Supporting Information for "Scaling laws in Aeolian sand transport under low sand availability"

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Introduction

In this Supplemental Material, we briefly review the features of the Discrete-Element-Method referred to in the main document, including the complete set of the equations of motion, the details of the numerical integration of these equations, and the models of particle-particle interactions adopted in the simulations of Aeolian sand transport. Furthermore, we present the results of our numerical simulations performed to verify our model, the vertical profiles of the wind velocity and grain-borne shear stress during steady-state transport, and the behavior of the transport layer thickness as a function of the thickness of mobile sand layer, as mentioned in the main document.

S1 Discrete-Element-Method

In the Discrete-Element-Method, the equations of motion are solved for every particle in the system under consideration of the main forces acting on them. These forces are, in the process of non-suspended Aeolian transport of cohesionless particles, the drag force, the inter-particle contact forces and the gravitational force.

S1.1 Equations of motion and contact force model for the sand particles

The equation of translational motion for a particle of mass m_i at position $\mathbf{r}_i$ reads,

$$m_i\ddot{\mathbf{r}}_i = \mathbf{F}_i^{\mathrm{d}} + m_i\mathbf{g} + \sum_{\substack{1\leq j\leq N_{\mathrm{p}}\\ j\neq i}} \mathbf{F}_{ij}^{\mathrm{c}} \tag{1}$$

where $\mathbf{F}_i^{\mathrm{d}}$ is the drag force on particle i, computed with the model described in the main document, $\mathbf{g}$ is gravity, N_{p} is the number of particles in the system, j denotes the index of a neighbouring particle that is in contact with particle i, and $\mathbf{F}_{ij}^{\mathrm{c}}$ denotes the contact force exerted by particle j on i (with $\mathbf{F}_{ij}^{\mathrm{c}} = -\mathbf{F}_{ji}^{\mathrm{c}}$).

Contact between particles j and i occurs with their center-to-center distance is smaller than the sum of their radii, i.e., the contact force acts only if the particles overlap. To model the

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contact force, the following equation is used to define the overlap,

$$\delta_{ij,\mathrm{n}} = \min\left\{0, \frac{1}{2}\left[d_i + d_j\right] - (\mathbf{r}_i - \mathbf{r}_j)\cdot\mathbf{e}_{ij,\mathrm{n}}\right\} \tag{2}$$

where d_i and d_j are the diameters of particles i and j, respectively, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, with $\boldsymbol{r}_j$ standing for the position of particle j, and $\mathbf{e}_{ij,\mathrm{n}} = \mathbf{r}_{ij}/r_{ij}$ denotes the normal unit vector pointing from the center of particle j to the center of particle i, with $r_{ij} = |\mathbf{r}_{ij}|$.

There are various contact force models for application in DEM simulations, and the modelling of these forces is still an active matter of research (Cundall & Strack, 1979; Schäfer et al., 1996; Brilliantov et al., 1996; Silbert et al., 2001; Di Renzo & Di Maio, 2004; Pöschel & Schwager, 2005; Kruggel-Emden et al., 2007; Luding, 2008; Machado et al., 2012; Parteli et al., 2014; Fan et al., 2017; Schmidt et al., 2020; Santos et al., 2020). In our simulations, we adopt the linear spring-dashpot model, because this model has been employed in previous simulations of wind-blown sand that reproduced the scaling laws associated with Aeolian transport over fully erodible beds (Carneiro et al., 2011, 2013; Durán et al., 2012; Comola et al., 2019).

Specifically, $\mathbf{F}^{\mathrm{c}}_{ij}$ can be described as the sum of a normal component, $\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{n}}$, and a tangential component, $\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{t}}$. Each of these components encodes an elastic term and a dissipative term, while the magnitude of the tangential force is bounded by the Coulomb friction criterion. The equations for $\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{n}}$ and $\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{t}}$ read (Cundall & Strack, 1979; Silbert et al., 2001; Santos et al., 2020)

$$\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{n}} = k_{\mathrm{n}}\delta_{ij,\mathrm{n}}\mathbf{e}_{ij,\mathrm{n}} - \gamma_{\mathrm{n}} m_{\mathrm{eff}}\mathbf{v}_{ij,\mathrm{n}} \tag{3}$$

$$\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{t}} = -\min\left\{\mu_{\mathrm{s}}|\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{n}}|, k_{\mathrm{t}}\xi_{ij,\mathrm{t}} + \gamma_{\mathrm{t}} m_{\mathrm{eff}}|\mathbf{v}_{ij,\mathrm{t}}|\right\}\frac{\mathbf{v}_{ij,\mathrm{t}}}{|\mathbf{v}_{ij,\mathrm{t}}|} \tag{4}$$

where $m_{\mathrm{eff}} = m_i m_j/(m_i + m_j)$, with m_i and m_j denoting the masses of particles i and j, respectively, k_{n}, k_{t}, γ_{n}, γ_{t} and μ_{s} are model parameters, discussed in Section S1.3 below, while the relative normal velocity $\mathbf{v}_{ij,\mathrm{n}}$ and the relative tangential velocity $\mathbf{v}_{ij,\mathrm{t}}$ between particles i and j are computed via

$$\mathbf{v}_{ij,\mathrm{n}} = (\mathbf{v}_{ij}\cdot\mathbf{e}_{ij,\mathrm{n}})\mathbf{e}_{ij,\mathrm{n}} \tag{5}$$

$$\mathbf{v}_{ij,\mathrm{t}} = \mathbf{v}_{ij} - \mathbf{v}_{ij,\mathrm{n}} - \frac{1}{2}(\boldsymbol{\omega}_i + \boldsymbol{\omega}_j)\times(\mathbf{r}_i - \mathbf{r}_j) \tag{6}$$

with $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$ denoting the difference between the velocities of particles i and j ($\mathbf{v}_i$ and $\mathbf{v}_j$, respectively), and $\boldsymbol{\omega}_i$ and $\boldsymbol{\omega}_j$ standing for their respective rotational velocities. Moreover, in Eq. (4), $\xi_{ij,\mathrm{t}}$ is the tangential displacement accumulated as the particles are in contact. The displacement is set as zero at initiation of the contact and is computed in the reference frame of the rotating particle pair to compensate for the effect of rigid body rotations, as described in detail in previous work (Silbert et al., 2001; Santos et al., 2020).

The equation of rotational motion for particle i reads

$$I_i\boldsymbol{\omega}_i = \sum_{\substack{1\le j\le N_{\mathrm{p}}\\ j\ne i}} \mathbf{M}_{ij} \tag{7}$$

with $I_i = m_i d_i^2/10$ and $\boldsymbol{\omega}_i$ denoting the moment of inertia and the angular velocity of particle i, respectively, and $\mathbf{M}_{ij}$ corresponding to the torque on particle i associated with $\mathbf{F}^{\mathrm{c}}_{ij,\mathrm{t}}$.

Table S1: Parameters of the Discrete-Element-Method.

parameter	symbol	value
elastic constant for normal contact	k_{n}	$157\,\mathrm{N\,m^{-1}}$
elastic constant for tangential contact	k_{t}	$52\,\mathrm{N}\,m^{-1}$
viscoelastic damping constant for normal contact	γ_{n}	$0.2\,\mathrm{kg}\,s^{-1}$
viscoelastic damping constant for tangential contact	γ_{t}	$0.2\,\mathrm{kg}\,s^{-1}$
Coulomb coefficient of friction	μ_{s}	0.3
particle diameter	d_i	$[160, 240]\,\mu\mathrm{m}$
particle density	ρ_{p}	$2650\,\mathrm{kg\,m^{-3}}$

S1.2 Contact forces between mobile and rigid particles (non-erodible elements)

The contact forces between mobile sand particles and the rigid particles constituting the roughness elements of the bed are computed using the same model as in the previous section, but considering that the rigid particles have an infinite mass (Verbücheln et al., 2015). Specifically, the normal and tangential components of the contact force from a rigid particle j on a mobile particle i are computed with Eqs. (3) and (4), respectively, by setting $m_{\mathrm{eff}} = m_i$. Furthermore, contact forces between rigid particles are not considered.

S1.3 Model parameters

Table S1 displays the values of the parameters in Eqs. (3) and (4), i.e., the elastic constants k_{n} and k_{t}, the damping coefficients γ_{n} and γ_{t}, and the Coulomb friction coefficient, μ_{s}. The elastic and damping constants are taken from previous models for Aeolian sand transport over fully erodible beds (Carneiro et al., 2011, 2013; Comola et al., 2019). In particular, the elastic constant for normal contact, k_{n}, is estimated using $k_{\mathrm{n}} = \pi d_{\mathrm{m}} Y/4$, where $d_{\mathrm{m}} = 200\,\mu\mathrm{m}$ is the mean particle size adopted in our simulations, while $Y = 1\,\mathrm{MPa}$ is the Young's modulus adopted in previous work (Carneiro et al., 2011; Comola et al., 2019) and in our computations. Furthermore, for the elastic constant for tangential constant, we use $k_{\mathrm{t}} = k_{\mathrm{n}}/3$, while the friction coefficient is consistent with values adopted previously (Comola et al., 2019).

S1.4 Numerical implementation and particle-wind coupling

To solve the equations of motion of the granular phase, we employ LAMMPS (Large-scale Atomic/Molecular Massively Parallel Simulator), which is an open source DEM solver based on MPI implementation (Plimpton, 1995). Furthermore, we have extended this solver to incorporate the hydrodynamic description of the turbulent wind flow over the granular surface, developed in previous work (Carneiro et al., 2011; Durán et al., 2012) and briefly reviewed in the main document. To this end, we have included new modules (LAMMPS "fixes") into the granular package of the DEM solver, to set the initial (logarithmic) vertical profile of the mean horizontal wind speed, to compute the drag force, and to update this drag force and the wind profile owing to the process of momentum exchange between the particles and the wind. These modules are available from the corresponding author upon request.

S2 Validation of our simulations over fully erodible beds

To verify our numerical simulations, we compare our numerical predictions for the height-integrated mass flux (Q) of wind-blown particles over a fully erodible bed with corresponding wind-tunnel observations of this flux as a function of the wind shear velocity, u_*. We compute Q using the following equation,

$$Q = \frac{\sum_i^N m_i v_i^x}{A} \tag{8}$$

where N is the number of particles in the system, m_i and v_i^x denote the mass and horizontal speed of the $i-$th particle, respectively, and $A = L_x \cdot L_y$ is the horizontal area of the simulation domain. To measure this flux, we start the wind-blown transport process as described in the main document and wait until this transport achieves steady state. The typical (physical) time separating the begin of wind-blown transport from the steady state is about 2-3 seconds and independent of u_* (Carneiro et al., 2011; Durán et al., 2012; Pähtz et al., 2014; Comola et al., 2019). All results reported in the present work refer to the characteristics of steady-state wind-blown transport, and denote mean quantities obtained from averaging over about 5-10 seconds during steady-state transport.

Furthermore, by suitably normalizing the steady-state flux Q, we obtain the following non-dimensional quantity,

$$\hat{Q} = \frac{Q}{\rho_\mathrm{p}\sqrt{(s-1)gd_\mathrm{m}^3}}, \text{ with } s = \frac{\rho_\mathrm{p}}{\rho_\mathrm{f}}, \tag{9}$$

which we plot in Fig. S1 as a function of the Shields number,

$$\Theta = \frac{u_*^2 \rho_\mathrm{f}}{(\rho_\mathrm{p} - \rho_\mathrm{f}) g d_\mathrm{m}} \tag{10}$$

where $\rho_\mathrm{p} = 2650\,\mathrm{kg/m^3}$ and $\rho_\mathrm{f} = 1.225\,\mathrm{kg/m^3}$ denote the densities of the particles and the air, respectively, while $d_\mathrm{m} = 200\,\mu\mathrm{m}$ is the mean particle diameter and $g = 9.81\,\mathrm{m/s^2}$ is gravity.

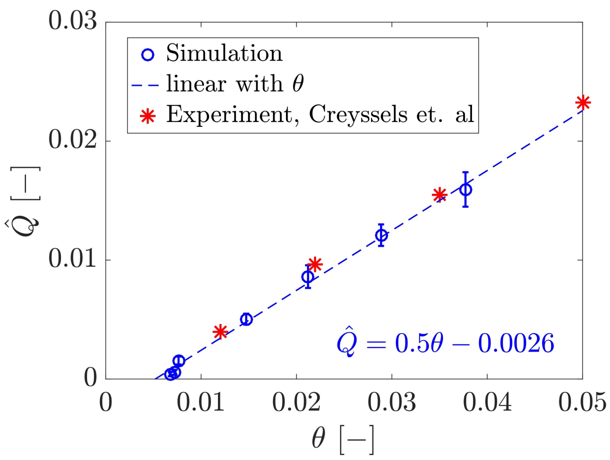

Figure S1: Normalized steady-state flux $\hat{Q}$ as a function of the Shields number Θ, considering a fully erodible bed ($\delta_0 \approx 15\,d_\mathrm{m}$).

We see in Fig. S1 that our numerical predictions for $\hat{Q}(\Theta)$ (circles) agree quantitatively well with observations from wind-tunnel experiments (Creyssels et al., 2009), denoted by the stars. The best fit to our simulation results using $\hat{Q} = a\Theta + b$ yields $a \approx 0.5$ and $b \approx 0.0026$ (dashed line in Fig. S1), from which obtain the minimal threshold $\Theta_\mathrm{t} \approx 0.0064$ below which no transport occurs ($\hat{Q} = 0$). From Eq. (10), this value of Θ_t leads to the minimal threshold wind shear velocity for sustained transport, $u_{*\mathrm{t}} \approx 0.165\,\mathrm{m/s}$.

We note that the value of $u_{*\mathrm{t}}$ predicted from our simulations is consistent with the prediction that $u_{*\mathrm{t}}$ is about 80% of the minimal threshold wind shear velocity $u_{*\mathrm{ft}}$ required to initiate transport,

$$u_{*\mathrm{ft}} = A_\mathrm{ft}\sqrt{\frac{\rho_\mathrm{p} - \rho_\mathrm{f}}{\rho_\mathrm{f}} g d_\mathrm{m}}, \tag{11}$$

with $A_\mathrm{ft} \approx 0.1$(Bagnold, 1941; Shao & Lu, 2000). Indeed, by applying the mean particle size $d_\mathrm{m} = 200\,\mu\mathrm{m}$ of our simulations in Eq. (11), we obtain $u_{*\mathrm{ft}} \approx 0.206\,\mathrm{m/s}$, i.e., our model is consistent with the relation $u_{*\mathrm{t}} \approx 0.8\,u_{*\mathrm{ft}}$ predicted for wind-blown transport.

S3 The modified wind profiles for varying δ_0

The initial vertical profile of the horizontal downstream wind velocity u_x is logarithmic and follows Eq. (3) of the main document. However, this wind velocity profile is updated every time-step, since the acceleration of the grains extracts momentum from the air thus creating a negative feedback on the wind. The modification of the wind velocity profile is computed using Eqs. (4) and (5) in the main document. The vertical profiles of the modified wind velocity u_x and the grain-borne shear stress τ_p are shown for different values of the mobile layer thickness δ_0 in Fig. S2.

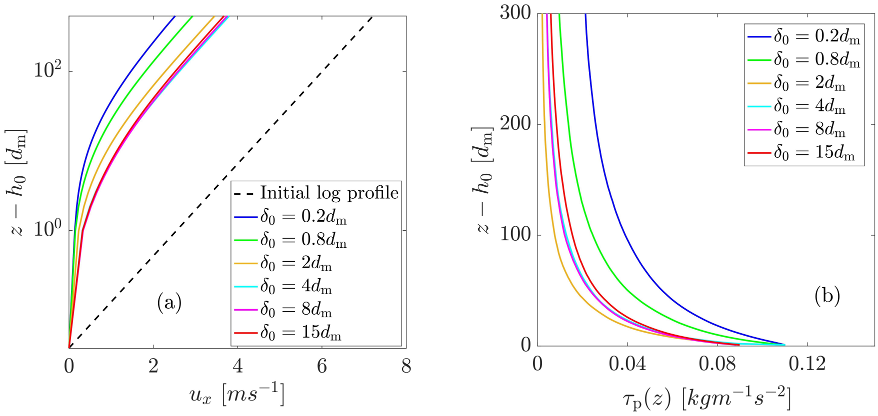

Figure S2: (a) The modified wind profiles for different values of δ_0 alongside the initial logarithmic profile; (b) grain-borne shear stress profile as a function of the height above the bed for different δ_0. The results were obtained with $u_* = 0.30\,\mathrm{m/s}$.

S4 Computation of the mean hop length and the mean horizontal impact and lift-off velocities

To compute the mean hop length ℓ_hop and the average horizontal impact and lift-off velocities, $u_{0\downarrow}$ and $u_{0\uparrow}$, respectively, we consider only the grains with a minimum vertical lift-off velocity of $\sqrt{6gd}$, i.e., the grains that achieve a minimum height of $3\,d_\mathrm{m}$ above the bed height. The values of ℓ_hop, $u_{0\downarrow}$ and $u_{0\uparrow}$, are then averaged over a time window of 5 seconds during steady-state sand transport.

The mean horizontal grain velocity or slip velocity u_0 is then computed using

$$u_0 = (u_{0\downarrow} + u_{0\uparrow})/2. \tag{12}$$

Furthermore, to obtain the mean hop length, we start from the mean hop time, which is given by (Ho et al., 2011),

$$t_{\text{hop}} \approx \frac{2v_{0\uparrow}}{g}, \tag{13}$$

where $v_0 \uparrow$ is the mean ascending vertical velocity of the grains (also averaged over 5 seconds in the steady state). Furthermore, the horizontal acceleration of the grains is given by,

$$a_{\text{hor}} \approx \frac{(u_{0\downarrow} - u_{0\uparrow})}{t_{\text{hop}}}, \tag{14}$$

so that the mean hop length is approximated as,

$$\ell_{\text{hop}} \approx (u_{0\downarrow} - u_{0\uparrow})\, \frac{v_{0\uparrow}}{g}. \tag{15}$$

S5 Relation between the steady-state bed layer thickness, δ_{s}, and the initial bed layer thickness, δ_0

As mentioned in the main document, once the sand transport process begins, the initial thickness of the mobile sand bed applied in the numerical simulations, δ_0, decreases toward a smaller value δ_{s}, which is achieved when transport conditions have reached steady state. As depicted in Fig. S3, δ_{s} and δ_0 are linearly related to each other, and the difference between both values of bed thickness displays a slight increase with u_* owing to the effect of wind shear velocity on enhancing erosion. However, we find that the scaling laws reported in the main document are valid whatever value of bed thickness is chosen, while the following relation applies,

$$\frac{\delta_{\mathrm{s}}}{d_{\mathrm{m}}} = \left(\frac{\delta_0}{d_{\mathrm{m}}} - C_{\mathrm{b}} \frac{u_*}{\sqrt{g d_{\mathrm{m}}}}\right) \cdot \Theta\left(\frac{\delta_0}{d_{\mathrm{m}}} - C_{\mathrm{b}} \frac{u_*}{\sqrt{g d_{\mathrm{m}}}}\right) \tag{16}$$

where $C_{\mathrm{b}} \approx 0.02$ is an empirical parameter and $\Theta(x)$ denotes the Heaviside function, i.e., $\Theta(x) = 1$ if $x \geq 0$ and $\Theta(x) = 0$ if $x < 0$. Therefore, the term $C_{\mathrm{b}} u_*/(\sqrt{g d_{\mathrm{m}}})$ denotes the thickness of the total eroded layer, relative to the particle size, from the beginning of transport until steady state (i.e., as the bed thickness evolves from δ_0 toward δ_{s}).

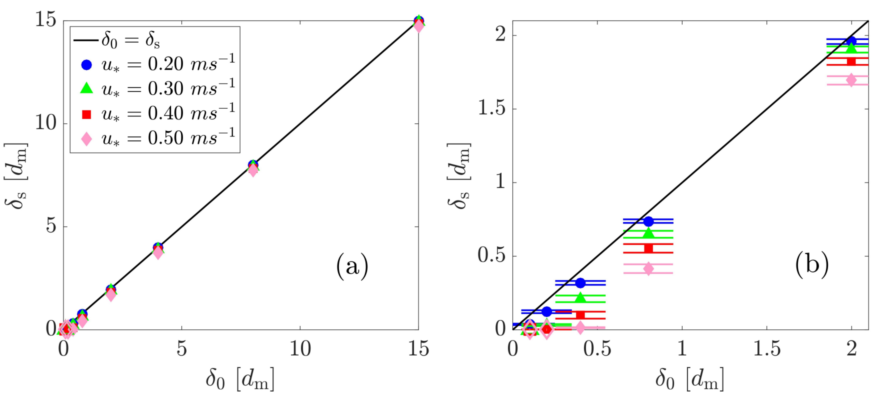

Figure S3: (a) Values of the bed layer thickness at steady-state transport, δ_{s}, plotted against the initial values of bed layer thickness, δ_0, for different values of the wind shear velocity u_*. The plot in (b) denotes a zoom into the region of bed layer thickness comparable to the particle size. Filled symbols correspond to saturated transport conditions, while empty symbols denote under-saturated scenarios (the same color code used for the filled symbols in the legend applies to specify u_* in these empty symbol scenarios).

In Fig. S3, the filled symbols correspond to numerical simulations in which the wind is carrying the maximum possible number of particles, i.e., the flux is saturated. Starting with $\delta_0 = 15\,d_\mathrm{m}$, for instance, and under a given value of $u_* - u_{*\mathrm{t}}$, we observe no change in the average number of particles in the Aeolian layer (or, equivalently, the mass density of dragged particles) upon a decrease in the initial bed thickness δ_0, as long as the scenarios associated with the filled symbols in Fig. S3 are considered. However, the empty symbols in this figure constitute scenarios where the bed thickness is so small, that the wind flow does not dispose of enough particles on the ground to drive transport toward the saturated flux. These empty symbols are associated with a value of steady-state bed thickness equal to 0 and are referred to as *under-saturated*. Specifically, for these empty symbols, the wind eroded the entire sand bed and still the amount of sand transport is not enough to saturate the sand flux (the most extreme, non-vanishing flux scenario of such under-saturated regime in our simulations would be, in particular, the case of one single grain hopping downwind).

We have thus not considered these under-saturated scenarios in our analysis of $Q(u_*)$ in the main document, since we are interested here in an expression for the saturated flux that accounts for the bed erodibility. Moreover, it is a straighforward conclusion that, in the under-saturated regime, the mass density of the transport layer decreases upon a reduction of the initial bed thickness δ_0/d_m. Nevertheless, we note that under-saturated transport scenarios constitute an interesting topic to be investigated in future work. For instance, such scenarios have applications to areas that are devoid of any sand availability but subjected to an upwind flux that is under-saturated (for instance in the presence of upwind vegetation or moisture), or over inter-dune bedrock areas within fields of sparsely distributed dunes (Fryberger et al., 1984)).

S6 Transport layer thickness as a function of the bed thickness

As explained in the main document, the transport layer expands gradually as the soil erodibility conditions change from fully erodible to rigid. To quantify this process, we compute the characteristic length-scale l_ν associated with the nearly exponential decay of the particle concentration $\nu(z)$ with the height z above the ground, i.e.,

$$\nu(z) = \nu_0 \exp(-z/l_\nu) \tag{17}$$

where ν_0 is the particle concentration extrapolated to the bed ($z = 0$). Fig. S4 shows the behavior of l_ν as a function of the thickness of mobile sand layer on the ground, δ_0. We see that, when the rigid ground is flat, l_ν decreases monotonically as δ_0 increases toward $15\,d_\mathrm{m}$ (the fully erodible bed scenario). However, when the rigid surface is armoured with non-erodible elements (which in our simulations have the same size as the mobile particles), a minimum in l_ν is observed near $\delta_0 = 2\,d_\mathrm{m}$. This minimum can be explained by the prevailing occurrence of backward ejecta in the range $\delta_0 \lesssim 2\,d_\mathrm{m}$ (as described in the main document), which leads to lower values of l_ν in this range when non-erodible particles cover the ground, compared to the flat ground scenario. As δ_0 becomes much larger than the particle size, the effect of the immobile particles on the Aeolian transport thickness becomes negligible, and l_ν approaches asymptotically the value corresponding to the fully erodible bed as shown in Fig. S4.

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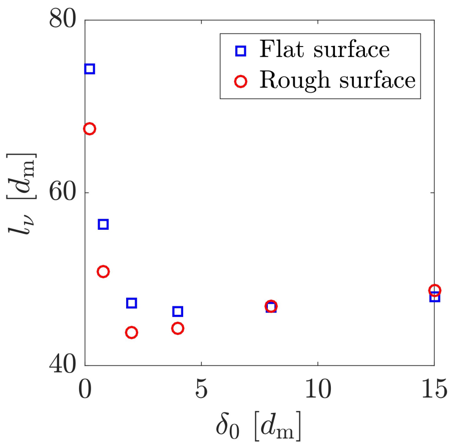

Figure S4: Thickness l_ν of the transport layer as a function of the thickness δ_0 of the mobile sediment layer covering the non-erodible surface, which consists of a flat horizontal surface (blue symbols) and a sheet of immobile particles (red symbols). The results were obtained with $u_* = 0.30\,\mathrm{m/s}$.

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