

1 **Strongly nonlinear effects on determining internal solitary wave parameters**
2 **from remote sensing signatures**

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10 **Key Points:**

- 11 • The relationship between the surface and internal characteristics of internal solitary
12 waves is established in laboratory experiments.
- 13 • A fully nonlinear model is used to determine wave parameters from surface features and
14 has been well verified.
- 15 • Stratification conditions differentiate the relationship between remote sensing signatures
16 and wave parameters in deep and shallow seas.

17

Abstract

19 The inversion of remote sensing signatures of internal solitary waves (ISWs) can retrieve
20 dynamic characteristics in the ocean interior. The ubiquitous large-amplitude ISWs limit the
21 weakly nonlinear methods commonly used to retrieve wave parameters. We establish the
22 relationship between surface features and internal characteristics of ISWs in laboratory
23 experiments through the correspondence of the remote sensing signatures and the surface
24 velocities of ISWs. The results show that the strong nonlinearity makes the solution of wave-
25 induced velocity inseparable, and ISW theories under the weakly nonlinear assumption are
26 inappropriate to describe strongly nonlinear ISWs from the surface. Therefore, the fully
27 nonlinear model Dubreil–Jacotin–Long equation is used in the retrievals and has been well
28 verified in both the laboratory and oceans. Mooring observations and the model show that
29 stratification conditions differentiate the relationship between remote sensing signatures and ISW
30 parameters in deep and shallow seas.

31

Plain Language Summary

33 Internal solitary waves (ISWs), as nonlinear internal waves, play an essential role in oceanic
34 human activities and ocean mixing. The surface current induced by ISWs can create rough and
35 smooth regions on the sea surface due to the modulated roughness, hence presenting alternating
36 bright and dark stripes in satellite images. Satellites can observe ISWs over a wide range via
37 surface manifestations, and the internal dynamics can be calculated from surface features using
38 retrieval methods. However, the availability of retrieval methods still needs to be verified, facing
39 the difficulty of matching mooring observations and satellite images of the same ISW in a short
40 time interval. According to the proportional relation of remote sensing signatures and wave-
41 induced velocities, this study establishes the relationship between surface features and internal
42 characteristics of ISWs in laboratory experiments. Different from the weakly nonlinear models
43 used in previous studies, a fully nonlinear model is used in the retrieval and has achieved good
44 results in the laboratory and oceans. The model and mooring observations show the critical role
45 of stratifications in the retrieval. This work provides a reliable dynamics model for the inversion
46 of remote sensing signatures of ISWs into characteristics in the ocean interior.

47 **1 Introduction**

48 Internal solitary waves (ISWs) are widespread in marginal seas (Jackson, 2007), and are
49 usually generated due to the nonlinear steepening of internal tides (Ramp et al., 2010). With
50 large amplitudes and strong currents, ISWs can damage ocean engineering and underwater
51 vehicles (Huang et al., 2016; Klymak et al., 2006; Osborne & Burch, 1980). The breaking of
52 ISWs during shoaling can cause enhanced mixing, change underwater acoustic transmission, and
53 affect nearshore ecosystems (Alford et al., 2015; Chiu et al., 2013; Moum et al., 2003; Wang et
54 al., 2007).

55 ISWs create convergence and divergence zones on the sea surface, resulting in rough and
56 smooth regions on the sea surface due to the modulated roughness (Lenain & Pizzo, 2021; Yue
57 et al., 2022) and presenting alternating bright and dark stripes in satellite images. The
58 characteristics of ISWs, such as wave crest length, propagation direction and propagation speed
59 can be acquired from the images (Jackson, 2007; Liu et al., 2014; Zhao et al., 2004), while other
60 essential characteristics such as amplitude and wavelength cannot be directly obtained.
61 Therefore, retrieval methods of ISW parameters based on physical models have been proposed,
62 which establish the relation between the remote sensing characteristics and other wave
63 parameters in the ocean interior. A variety of retrieval methods were established using different
64 ISW theories and wave-induced velocity calculation methods.

65 A retrieval method based on the Korteweg-de Vries (KdV) equation was first proposed
66 (Small et al., 1999; Zheng et al., 2001) and widely used for determining ISW parameters (Fan et
67 al., 2015; Gong et al., 2021; Hong et al., 2016; Phaniharam et al., 2020; Wang et al., 2022; Xie et
68 al., 2022). This method established the relation between the ISW distribution of stripes and their
69 wavelengths. Then, the waveform and amplitude could be determined with stratifications.
70 Retrieval methods using the Joseph–Kubota–Ko–Dobbs (JKKD) equation for finite depth (Pan et
71 al., 2007), the Benjamin–Ono (BO) equation for infinite depth (Chen et al., 2011; C. Wang et al.,
72 2019) and the extended KdV (eKdV) equation (Jia et al., 2019; Xue et al., 2013) were also
73 proposed to analyze satellite images similarly. These methods above were based on a weakly
74 nonlinear assumption, which assumes that the amplitude of ISWs is small compared with the
75 intrinsic vertical scale (Choi & Camassa, 1999). Under the weakly nonlinear assumption, the
76 solution of the wave-induced velocity is considered to be separable in the horizontal and vertical
77 directions.

78 However, strongly nonlinear ISWs that do not satisfy the weak nonlinearity assumption
79 account for a large proportion of in-situ oceanic observations. (Chang et al., 2021; Huang et al.,
80 2022; Stanton & Ostrovsky, 1998; Yang et al., 2021). Although weakly nonlinear theories have
81 been used outside their formal range of validity in some cases (Helfrich & Melville, 2006), for a
82 detailed description of the dynamics of large-amplitude ISWs observed in the real ocean,
83 theoretical models with strong nonlinearity are needed (Vlasenko et al., 2000). The Miyata–
84 Choi–Camassa (MCC) equation, a fully nonlinear theory of long waves was proposed and agreed
85 well with laboratory experiments (Camassa et al., 2006; Kodaira et al., 2016). Meanwhile,
86 without any assumptions, the fully nonlinear Dubriel–Jacotin–Long (DJL) equation has been
87 applied effectively in the laboratory and the ocean (Camassa et al., 2018; Chang et al., 2021;
88 Lien et al., 2014; Luzzatto-Fegiz & Helfrich, 2014). Different from the separable solution of
89 velocity under weakly nonlinear approximation, the solution cannot be separated with strong
90 nonlinearity. To date, strongly nonlinear effects on the retrievals of wave parameters have not
91 been fully considered in the following two aspects: (1) the calculation of wave-induced velocities
92 and (2) the applicability of ISW theories. The adequate modeling of ISW dynamics should be
93 determined in the retrieval, as mentioned by Romeiser and Graber (2015).

94 To address the difficulty of matching mooring observations and satellite images of the
95 same ISW in a short time interval, this paper establishes the relationship between surface
96 features and internal characteristics of ISWs in laboratory experiments. Then the strongly
97 nonlinear effects on retrieving ISW parameters are evaluated in terms of wave-induced velocities
98 and ISW theories. Finally, we assess the retrieval of wave parameters in different oceanic
99 environments.

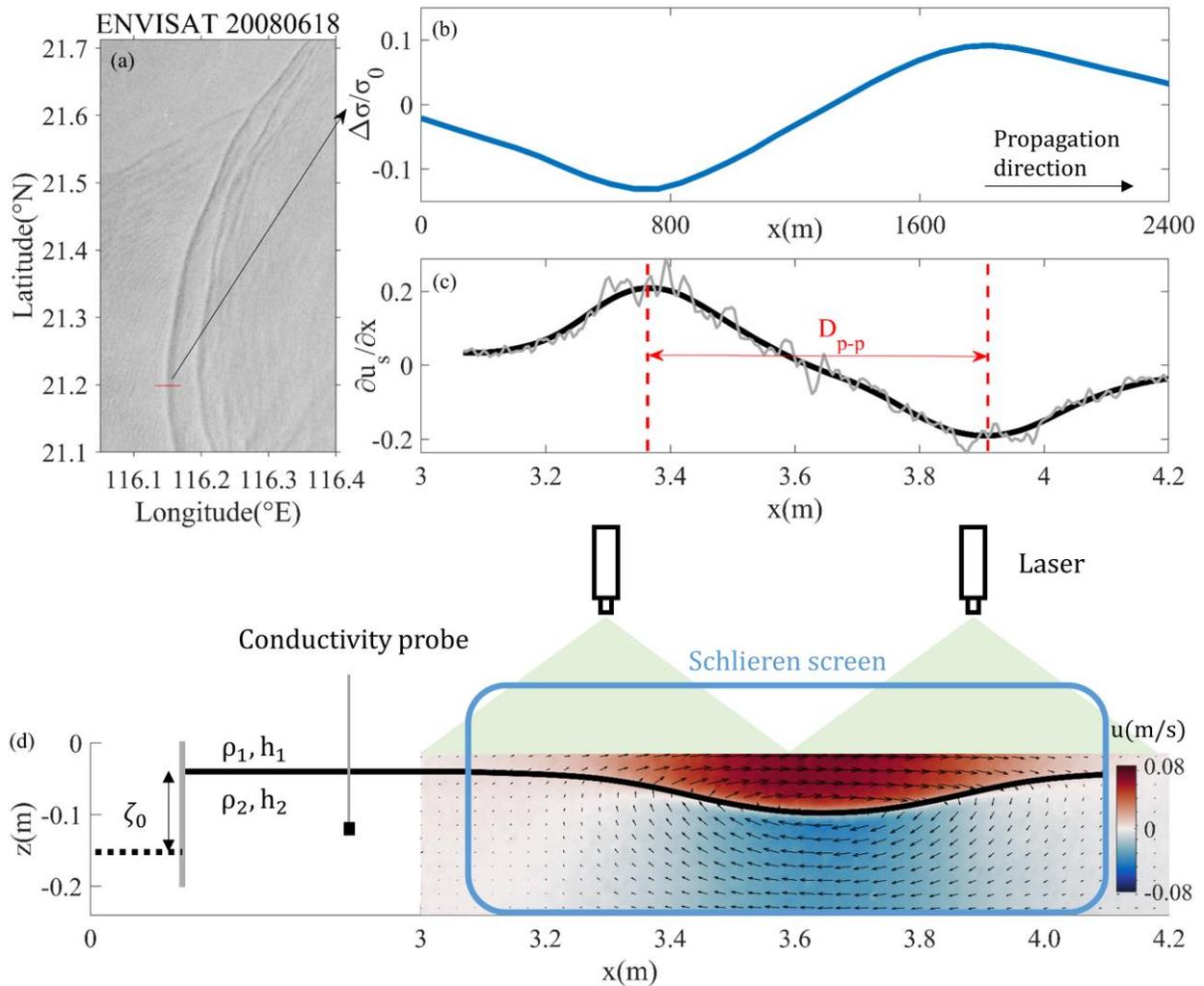
100 **2 Methodology**

101 2.1 Laboratory experiments and data processing

102 Experiments are conducted in the Key Laboratory of Physical Oceanography, Ocean
103 University of China. Two layers of fluid with thickness of h_1 and h_2 and densities of 1020kg/m^3
104 and 1040kg/m^3 are injected into the tank. The depth ratio of the lower and upper layers
105 h_2/h_1 vary from 3 to 10. The waves are generated by the lock–release method (Sutherland et al.,
106 2015), with nondimensional amplitudes η_0/h_1 ranging from 0.18 to 2.50; see Table S1 in the
107 supporting information for the detailed conditions. Two synchronous charge coupled device

108 (CCD) cameras with bandpass filters are set in front of the tank. The wave-induced velocity is
 109 measured by particle image velocity (PIV) (Thielicke & Stamhuis, 2014; S. Wang et al., 2019),
 110 and the waveform $\eta(x)$ is determined by identifying the depth where the maximal vertical
 111 density gradient is located using the synthetic Schlieren technique (Dalziel et al., 2007). The
 112 wavelength of ISWs is defined as

$$L_w = \frac{1}{2\eta_0} \int_{-\infty}^{\infty} \eta(x) dx. \quad (1)$$



114

115 **Figure 1.** Schematic of experiments. (a) ISWs captured by ENVISAT on June 18, 2008, UTC.

116 (b) Relative image intensity along the direction of wave propagation (the red line in subfigure a).

117 (c) The surface horizontal velocity divergence of the ISW in the laboratory experiment. The gray
 118 and black lines present the original and smoothed results, respectively. Red dashed lines indicate
 119 the horizontal position corresponding to the maximum and minimum values. (d) Schematic
 120 diagram of the experiment in the same case as subfigure c.

121 We consider the radar imaging theory which describes the influence of surface currents
 122 on remote sensing imaging (Alpers, 1985) as

$$123 \quad \frac{\Delta\sigma}{\sigma_0} \propto -\frac{\partial u_s}{\partial x}, \quad (2)$$

124 where $\Delta\sigma = \sigma - \sigma_0$ denotes the deviation of the normalized radar cross-section intensity from its
 125 mean value, u_s denotes the velocities of surface currents, and the x direction is defined as the
 126 direction of wave propagation. This means that the signal in satellite images (Figure 1a and 1b)
 127 can be calculated from the divergence of surface velocities induced by ISWs in an equilibrium
 128 and steady environment. The distance between the positive peak and the adjacent negative peak
 129 D_{p-p} in satellite images of ISWs is less affected by winds in most cases (Brandt et al., 1999;
 130 Xue et al., 2013). Hence the peak-to-peak distance D_{p-p} is chosen to characterize the surface
 131 features of ISWs in our experiments (Figure 1c and 1d), which is expressed as

$$132 \quad D_{p-p} = \left| x \Big|_{\frac{\partial u_s}{\partial x} = \min(\frac{\partial u_s}{\partial x})} - x \Big|_{\frac{\partial u_s}{\partial x} = \max(\frac{\partial u_s}{\partial x})} \right|. \quad (3)$$

133 2.2 Theories

134 The weakly nonlinear ISW models we used are the KdV, JKKD, BO, and eKdV
 135 equations. For strongly nonlinear ISWs models, we consider the MCC and DJL equations (see
 136 Text S1).

137 The wave-induced velocity used for quantifying surface divergence can be determined by
 138 the stream function (Stastna & Peltier, 2005) :

$$139 \quad \psi(x, z) = c\eta(x, z). \quad (4)$$

140 For a given waveform $\eta(x)$, the solution of the induced horizontal velocity under weakly
 141 nonlinear conditions has the following separable form, in which the vertical modes are
 142 independent of the horizontal location:

143
$$u(x, z) = \frac{\partial \psi(x, z)}{\partial z} = c_0 \eta(x) \frac{\partial \phi_b(z)}{\partial z}, \quad (5)$$

144 where c_0 is the linear phase speed and $\phi_b(z)$ is obtained by solving the Sturm–Liouville
145 equation, written as:

146
$$\left(\frac{d^2}{dz^2} + \frac{N_b^2(z)}{c_0^2} \right) \phi_b(z) = 0 \quad \phi_b(-H) = \phi_b(0) = 0, \quad (6)$$

147 where $N_b(z)$ is the given background buoyancy frequency, calculated by:

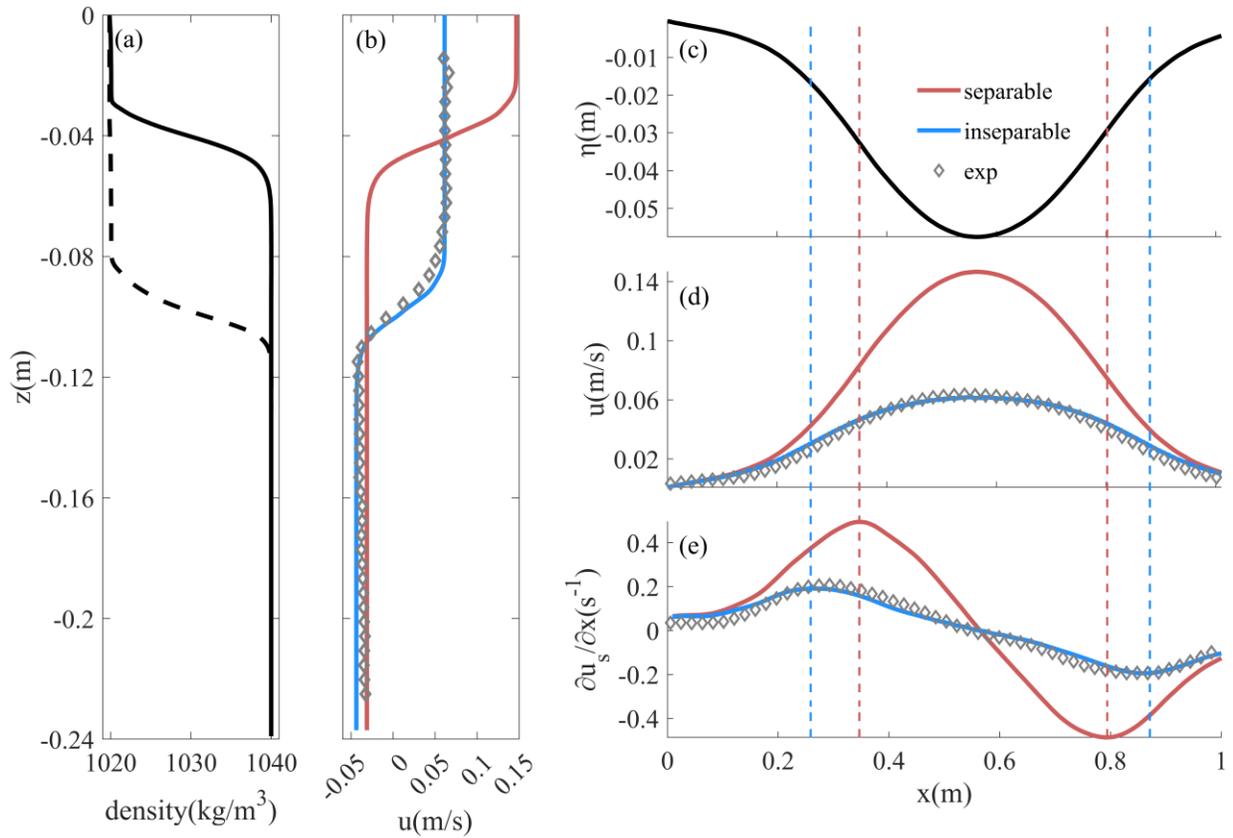
148
$$N_b^2(z) = -\frac{g}{\rho_0} \frac{d\rho(z)}{dz}, \quad (7)$$

149 where g is the gravitational acceleration, $\rho(z)$ is the density profile, and ρ_0 is the reference
150 density.

151 If strong nonlinearity is considered, the isopycnal displacement caused by ISWs should
152 be considered when calculating the vertical structure. Therefore, the vertical structure function
153 should depend on x as well (Apel, 2003). The solution of the induced horizontal velocity is
154 calculated in the following inseparable form:

155
$$u(x, z) = c\eta(x) \frac{\partial \phi_{wave}(x, z)}{\partial z}, \quad (8)$$

156 where $\phi_{wave}(x, z)$ is the vertical structure function of strongly nonlinear ISWs, calculated by
157 iterating the changed stratification and $\eta(x, z)$ with the initial value of $\eta(x)$ (see Text S2).

158 **3 Results**159 **3.1 Separability of the solution of wave-induced velocity**

160

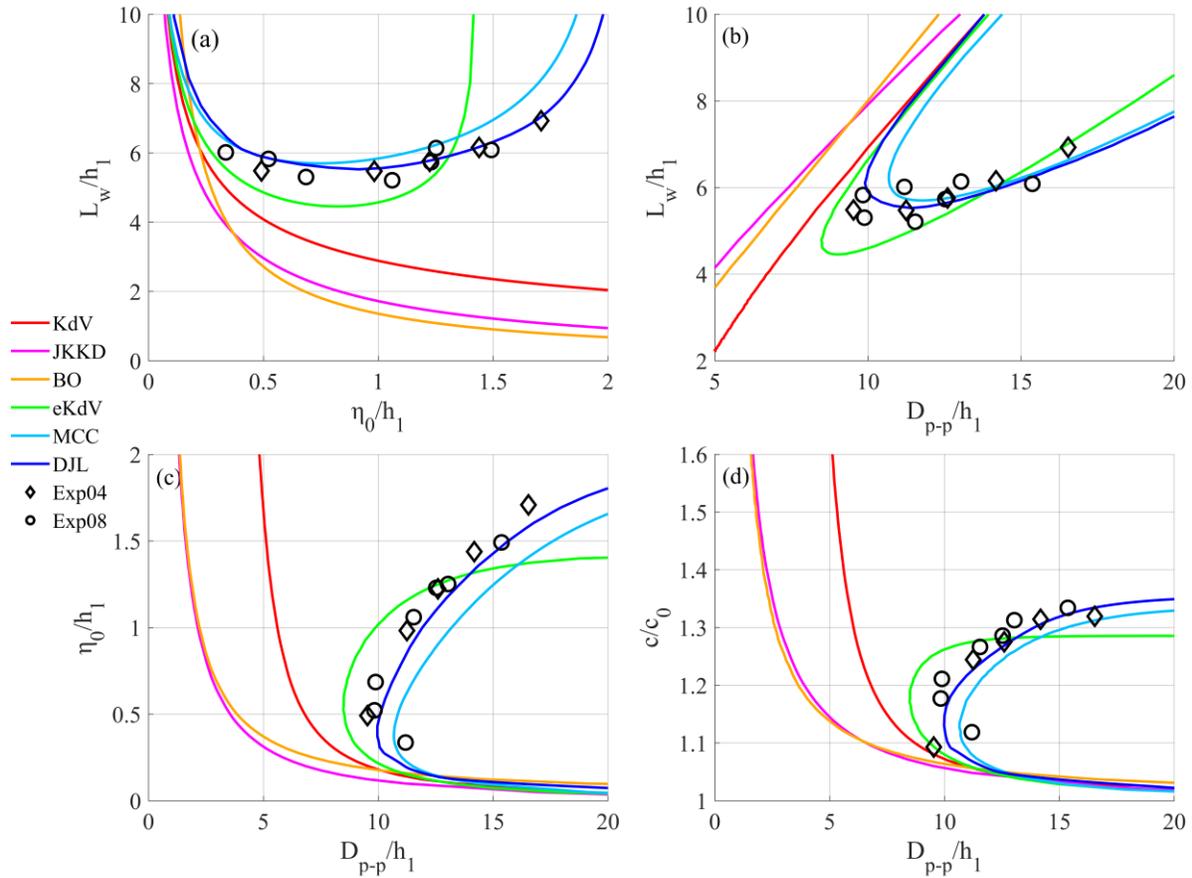
161 **Figure 2.** Experimental results with $h_2/h_1=5$, $\eta_0/h_1 = 1.44$. (a) The solid black line is the
 162 background density profile measured by the conductivity probe, and the dashed line is the
 163 density profile at the wave crest calculated by iteration. (b) Horizontal velocity profile at the
 164 crest, the red line, blue line, and diamond are the velocities in the separable form, inseparable
 165 form, and experiment, respectively. The legend is the same in (b, d, e). (c) Waveform. (d)
 166 Surface horizontal velocity, (e) Divergence of horizontal velocity at the surface. The red and blue
 167 dashed lines indicate the horizontal positions where the divergences reach peaks.

168 To visually show the influence of strong nonlinearity on the structure of wave-induced
 169 velocities, here we take a case of $h_2/h_1=5$ and $\eta_0/h_1 = 1.44$ as an example in which the
 170 amplitude is of the same magnitude as h_1 and shows significant nonlinearity. In terms of
 171 horizontal velocity magnitude and structure, there are differences between the velocity solutions
 172 in separable (Eq.5) and inseparable (Eq.8) forms (Figure 2b and 2d). The PIV measurements

173 match the velocity solution in inseparable form well in both the magnitude and the structure. In
174 the upper layer, the solution of velocity in separable form is more than twice the inseparable
175 form, while they are closer in the lower layer. This difference between observations and
176 theoretical solutions in separable form was also observed in the ocean, see Fig.13 of Rong et al.
177 (2023). This can be explained by the flow conservation that the influence of ISWs on the
178 stratification changed the thickness of the upper and lower layers. Figure 2e shows the distances
179 of divergence peaks, where the distance in inseparable form is larger. The differences reach a
180 maximum of 33% in the case of $\eta_0/h_1 = 2.20$. In previous studies, the relationship between
181 D_{p-p} and L_w can be obtained directly by calculating velocity in separable form, as $D_{p-p} = 1.32$
182 L_w in the KdV equation, and the ratio is independent of the amplitude. However, the
183 experimental results show that the ratio varies from 1.4 to 2.7 with increasing amplitude. The
184 results calculated in the inseparable form are consistent with the experiments (see Figure S1).
185 Therefore, the strong nonlinearity cannot be neglected in the velocity calculation, especially with
186 increasing wave amplitude. In Section 3.2, only the solution in inseparable form will be used to
187 calculate velocities.

188 3.2 Remote sensing characteristics of ISWs

189 The relationship between remote sensing characteristics and wave parameters will be
190 established in this section, and the applicability of each ISW theory will be evaluated by
191 comparing it with experimental results. Here we take two cases of $h_2/h_1=5$, $h_1=0.04$ m, and 0.08
192 m as examples. The results under the other stratifications are shown in Figures S2–S4. Our
193 experiments mainly focus on the ISWs under strong nonlinearity, and the small amplitude ISWs
194 have been fully proven to be consistent with the theoretical and experimental results in previous
195 studies (Ostrovsky & Stepanyants, 2005).



196

197 **Figure 3.** Theoretical and experimental results with $h_2/h_1 = 5$, (a) the variation of L_w with η_0 .
 198 (b), (c), and (d) are the relationships between D_{p-p} and wavelength, amplitude, and phase speed,
 199 respectively. The red, magenta, yellow, green, light blue, and dark blue lines represent the KdV,
 200 JKKD, BO, eKdV, MCC, and DJL equations respectively. The black diamond (Exp04) and the
 201 circle (Exp08) represent the experimental results for $h_1 = 0.04$ m and 0.08 m, respectively.

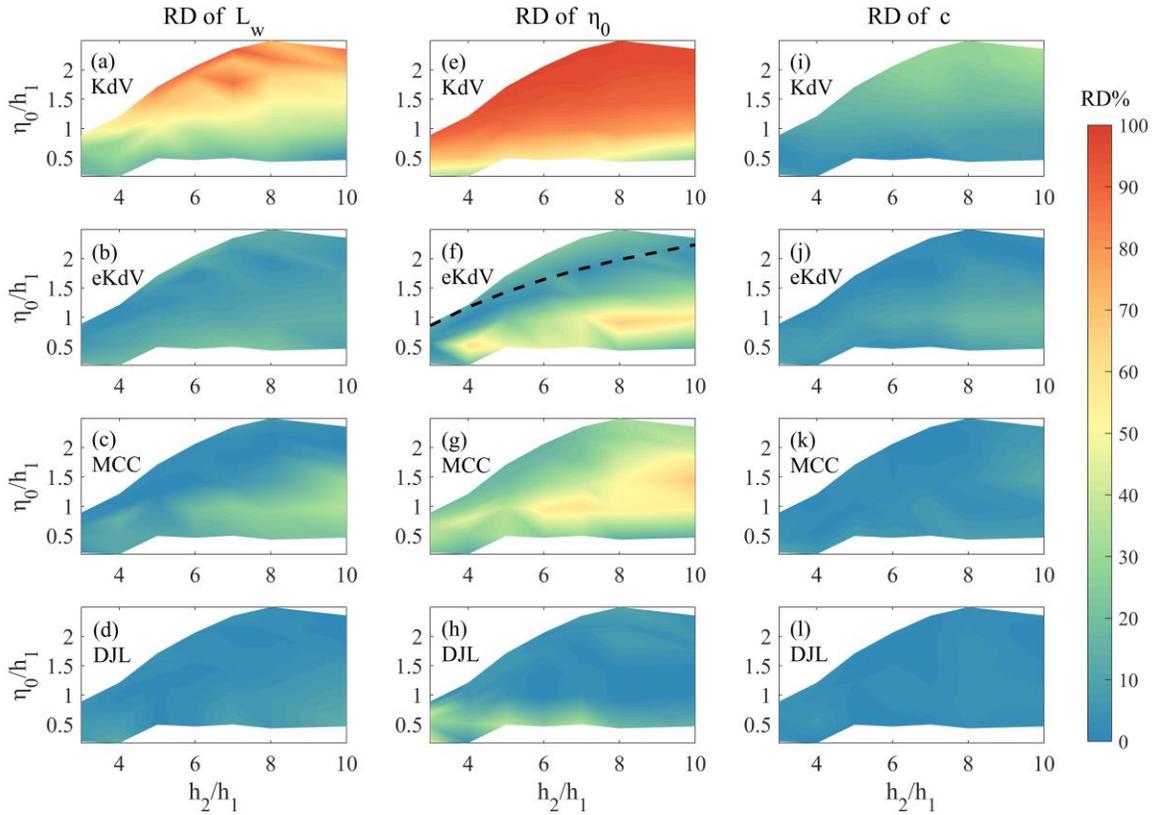
202 Figures 3b-3d show the variation in wave parameters with the peak-to-peak distance
 203 D_{p-p} . The results from the KdV, JKKD, and BO equations show a similar pattern, in which
 204 parameters change monotonically with D_{p-p} . However, for the eKdV, MCC, and DJL equations,
 205 the relationship between the wave parameters and D_{p-p} is no longer monotonic. Its typical
 206 feature is the existence of a turning point, which means that one D_{p-p} will correspond to two
 207 parameters, that is, the existence of double solutions. The selection method for double solutions
 208 can be determined by the properties of wave packets that the leading wave reaches the maximum
 209 amplitude (Xue et al., 2013). In addition, both the eKdV equation and the fully nonlinear

210 equations have a limiting amplitude, which varies with different stratifications (Cui et al., 2021),
 211 and the limiting amplitude of the eKdV equation is smaller than that of the MCC and DJL
 212 equations. The comparison between the theoretical and experimental results shows that it is
 213 feasible to describe the remote sensing characteristics of ISWs with theoretical models.

214 To quantitatively compare the differences between ISW theories and experimental results
 215 in retrievals, we define the relative deviation (RD) as follows

$$216 \quad \text{RD} = \left| \frac{\lambda_{theory} - \lambda_{exp}}{\lambda_{exp}} \right| * 100\%, \quad (9)$$

217 where λ_{theory} and λ_{exp} are any theoretical and experimental parameters under a specific D_{p-p} . If
 218 the D_{p-p} of some experiments are smaller than the theoretical minimum value, the theoretical
 219 turning point is used to calculate its RD. In Figure 4, the KdV equation is only applicable in
 220 relatively small amplitudes at any stratification. For the eKdV equation, significant RD is caused
 221 in the retrieval with dimensionless amplitude ranging between 0.5–1, corresponding to the
 222 turning point with smaller D_{p-p} in this range. In addition, its limiting amplitude restricts its
 223 application at large amplitudes. Therefore, the eKdV equation is applicable to ISWs with small
 224 or nearly limiting amplitudes. For the MCC equation, the turning point gradually shifts to the
 225 direction of D_{p-p} lengthening with increasing depth. The RD becomes larger with the gradual
 226 failure of the long wave assumption. The DJL equation has great applicability in all conditions.
 227 In laboratory conditions, the RD of wavelength and phase speed are less than 10% in retrievals,
 228 and the RD of small amplitude is approximately 30% which is mainly caused by the relative
 229 magnitude of variation, measurement errors, and dissipations, while RD of other amplitude is
 230 reduced to 10%.

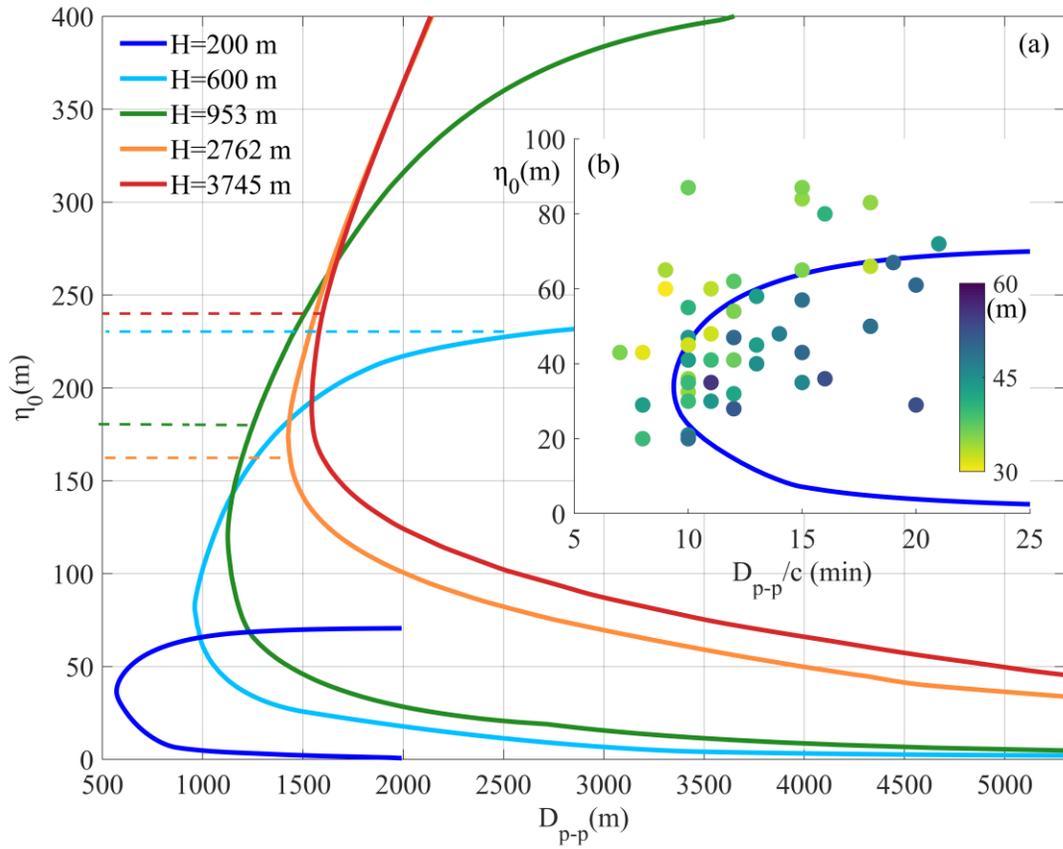


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232 **Figure 4.** RD between theoretical and experimental results in retrievals using D_{p-p} . (a)-(d) are RD
 233 of KdV, eKdV, MCC, and DJL equations in retrieving wavelength, (e)-(h) are RD in retrieving
 234 amplitude, and (i)-(l) are RD in retrieving phase speed. The color bar indicates RD% in each
 235 condition, and the black dashed line in (f) indicates the limiting amplitude.

236 4 Discussion

237 The above work explores the retrieval of wave parameters in a quasi-two-layer procedure
 238 under laboratory conditions. However, the complex environments in the ocean introduce
 239 difficulties and variety to retrieval. Therefore, the applicability of the established method is
 240 further tested with oceanic observations.



241

242 **Figure 5.** Relationship between D_{p-p} and amplitude of each location calculated by the DJL
 243 equation. (a) The dark line, light blue, green, orange and red lines are the results of mooring
 244 stations with depths of 200 m, 600 m, 953 m, 2762 m, and 3745 m. Dashed lines of
 245 corresponding colors indicate the maximum amplitude of the observed ISW at each station. (b)
 246 The dark blue line is the results of stations with a depth of 200m, the dots represent the measured
 247 ISWs, and their colors indicate the upper layer depth before the arrival of ISWs. The horizontal
 248 axis represents the time interval of peak-to-peak in the mooring observation.

249

250 The results at several locations with long-term mooring are shown in Figure 5. The 200 m
 251 depth is S5 during the Asian Seas International Acoustics Experiment (ASIAEX) (Duda et al.,
 252 2004), the 600 m depth station is LR1 of Chang et al. (2021), and the 953 m, 2762 m, and 3745
 253 m depth stations are M1, M6, and M10 of Huang et al. (2022). The yearly mean climatological
 254 WOA18 dataset is used in the DJL equation except for the location of 200 m. The stratification
 in 200 m depth is given by the average results of temperature profiles in observations before each

255 ISW arrives. In Figure 5b, 49 ISWs were observed during ASIAEX. The amplitude is defined as
 256 the maximum isotherm displacement. Using the time series of upward-looking ADCP with an
 257 interval of 1 min, we estimate D_{p-p} by

$$258 \quad D_{p-p} = c \left| t \Big|_{\frac{\partial u_s}{\partial t} = \min(\frac{\partial u_s}{\partial t})} - t \Big|_{\frac{\partial u_s}{\partial t} = \max(\frac{\partial u_s}{\partial t})} \right|, \quad (10)$$

259 and the axis is set to D_{p-p}/c due to the absence of the phase speed c . Different from the
 260 laboratory results, the relationship between surface features and amplitudes is relatively scattered
 261 in the ocean. The amplitude will be greatly different under one condition. Even in a short period
 262 of 15 days, the upper layer depth will change between 30 m and 60 m corresponding to the
 263 amplitude difference, and this short-period change may be caused by surface forcing (Font et al.,
 264 2022). As seen from the scatter, thicker and thinner upper layers usually correspond to smaller
 265 and larger amplitudes, respectively. The amplitudes are affected by this variation in stratification
 266 over a short period, as observed by Small et al. (1999) and Lien et al. (2014). The difficulty in
 267 obtaining real-time stratification will cause errors in retrieving the parameters of a specific ISW.
 268 Nevertheless, the DJL equation using an average stratification can still reveal the trend of wave
 269 parameters with remote sensing characteristics, which will play an important role in the
 270 statistical work of the properties of ISWs using satellite images.

271 The curves under the measured maximum amplitude show different patterns. The
 272 amplitudes decrease approximately with the increase in D_{p-p} in a deep sea of more than 2700 m.
 273 At approximately 1000 m, the amplitudes decrease monotonically except for a small area near
 274 the turning point. For the 600 m and 200 m stations, the amplitudes above the turning point will
 275 increase with increasing of D_{p-p} while those below the turning point will decrease. The
 276 selection rule in the retrieval has been mentioned in Section 4.2. Different from other stations,
 277 the maximum amplitude observed at 200 m exceeds the limitation calculated by averaged
 278 stratification, which may be caused by variations in the upper layer and the shoaling of ISWs.
 279 The relationship between the remote sensing characteristics and the wave parameters of ISWs is
 280 different at those stations. In fact, the calculation results of the DJL equation under several
 281 stratifications indicate that the different stratification characteristics in different areas are the
 282 main factors that cause the differences in the retrievals, such as shallow waters with quasi-two-
 283 layer stratifications and deep seas with a main thermocline that spans hundreds of meters.

284 Stratification plays a critical role in the retrieval of ISW parameters from satellite images.
285 In shallow seas, the climatological dataset cannot reflect the short-period variation in
286 stratification, and the retrieval of parameters of a specific ISW needs additional information to
287 improve. For the deep seas, the relatively stable stratification provides us with more possibilities
288 for accurate retrieval.

289 **5 Conclusions**

290 In this study, we establish the relationship between the surface and internal characteristics
291 of ISWs in dimensionless laboratory experiments. The strongly nonlinear effects of ISWs in the
292 retrieval of wave parameters are evaluated, and a fully nonlinear model is applied in oceans.
293 Strong nonlinearity causes the solution of the wave-induced velocity to be inseparable, and an
294 iterative method for calculating the velocity solution in inseparable form is proposed and fits
295 well with the experimental results. The difference in the retrieval between velocity solutions in
296 separable and inseparable forms reaches a maximum of 33% in the range of our experimental
297 parameters. ISW theories under weakly nonlinear assumptions have difficulty describing
298 strongly nonlinear ISWs from the surface. The fully nonlinear DJL equation is used in the
299 retrieval and compared with experimental results. The relative deviation of the retrieval of
300 wavelength, amplitude, and phase speed is less than 10% in laboratory experiments except in
301 small amplitude conditions.

302 The determination of ISW parameters from satellite images in oceans is significantly
303 affected by in-situ stratifications. The variation in stratification over a short period can bring
304 errors in retrieving the parameters of a specific ISW. However, the comparison of observations
305 and theory shows that the DJL equation using background stratifications can reveal reasonable
306 internal characteristics of ISWs from surface features. Stratification conditions make the
307 relationship between remote sensing signatures and ISW parameters differ in deep and shallow
308 seas. The amplitudes of ISWs decrease monotonically with increasing D_{p-p} in deep seas, but
309 double solutions should be considered in shallow seas.

310 This work provides a reliable hydrodynamics model for the inversion of remote sensing
311 signatures of ISWs into characteristics in the ocean interior. With the combination of the fully
312 nonlinear model and the satellite imaging mechanism, more underwater information can be
313 interpreted from remote sensing in further work.

314

315

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322

323 **Open Research**

324 The dataset of laboratory experiments is available at: <https://doi.org/10.5281/zenodo.7636122>.

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