Supporting Information for “Modeling Multiphase Flow Within and Around Deformable Porous Materials: A Darcy-Brinkman-Biot Approach”

Francisco J. Carrillo[1], Ian. C. Bourg[2,3]

[1]Department of Chemical and Biological Engineering, Princeton University, Princeton, NJ, USA

[2]Department of Civil and Environmental Engineering, Princeton University, Princeton, NJ, USA

[3]Princeton Environmental Institute, Princeton University, Princeton, NJ, USA

Contents of this file

1. Text S1 to S2
2. Figures S1 to S4

Introduction

Here we present the simulation and semi-analytical solution to the pressure behaviour of an oscillating porelastic core. We also present alternative representations of Figures 5 and 6 from the main manuscript.

Text S1. Pressure Oscillation in Poroelastic Core

This additional verification quantifies the effects of the seismic stimulation of a poroelastic core saturated with water and trichloroethene (TCE). Our simulations follow the experimental and numerical set up described in Lo, Sposito, and Huang (2012), where a horizontal one-dimensional sand core (0.3 m long, 30 $\times$ 1 grid cells, $\phi_f = 0.5$, $\alpha_w = 0.9$, $k_0 = 1.1 \times 10^{-11}$ m^2) is subjected to constant uniaxial compression and oscillatory pore pressure variations imposed by time-dependent boundary conditions (Figure S1). In this case, flow is allowed through both boundaries, which results in a system that continuously undergoes a relaxation-compression cycle. The ensuing cyclical change in the core's fluid content as a function of time can be described by a semi-analytical solution first derived in Lo et al. (2012) and reproduced in the next section.

For our matching simulations, the porous structure's Young's modulus was set to $E =$ 53 MPa and its Poisson ratio to $\nu = 0.32$. Here, water density was $\rho_w = 1000$ kg/m^3, water viscosity was 1 cp, TCE density was $\rho_{TCE} = 1480$ kg/m^3, and TCE viscosity was $\mu_{TCE} = 0.57$ cp. Furthermore, the pressure at the left boundary was held at $p = 1$ kPa while the pressure at the right boundary was set by $p = p_0 sin\,(2\pi f t)$, with $p_0 = 1$ - 2 MPa and $f = 35$ - 70 Hz. Lastly, the core was uniaxially compressed through a constant stress of 1 kPa applied at both boundaries. A comparison between our numerical solutions and Lo's semi-analytical solution is presented in Figure S1, yielding excellent agreement for all tested cases.

Lastly, we note that the Multiphase DBB formulation should be able to describe 'Slow" Biot pressure waves caused by the relative motion of the solid and fluid phases which occurs at much higher frequencies than the ones simulated here (i.e. 10 MHz). However, capturing these effects and modelling "Fast/Compressional" pressure waves would require the implementation of a pressure-velocity coupling algorithm that allows for compressible flow (Lo et al., 2012). Such an endeavour is outside the scope of this paper.

Text S2. Semi-Analytic Solution for the Seismic Stimulation of a Poroelastic Core

Here we present the analytical solution used to describe the system in the previous section. Given a Biot coefficient of unity and incompressible fluids, the fractional change in an oscillating poroelastic core's fluid content Ω as a function of time t is given by (Lo et al., 2012)

$$\Omega\left(t\right) = -a_1\upsilon + a_2\alpha_w p_a + 0.5\left(a_2 p_0 sin(\omega t) - a_2\alpha_w p_a\right) + \sum_{n=1}^{\infty} A$$

$$A = \left(n\pi\right)^{-2} 2cos\left(n\pi\right)\, a_2 p_0 \left(sin\left(\omega t\right) + \frac{\omega_n^2 sin\left(\omega t + \delta_n\right)}{\left(\left(\omega^2 - \omega_n^2\right)^2 + D^2\omega^2\right)^{0.5}}\right)\left(1 - cos\left(n\pi\right)\ \right)$$

where υ is the uniaxial confining pressure, p_a is the fixed pressure at the left boundary, p_0 is the amplitude of the oscillating pressure at the right boundary, and $\omega = 2\pi f$ is the angular frequency of the pressure variation. The summation terms ω_n and $sin\left(\delta_n\right)$ are defined as

$$\omega_n = \left(\frac{Cn\pi}{Length}\right)^2$$

$$sin\left(\delta_n\right) \ = \frac{D\omega}{\left(\left(\omega^2 - \omega_n^2\right)^2 + D^2\omega^2\right)^{0.5}}$$

$$cos\left(\delta_n\right) \ = \frac{\omega^2 - \omega_n^2}{\left(\left(\omega^2 - \omega_n^2\right)^2 + D^2\omega^2\right)^{0.5}}$$

Furthermore, the dissipation constant D, the wave speed C, and the compressibility constants a_1 and a_2 are defined as follows

$$D = \frac{1}{k_0}\ \frac{1}{\frac{T}{\phi_f}\left(\frac{\rho_w M_w}{\alpha_w} + \frac{\rho_n M_n}{\alpha_n}\right) - \left(\rho_w M_w + \rho_n M_n\right)}$$

$$C^2 = \left(K_b + \frac{4}{3}G\right) \frac{M}{\frac{T}{\phi_f}\left(\frac{\rho_w M_w}{\alpha_w} + \frac{\rho_n M_n}{\alpha_n}\right) - \left(\rho_w M_w + \rho_n M_n\right)}$$

$$a_1 = (3K_b)^{-1}$$

$$a_2 = K_b^{-1}$$

where $T = 0.5\left(1 + \phi_f^{-1}\right)$ is the tortuosity, K_b is the bulk modulus of the solid matrix, G is the shear modulus of the solid matrix, and the rest of the variables are defined as in the main manuscript. The infinite sum was calculated through a python script, where it was truncated at the point where the last sum term represented 0.01% of the previous term.

References

Barros-Galvis, N., Fernando Samaniego, V., & Cinco-Ley, H. (2017). Fluid dynamics in naturally fractured tectonic reservoirs. *Journal of Petroleum Exploration and Production Technology*, *8*(1). Retrieved from `http://www.` doi: 10.1007/s13202-017-0320-8

Lo, W. C., Sposito, G., & Huang, Y. H. (2012). Modeling seismic stimulation: Enhanced non-aqueous fluid extraction from saturated porous media under pore-pressure pulsing at low frequencies. *Journal of Applied Geophysics*, *78*, 77–84. doi: 10.1016/j.jappgeo.2011.06.027

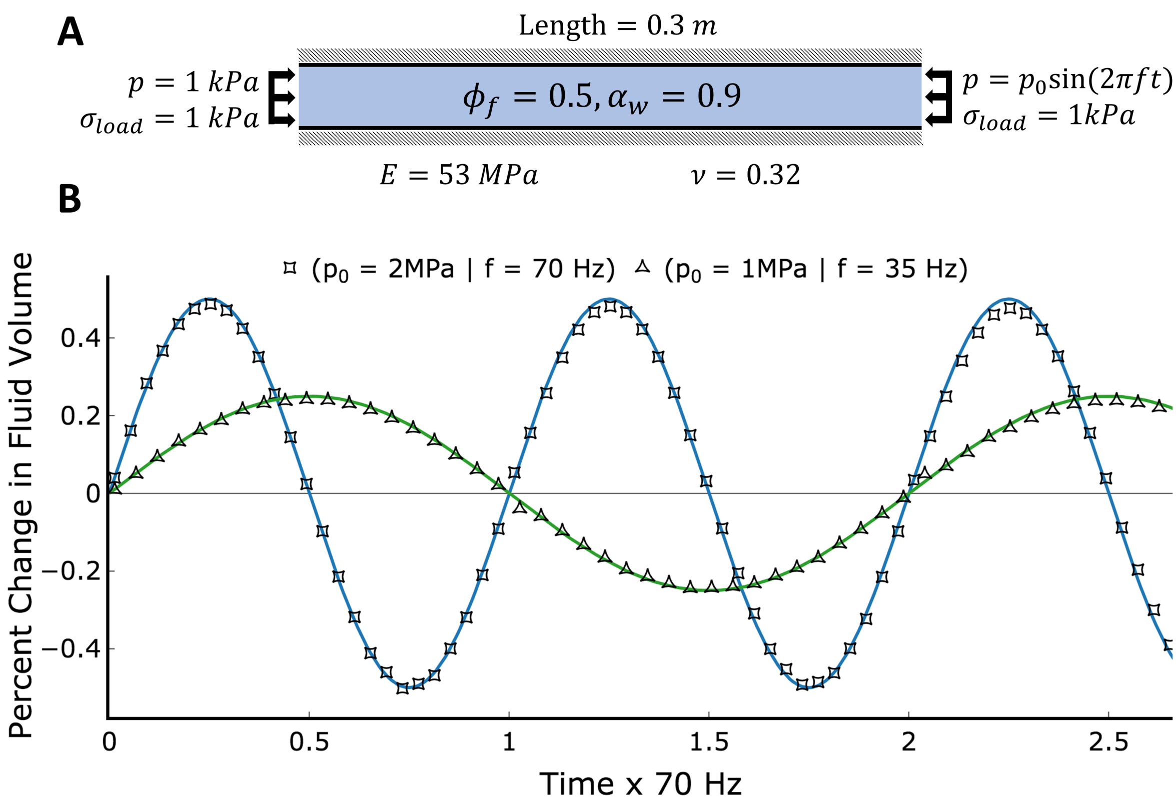

Figure S1. Change in fluid content of an oscillating poroelastic core. (A) Simulation setup. (B) Semi-analytical (solid lines) and numerical solutions (symbols) for the percent change in the core's fluid volume as a function of time.

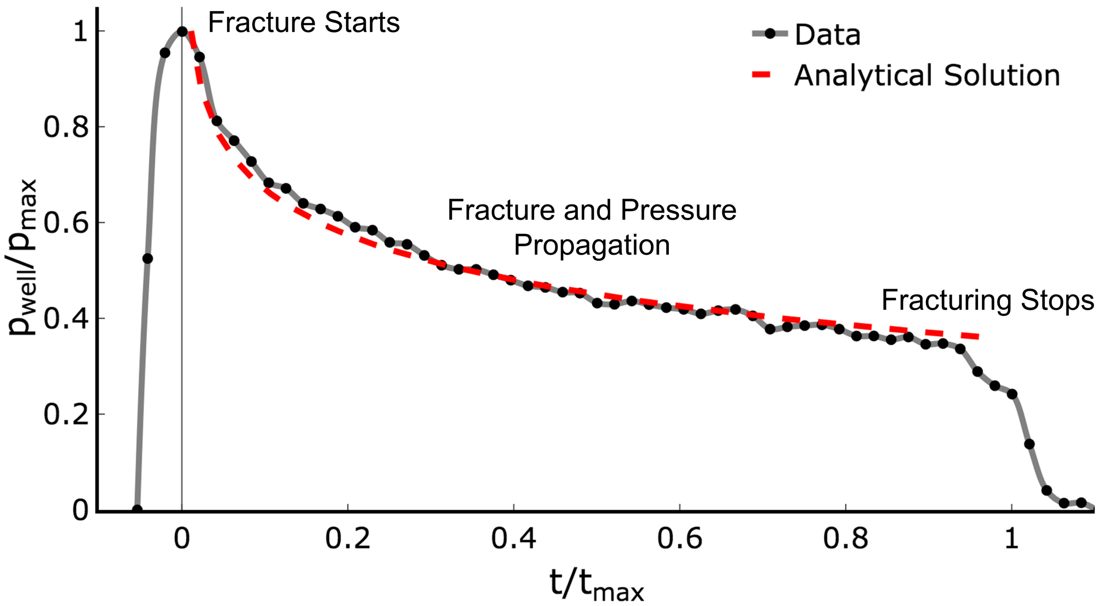

Figure S2. Wellbore pressure evolution during fluid-induced fracturing of low permeability rocks. Here we show how we sync the fracturing analytical solution shown in (Barros-Galvis et al., 2017) together with raw numerical data through non-dimensionalization of time.

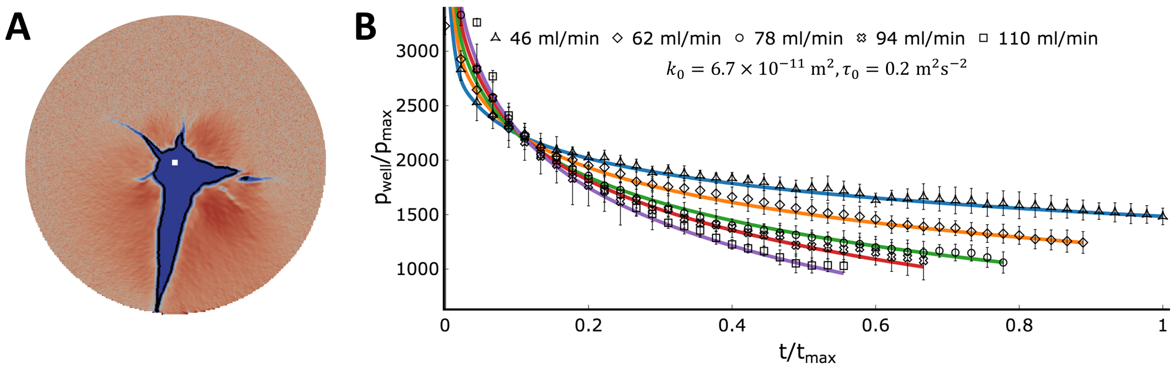

Figure S3. Alternative to Figure 6 depicting dimensionalized wellbore pressure as a function of injection rate and time. (A) A fractured system, where the thin black line represents the position of the advancing glycerin-air interface. (B) The wellbore pressure as a function of time for different flow rates and different yield stress-permeability pairs. Solid curves represent analytical solutions, while symbols represent the simulated data points. The color scheme in A is the same as in Figure 4 in the main manuscript.

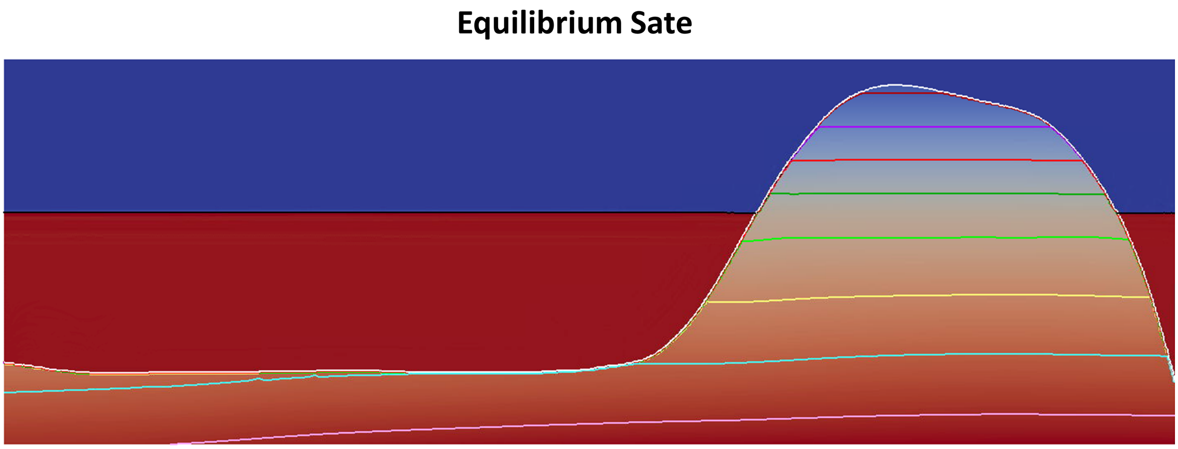

Figure S4. Equilibrium state of the coastal barrier case shown in Figure 8 within the main manuscript. The color scheme and simulation setup is the same as in said figure.