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The effect of New Integral transform on the establishment of solutions for fractional mathematical models†

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Summary

In intention of this research is to established the truncated solutions of time-space fractional partial differential equations (FPDEs) by employing the combination of Shehu transform and iterative method. The numerical solutions reveals the effectivity and accuracy of the technique.

KEYWORDS:
Shehu transform, time-space fractional partial differential equation, iterative method

1 | INTRODUCTION

In the modelling of various processes in mathematics physics, biology, dynamical systems, control systems, engineering and so on [1,2,3,4,5,6,7,8], fractional partial differential equations are utilized since the results of fractional mathematical models have better results than other mathematical models. Therefore quite a few research such as existence, uniqueness and regularity of solutions, on the processes of heat diffusion, modelled by fractional differential equations [9,10,11]. The contribution of fractional differential equations plays an important role in science and technology. In the establishment of the solutions for fractional nonlinear problems, integral transform techniques are the common ones [2,12]. This result lead us to establish numerical solutions of nonlinear FPDEs by means of the combination of Daftardar-Jafari method (DJM) and Shehu transform.The encountered fractional nonlinear problems, modelling real life pheonema are analyzed and solved by taking physical knowledge and physical properties of the nonlinear problem into consideration. Shehu transform, introduced by Shehu Maitama and Weidong Zhao [13], is an integral transformation converting the ordinary and partial differential equations into simpler equations. It is obtained by generalizing Laplace transformation. Moreover it is a linear transformation like Laplace and Sumudu transformations. Laplace and Yang integral transformations are obtained from Shehu transformation by taking $q = 1$ and $p = 1$ respectively. From this point of view, it could be better to use Shehu transform instead of Laplace or Yang transforms [13].

The aim of this research is to extend Shehu Transform iterative method (STIM) to construct truncated solutions of time-space FPDEs. The STIM method is applied to solve a various linear and nonlinear FPDEs. The truncated solutions are established in terms of Mittag-Leffler functions and fractional trigonometric functions. The advantages of this method can be listed as follows:
1. CPU time is shorter,
2. Robust method for nonlinear and linear FPDEs,
3. Less calculation time,
4. Less margin of error.
As a results, it is clear that utilizing STIM is one of the best choice for the establishment of the solutions for linear and nonlinear mathematical models including FPDEs.

†This is an example for title footnote.
[0]**Abbreviations:** FPDEs, fractional partial differential equations; DJM, Daftardar-Jafari method; STIM, Shehu Transform iterative method

2 | PRELIMINARIES

Essential knowledge such as notations and features of the fractional calculus are presented in this subsection[2,4]. The definition of Riemann-Liouville time-fractional integral of a real valued function $u(x,t)$ is given as

$$I_t^\alpha u(x,t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} u(x,s)ds \tag{1}$$

where $\alpha > 0$ represents the order of the integral.

α^{th} order the Caputo time-fractional derivative operator of $u(x,t)$ is defined as

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = I_t^{m-\alpha}\left[\frac{\partial^m u(x,t)}{\partial t^m}\right] = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-y)^{m-\alpha-1} \frac{\partial^m u(x,y)}{\partial y^m} dy, & m-1 < \alpha < m, \\ \frac{\partial^m u(x,t)}{\partial t^m}, & \alpha = m \end{cases} \tag{2}$$

Mittag-Leffler function with two parameters is defined as

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)}, \; Re(\alpha) > 0, \; z, \beta \in \mathbb{C} \tag{3}$$

where α and β are parameters.

The following set of functions has Shehu transformation

$\left\{ f(t) | \exists P, \tau_1, \; \tau_2 > 0, |f(t)| < Pe^{\frac{|t|}{\tau_j}}, if \; t \in (-1)^j \times [0,\infty) \right\}$

and it is defined as

$$\mathbb{S}[f(t)] = F(p,q) = \int_0^\infty e^{-\frac{p}{q}t} f(t)\,dt \tag{4}$$

which has the following property

$$\mathbb{S}\left[t^\alpha\right] = \int_0^\infty e^{-\frac{pt}{q}} t^\alpha dt = \Gamma(\alpha+1)\left(\frac{q}{p}\right)^{\alpha+1}, \; Re(\alpha) > 0 \tag{5}$$

inverse Shehu inverse transform of $\left(\frac{q}{p}\right)^{n\alpha+1}$ is defined as

$$\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{n\alpha+1}\right] = \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \; Re(\alpha) > 0 \tag{6}$$

where $n > 0$[13].

For α^{th} order the Caputo time-fractional derivative of $f(x,t)$, the Shehu transformation has the following form[14]:

$$\mathbb{S}\left[\frac{\partial^\alpha f(x,t)}{\partial t^\alpha}\right] = \left(\frac{p}{q}\right)^\alpha \mathbb{S}[f(x,t)] - \sum_{k=0}^{n-1}\left[\left(\frac{p}{q}\right)^{\alpha-k-1} \frac{\partial^k f(x,0)}{\partial t^k}\right], n-1 < \alpha \le n, n \in \mathbb{N} \tag{7}$$

3 | THE IMPLEMENTATION OF STIM

The implementation of STIM for mathematical models, including space time fractional differential equations, are presented in this section. Now take the following problem

$$\frac{\partial^\zeta f}{\partial t^\zeta} = \mathbb{F}\left(x, f, \frac{\partial^\eta f}{\partial x^\eta}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right), j-1 < \zeta \le \; j, i-1 < \eta \le \; i, l, j, i \in \mathbb{N} \tag{8}$$

with the initial restrictions

$$\frac{\partial^m f(x,0)}{\partial t^m} = h_m(x,)k = 0, 1, 2, \dots, j-1, \tag{9}$$

into consideration where $f = f(x,t)$ is the function to be determined and $\mathbb{F}\left(x, f, \frac{\partial^{\eta} f}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right)$ denotes linear or nonlinear function. After carrying out the Shehu transform to Eq. (8) and rearrangement, we have

$$\mathbb{S}\left[f(x,t)\right] = \sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^{m} f(x,0)}{\partial t^{m}}\right] + \left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, f, \frac{\partial^{\eta} f}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right)\right]. \tag{10}$$

Enforcing the inverse Shehu transform of Eq. (10) leads to

$$f(x,t) = \mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^{m} f(x,0)}{\partial t^{m}}\right]\right] + \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, f, \frac{\partial^{\eta} f}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right)\right]\right]. \tag{11}$$

After rearrangement, we have

$$f(x,t) = g(x,t) + \boldsymbol{G}\left(x, f, \frac{\partial^{\eta} f}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right), \tag{12}$$

where

$$\begin{cases} g(x,t) = \mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^{m} f(x,0)}{\partial t^{m}}\right]\right] \\ \boldsymbol{G}\left(x, f, \frac{\partial^{\eta} f}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right) = \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, f, \frac{\partial^{\eta} f}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f}{\partial x^{l\eta}}\right)\right]\right] \end{cases} \tag{13}$$

Here $\boldsymbol{G}$ represent a linear or nonlinear operator and the function g is given.
In order to construct the solution of Eq. (12), the DJM introduced by Daftardar-Gejji and Jafari [15] is employed. The solution is established in the series form as follows:

$$f = \sum_{n=0}^{\infty} f_n, \tag{14}$$

where the terms f_n are determined recursively.After decomposition of the operator $\boldsymbol{G}$, we have

$$\begin{aligned} &\boldsymbol{G}\left(x, \textstyle\sum_{n=0}^{\infty} f_n, \frac{\partial^{\eta}\left(\sum_{n=0}^{\infty} f_n\right)}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{\infty} f_n\right)}{\partial x^{l\eta}}\right) = \\ &\boldsymbol{G}\left(x, f_0, \frac{\partial^{\eta} f_0}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f_0}{\partial x^{l\eta}}\right) \\ &+ \textstyle\sum_{c=1}^{\infty}\left(\boldsymbol{G}\left(x, \sum_{n=0}^{c} f_n, \frac{\partial^{\eta}\left(\sum_{n=0}^{c} f_n\right)}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c} f_n\right)}{\partial x^{l\eta}}\right)\right) \\ &- \textstyle\sum_{c=1}^{\infty}\left(\boldsymbol{G}\left(x, \sum_{n=0}^{c-1} f_n, \frac{\partial^{\eta}\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^{l\eta}}\right)\right) \end{aligned} \tag{15}$$

$$\begin{aligned} &\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \textstyle\sum_{n=0}^{\infty} f_n, \frac{\partial^{\eta}\left(\sum_{n=0}^{\infty} f_n\right)}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{\infty} f_n\right)}{\partial x^{l\eta}}\right)\right]\right] = \\ &\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, f_0, \frac{\partial^{\eta} f_0}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta} f_0}{\partial x^{l\eta}}\right)\right]\right] \\ &+ \textstyle\sum_{c=1}^{\infty}\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \sum_{n=0}^{c} f_n, \frac{\partial^{\eta}\left(\sum_{n=0}^{c} f_n\right)}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c} f_n\right)}{\partial x^{l\eta}}\right)\right]\right] \\ &- \textstyle\sum_{c=1}^{\infty}\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \sum_{n=0}^{c-1} f_n, \frac{\partial^{\eta}\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^{\eta}}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^{l\eta}}\right)\right]\right] \end{aligned} \tag{16}$$

Plugging Eqs. (14), (16) into Eq. (12) leads to

$$\sum_{n=0}^{\infty} f_n = \mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^m f(x,0)}{\partial t^m}\right]\right] + \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, f_0, \frac{\partial^\eta f_0}{\partial x^\eta}, \dots, \frac{\partial^{l\eta} f_0}{\partial x^{l\eta}}\right)\right]\right] + \sum_{c=1}^{\infty}\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \sum_{n=0}^{c} f_n, \frac{\partial^\eta\left(\sum_{n=0}^{c} f_n\right)}{\partial x^\eta}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c} f_n\right)}{\partial x^{l\eta}}\right)\right]\right] - \sum_{c=1}^{\infty}\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \sum_{n=0}^{c-1} f_n, \frac{\partial^\eta\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^\eta}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^{l\eta}}\right)\right]\right].$$

The following recurrence relation is established:

$$\begin{aligned} f_0 &= \mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^m f(x,0)}{\partial t^m}\right]\right] \\ f_1 &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, f_0, \frac{\partial^\eta f_0}{\partial x^\eta}, \dots, \frac{\partial^{l\eta} f_0}{\partial x^{l\eta}}\right)\right]\right] \\ f_{r+1} &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \sum_{n=0}^{c} f_n, \frac{\partial^\eta(\sum_{n=0}^{c} f_n)}{\partial x^\eta}, \dots, \frac{\partial^{l\eta}(\sum_{n=0}^{c} f_n)}{\partial x^{l\eta}}\right)\right]\right] \\ &-\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x, \sum_{n=0}^{c-1} f_n, \frac{\partial^\eta\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^\eta}, \dots, \frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1} f_n\right)}{\partial x^{l\eta}}\right)\right]\right] \end{aligned} \tag{17}$$

The truncated solutions of r- terms of Eqs. (8), (9) is established as $f \approx f_0 + f_1 + \dots + f_{r-1}$. For the convergence of DJM, we refer the reader to [16].

4 | ILLUSTRATIVE EXAMPLE

Take the following mathematical model

$$\frac{\partial^\zeta f}{\partial t^\zeta} = \left(\frac{\partial^\eta f}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta f}{\partial x^\eta}\right),\ t >, 0\zeta, \eta \in (0,1], \tag{18}$$

with the initial condition

$$u(x,0) = 3 + \frac{5}{2}E_\eta(x^\eta). \tag{19}$$

into account. Applying the Shehu transform to Eq. (18) leads to

$$\mathbb{S}\left[\frac{\partial^\zeta f}{\partial t^\zeta}\right] = \mathbb{S}\left[\left(\frac{\partial^\eta f}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta f}{\partial x^\eta}\right)\right].$$

The property (7) allow us to have the following

$$\mathbb{S}[f(x,t)] = \left(\frac{q}{p}\right) f(x,0) + \left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^\eta f}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta f}{\partial x^\eta}\right)\right]\right). \tag{20}$$

Carrying out the inverse Sumudu transform to Eq. (20) leads to

$$f(x,t) = \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right) f(x,0)\right] + \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^\eta f}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta f}{\partial x^\eta}\right)\right]\right)\right]$$

Utilizing the recurrence relation (17), we have

TABLE 1 Comparison of the exact solution with the 10-term truncated solutions by SVIM for various ζ and η for example 1.

	SVIM	Exact	SVIM	Exact	SVIM	Exact
t	ζ,η=0.9	ζ,η=0.9	ζ,η=0.95	ζ,η=0.95	ζ,η=1	ζ,η=1
	time= 0.1139s		time= 0.1173s		time= 0.1125s	
0	9.9292	9.9292	9.8664	9.8664	9.7957	9.7957
0.1	7.7178	8.0988	7.8855	8.0681	8.0344	8.0344
0.2	6.4308	6.7935	6.5797	6.7610	6.7296	6.7296
0.3	5.5702	5.8563	5.6615	5.8082	5.7629	5.7629
0.4	4.9704	4.9495	5.0029	4.3881	5.0468	5.0468
0.5	4.5413	4.5055	4.5245	4.3661	4.5163	4.5163
0.6	4.2280	4.2772	4.1736	3.9080	4.1233	4.1233
0.7	3.9958	4.0783	3.9139	3.9112	3.8322	3.8322
0.8	3.8225	3.8596	3.7204	3.9075	3.6166	3.6165
0.9	3.6965	3.6961	3.5763	3.5912	3.4571	3.4567
1	3.6187	3.5796	3.4712	3.4481	3.3398	3.3383

$$
\begin{aligned}
f_0 &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right) f(x,0)\right] = 3+\frac{5}{2}E_\eta(x^\eta)\\
f_1 &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^\eta f_0}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta f_0}{\partial x^\eta}\right)\right]\right)\right] = -\frac{15t^\zeta E_\eta(x^\eta)}{2\Gamma(\zeta+1)}\\
f_2 &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^\eta (f_0+f_1)}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta (f_0+f_1)}{\partial x^\eta}\right)\right]\right)\right]\\
&\quad -\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^\eta f_0}{\partial x^\eta}\right)^2 - f\left(\frac{\partial^\eta f_0}{\partial x^\eta}\right)\right]\right)\right] = \frac{45t^{2\zeta} E_\eta(x^\eta)}{2\Gamma(2\zeta+1)}\\
f_3 &= -\frac{135t^{3\zeta} E_\eta(x^\eta)}{2\Gamma(3\zeta+1)}\\
f_4 &= \frac{405t^{4\zeta} E_\eta(x^\eta)}{2\Gamma(4\zeta+1)}
\end{aligned}
$$

Finally, the solution of the problem (18)-(19) is obtained in series form as follows: $f(x,t) = f_0 + f_1 + f_2 + f_3 + \ldots = 3 + \left[\frac{5}{2}E_\zeta\left(-3t^\zeta\right)\right] E_\eta(x^\eta)$, which is the same exact solution obtained in [17].

Notice that the values of the solution for $\zeta = \eta = 1$ and exact solution are almost the same which implies that the method implemented in this study is one of the best one for the solution of space-time fractional differential equations of any order. The programming language is MATLAB 2016b. The computer used has an Intel (R) Core (TM) i3 CPU M 370.

5 | CONCLUSION

In this study we build numerical or analytical solutions of mathematical models including nonlinear space time FPDEs by means of STIM which is formed by the combination of DJM [15] and Shehu transform. It is proved that this approach is more convenient and effective for nonlinear FPDEs than the methods obtained by taking the combinations of Laplace transformation and homotopy, Sumudu or Adomian polynomials. This result is verified by the illustrative example.

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Author contributions

All authors contributed equally to this work.

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Conflict of interest

This work does not have any conflicts of interest.

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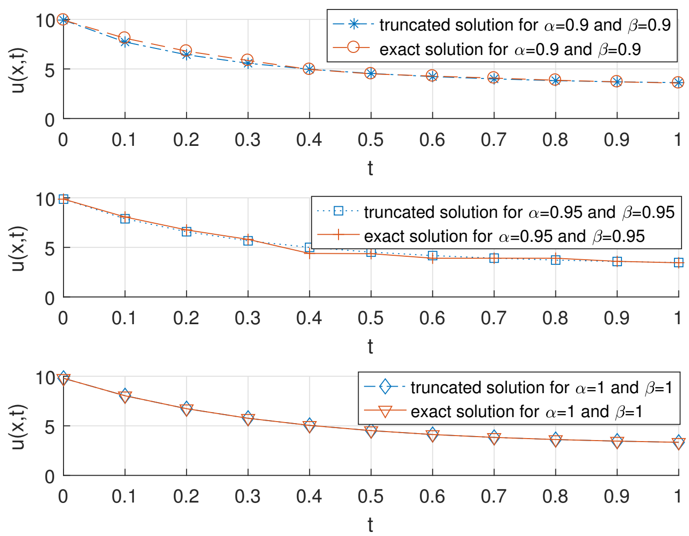

FIGURE 1 Figures of 10-term truncated solution and exact solution for different ζ and η at $x = 1$.