Lagrangian Statistics for Dispersion in Magnetohydrodynamic Turbulence

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Key Points:

- turbulence in the heliosphere
- magnetohydrodynamics
- Lagrangian statistics

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Abstract

Measurements in the solar wind and high-resolution fluid simulations give clear indications for the anisotropy of MHD turbulence in the presence of magnetic fields. How this anisotropy affects transport processes like diffusion and dispersion remains an open question. The first efforts to characterize Lagrangian single-particle diffusion and two-particle dispersion in incompressible MHD turbulence were performed a decade ago. We revisit those pioneering results through updated simulations performed at higher Reynolds number. We present new investigations that use the dispersion of many Lagrangian tracer particles to examine the extremes of dispersion and the anisotropy in direct numerical simulations. We then point out directions in which Lagrangian statistics need to be developed to address the fundamental problem of anisotropic MHD turbulence and transport in solar and stellar winds.

1 Introduction

To understand the solar wind or stellar winds, we first study the fundamental behavior of a turbulent plasma. Understanding the anisotropic dynamics of a turbulent plasma is key to producing predictions and models for the scattering of energetic particles in a solar wind. In his famous book on turbulence (Lesieur, 1987), Marcel Lesieur wrote "Turbulence is a dangerous topic which is often at the origin of serious fights in the scientific meetings devoted to it since it represents extremely different points of view, all of which have in common their complexity, as well as an inability to solve the problem." One point of view follows from adopting the Lagrangian frame of reference, the natural point of view for studying diffusive processes such as turbulence in the solar wind. This work develops Lagrangian statistics to quantify anisotropic magnetohydrodynamic (MHD) turbulence.

Over the last two decades, Lagrangian statistics of turbulent flows have attracted increasing attention, motivated by new experimental techniques, e.g. La Porta, Voth, Moisy, and Bodenschatz (2000); Mordant, Lévêque, and Pinton (2004); Xu, Bourgoin, Ouellette, Bodenschatz, et al. (2006); Biferale et al. (2008); Bourgoin, Pinton, and Volk (2014); Bourgoin and Xu (2014); Lawson, Bodenschatz, Lalescu, and Wilczek (2018); Polanco, Vinkovic, Stelzenmuller, Mordant, and Bourgoin (2018); Lawson, Bodenschatz, Knutsen, Dawson, and Worth (2019). High-resolution numerical simulations have also enabled increasingly detailed studies of the dynamics of Lagrangian tracer particles, e.g. Yeung and Borgas (2004); Biferale et al. (2005); Buaria, Yeung, and Sawford (2016); Sawford and Yeung (2015); Bianchi, Biferale, Celani, and Cencini (2016); Schneide, Pandey, Padberg-Gehle, and Schumacher (2018). This explosion of work using Lagrangian tracer particles to explore the statistics of turbulence in neutral fluids have been summarized in several reviews, including S. Pope (1994); Wilson and Sawford (1996); Yeung (2002); Toschi and Bodenschatz (2009); Meneveau (2011) including a comprehensive review dedicated to two-particle dispersion, Salazar and Collins (2009).

The first program to investigate MHD turbulence from the Lagrangian point of view began over a decade ago as a collaboration between a group at the Max Planck Institute for Plasma Physics and a group at the University of Bochum (Müller & Busse, 2007b; Busse et al., 2007; Müller & Busse, 2007a; Homann, Grauer, et al., 2007; Busse & Müller, 2008; Homann et al., 2009; Busse et al., 2010). This work was based on direct numerical simulations of three-dimensional incompressible homogeneous MHD turbulence. Here we revisit some of the fundamental results of those earlier studies, using new simulations at higher Reynolds number. We also expand on progress that has been made more recently to use Lagrangian statistics for anisotropic turbulence convection and magnetoconvection (Pratt et al., 2017). We discuss further directions for future work.

2 Direct Numerical Simulations for Lagrangian single-particle diffusion and two-particle dispersion

To study diffusion and dispersion we produce simulations of statistically stationary, forced homogeneous magnetohydrodynamic turbulence, in the presence of a static mean magnetic field. In each direct numerical simulation presented in this work, we solve the non-dimensional equations for incompressible MHD turbulence:

$$\frac{\partial \vec{\omega}}{\partial t} - \nabla \times (\vec{v} \times \vec{\omega} + \vec{j} \times \vec{B}) = \hat{\nu} \nabla^2 \vec{\omega} + \vec{f}^{\omega} \ , \tag{1}$$

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{v} \times \vec{B}) = \hat{\eta} \nabla^2 \vec{B} + \vec{f}^{b} \ , \tag{2}$$

using a pseudospectral method in a three-dimensional rectangular simulation volume with periodic boundary conditions. These equations include the solenoidal velocity field $\vec{v}$, vorticity $\vec{\omega} = \nabla \times \vec{v}$, magnetic field $\vec{B}$, and current density $\vec{j} = \nabla \times \vec{B}$. Each of the quantities in eqs. (1) and (2) has been non-dimensionalized using relevant time and length scales, commonly referred to as Alfvénic units. Two dimensionless parameters, $\hat{\nu}$ and $\hat{\eta}$, appear in the equations. They derive from the kinematic viscosity ν and the magnetic diffusivity η. A fixed time-step and a low-storage third-order Runge–Kutta method (Williamson, 1980) are used for the time-integration. A static macroscopic magnetic field B_0 pointing in the positive z-direction may be imposed.

Table 1 provides a summary of the fundamental parameters of the simulations we consider. Each simulation is performed on a grid of 1024^3 collocation points. In the table, we record the strength of the macroscopic magnetic field B_0 imposed in the z direction, as well as the root-mean-square of the magnetic fluctuations B_{RMS}, averaged over the simulation time. We measure length in units of the Kolmogorov microscale $\eta_{\mathsf{kol}} = (\hat{\nu}^3/\epsilon_{\mathrm{v}})^{1/4}$ and time in units of the Kolmogorov time-scale $\tau_\eta = (\hat{\nu}/\epsilon_{\mathrm{v}})^{1/2}$; these are the smallest length and time scales that characterize turbulent flows. All of our simulations fulfill the classic criterion of S. B. Pope (2000), i.e. $k_{\mathrm{max}}\eta_{\mathsf{kol}} > 1.5$, that has been commonly used to evaluate whether homogeneous isotropic turbulence is sufficiently resolved (see also Yeung, Sreenivasan, and Pope (2018) for a recent study of the effect of resolution on homogeneous isotropic turbulence).

To maintain a turbulent steady state, both the vorticity and magnetic fields are forced on the largest scales of the simulation volume using a method that allows the largest scale motions of the system to evolve. In eqs. (1) and (2) forcing terms $\vec{f}^{\omega}$ and $\vec{f}^{b}$ are introduced. For the simulations in Table 1, these forcing terms are non-zero only for the wave-vector shell $1 \leq |\vec{k}| \leq 2.5$. A deterministic homogeneous method of forcing that establishes a constant injection of energy at large scales is used (see Busse (2009) for a detailed discussion of this forcing method). Large-scale Alfvén waves are permitted and are observed for this forcing method.

For simulations of MHD turbulence that are anisotropic because of the effect of a mean magnetic field, a box that is elongated in the z-direction has been used in many earlier works, including for example Mason, Cattaneo, and Boldyrev (2006). To determine the necessary elongation of the simulation volume in the z-direction, we consider the correlation length of the velocity field in each direction. We measure a correlation length, $L_{\mathsf{c},\parallel}$, of the velocity field in the z-direction that is larger than in the x and y directions, in agreement with previous studies, e.g. Chandran (2008); Boldyrev (2005); Cho, Lazarian, and Vishniac (2002). To accommodate this larger $L_{\mathsf{c},\parallel}$ within our simulation volume, the simulation volume can be elongated in the z-direction, so that the condition on the box length $L_z \gg L_{\mathsf{c},\parallel}$ is satisfied. The elongation of the simulation box, L_z/L_x, is listed in the table for each simulation. The simulation volume has sides of length 2π in the x and y directions.

2.1 Lagrangian tracer particles

For the simulations in Table 1, the positions of Lagrangian tracer particles are initialized in a homogeneous random distribution at a time when the turbulent flow is in a statistically stationary steady state. The total number of particles in each simulation is $\mathsf{n_p} \approx 8.3$ million. This is a high density of tracer particles, comparable to earlier works (Müller & Busse, 2007b; Busse et al., 2007; Müller & Busse, 2007a; Homann, Grauer, et al., 2007; Busse & Müller, 2008; Homann et al., 2009; Busse et al., 2010). The Lagrangian statistics produced have been tested and found to be well-resolved in space.

At each time step the particle velocities are interpolated from the instantaneous Eulerian velocity field using a tricubic polynomial interpolation scheme (for a clear analysis of the impact of the interpolation scheme on Lagrangian statistics, see Homann, Dreher, and Grauer (2007)). Particle positions are calculated by numerical integration of the equations of motion using a low-storage third-order Runge–Kutta method. Each simulation is run for a sufficient time that Lagrangian particle pair-separations exhibit a clear diffusive trend; this length of time is approximately 400 τ_η.

2.2 Reynolds number for anisotropic MHD turbulence

For an isotropic system with $B_0 = 0$, we define the Reynolds number in the standard way as

$$\mathsf{Re} = \langle \mathsf{E_v^{1/2} L_E} \rangle / \hat{\nu} \ , \tag{3}$$

from the kinetic energy, a characteristic length scale, and the viscosity. For statistically homogeneous isotropic turbulent flows, the characteristic length scale $\mathsf{L_E}$ is commonly defined as a dimensional estimate of the size of the largest eddies, $\mathsf{L_E} = \mathsf{E_v}^{3/2}/\epsilon_\mathrm{v}$, where $\epsilon_\mathrm{v} = \hat{\nu}\langle \sum_k k^2 \vec{v}^2 \rangle$ is the time-averaged rate of kinetic energy dissipation. This length scale $\mathsf{L_E}$ is included in Table 1 for our simulations.

To compare isotropic and anisotropic turbulent flows, we use a more basic definition of the Reynolds number (see chapter 6.1.2 of S. B. Pope (2000))

$$\mathsf{Re} = c \ (\eta_\mathsf{kol}/\mathsf{L_F})^{-4/3} \ . \tag{4}$$

This definition requires knowledge of the forcing length scale $\mathsf{L_F}$, and a constant c. Our method of forcing affects a minimum length scale

$$\mathsf{L_F} = 2\pi / k_{f,\mathsf{max}} = 2\pi/3 \ . \tag{5}$$

We determine the constant c by comparison with the definition of the Reynolds number in the isotropic case given in eq. (3). The Reynolds number is calculated in this way for simulation ANOVIS3 in Table 1. The magnetic Reynolds number is defined from the Reynolds number and the magnetic Prandtl number, i.e. $\mathsf{Re_m} = \mathsf{Pr_m Re}$. In all simulations in this work, the magnetic Prandtl number $\mathsf{Pr_m} = 1$ so that the magnetic Reynolds number is equal to the Reynolds number.

3 Single-particle diffusion and two-particle dispersion in MHD turbulence

Perhaps the most fundamental result from studies of Lagrangian statistics in MHD turbulence is a comparison of single-particle diffusion and two-particle dispersion. While single-particle diffusion exhibits the same essential behavior in Navier-Stokes turbulence and MHD turbulence, two-particle relative dispersion in MHD turbulence differs significantly from the Navier-Stokes behavior (Busse et al., 2007).

In Figure 1, we have produced single-particle diffusion curves for isotropic Navier-Stokes turbulence (simulation HNOVIS) and isotropic MHD turbulence (simulation ANOVIS0). These two curves exhibit nearly identical behavior. Each of the curves in Figure 1 exhibits a clear ballistic scaling as t^2 at early times, and diffusive scaling as t at late times. This figure also includes the curves for an anisotropic MHD turbulence simulation, ANOVIS3, which is significantly different from either isotropic case. The results from ANOVIS3 have been split into diffusion along the magnetic field direction, and diffusion perpendicular to the mean magnetic field. Diffusion in both these distinct directions takes a similar shape. At intermediate times, both the diffusion parallel and perpendicular to the mean magnetic field grow more slowly in anisotropic MHD than in either isotropic simulation. This slow-down is larger in the direction parallel to the mean magnetic field. At long times, the separation process accelerates until the diffusive regime is reached. These observations agree with results first reported in Busse and Müller (2008); Busse (2009).

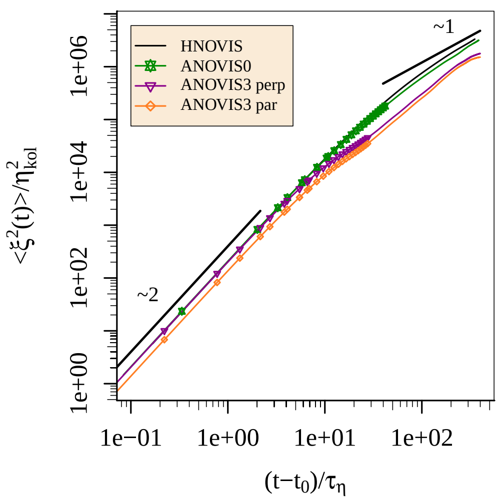

Figure 1. Evolution of mean-square distance from the initial position for the three simulations described in Table 1. The mean-square distance for the anisotropic MHD simulation ANOVIS3 is separated into distances parallel and perpendicular to the mean magnetic field. Each curve is produced from the average of at least 4 independent initial times.

The velocity autocorrelation function has a differential relation to diffusion:

$$\frac{d}{dt}\langle dr(0)dr(t)\rangle \quad = \quad 2\int_0^t \langle v(0)v(\tau)\rangle d\tau \ . \qquad (6)$$

Because of this relationship the velocity autocorrelation function is typically used to shed new light on diffusion by providing information about the relaxation of fluctuations over long times and distances. For Brownian motion, the velocity autocorrelation function is fit well by a single decaying exponential. A single decaying exponential has also been shown to be a good fit for hydrodynamic turbulence in both experimental and numerical studies (Sato & Yamamoto, 1987; Yeung & Pope, 1989). A single decaying exponential is an excellent fit for the velocity autocorrelation function of simulation HNOVIS (see Figure 2). However for the MHD simulations ANOVIS0 and ANOVIS3, it is not clear whether a single decaying exponential is a reasonable model. Isotropic MHD turbulence leads to a swifter overall decay of the velocity autocorrelation function than hydrodynamic turbulence. In anisotropic MHD turbulence, this swift decay is further exaggerated, fundamentally changing the initial shape of the decay of the velocity autocorrelation function so that a decaying exponential produces a poor fit. In the anisotropic case, large scale Alfvénic fluctuations are also present. The characteristic time of decay for the velocity autocorrelation function is smaller in the anisotropic case than in the isotropic case. In the direction aligned with the mean magnetic field this decay time is slightly shorter than in the direction perpendicular. Small-scale fluctuations in the velocity are therefore more probable in the direction perpendicular to the magnetic field. Over long times, the absolute diffusion parallel to the mean magnetic field is therefore smaller because of the different prevalence in velocity fluctuations.

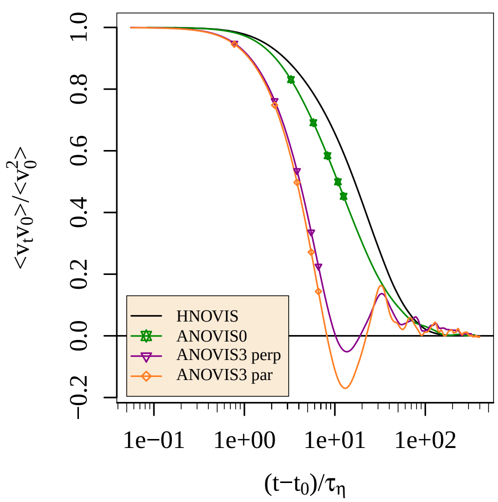

Figure 2. Velocity autocorrelation function for the three simulations described in Table 1. The velocity autocorrelation function for the anisotropic MHD simulation ANOVIS3 is separated into velocities in the direction parallel and perpendicular to the mean magnetic field. Each curve is produced from the average of at least 4 independent initial times.

For two-particle dispersion, Busse et al. (2007) find considerable differences between isotropic Navier-Stokes turbulence and isotropic MHD turbulence. We examine the separation of pairs of particles that are initially separated by the smallest initial separation that is resolved by our grid, $2\eta_{\mathsf{kol}}$ (see Figure 3). Each of these curves exhibits a clear ballistic scaling as t^2 at early times, and diffusive scaling as t at late times. We find that the rate of dispersion is slower for MHD turbulence than for Navier-Stokes turbulence. This rate of dispersion first slows down at intermediate times, and the slow-down is more significant for the anisotropic MHD simulation ANOVIS3. This common behavior between isotropic and anisotropic MHD simulations may be explained as an effect of the local, fluctuating magnetic field. This field appears to be sufficient to produce a degree of anisotropy in the relative dispersion process, even in a globally isotropic simulation.

4 Direct Numerical Simulations for Lagrangian many-particle dispersion

To examine many-particle dispersion, we conduct a separate series of simulations that use a significantly different initial set-up of Lagrangian tracer particles. The positions of the Lagrangian tracer particles are initialized into spherical volumes of a

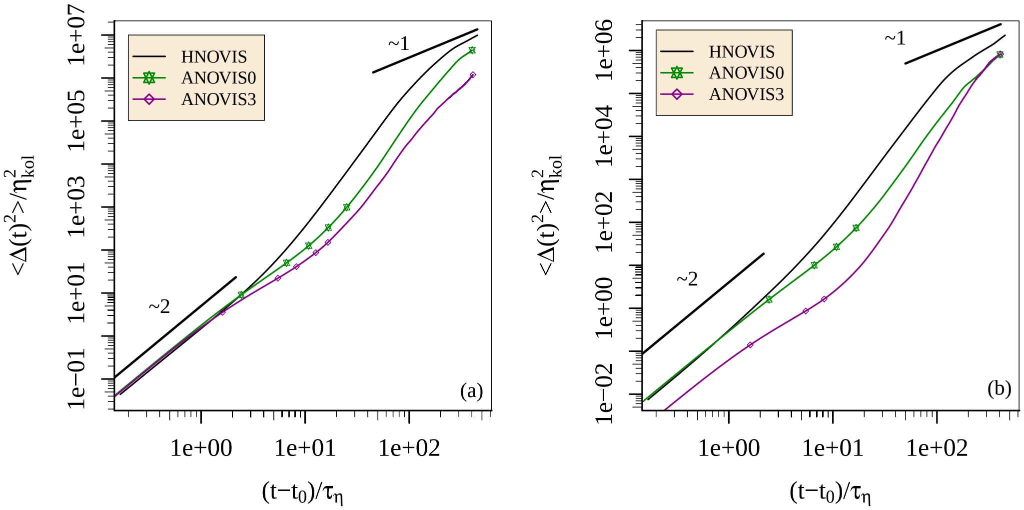

Figure 3. Evolution of mean-square relative dispersion in homogeneous isotropic Navier-Stokes turbulence (simulation HNOVIS), homogeneous isotropic MHD turbulence (simulation ANOVIS0), and homogeneous anisotropic MHD turbulence (simulation ANOVIS3). The behavior for pairs with an initial separation of $2\eta_{\mathsf{kol}}$ is shown (a) for particles initially separated in the direction perpendicular to the mean magnetic field and separation distance perpendicular to the field measured, and (b) for particles initially separated in the direction parallel to the mean magnetic field, separation distance aligned with the mean magnetic field is measured.

given size and density of tracer particles, henceforth called *droplets*. The droplets are homogeneously and randomly distributed throughout the simulation volume at a time when the turbulent flow is in a statistically stationary steady state. A visualization of such a droplet dispersing is shown in Figure 4. Simulations using this initial droplet set-up for Lagrangian particles were published in Pratt et al. (2017) for a study of convection, and a similar initial set-up was used to study Navier–Stokes turbulence in Bianchi et al. (2016). Each simulation described in Table 2 has this initial set-up, and a total number of tracer particles of $\mathsf{n_p} \approx 4.5$ million.

For the simulations in Table 2, a stochastic forcing method is used. For simulations HPUFF and APUFF0, the forcing terms are non-zero only for the wave-vector shell $1 \leq |\vec{k}| \leq 2.5$. For simulation APUFF3, this forcing wave-vector shell is shifted to $2.5 \leq |\vec{k}| \leq 3.5$; this adjustment was made because forcing wave-vectors in the lower k shell in combination with the mean magnetic field was found to lead to a build-up of energy at large scales, significantly changing the energy spectra. As with the previous series of simulations, large-scale Alfvénic fluctuations are permitted by this forcing method and are observed in our simulations. Aside from the different initial set-up of Lagrangian tracer particles and the use of a stochastic forcing method, the simulations in Table 2 follow a similar set-up to the simulations in Table 1.

5 Many-particle dispersion in MHD turbulence

Lagrangian statistics are known to be sensitive to extreme events in the fluctuating turbulent fields, e.g. Yeung and Borgas (2004); Boffetta and Sokolov (2002). In Pratt et al. (2017), we developed an analysis using the convex hull to calculate the extremes of dispersion of a group of many Lagrangian tracer particles. The simplest diagnostic resulting from this approach is the maximal ray internal to a convex hull.

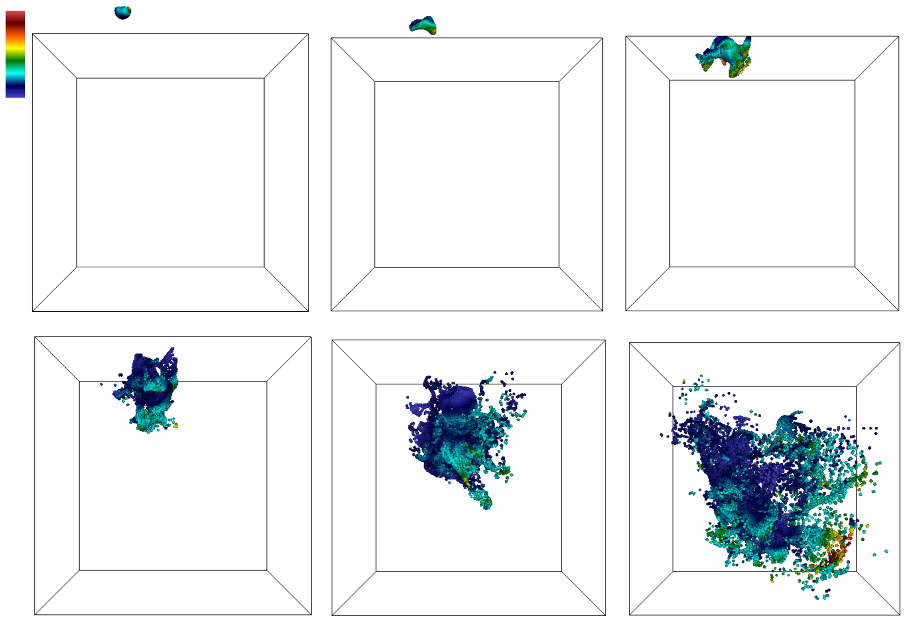

Figure 4. Diffusion of a single droplet with initial diameter $14\eta_{\mathsf{kol}}$, composed of approximately 16 thousand Lagrangian tracer particles, in simulation HPUFF. The series of 6 snapshots documents the dispersion of the particles over approximately 40 τ_η. Particles are colored by the kinetic energy of the flow.

For a group of particles G, the maximal ray can be calculated:

$$r = \max_{i,j \in G} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}\ . \tag{7}$$

This measure of separation is based on pairs of particles within the droplet that are furthest apart at a given point in time, so that it always is defined as the largest extent of dispersion of the group. Thus different particles may be used to determine the maximal ray as the hull evolves in time; in contrast, two-particle dispersion always considers a fixed pair of particles. The maximal ray evolves with similar ballistic, intermediate, and diffusive phases to two-particle dispersion (see Figure 5). However, because it measures the extreme of dispersion, during the initial short ballistic regime the maximal ray dispersion curve grows slightly more quickly than t^2. In the intermediate regime, we observe that even the extremes of dispersion are slowed by the presence of magnetic fields, with dispersion in the anisotropic MHD simulation APUFF3 growing more slowly than in the isotropic case APUFF0. This has interesting overlap with the earlier observation (Busse, 2009) that intermittency in particle accelerations is lower in isotropic MHD turbulence than in isotropic Navier-Stokes turbulence, and lower still in anisotropic MHD turbulence. The extremes of dispersion, not just the averages, are suppressed by the anisotropy of the magnetic fluctuations.

In Pratt et al. (2017), we used the surface area s and volume v of the convex hull surrounding the Lagrangian tracer particles to quantify the anisotropy in a given simulation. For a perfect sphere, the non-dimensional ratio $s/v^{2/3}$ takes a value of $(36\pi)^{1/3} \approx 4.8$; for an anisotropic shape like a pancake or needle, this ratio will be larger. How much greater this ratio is, compared with 4.8, indicates how anisotropic the convex hull around a group of Lagrangian tracer particles is. During intermediate times, the anisotropy ratio grows dramatically for all simulations, before falling again to smaller levels (see Figure 6). The anisotropy ratio is larger in both isotropic and

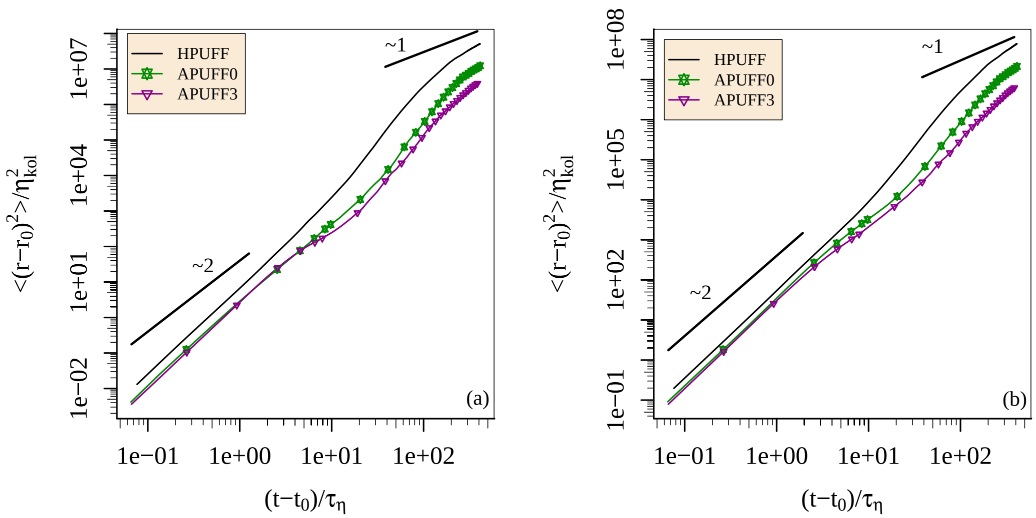

Figure 5. Evolution of mean-square maximal ray r of the convex hulls for the simulations described in Table 2. Droplets with (a) an initial diameter of $4\eta_{\text{kol}}$, and (b) an initial diameter of $14\eta_{\text{kol}}$.

anisotropic MHD than in our hydrodynamic simulation. It is also slightly larger in isotropic MHD turbulence than in anisotropic MHD turbulence. However the strong overlap of the shaded regions in Figure 6, which indicate one standard deviation above and below the average line, indicate that this difference between the average lines is not statistically significant. The difference in the anisotropy ratio between Navier-Stokes turbulence and MHD turbulence is clearly statistically significant. A comparison between panels (a) and (b) of this figure demonstrates that, for initially smaller groups of particles, a larger anisotropy ratio is achieved. This gives an indication that intermittency is different on different length scales of a turbulent flow.

6 Summary and Discussion

Lagrangian tracer particles provide a powerful tool for quantifying the diffusion and dispersion in MHD turbulence. In the simulations presented here, we have confirmed that single-particle diffusion curves are similar for isotropic hydrodynamic and magnetohydrodynamic turbulence. However, for anisotropic MHD turbulence the single-particle diffusion curves exhibit characteristic differences; they show slower diffusion, at intermediate separation times. Two-particle dispersion curves for isotropic MHD turbulence have clear differences from isotropic hydrodynamic turbulence. Anisotropic MHD turbulence exaggerates those differences. We obtain these results from three simulations that have identical resolution and closely comparable Reynolds numbers and general set-up. These simulations have higher Reynolds numbers and use different forcing methods, but the results agree with the earlier work available (Busse et al., 2007; Busse & Müller, 2008; Busse, 2009).

Single-particle diffusion and two-particle dispersion curves provide critical information, however they do not provide a complete picture of transport processes for anisotropic MHD turbulence. We therefore use the methods developed in Pratt et al. (2017) for convection simulations to examine anisotropic MHD turbulence, a new application of these statistics. The maximal ray, which represents the extremes of dis-

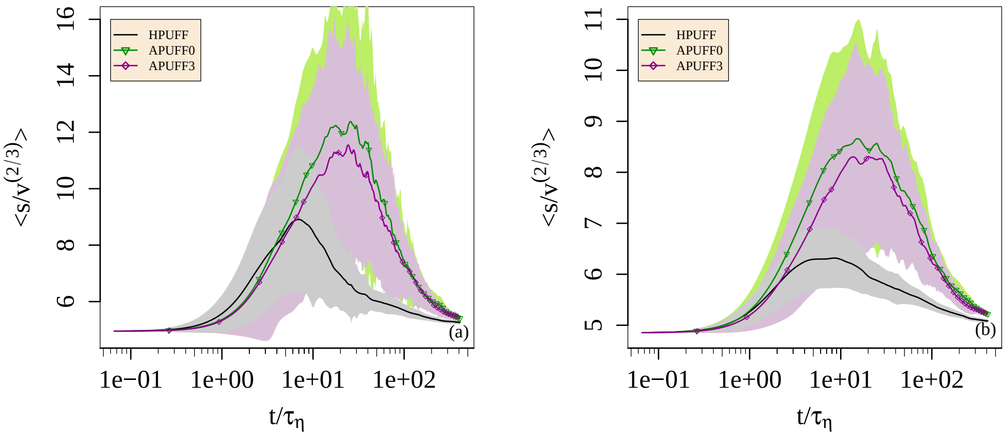

Figure 6. Average anisotropy ratio for the simulations described in Table 2, for droplets with (a) an initial diameter of $4\eta_{\mathrm{kol}}$, and (b) an initial diameter of $14\eta_{\mathrm{kol}}$. The density of particles in both sizes of droplet is the same. A shaded area indicating one standard deviation above and below each line is provided.

persion, is found to follow a pattern similar to the two-particle dispersion: the speed of dispersion can be ordered with hydrodynamic turbulence producing the fastest dispersion, then isotropic MHD turbulence, and then anisotropic MHD turbulence. When we examine the anisotropy ratio for groups of many Lagrangian tracer particles, isotropic MHD turbulence possesses a local anisotropy that is similar to, or greater than, anisotropic MHD turbulence. Both types of MHD turbulence are more anisotropic than Navier-Stokes turbulence.

In order to fully understand how magnetic fields affect MHD turbulence and anisotropy, further development of Lagrangian statistics is needed. One aspect in particular that would be helpful to clarify transport processes is the analysis of particle trajectories in MHD turbulence; studies of particle trajectories in neutral fluid turbulence have produced interesting results in recent years (Ouellette & Gollub, 2007; Xu et al., 2007; Choi et al., 2010; Siu & Taylor, 2011; Bos et al., 2015). Quantifying the trajectories involves calculating the curvature and torsion that a particle experiences along its path. To better understand these trajectories, studies of the extremes of acceleration (Homann, Grauer, et al., 2007) need to be further developed for anisotropic MHD turbulence. Intriguing experimental results for the acceleration PDF of neutral-fluid turbulence (e.g. Mordant, Crawford, and Bodenschatz (2004); Liot, Gay, Salort, Bourgoin, and Chillà (2016)) make it likely that there is a great deal more to understand about MHD turbulence through the peculiar behavior of the Lagrangian acceleration. In addition, studies of anisotropic MHD turbulence are often based on relatively few simulations, and therefore do not reveal the role of the mean magnetic field in generating anisotropy and changing transport properties. A larger range of anisotropic simulations needs to be studied in conjunction so that the influence of a mean magnetic field can be established. These areas of study are included in our ongoing and planned work.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. 1907876. Simulations were performed on the Konrad and Gottfried computer systems of the Norddeutsche Verbund zur Förderung des Hoch- und

Höchstleistungsrechnens (HLRN) by the project bep00051 "Lagrangian studies of incompressible turbulence in plasmas".

In accordance with the Enabling FAIR data Project guidelines, simulation data associated with this work is made permanently available on the author's FigShare repository `https://figshare.com/authors/Jane_Pratt/2136082`.

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Table 1. Simulation parameters: magnitude of the static macroscopic magnetic field B_0, the root-mean-square of magnetic fluctuations B_{RMS} averaged over the full simulation time, Kolmogorov time scale τ_η, the large-eddy length scale $\mathsf{L_E}$, and the Kolmogorov microscale η_{kol}. Each simulation is performed on a grid of $N^3 = 1024^3$, and uses 8.3 million Lagrangian tracer particles in a homogeneous random distribution.

	B_0	B_{RMS}	$\tau_\eta(10^{-2})$	$\mathsf{L_E}$	η_{kol} (10^{-3})	Re	elongation
HNOVIS	–	–	4.47	3.56	3.46	7571	1
ANOVIS0	0.	1.10	5.15	2.81	3.59	7230	1
ANOVIS3	3.	1.09	5.74	5.56	3.79	7570	2

Table 2. Simulation parameters: magnitude of the static macroscopic magnetic field B_0, the root-mean-square of magnetic fluctuations B_{RMS} averaged over the full simulation time, Kolmogorov time scale τ_η, the large-eddy length scale $\mathsf{L_E}$, and the Kolmogorov microscale η_{kol}. Each simulation is performed on a grid of $N^3 = 1024^3$ and uses 4.5 million particles initialized into droplets.

	B_0	B_{RMS}	$\tau_\eta(10^{-2})$	$\mathsf{L_E}$	η_{kol} (10^{-3})	Re	elongation
HPUFF	–	–	4.20	3.70	3.55	10570	1
APUFF0	0.	1.78	4.89	2.49	3.83	5640	1
APUFF3	3.	1.17	4.82	2.57	3.80	5940	2