the k field; (b-c) two realizations of the k field, and (d-e) the mean and standard deviation (std) of the k field.

275 The direct sampling method, as described by Mariethoz et al. (2010), is utilized to generate
 276 random realizations of the $k(x, y)$ field, each field having a dimension of $N_k = 41 \times 41$. These
 277 realizations are generated based on a training image that includes background information about
 278 fractures, such as potential orientation and density (as shown in Figure 2a). In the generated k
 279 realizations, black cells are designated as k_m , representing the matrix category, while white cells
 280 are designated as k_f , signifying the fracture category. In Figures 2(b-c), we present two random
 281 samples of the generated k fields. The sample mean and standard deviation (std) fields calculated
 282 from all the realizations are depicted in Figures 2(d-e).

283 3.1.2 Results from Four DA Methods

284 Four DA methods, namely ESMDA, ILUES, ESDL, and PEDL, are applied to estimate the k
 285 field across the flow domain. All methods, except for PEDL (without iteration, $N_e = 5000$),
 286 undergo five iterations with an ensemble size of $N_e = 5000$. In the case of ILUES, the hyper-
 287 parameter β , which dictates the selection of the local ensemble, is set to 0.1. For both ESDL and
 288 PEDL, we have adopted the U-net architecture initially proposed by Ronneberger et al. (2015),
 289 as depicted in Figure 7(a). Given the presence of three distinct types of measurement data—
 290 permeability (k), flowrate (Q), and pressure (p)—each with varying dimensionalities, we employ
 291 three U-net blocks to process each data type. Subsequently, we concatenate and further process
 292 these data streams until the target (the 41×41 k field) is reached. In both ESDL and PEDL, the

293 DL model is trained using the Adam optimizer with a consistent learning rate of 0.001. The
 294 training process consists of 100 epochs, with a batch size of 256.

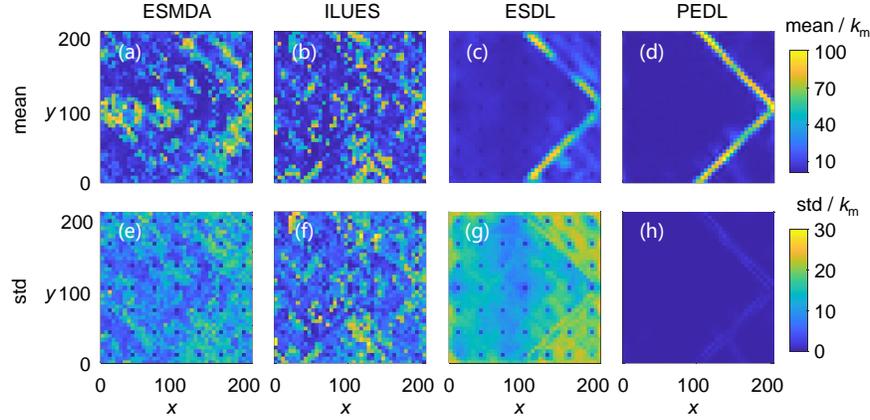


Figure 3. (a-d) Mean and (e-h) standard deviation (std) of the estimated k fields obtained by ESMDA, ILUES, ESDL, and PEDL, respectively. Note that all the means and stds are expressed as multiples of k_m .

295 The normalized root-mean-square errors (NRMSEs) relative to the matrix permeability k_m ,
 296 between the estimated mean (\bar{k}) and the reference (k^*) fields, i.e.,

$$\text{NRMSE} = \frac{1}{k_m} \sqrt{\frac{1}{N_k} \sum_{i=1}^{N_k} (\bar{k}_i - k_i^*)^2}, \quad (13)$$

297 are obtained as follows: ESMDA (32.31), ILUES (30.03), ESDL (15.89), and PEDL (12.11). It is
 298 worth noting that the NRMSE value between the initial mean k field and the reference k field is
 299 22.35. The NRMSE values for ESMDA and ILUES exceed that of the initial mean field,
 300 indicating the occurrence of filter divergence in these two methods. With the DL-based update,
 301 ESDL can obtain improved estimation of k , but the reduction in NRMSE from the initial field is
 302 not very large. Without requiring iterations, PEDL can obtain the best match of the highly
 303 complex parameter field.

304 Figure 3 illustrates the mean and standard deviation (std) of the k fields estimated by these four
 305 DA methods. It is evident that PEDL effectively captures the distribution of fracture cells (highly
 306 permeable zones) with low uncertainty, albeit with some minor details missing. Conversely,

307 neither ESMDA nor ILUES provide a satisfactory estimation of k . On the other hand, ESDL
 308 partially recognizes the fracture structure. Specifically, ESDL tends to overestimate the k values
 309 in non-fractured areas near the upper-right and lower-right corners, and the standard deviation of
 310 the k field obtained by ESDL is significantly higher than those obtained using other methods.

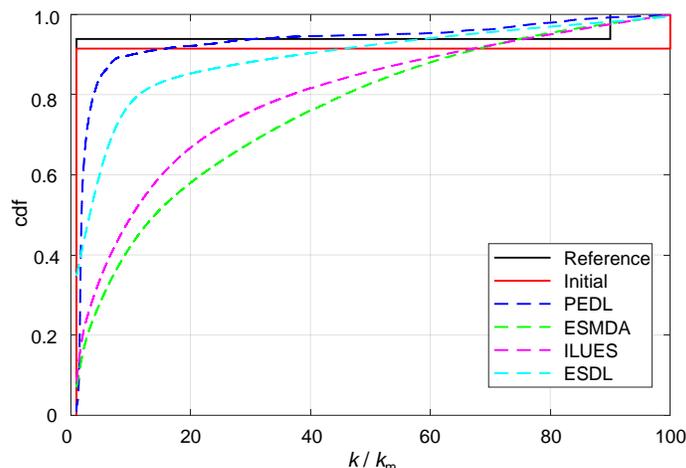


Figure 4. Cumulative distribution functions (cdf) for the reference k field, initial k realizations, and estimated k fields from PEDL, ESMDA, ILUES, and ESDL, respectively.

311 Given the strong non-Gaussian nature of this problem, it is necessary to consider more
 312 information beyond the mean and standard deviation to thoroughly evaluate the results. Figure 4
 313 displays the cumulative distribution functions (cdf) of the updated k fields for all four methods,
 314 as well as the reference and initial k fields. It is evident that PEDL produces a cdf of k field that
 315 closely resembles the stepwise cdf of the reference k field, demonstrating PEDL's ability to
 316 handle strong non-Gaussianity.

317 Moreover, to monitor the evolution of individual k field samples before and after the various
 318 updates, we compare four randomly selected realizations in Figure 5. It reveals that initially
 319 distinct realizations (the first row of Figure 5) tend to converge and approach the reference k
 320 field after the update with PEDL (the last row of Figure 5). This consistency aligns with the
 321 observed low k standard deviation values across the entire domain (as seen in Figure 3h).
 322 Conversely, the first three rows of Figure 5 demonstrate that in both ESMDA and ILUES, the
 323 updated realizations do exhibit some degree of convergence among them, but they do not
 324 resemble the reference k field. This suggests the occurrence of filter divergence. In the case of

325 ESDL (the fourth row), while high-permeability stripes akin to the reference field do emerge in
 326 the updated k realizations, the initial random structures of fracture in the four realizations largely
 327 persist, resulting in substantial irregularities within the final realizations. One possible
 328 explanation for this behavior is that the $\{\Delta \mathbf{m}^t, \Delta \mathbf{d}^t\}$ pairs may vary significantly for different
 329 $\{\mathbf{m}^{t-1}, \mathbf{d}^{t-1}\}$ in nonlinear problems, yet they are not considered during the model training
 330 process. The high values in \mathbf{m}^{t-1} only partially cancel out the values at the same locations in
 331 $\Delta \mathbf{m}^t$, causing the unwanted irregularities in each realization. Although these irregularities can be
 332 averaged out in the ensemble mean, as shown in Figure 3(c), the large uncertainty do exist in the
 333 standard deviation field (Figure 3g).

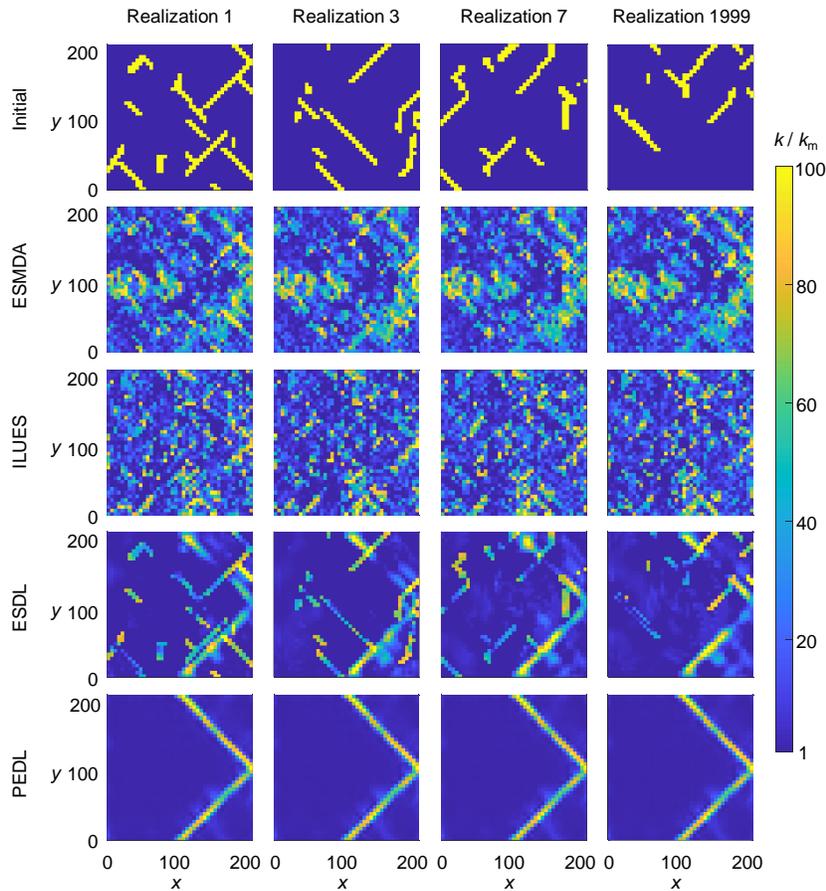


Figure 5. Four randomly sampled initial k fields and the corresponding updated ones with ESMDA, ILUES, ESDL, and PEDL, respectively. All k fields are expressed as multiples of k_m .

334 From the results above, it is concluded that while ESMDA, ILUES and ESDL encounter
 335 significant difficulties in this problem, PEDL yields much better estimation results from the
 336 perspective of ensemble behavior and individual realization performance.

337 3.1.3 Impact of Ensemble Size and Network Architecture

338 To assess the influence of ensemble size on PEDL’s performance, we further conduct
 339 experiments using ensembles with various sizes ($N_e = 10^2$ to 10^5 , as depicted in Figure 6). To
 340 ensure the statistical reliability of the results, we carried out twenty parallel tests for each
 341 ensemble size. Notably, Figure 6 highlights that the NRMSE values begin to decline when N_e
 342 exceeds 500 and stabilize as N_e surpasses 5,000. This observation is intriguing, as deep neural
 343 networks (DNNs) are typically associated with a demand for an extensive volume of training
 344 data.

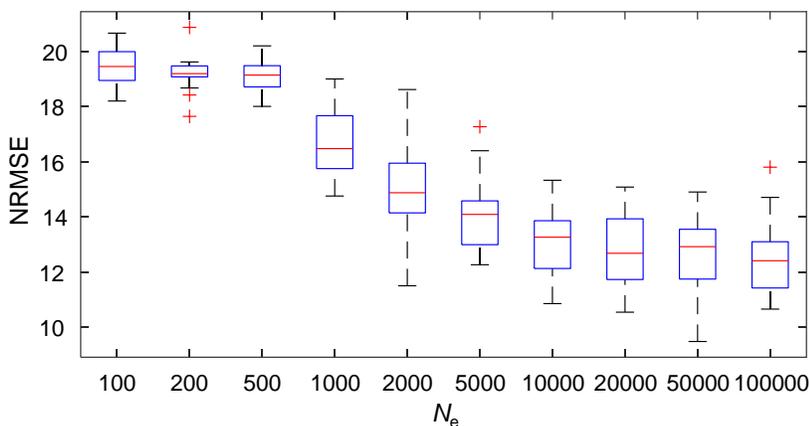


Figure 6. NRMSEs of results obtained by PEDL with various ensemble sizes.

345 In our prior experiment, PEDL utilized a U-net architecture for DL. However, it’s natural to
 346 question whether alternative types of adequate DNNs can deliver comparable results within the
 347 PEDL framework. To investigate the impact of network architecture, we decide to substitute the
 348 U-net with a residual neural network (ResNet, shown in Figure 7b), while maintaining the
 349 ensemble size of $N_e = 5000$. The training settings for the ResNet model remain identical to
 350 those applied to U-net. Remarkably, the resulting NRMSE in this instance is 12.66, a value that
 351 closely aligns with the NRMSE of 12.11 achieved by PEDL with U-net.

352

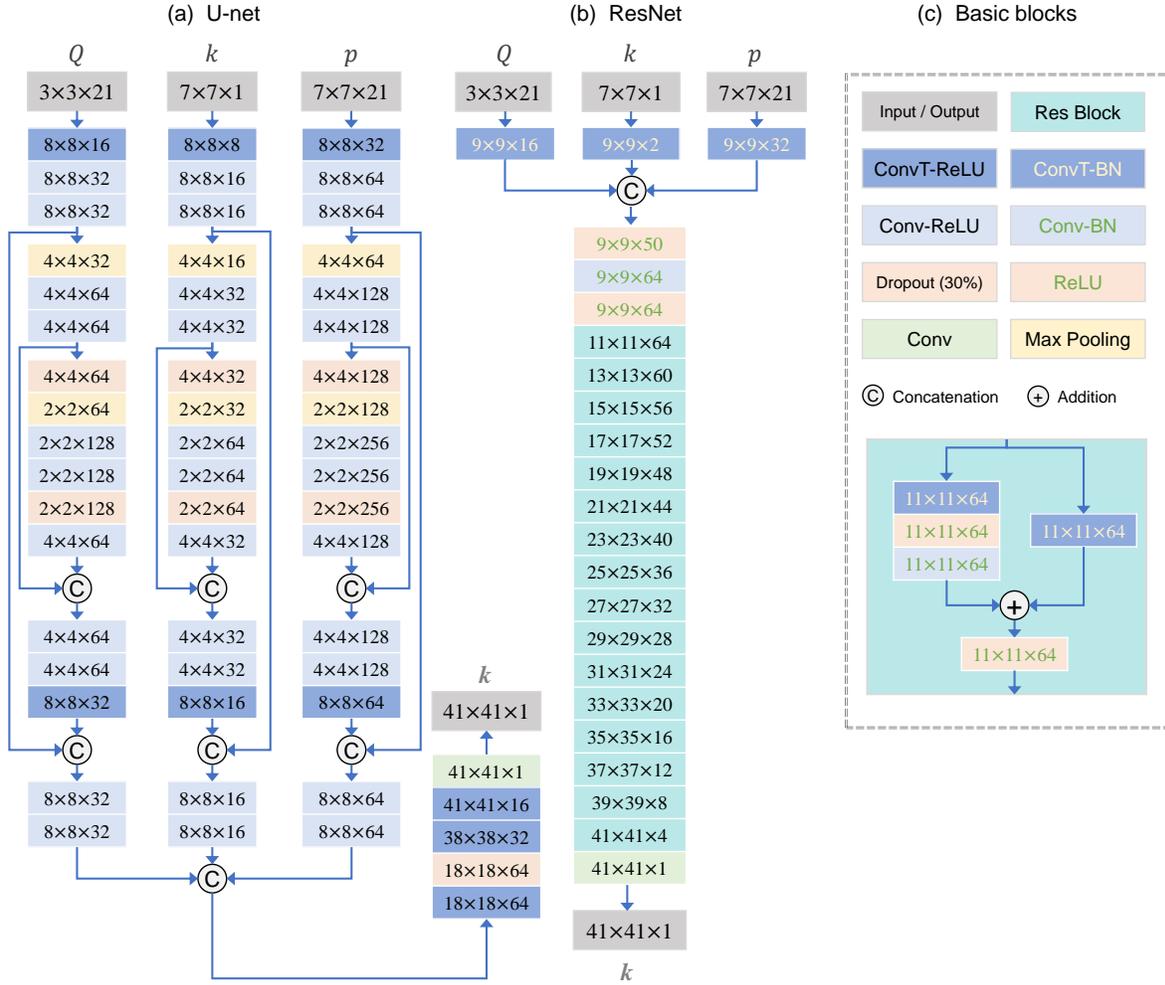


Figure 7. DL model architectures used in this study: (a) U-net, (b) ResNet, and (c) basic blocks used in U-net and ResNet, respectively. Here, Q , p , k represent flowrate, pressure, and permeability, as input or output, respectively. Conv and ConvT mean the 2-D convolution and the transposed 2-D convolution layers, ReLU denotes the rectified linear unit, and BN signifies the batch normalization layer, respectively.

353 In Figure 8, we present the spatial distributions of the mean and standard deviation of k fields
 354 obtained through the ResNet-based PEDL, along with four random realizations of updated k
 355 field. The results closely resemble the PEDL approach applied with the U-net (as seen in Figures
 356 3 and 5). This observation underscores the fact that PEDL demonstrates satisfactory performance
 357 with both U-net and ResNet architectures. It is likely that using a better designed DL model can
 358 produce enhanced estimation of fractured media properties. Yet, the searching for the optimal
 359 DL model and training options are beyond the scope of the current work.

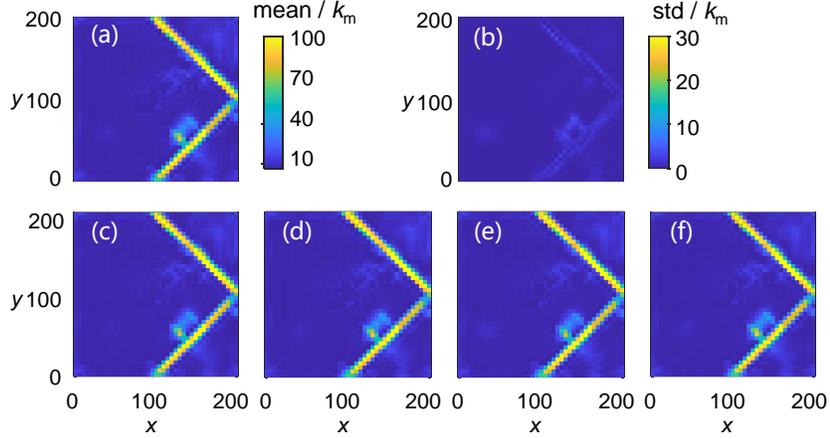


Figure 8. Results obtained by PEDL using ResNet: (a) mean of the updated k fields, (b) standard deviation of the updated k fields, and (c-f) four random realizations of the updated k fields.

360 **3.2 Case 2: Data Assimilation Considering Model Structural Error**

361 In the previous section, the model structural error is not considered, and both the reference model
 362 and forecast model are based on the SC model. However, fractured media in the natural world
 363 often showcase significantly more pronounced complex and non-Gaussian behaviors than what
 364 can be accurately represented by the simplistic SC model. In this section, we shift our focus to
 365 the more accurate EDFM model, which serves as the reference model, to thoroughly assess the
 366 effectiveness of PEDL in addressing more complex scenarios. It's worth noting that when
 367 estimating the properties of the media through DA, we still employ the SC model, albeit
 368 introducing some degree of structural error into the model. In this section, we will subject PEDL
 369 to further testing within the context of model structural error to evaluate its robustness.

370 As the name implies, the EDFM model incorporates discrete fractures into the matrix as lower-
 371 dimensional entities, such as fracture lines within a 2-D matrix or fracture planes within a 3-D
 372 matrix. The EDFM model describes the flow within the matrix as follows:

$$\frac{\partial(\phi\rho)}{\partial t} = \nabla \cdot \left(\frac{\mathbf{k}}{\mu} \nabla(p - \rho g z) \right) + q^{(m)} - \sum_j q_j^{(mf)}, \quad (14)$$

373 where ρ means the density of water, g is the gravitational constant, z signifies the depth, $q^{(m)}$
 374 denotes the source/sink term in the matrix cell, and $q_j^{(mf)}$ represents the exchange term from the
 375 matrix cell to the j -th fracture (set as zero if matrix cell and fracture are nonadjacent),
 376 respectively. The flow equation in the j -th fracture can be expressed as:

$$\frac{\partial(\phi_j^{(f)}\rho)}{\partial t} = \nabla \cdot \left(\frac{\mathbf{k}_j}{\mu} \nabla (p_j^{(f)} - \rho g z) \right) + \frac{1}{a_j} \left(q^{(f)} - q_j^{(fm)} - \sum_{i \neq j} q_{j,i}^{(ff)} \right), \quad (15)$$

377 where $\phi_j^{(f)}$ and \mathbf{k}_j are the porosity and permeability of the j -th fracture, $p_j^{(f)}$ is the hydraulic
 378 pressure in the current cell of the j -th fracture, $q^{(f)}$ is the source/sink term in the fracture cell,
 379 $q_j^{(fm)}$ is the exchange from the j -th fracture to the surrounding matrix cell, $q_{j,i}^{(ff)}$ is the exchange
 380 from the j -th fracture to other fractures (set as zero if they don't intersect in the current cell), and
 381 a_j is the aperture of the j -th fracture, respectively.

382 The matrix-fracture exchanges $q_j^{(mf)}$ and $q_j^{(fm)}$, when calculated on cells, can be expressed as:

$$Q_j^{(mf)} = \int_{V_m} q_j^{(mf)} dV = T^{(mf)} (p - p_j^{(f)}), \quad (16)$$

383 and

$$Q_j^{(fm)} = \int_{A_m} q_j^{(fm)} dA = T^{(mf)} (p_j^{(f)} - p) = -Q_j^{(mf)}, \quad (17)$$

384 respectively. In the above equations, $T^{(mf)} = \frac{k^{(mf)} A^{(mf)}}{d^{(mf)}}$ is the transmissivity between matrix and
 385 fracture, $k^{(mf)}$ is the volume-weighted harmonic mean of fracture and matrix permeability, $A^{(mf)}$
 386 is the exchange area between matrix and fracture cells, and $d^{(mf)}$ is the characteristic distance
 387 between matrix cell and fracture plane, respectively. The matrix-fracture exchanges calculated
 388 with equations (15-17) are sometimes called “non-neighboring connections” since the matrix and
 389 fracture cells are not neighbors in gridding but connected by extra relations. For more details
 390 about the EDFM model, interested readers can refer to (Lie & Møyner, 2021; Moinfar et al.,
 391 2014).

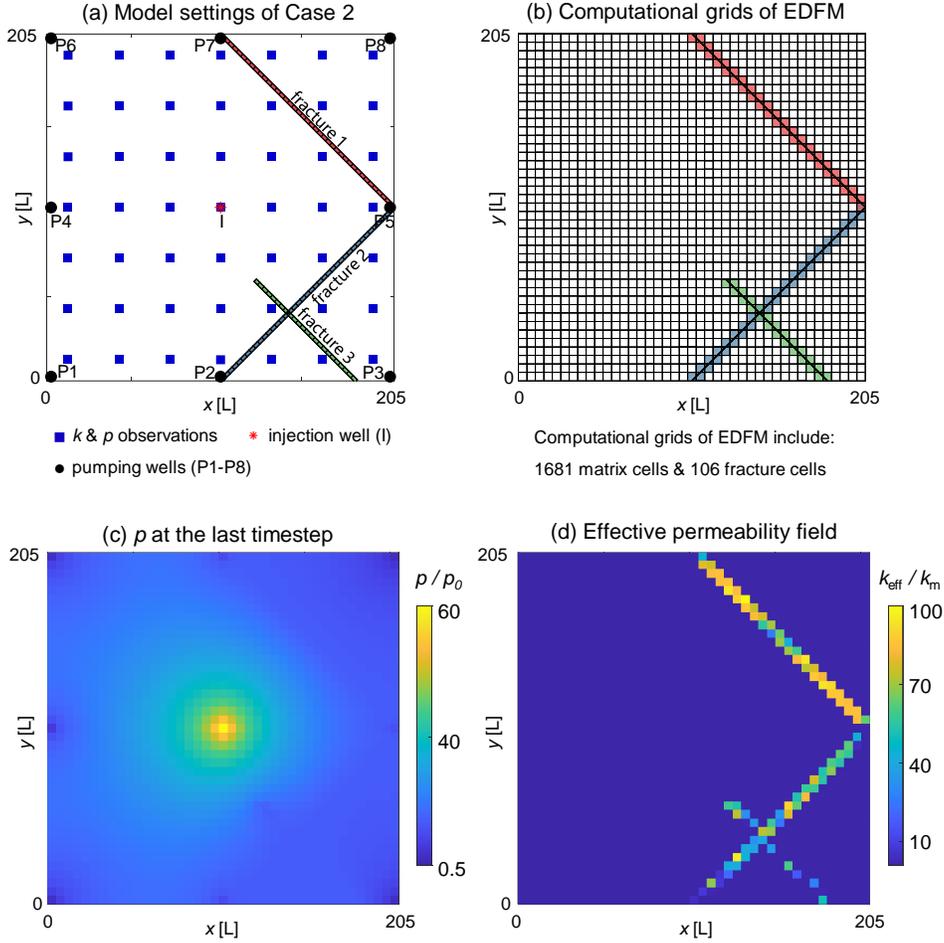


Figure 9. (a) Model settings of Case 2; (b) Computational grids of the EDFM model (colored cells are linked to fracture cells appended through non-neighboring connections); (c) Spatial distribution of pressure calculated with the EDFM model at the last timestep; (d) Effective permeability field found by an optimization method based on p and Q at all timesteps in the EDFM model.

392 In Case 2, we examine a fractured aquifer similar to the one in Case 1 (depicted in Figure 1),
 393 with the exception of the fracture part. In this aquifer, there are no high-permeability matrix cells.
 394 Instead, there are three fractures with permeability of $k_f^* = 100k_m$, porosity of $\phi = 0.8$, and
 395 aperture of $a = 0.2[L]$ (as illustrated in Figure 9a). As in Case 1, the simulation period is set to
 396 be $T_s = 86400 [T]$, evenly divided into 20 timesteps. At $t = 0[T]$, the pressure is uniformly set as
 397 $p_0 = 107[ML^{-1}T^{-2}]$ across the domain. The injection and pumping wells are managed in the
 398 same manner as Case 1, i.e., one injection well located at the center of the domain with the rate
 399 of $0.1[L^2T^{-1}]$, and eight pumping wells (P1-P8) extracting water at a fixed pressure of $0.5p_0$. As

400 shown in Figure 9(b), the matrix is discretized into 41×41 uniform grids, which divides the
 401 fractures into 106 fracture cells. In total, there are 109 identified non-neighboring connections,
 402 including three fracture-fracture connections. The pressure distribution calculated with the
 403 EDFM model at the last timestep is displayed in Figure 9(c). In this case, the EDFM model,
 404 acting as the reference model, is used to generate measurement data.

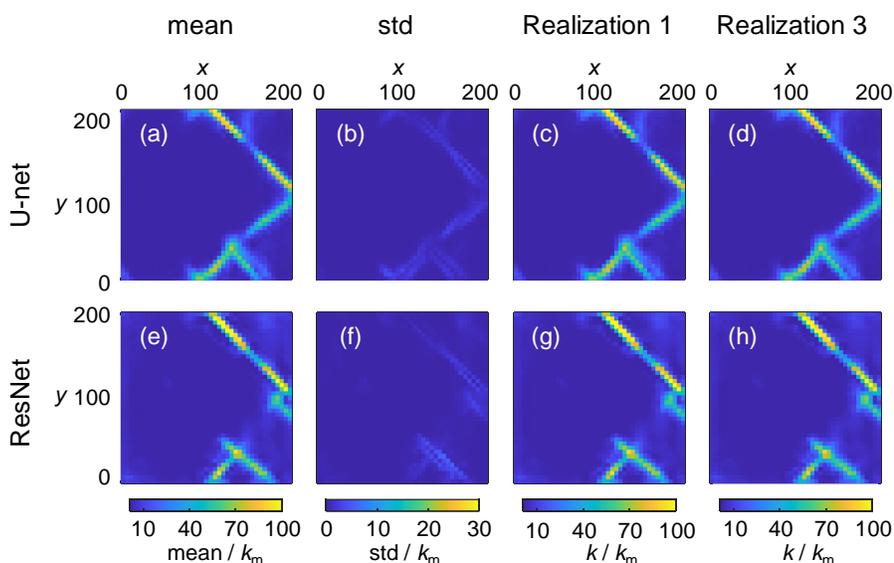


Figure 10. Results of permeability estimation in Case 2 using U-net-based PEDL (top row) and ResNet-based PEDL (bottom row), respectively. The mean estimates for the two methods are presented in subfigures (a) and (e), the estimation uncertainty measured by the standard deviation (std) are given in (b) and (f), and some random realizations are depicted in (c-d) and (g-h), respectively.

405 In the implementation of DA, the simpler SC model with 41×41 grids is adopted to calculate the
 406 model responses, due to its ease of implementation and prevalence in fractured aquifer
 407 characterization practices (National Academies of Sciences, 2020). Unlike the EDFM model, the
 408 SC model has no fracture cells or non-neighboring connections. The impact of actual fractures is
 409 reflected in the effective permeability field (EPF) of matrix. To evaluate the performance of
 410 PEDL, a “pseudo-reference” of EPF for the SC model is found by solving a nonlinear data fitting
 411 problem with the trust-region-reflective algorithm (Coleman & Li, 1996) based on the complete
 412 data of p and Q at all grids and timesteps from the EDFM model simulation. At each timestep,
 413 there are 1681 p values and nine Q values. The obtained EPF is shown in Figure 10(d), which is
 414 used to calculate the NRMSE values of the updated k fields obtained by PEDL.

415 The measurement data from the same observation network (Figure 9a) as in Case 1 are used in
416 the estimation of k field here, i.e., 49 k values and 49×21 p values at the 7×7 blue nodes, and
417 9×21 Q values at the single injection well and eight pumping wells. Figure 10 shows the
418 estimated mean, std and two arbitrarily picked realizations of k field by U-net-based PEDL (top
419 row) and ResNet-based PEDL (bottom row). Evidently, PEDL demonstrates the capacity to
420 capture the fundamental structure of fractures, employing either the U-net or ResNet architecture.
421 Notably, there are no remarkable distinctions between the mean estimate and individual
422 realizations, underscoring the consistency of PEDL's performance. Additionally, PEDL exhibits
423 NRMSE values of 16.02 for U-net and 15.07 for ResNet. While these NRMSEs may not reach
424 the levels observed in Case 1 (with NRMSE values close to 12), they remain quite acceptable,
425 given the presence of model structural error introduced by adopting the simplified SC model in
426 DA.

427 **4 Conclusions and Discussions**

428 Water flow and solute transport in fractured media exhibit distinct characteristics, characterized
429 by the notable non-Gaussian distribution of media properties and the substantial non-linearity in
430 the underlying processes. Accurately estimating the heterogeneous hydraulic parameters of
431 fractured media is of utmost importance for reliable predictions and well-informed decision-
432 making. Nevertheless, the complexities arising from the high dimensionality and non-Gaussian
433 nature of fractured media present substantial challenges for traditional DA methods, such as
434 MCMC and EnKF, when aiming to achieve a robust estimation of these intricate properties.
435 Previous attempts to facilitate the effective application of these DA methods have primarily
436 relied on reparameterization techniques, involving the transformation of non-Gaussian variables
437 into Gaussian ones. However, the use of these techniques can result in the loss of essential
438 information related to the true nature of fractured media.

439 DL, known for its proficiency in modeling complex, nonlinear relationships and identifying
440 intricate patterns within data, can improve DA by addressing the challenges arising from high-
441 dimensionality and non-Gaussianity concurrently. J Zhang et al. (2020) proposed the innovative
442 use of DL to create a nonlinear updating scheme, replacing the traditional linear Kalman
443 formulation widely employed across various research domains. This DA method, named ESDL,

444 offers a superior capacity to capture the non-Gaussian characteristics of subsurface media's
445 parameter fields compared to its Kalman-based counterparts. In our study, we applied ESDL for
446 the first time to estimate hydraulic parameter fields within fractured media, comparing it with
447 two Kalman-based DA methods, ESMDA (Emerick & Reynolds, 2013) and ILUES (J Zhang et
448 al., 2018). Our findings demonstrate that ESDL significantly enhances the characterization of
449 fractured media over ESMDA and ILUES. However, we observed some irregularities in the
450 fracture networks inferred by ESDL, potentially attributable to the transition process from the
451 innovation vector (the difference between observations and model predictions) to the update
452 vector (the difference between posterior and prior parameters). Based on this insight, we
453 enhanced ESDL by introducing a new updating scheme. The resulting DA method, named
454 parameter estimator with DL (PEDL), establishes a direct update from the multi-source
455 observation vector to the posterior parameter vector. PEDL surpasses ESDL in the same fracture
456 characterization problem. In our sensitivity analysis regarding ensemble size, we observed that
457 PEDL improves permeability estimates when utilizing ensemble sizes up to 5000. This is
458 noteworthy, as DL models typically require a substantial volume of training data. Thus, an
459 ensemble size not exceeding 10,000 appears as a reasonable compromise between inference
460 accuracy and computational efficiency. Furthermore, PEDL's adaptability is evident in its
461 flexibility regarding the architecture of the DL model employed. We verified this by testing two
462 distinct model architectures, U-net and ResNet, within the PEDL framework, both of which yield
463 similar estimation results in two case studies with increasing complexity. Additionally, we
464 evaluated PEDL's performance in the presence of structural model error. Specifically, we used
465 the sophisticated EDFM model as the reference model to generate measurement data but adopted
466 the simplified SC model as the forecast model in DA. In this scenario, the ground truth exhibited
467 heightened non-Gaussian and nonlinear characteristics, mirroring the complexities typically
468 encountered in practical applications. Although PEDL, utilizing either U-net or ResNet, could
469 not achieve an exact match between the forecast model and the ground truth, it displayed the
470 ability to discern the fundamental structure of actual fractures, albeit with slightly reduced
471 performance in comparison to Case 1, due to the presence of model structural error.

472 In the current study, we investigate two cases featuring only three fractures. While our tests have
473 unequivocally demonstrated the effectiveness of PEDL, it is imperative to address more intricate
474 scenarios in future research endeavors. The presence of more fractures introduces formidable

475 challenges to both modeling and DA techniques. To construct an adequate numerical model, it is
 476 essential to enhance the discretization in both spatial and temporal domains. Naturally, this will
 477 lead to a substantial increase in simulation time. To bolster simulation efficiency, one can
 478 consider adopting surrogate models or low-fidelity approximations. Striking a balance between
 479 efficiency and accuracy, a promising solution may involve leveraging multi-fidelity simulations
 480 underpinned by DL techniques, as suggested by Chakraborty (2021). Furthermore, to
 481 comprehensively characterize more complex geological media, we must expand our data
 482 collection efforts to improve information content. This entails considering various types of
 483 measurements, including electromagnetic and electrical resistivity tomography data. The
 484 intricate nature of the models and the data necessitates the development of a more sophisticated
 485 DL model and an updated formula for PEDL.

486 **Open Research**

487 The data and codes on which this article is based are available in (Nan & Zhang, 2023).

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