# Accelerating subglacial hydrology for ice sheet models with deep learning methods

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**Key Points:**

- We develop a deep learning emulator to simulate evolving subglacial hydrology in response to meltwater input for ice sheet simulations.
- The emulator shows generalization capabilities, large computational savings, and can be used to force numerical ice sheet models.
- We demonstrate that machine learning has substantial potential in improving ice sheet models, through using information-rich data sets.

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## Abstract

Subglacial drainage networks regulate the response of ice sheet flow to surface meltwater input to the subglacial environment. Simulating subglacial hydrology evolution is critical to projecting ice sheet sensitivity to climate, and contribution to sea-level change. However, current numerical subglacial hydrology models are computationally expensive, and, consequently, evolving subglacial hydrology is neglected in large-scale ice sheet simulations. We present a deep learning emulator of a state-of-the-art subglacial hydrology model, trained at multiple Greenland glaciers. Our emulator performs strongly in both temporal ($R^2$ >0.99) and spatial ($R^2$ >0.96) generalization, offers high computational savings, and can be used to force numerical ice sheet models. This will enable century- and large-scale ice sheet model simulations, including interactions between ice flow and increased meltwater input to the subglacial environment. Generally, our work demonstrates that machine learning can further improve ice sheet models, reduce computational bottlenecks, and exploit information from high-fidelity models and novel observational platforms.



## Plain Language Summary

Meltwater at the surface of ice sheets can drain to the subglacial environment, lubricate the bed, and influence ice sheet flow. Complex numerical subglacial hydrology models represent the subglacial drainage system, but are too computationally expensive to be included in large-scale and long-term ice sheet simulations. Consequently, model predictions of future ice sheet contribution to sea-level rise ignore ice flow modulation by evolving subglacial hydrology. Here, we use deep learning to emulate a state-of-the-art subglacial hydrology model. The emulator can directly force large-scale ice sheet models to capture ice flow sensitivity to subglacial hydrology. The computational speed and accuracy of our emulator show the potential to use machine learning to efficiently incorporate previously neglected processes into ice sheet models.


## 1 Introduction

The Greenland ice sheet has experienced accelerating mass loss since the early 1990s (Otosaka et al., 2023). Ice loss has been driven by increasing surface melt (Fettweis et al., 2016) and accelerating ice flow into the ocean (King et al., 2020). These two processes are linked by surface meltwater drainage into the subglacial environment. The evolution of the subglacial drainage system in response to meltwater input determines the subglacial water pressure, which regulates the speed of ice sliding over the bed (Nienow et al., 2017). Observations have demonstrated a strong sensitivity of ice flow speed to meltwater supply to the bed on timescales ranging from days to months (Zwally et al., 2002; Shepherd et al., 2009; Smith et al., 2021). The subglacial drainage system modulates this sensitivity, and there is no simple relationship between meltwater forcing and ice flow speed, due to the complexities of subglacial hydrology (Bartholomew et al., 2011; van de Wal et al., 2015).

Subglacial hydrology models simulate the evolution of subglacial water pressure under different conditions of ice sheet geometry and meltwater input (Werder et al., 2013; de Fleurian et al., 2018). They represent the transient evolution of the subglacial drainage system, are highly complex, and require many parameters and substantial computational expense (Werder et al., 2013; Hoffman et al., 2016). When applied to individual Greenland glaciers, studies have shown that accurately representing the evolution of the subglacial hydrology system with such complex models is necessary to explain observed variability in ice flow (Hewitt, 2013; Hoffman et al., 2016; Ehrenfeucht et al., 2022). However, their computational expense prohibits long, ice sheet scale simulations. In contrast, simple first-order formulations of the growth and decay of subglacial water flux do exist, facilitating large-scale simulations (Kazmierczak et al., 2022). But these formulations

do not represent the different drainage components and assume a constant transmissivity. As such, they have critical limitations when considering high meltwater input scenarios, complex topographies, or reproducing realistic variability in subglacial hydrology and ice flow (de Fleurian et al., 2018).

Most subglacial hydrology models divide subglacial drainage into distributed and channelized systems (Schoof, 2010; Werder et al., 2013; de Fleurian et al., 2018). The former is typical of the early melt season, when the subglacial hydrology system is inefficient and water pressure increases strongly with meltwater supply. Later in the melt season, the system develops into a channelized system, efficiently evacuating meltwater, and causes water pressure to decrease with increasing meltwater input (Schoof, 2010; Bartholomew et al., 2011; Cowton et al., 2013). Ice flow variability is sensitive to the representation of, and transitions between different forms of drainage. Nevertheless, because of their computational expense, subglacial hydrology models are not included in model simulations at ice sheet scale (Goelzer et al., 2020; Seroussi et al., 2020). Consequently, current sea-level projections ignore a critical process regulating ice flow.

The climate modeling community has used machine learning techniques successfully to parameterize processes unresolved in coarse resolution global models (e.g., Brenowitz & Bretherton, 2018; Rasp et al., 2018). In particular, artificial neural networks (ANNs) are particularly powerful tools for parameterizing complex relationships between input and output variables, as they are capable of approximating any continuous function (the universal approximation theorem, Hornik et al., 1989). Furthermore, recent improvements in computational hardware, software, and optimization techniques have led to important ANN developments in multiple fields, including the Earth sciences (LeCun et al., 2015; Reichstein et al., 2019). Once trained, ANN models are computationally efficient, and ANNs have been used previously to emulate glacier flow models (Brinkerhoff et al., 2021; Jouvet et al., 2021). In this study, we use deep learning to enable representation of subglacial hydrology in large-scale ice sheet model simulations. Specifically, we develop an ANN emulator of the Glacier Drainage System model (GlaDS, Werder et al., 2013), an advanced and computationally expensive subglacial hydrology model. More generally, our work is a proof of concept for an important advancement in ice sheet modeling: we demonstrate that deep learning techniques can replace computationally-demanding or poorly constrained processes in large-scale ice sheet models.

## 2 Methods

We run GlaDS for 40 years at eight major Greenland glaciers (Petermann, Jakobshavn, Helheim, Kangerlussuaq, Humboldt, Koge Bugt, Russell, and Upernavik, see Fig. S1 for locations). The glacier geometries and ice flow velocities are taken from present-day observations (Joughin et al., 2017; Morlighem et al., 2017), and the meltwater runoff forcing from the 1970-2009 output of the diurnal Energy Balance Model (Krebs-Kanzow et al., 2020) (see Supporting Information). GlaDS simulates the evolution of the subglacial hydraulic potential, $\phi$, in time and space by representing both channelized and distributed drainage. Accurate representation of $\phi$ in ice sheet models is critical, as it directly determines the subglacial water pressure, $p_w$. In turn, $p_w$ determines the effective pressure at the ice-bed interface, $N$:

$$\begin{cases} N = p_{ice} - p_w \\ p_w = \phi - g\rho_w B \\ p_{ice} = g\rho_{ice} H_{ice} \end{cases}, \tag{1}$$

where $p_{ice}$ is ice pressure [Pa], $g$ is gravitational acceleration [m s$^{-2}$], $H_{ice}$ is thickness of the above-lying ice column [m], $B$ is bed elevation [m], and $\rho_w$ and $\rho_{ice}$ are water and ice density [kg m$^{-3}$], respectively. Critically, $N$ [Pa] is a key variable in basal sliding laws

for ice flow (Budd et al., 1984; Hewitt, 2013). We run GlaDS at a 2-hourly time step to preserve numerical stability.

Dynamics of $\phi$ are governed by the amount of surface meltwater draining through the ice sheet to the bed, followed by the routing of the water through the subglacial system. The goal of our ANN is to predict $\phi$ based on ice sheet state and meltwater runoff forcing, such that our $p_w$ emulation accounts for spatio-temporal evolution of subglacial hydrology. Specifically, our ANN uses as inputs ice thickness, ice velocities, bedrock topography, and meltwater runoff fields, as well as moulin locations where meltwater reaches the subglacial domain. All these variables are common variables, parameters, or inputs to typical ice sheet models. Similar to current subglacial hydrology models, our ANN is aimed at one-way coupling with ice sheet models, i.e., it needs to be run prior to the ice sheet model and the ANN output is subsequently used as a forcing to the ice sheet model. We discuss prospects for full two-way coupling in the *Discussion* section.

Our ANN is a convolutional neural network, based on the U-Net architecture (Ronneberger et al., 2015). Our ANN thus uses two-dimensional input fields, and outputs a two-dimensional $\phi$ field at any given time step (see Supporting Information). In total, the ANN has 259,953 trainable parameters that are optimized such that ANN predictions of $\phi$ match training targets with accuracy. We calibrate our ANN to the GlaDS 1975-2004 output at seven of the eight glaciers. We keep the last 5 years (2005-2009) of output at these seven glaciers as test data to evaluate the temporal generalization capabilities of the ANN. Furthermore, we keep all the output of the eighth glacier for test data in order to evaluate the ANN spatial generalization capabilities. All the GlaDS output test data were totally unseen by the deep learning algorithm or the authors during the calibration. The calibration data are separated into training data, used to optimize the ANN parameters, and validation data, used to optimize hyperparameters and to avoid overfitting (see Supporting Information). All the results presented in the next sections have been computed on the test data.

## 3 Results

### 3.1 Temporal Generalization Performance

To assess the ability of the ANN to reproduce GlaDS hydraulic potential ($\phi$) fields, we start by comparing their respective outputs at the 7 calibration glaciers over the last 5 years of simulations (2005-2009), which have not been used for the ANN training. Figures 1a and 1b show for one of the calibration glaciers (Helheim glacier) that the mean $\phi$ field of the ANN over these 5 test years reproduces the spatial patterns of GlaDS output well. In particular, the ANN captures the radial patterns of high $\phi$ values and variability centered at moulin locations, where meltwater runoff drains to the subglacial environment. The ANN performs well throughout the domain, as the root-mean-square error (RMSE) is mostly lower than 0.5 MPa (Fig. 1c). The temporal dynamics are also captured well, as $\phi$ time series show close correspondence between ANN and GlaDS outputs in most of the domain (Fig. 1d). The ANN captures the different ranges in seasonality and inter-annual variability of $\phi$. It also reproduces the asymmetry between early and late melt season, which is due to changing drainage efficiency (Nienow et al., 2017). In Figure 1d, the orange curve shows a time series selected within the high RMSE area (Fig. 1c). This example illustrates that the ANN still captures the temporal variability, but is biased low at this particular location.

Figure 1e shows the ANN performance over the 5 years of test data at all the seven calibration glaciers, demonstrating low bias and RMSE. Furthermore, the ANN explains >99% of the variance in GlaDS $\phi$ output, as quantified by the coefficient of determination ($R^2$). These results demonstrate that the ANN is able to predict $\phi$ with good accuracy over years, and thus meltwater input conditions, unseen during training.

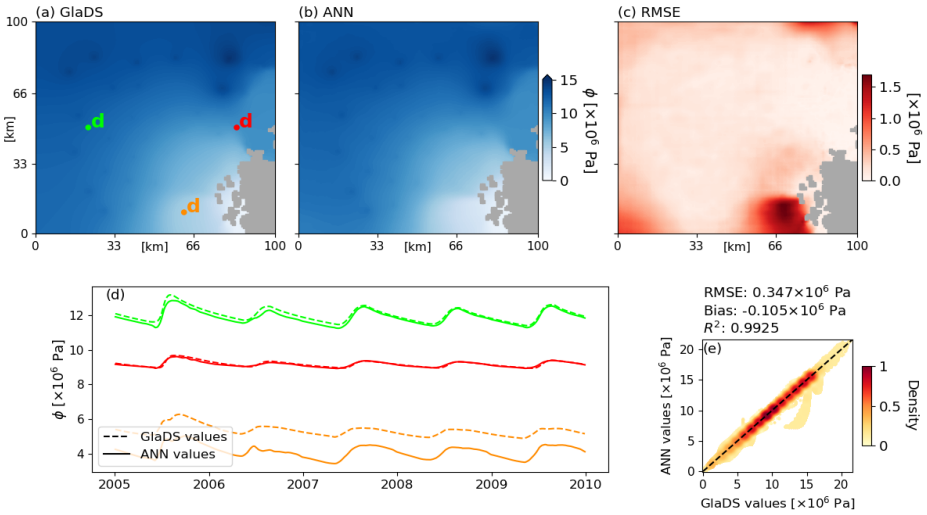


**Figure 1.** Temporal generalization performance of the ANN over the test period. Maps of mean 2005-2009 $\phi$ fields at one calibration glacier (Helheim, Fig. S1 for location) simulated by (a) GlaDS, and (b) the ANN. The Root-Mean-Square Error (RMSE) of the ANN with respect to GlaDS is shown in (c). Time series (d) of $\phi$ at specific grid points simulated by GlaDS (dashed lines) and the ANN (solid lines), with color-coded locations shown in (a). Performance statistics (e) of the ANN with respect to GlaDS evaluated at all the grid points of the 7 calibration glaciers for all the 2005-2009 time steps.

### 3.2 Spatial Generalization Performance

We now perform a similar evaluation, but at the test glacier (Upernavik), not included in the ANN calibration. This task presents a harder challenge, as both years of meltwater runoff and glaciological characteristics have not been used in calibration. The time-mean spatial patterns of $\phi$ at the test glacier are well-reproduced (Fig. 2a, 2b). The RMSE is low throughout the domain (Fig. 2c), except on a small portion near the glacier terminus. The time series shown in Figure 2d demonstrate that the ANN performance in reproducing temporal dynamics at seasonal and inter-annual scales are similar to its performance on calibration glaciers (compare with Fig. 1d). Here also, the orange lines in Fig. 2d show a time series corresponding to a low-performance location. Again, the ANN still captures temporal dynamics correctly, but has a consistent bias over the time series at this location. Finally, the metrics of performance for this test glacier show an RMSE lower than 0.56 MPa, a small positive bias of 0.24 MPa, and $R^2$ >0.96.

### 3.3 Sensitivity to number of calibration years

To calibrate the ANN, we have used 30 years (1975-2004) of GlaDS output at the calibration glaciers, and preserved 5 years (2005-2009) of output for testing (see Supporting Information). In this section, we investigate the sensitivity of ANN accuracy to the number of calibration years. Starting with all 30 years of available GlaDS output, we reduce the calibration data by 3-year increments, re-train the ANN at each increment, and evaluate performance metrics always on the 2005-2009 years.

Figure 3a shows the ratios in the $R^2$, RMSE and absolute bias metrics for each sensitivity experiment, with respect to the results computed when all the 30 calibration years are used in the ANN calibration. There is no decrease in accuracy for calibration data

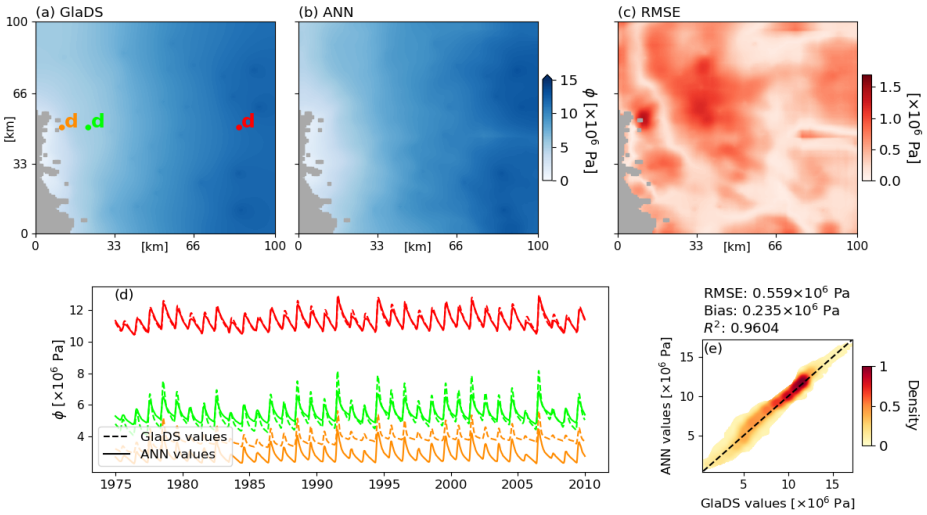


**Figure 2.** Spatial generalization performance of the ANN. Maps of mean 1975-2009 $\phi$ fields at the test glacier (Upernavik, Fig. S1 for location) simulated by (a) GlaDS, and (b) the ANN. The Root-Mean-Square Error (RMSE) of the ANN with respect to GlaDS is shown in (c). Time series (d) of $\phi$ at specific grid points simulated by GlaDS (dashed lines) and the ANN (solid lines), with color-coded locations shown in (a). Performance statistics (e) of the ANN with respect to GlaDS evaluated at all the grid points of the test glacier for all the 1975-2009 time steps.

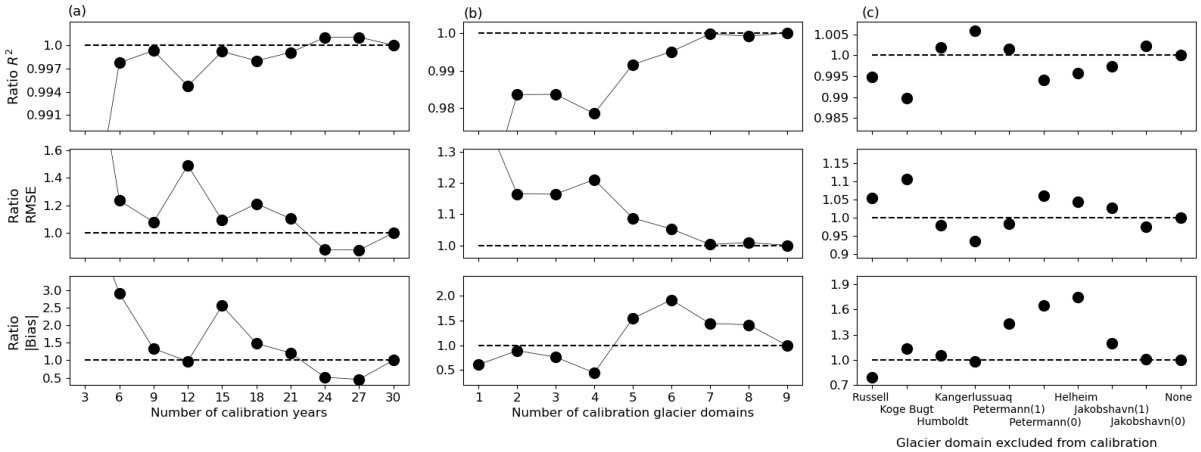


**Figure 3.** Sensitivity experiments (a) for number of years included in the calibration data, with evaluation on the 2005-2009 test years. Sensitivity experiments (b) for number of glacier domains included in the calibration data, with evaluation on the test Upernavik glacier domain. Leave-one-out experiments (c) in which each calibration glacier domain is separately excluded from the calibration data, with evaluation on the test Upernavik glacier domain. Dashed lines show no performance change with respect to the model calibrated with the default calibration data (30,9,'None' in a,b,c).

reduced down to 21 years. Small accuracy variations down to this limit are likely due to the inherent randomness in the training procedure of neural networks, through the parameter initialization method and the optimization algorithm. If calibration years are reduced to 18 years or less, performance metrics decrease and show more volatility. RMSE is increased by 21 % for 18 years, and by >250% for <6 years of calibration data.

### 3.4 Sensitivity to number of calibration glaciers

We also perform a sensitivity analysis to the number of glacier domains included in the calibration. Our initial calibration data set consists of seven glaciers, two of which have been split in two to match with the ANN domain input size, thus resulting in nine distinct glacier domains for calibration (see Supporting Information). Here, we sequentially drop one additional random glacier domain from the calibration data, and evaluate the ANN performance metrics always on the same test glacier (Upernavik).

Figure 3b shows the sensitivity of the performance metrics to the number of calibration glaciers. Accuracy decrease is minor when the calibration data are reduced to 8 and 7 domains, except for increases in the absolute bias. However, we observe a strong deterioration in accuracy for calibration data sets of 6 or fewer glacier domains. At 4 domains, the increase in RMSE reaches 21%. The ANN accuracy has levelled off for >6 glacier domains, showing that we have used sufficient calibration data.

Finally, we investigate if any single glacier domain is disproportionately important to the ANN accuracy at the test glacier. We repeat the ANN training with each one of the 9 calibration glacier domains left out of the calibration data, and then evaluate performance metrics on the test glacier (Fig. 3c). In agreement with the results from excluding only a single glacier domain shown above, changes in performance metrics are mostly small. For some of these leave-one-out experiments, performance metrics even improve slightly. The maximal increase in RMSE is 10.6 %, occurring when Koge Bugt glacier is left out (Fig. 3c). These results show that the ANN calibration is not excessively sensitive to any particular glacier. This verifies that the ANN does not predict at an out-of-sample glacier based only on the characteristics from the most similar glacier seen in training, but rather that it learns general relationships controlling $\phi$ patterns across different glaciological contexts.

### 3.5 Ice sheet model forcing

We now demonstrate that our ANN emulator can readily be used as forcing for an ice sheet model. We run the Ice-sheet and Sea-level System Model (Larour et al., 2012) at the test glacier of our data set: Upernavik (see Supporting Information for simulation details). We perform two 1975-2009 simulations: one forced with $p_w$ from GlaDS output (GlaDS-forced run), and the other with $p_w$ from the ANN (ANN-forced run). Except for the $p_w$ forcing, the two simulations share identical initial conditions, climatic forcing, and other parameterizations, thus isolating differences in ice thickness and ice flow caused by discrepancies in the ANN emulation of GlaDS.

Figures 4a and 4b show the change in ice thickness ($\Delta H_{ice}$) over the 35 years of simulations for the GlaDS-forced and ANN-forced runs, respectively. The patterns of $\Delta H_{ice}$ are very close between these two runs, which is confirmed at a grid point level (Fig. 4c, $R^2$=0.88). To quantify ice flow variability, we compute the temporal standard deviation in ice velocity ($\sigma(u_{ice})$) at each grid point. For this metric also, the GlaDS-forced and ANN-forced runs are in close agreement throughout the domain (Fig. 4d, 4e). However, $\sigma(u_{ice})$ is slightly underestimated by the ANN-forced run at the glacier terminus, especially at the northernmost branch where we observed the modest bias on $\phi$ of the ANN (Fig. 2c, 2d). At the two other branches, $\sigma(u_{ice})$ in our two simulations agree well. Throughout the domain, the ANN-forced run explains 78% of the variance in $\sigma(u_{ice})$ of the GlaDS-

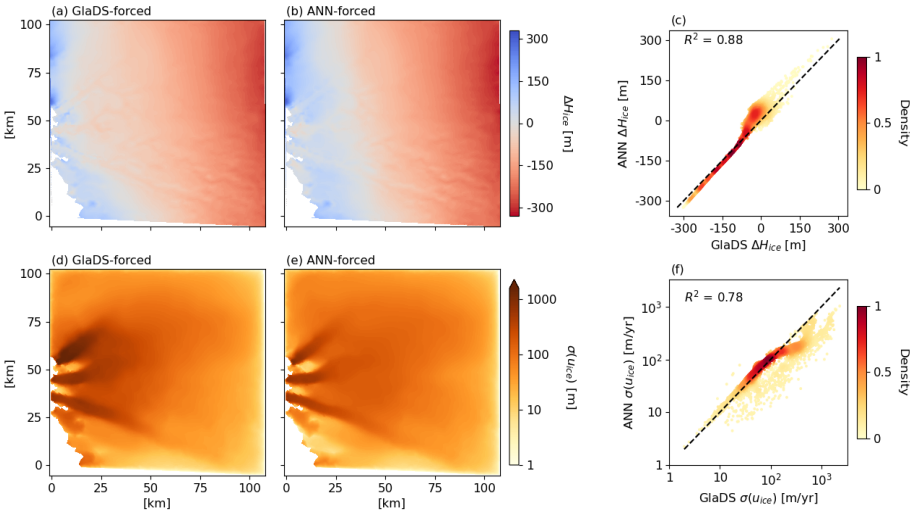


**Figure 4.** Results of 1975-2009 ice sheet model runs at the test glacier (Upernavik), with subglacial hydrology forcing from GlaDS (a,d) and from the ANN (b,e). Maps show ice thickness change (a,b, variable $\Delta H_{ice}$), and standard deviation in ice velocities (d,e, variable $\sigma\left(u_{ice}\right)$) over 1975-2009. Performance of the ANN-forced run with respect to the GlaDS-forced run in $\Delta H_{ice}$ (c) and $\sigma\left(u_{ice}\right)$ (f). Note the logarithmic colorbar in (d,e) and axes in (f).

forced run (evaluated on logarithmic scale). The previous sections demonstrated the high accuracy of the ANN in reproducing $\phi$ spatio-temporal evolution as modeled by GlaDS. This section shows that this accuracy translates into dynamical ice sheet model results being only weakly sensitive to substituting our ANN for GlaDS to prescribe the $p_w$ forcing.

In terms of computation, savings are large: simulating the 1975-2009 period in GlaDS over the Upernavik domain requires 859.9 CPU-hours, compared to 1.0 CPU-hour for predictions from our ANN on an identical core, i.e., close to $\mathcal{O}(10^3)$ faster. Finally, the 35-year GlaDS simulation required 268 times more CPU-hours than the ice sheet model simulation itself (3.2 CPU-hours), showing that subglacial hydrology models are a major computational bottleneck for large-scale ice sheet simulations.

## 4 Discussion

Our ANN produces realistic spatio-temporal patterns of subglacial hydraulic potential. It is skillful at temporal and spatial generalization on out-of-sample cases, when trained on as few as seven glacier domains and two decades of data. We do find small discrepancies between the ANN and GlaDS $\phi$ outputs, typically ranging between 0.2 and 1.5 MPa. Such values are smaller than discrepancies between subglacial hydrology models calculated in a recent intercomparison study (de Fleurian et al., 2018). Note that this comparison is not exact, because the intercomparisons used idealized configurations, whereas we use realistic Greenland glacier configurations. Still, because subglacial hydrology models are themselves an imperfect representation of real subglacial hydrology, the ANN output falling within typical inter-model spread reinforces our confidence that the ANN performs similarly to state-of-the-art numerical models.

Despite the demonstrated generalization capabilities of our ANN, we emphasize that deep learning models are prone to large errors, and possibly implausible behavior, when

used to extrapolate beyond their range of training conditions (Rasp et al., 2018; Reichstein et al., 2019). Training data should encompass the range of meltwater runoff and glaciological conditions that will be targeted for predictions of the subglacial hydrology deep learning model. For future Greenland ice sheet projections, training should include high-runoff forcing, as surface melting is predicted to increase (Fettweis et al., 2013). We have verified the quality of our ANN training through sensitivity analyses, demonstrating that calibration data are sufficient, and that the ANN does not overfit but has learned general spatio-temporal relationships inherent to subglacial hydrology.

The ANN presented in this study, and machine learning techniques more generally, provide solutions to the extreme computational expense of running subglacial hydrology models in realistic ice sheet simulations. In addition to subglacial hydrology, machine learning techniques could also potentially replace other inaccurate parameterizations of ice sheet processes, where sufficient observations and/or high-fidelity model output exist to use as training data. For example, the physics of iceberg calving remain challenging to simulate, but capturing observed temporal dynamics of calving rates could be the target of machine learning parameterizations. As another example, such parameterizations can aim to represent ice sheet surface mass balance at fine scales without the need for expensive climate model downscaling, as has already been demonstrated for Alpine glaciers (Bolibar et al., 2020) and for the Antarctic Peninsula (van der Meer et al., 2023).

Observations of subglacial water pressure are scarce, especially when considering the large data requirements for deep learning. Thus, our emulator has been calibrated exclusively with output from high-fidelity models, which may themselves be biased. The value of observations could be exploited through pre-training on model output followed by fine-tuning on existing, spatio-temporally sparse observations (e.g., Rasp & Thuerey, 2020). In addition, there are other possible future avenues for improving this deep learning emulator. Associating the convolutional nature of our ANN with recurrent neural networks would allow to simulate temporal dependencies explicitly, in addition to spatial patterns. Temporal dependencies are here accounted for in an ad-hoc manner through our processing of inputs (see Supporting Information). Also, here the coupling of the ANN and the ice sheet model is one-way; the ANN is run first, and its output used as forcing to the ice sheet model. This approach allows the subglacial hydrology emulator to be used directly with any ice sheet model. Tight two-way coupling would capture feedback processes between subglacial hydrology and changes in ice sheet geometry and velocities, but requires implementation of the ANN within the source code of an ice sheet model. The lack of deep learning libraries in low-level languages, which are the basis of most modern ice sheet and climate model architectures, makes such implementation challenging (Partee et al., 2022). Recent development of new ice sheet models within high-level languages (e.g., Shapero et al., 2021) hold promise for better integration of machine learning directly into ice sheet models.

## 5 Conclusion

Our study demonstrates that deep learning techniques enable simulation of subglacial hydrology for ice sheet model forcing. Our emulator reproduces output of a state-of-the-art subglacial hydrology model with great fidelity, strong generalization skills, and $\mathcal{O}(10^3)$ savings in computation time. This advance has the potential to enable coupled simulations of ice sheet flow and evolving subglacial hydrology over entire ice sheets on centennial and longer time scales. Our work also demonstrates how machine learning techniques can be adopted in the ice sheet modeling community to resolve current issues related to knowledge gaps and computational bottlenecks. This general methodology is not limited to emulating subglacial hydrology models, but can potentially improve the representation of many other ice sheet model processes. Recent advances in computa-

tional capabilities and machine learning will, in parallel with traditional ice sheet model development, bring key improvements in predictions of ice sheet response to climate change.

## 6 Open Research

Model code is openly available at: https://doi.org/10.5281/zenodo.8006962

The code includes all scripts to run GlaDS, to process data, to train the ANN, and to predict with the ANN. The input files, hydrology model results, trained ANN parameter files, and final ANN predictions at the 8 glaciers of this study are included. Detailed data and code descriptions are provided.

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