

# Supporting Information for “Impact of vertical mixing parameterizations on internal gravity wave spectra in regional ocean models”

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In this supporting information (SI) section, we first describe the details of the ocean simulations in section 1. We then describe the methodology of calculating “WKBJ-stretched” depths in section 2 and the methodology of computing vertical wave-number spectra in section 3.

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The vertical level thicknesses of the simulations are shown in Fig S1 and the relationship between actual ocean depth and the WKBJ-stretched depth is in Fig S2. We show the sensitivity of the kinetic energy (KE) spectra at three different vertical resolutions of the model to the background mixing components of the K-Profile Parameterization (KPP) by comparing them with the KE spectra from McLane moored profilers (MP) in Fig S3. The effect of KPP background components in different frequency bands of the modeled KE spectra for the 264-level simulation is seen in Fig S4 where we see that in all the frequency bands, the modeled KE spectra with the KPP background turned off have a higher variance in the IW continuum. We find that the deep ocean (1500–4000 m) modeled KE spectra are sensitive to model vertical resolution as they progressively include more variance in the IW continuum with an increase in model vertical resolution (Fig S5). In S6, the ratios of modeled strain to MP strain in different frequency bands show that the model always has lower high-wavenumber strain. In Fig S7, we show the deep ocean (1500–4000 m) modeled strain spectra and see improvement in strain variance with an increase in model resolution.

## 1. Global and regional simulations

The numerical simulations in this study are hydrostatic simulations of the Massachusetts Institute of Technology general circulation model (MITgcm). The global simulation which forces the regional simulations in this paper has previously been referred to as LLC4320 (Rocha et al., 2016) and as MITgcm48 (Savage et al., 2017). This global simulation has a horizontal grid spacing of  $1/48^\circ$  ( $\sim 2$  km near the equator) and 90 vertical levels with a surface level thickness of 1m which progressively increases downwards to 480m (Fig S1).

The model depth is approximately 6 km in the deepest parts of the domain. The model uses a latitude–longitude–polar cap (LLC) grid and has dynamic sea ice implemented into it. The global spinup is taken from a lower resolution ( $1/24^\circ$ ) simulation. The simulation time step for this  $1/48^\circ$  run is 25 s. For our purposes, we use hourly snapshots of the model output. The model setup of the global LLC4320 can be found at [http://mitgcm.org/viewvc/MITgcm/MITgcm\\_contrib/llc\\_hires](http://mitgcm.org/viewvc/MITgcm/MITgcm_contrib/llc_hires) and the model output at <https://data.nas.nasa.gov/ecco/>.

The regional simulations in this paper have three different vertical grid spacings (see Fig S1), while the horizontal grid spacing is equal to that in LLC4320. The regional simulations begin on 1 March 2012 with the initial state from LLC4320 interpolated onto the requisite vertical grid. These simulations are forced by realistic atmospheric and astronomical tidal forcing (Arbic et al., 2018). The wind and other surface variables like temperature and humidity are updated on a 6-hourly basis from  $0.14^\circ$  European Center for Medium-Range Weather Forecasts (ECMWF). The atmospheric fields are converted to surface fluxes using bulk formulae of Large and Yeager (2004). At every model time step, a hard boundary condition is enforced at the lateral boundaries of the regional model to match the fields of the regional simulation to that of the global LLC4320. The horizontal mixing is represented by a modified Leith scheme (Adcroft et al., 2018) and the vertical mixing by the K-Profile Parameterization (KPP) of Large, McWilliams, and Doney (1994). The model uses the ocean bathymetry of Smith and Sandwell (1997). With a change in the vertical level thicknesses of the regional simulations, the time steps also need to be adjusted. The highest vertical resolution (264-level) simulation has a

time step of 10 s, the 153-level simulation uses 15 s, and the lowest vertical resolution (109-level) simulation was run with a time step of 25 s, the latter being the same as the global LLC4320 time step. We use hourly snapshots of model output from these regional simulations in all of our analyses. The regional simulation output and the analysis codes can be found at <https://doi.org/10.7910/DVN/HOVAP0>.

## 2. WKBJ-stretched depths

Internal wave propagation is affected by changes in ocean stratification. To account for the effect of variable stratification in computing the vertical wavenumber spectra, both velocity and depths from model and MP observations need to be normalised by the buoyancy frequency. The horizontal velocities are scaled by  $(\overline{N}(z)/N_0)^{1/2}$ , where  $N(z)$  is the Brunt–Väisälä frequency with the overbar denoting time average for the model run time or the MP deployment time, and the constant  $N_0 = 3$  cph following Leaman and Sanford (1975). The “WKBJ-stretched” or buoyancy-frequency-stretched depths ( $z_s$ ) are estimated according to the differential equation  $dz_s = (\overline{N}(z)/N_0)dz$ . The vertical wavenumbers are then  $1/\Delta z_s$  in units of cycles per stretched-meter (cpm). An example relationship between the actual ocean depths and the stretched depths for the 264-level model is shown in Fig S2. The vertical wavenumber spectra presented in this paper are based on the stretched depths.

## 3. Spectral estimates

The vertical wavenumber spectra are calculated as time averages of individual one-dimensional spectra of the fields in the vertical direction. For the simulations, the spectra are averaged over approximately 73 days of model runs and for the deployment days of

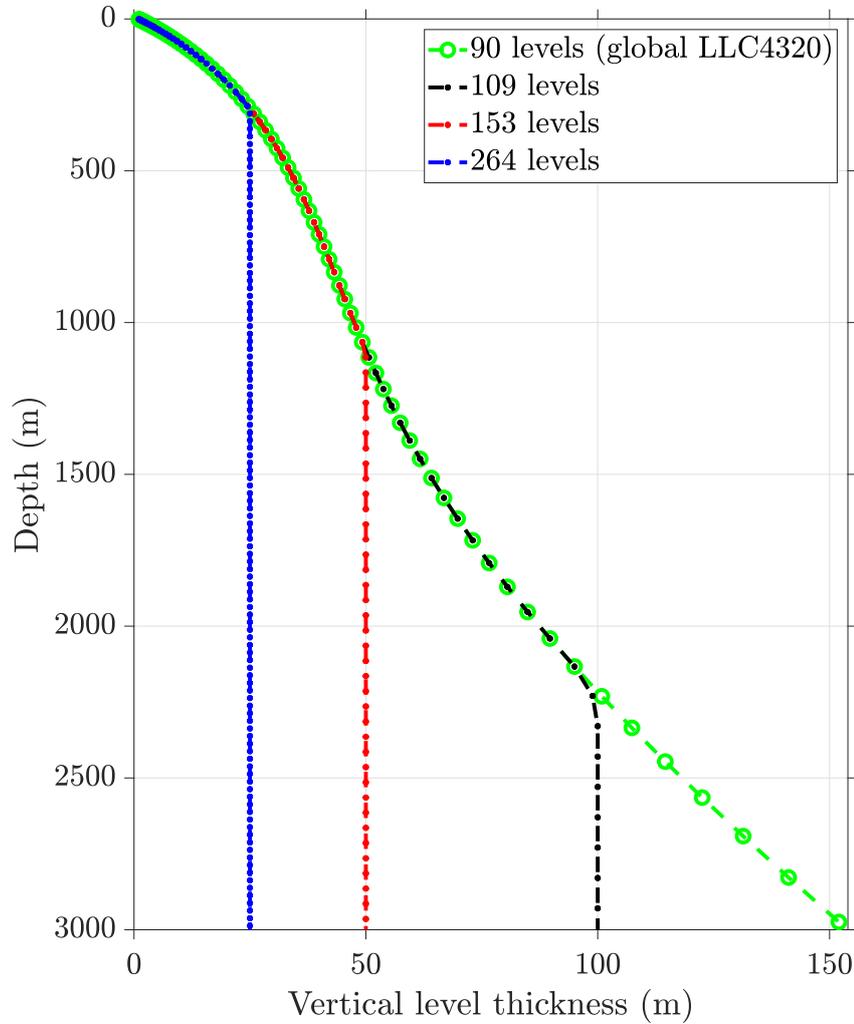
the MPs. The means and linear trends are removed from the model output and MP data before spectra calculations. A Hanning window is used to taper the velocities, temperature, and salinity at the vertical ends of the water column for all the spectral calculations. The reduced variances due to this tapering are added back into the spectral estimates. Ocean vertical velocities are generally smaller than horizontal velocities and hence not included here due to greater uncertainties in their measurements. Due to the relatively low number of grid points in the vertical direction (40 points for the 109-level and 58 points for the 264-level model in the 80–1400 m depth range), we use the whole water column to calculate the spectra for both model and MP, i.e., there is no segmenting of the velocities and scalar field involved unlike that used in traditional frequency spectra calculation using overlapping segments (e.g., the Welch periodogram method (Welch, 1967)). The highpass and the lowpass definition used in this paper have a cutoff of 11.5 hr and are hence the supertidal and subtidal frequency bands. The inertial period is taken as  $\pm 10\%$  above and below the local inertial period at each MP location. The inertial period is 27.84 hr at MP1 and 23.92 hr at MP4.

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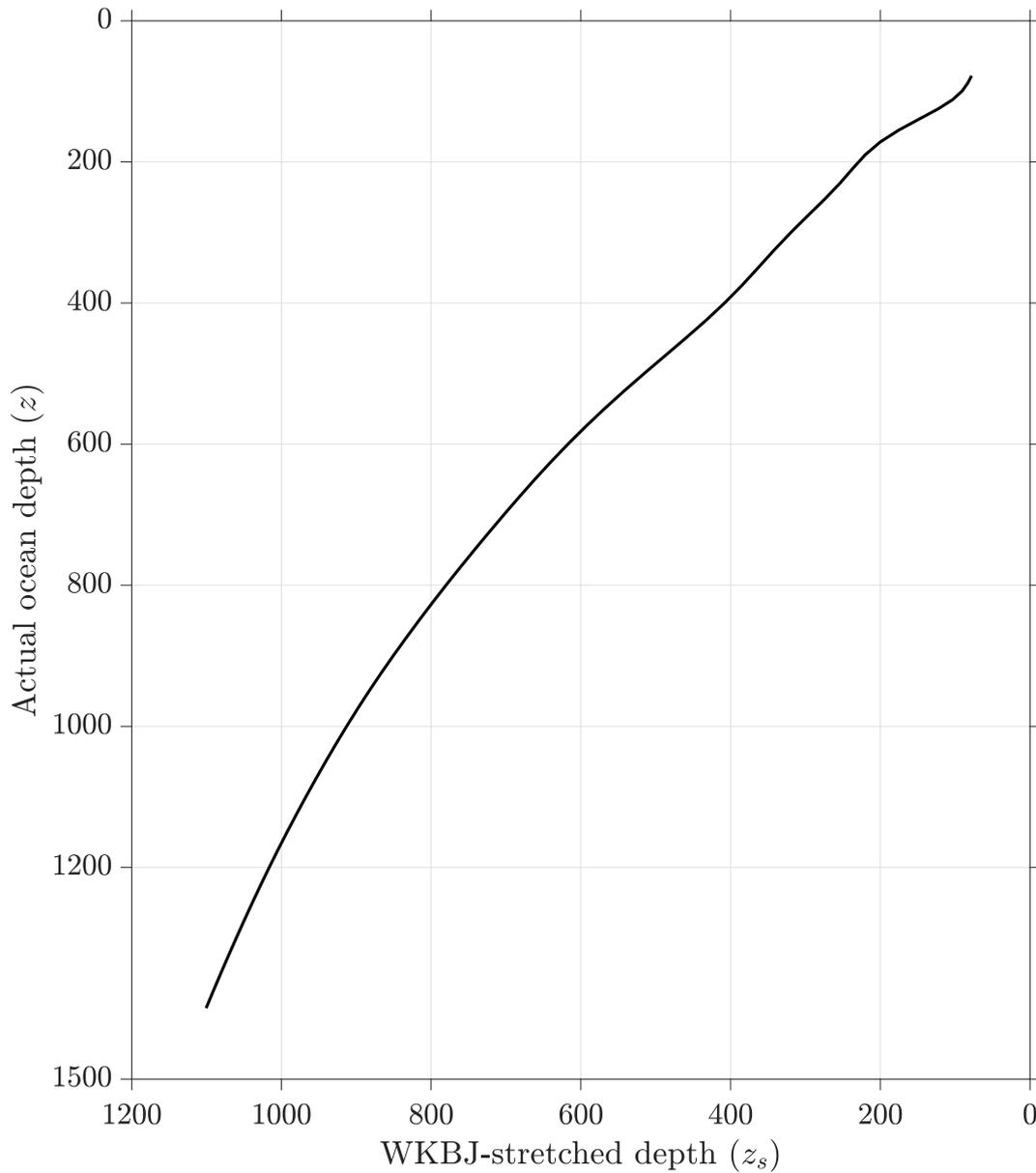
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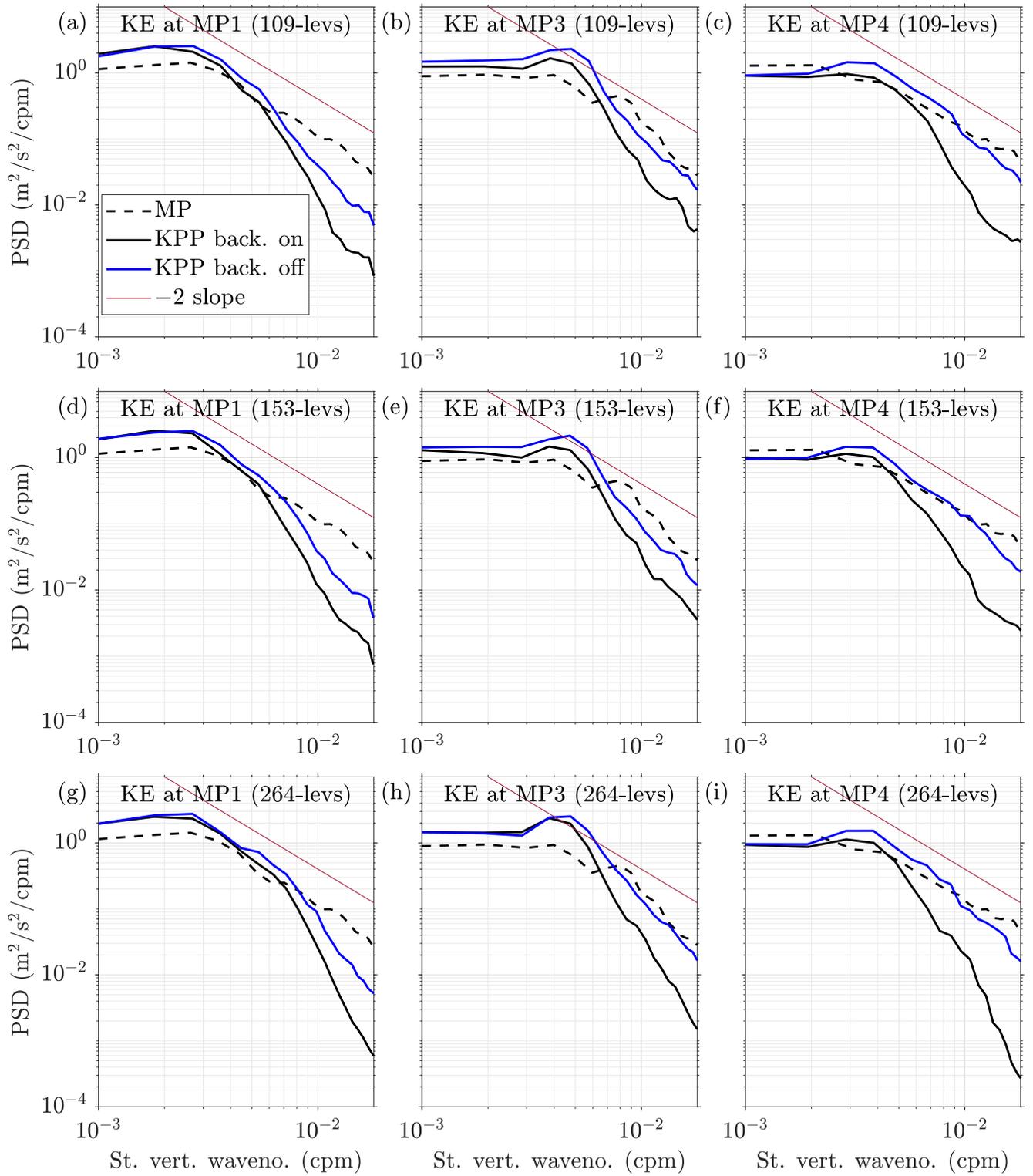
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**Figure S1.** Model vertical level thickness is plotted against ocean depth for the 109-level (black curve with dots), 153-level (red curve with dots), and 264-level (blue curve with dots) regional MITgcm simulations. The vertical level thickness of the global LLC4320 are shown with the green curve with circles. All have the same horizontal grid spacing of  $1/48^\circ$  ( $\sim 2$  km near the equator). The model depth is  $>6000$ m at this location but only the upper 3000m are shown.

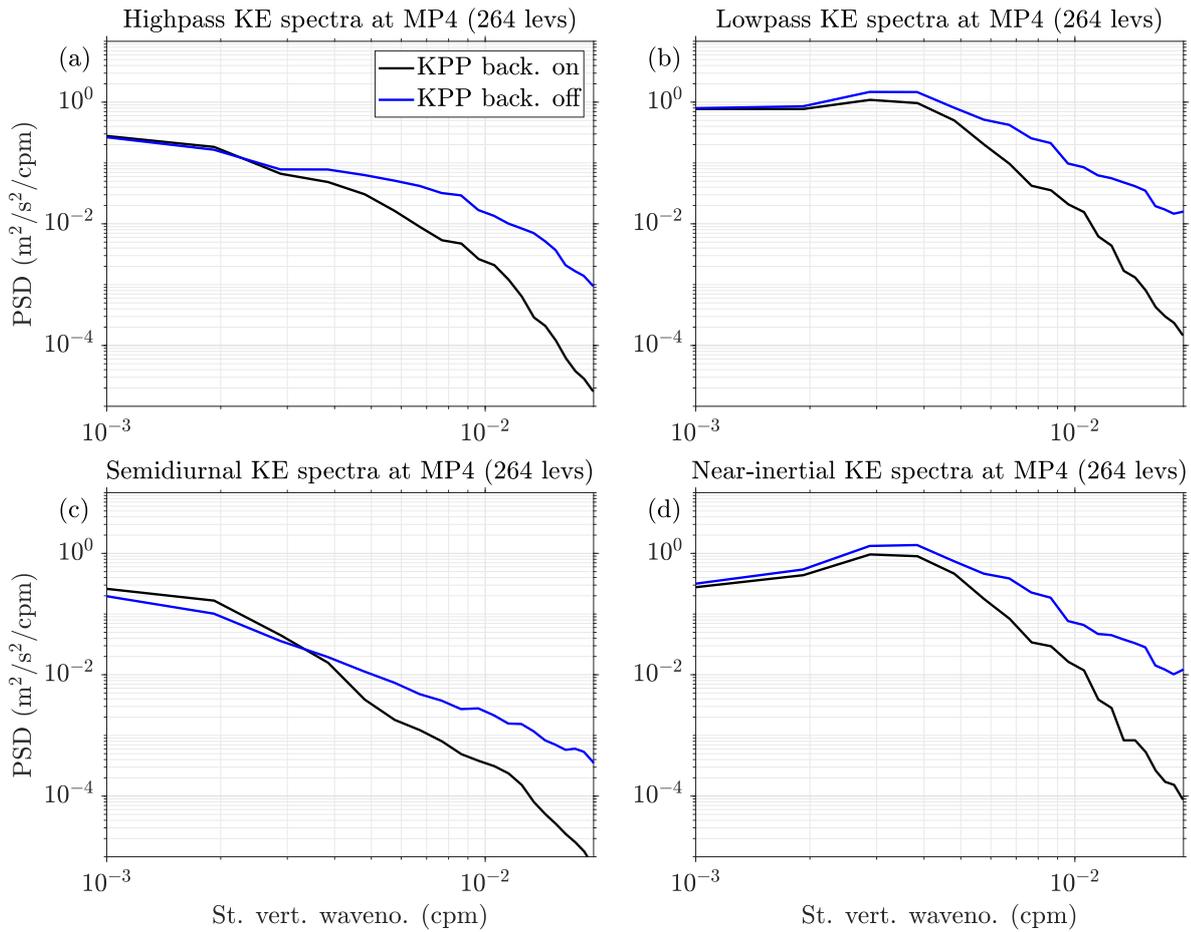


**Figure S2.** WKBJ-stretched or buoyancy-frequency-stretched depth is plotted against actual ocean depth from the 264-level regional simulation at the MP4 location. Despite being from a different year, MP observations have a similar stretched depth vs. actual depth relationship at this location. All vertical wavenumbers calculated in this paper are based on stretched depths. Only 80–1400m of actual ocean depth from the simulation is shown here because all spectra in the main paper are calculated in this depth range.

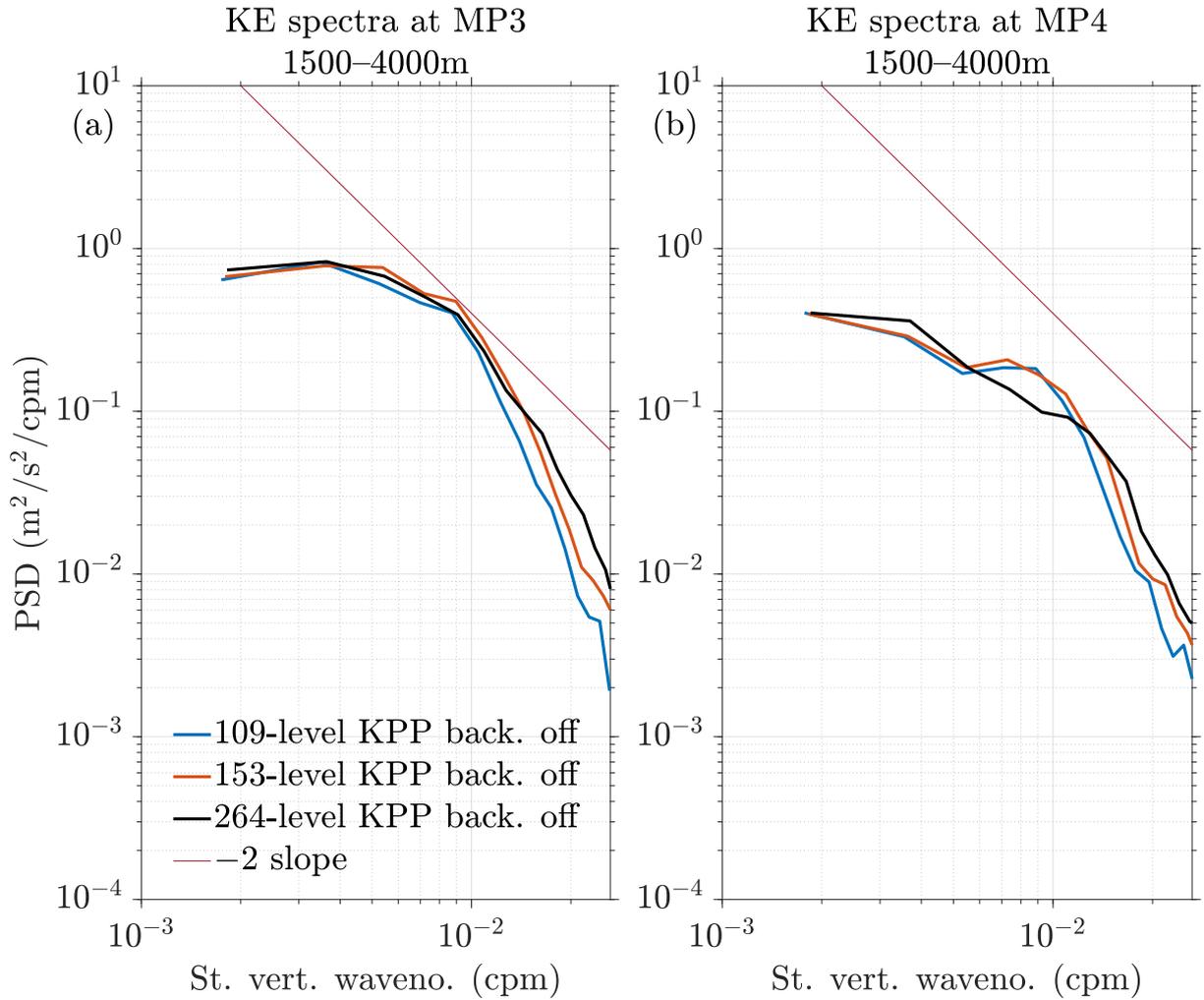


**Figure S3.** (Caption on next page)

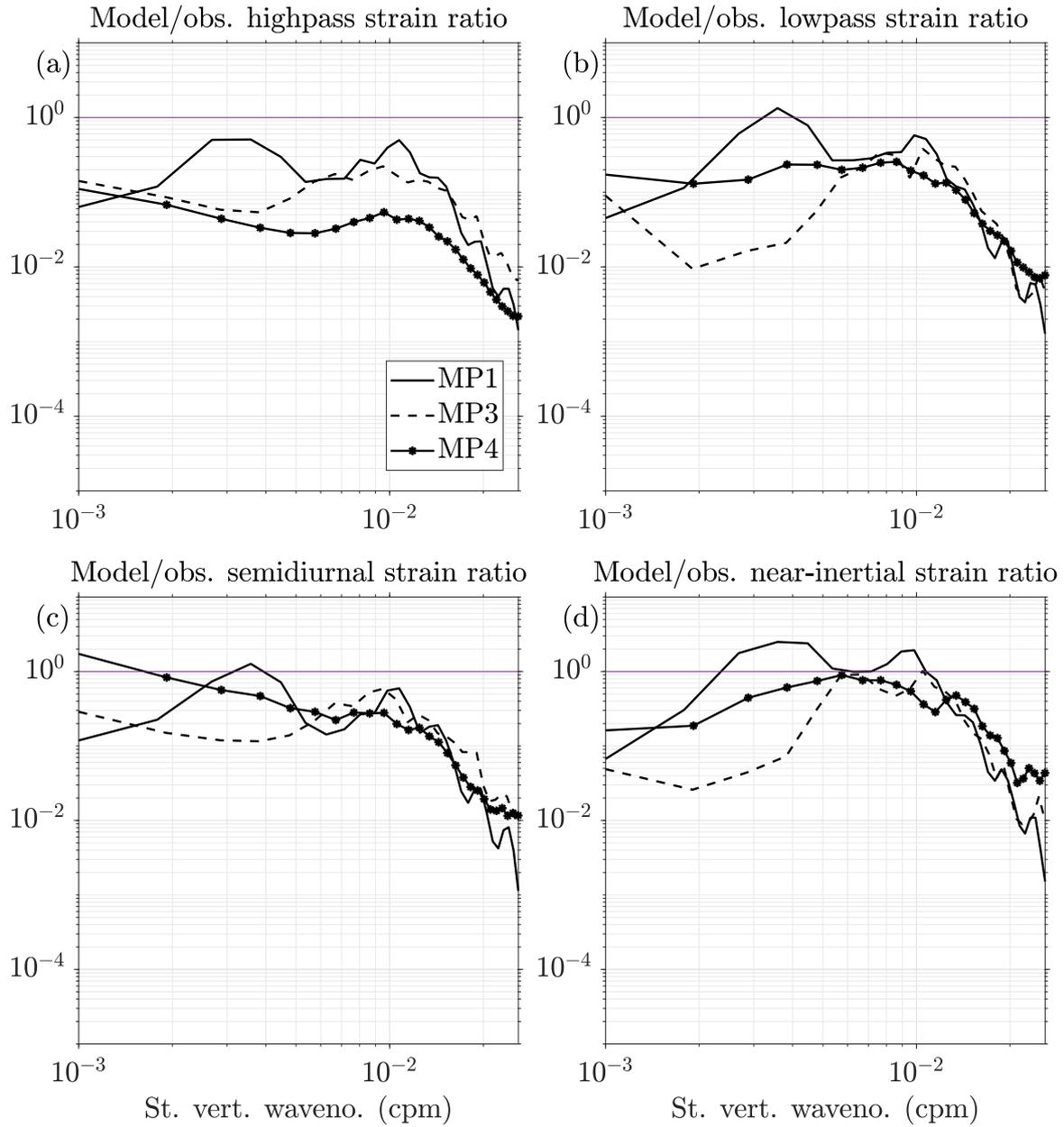
**Figure S3.** (Previous page) Vertical wavenumber spectra of modeled KE (solid curves) for simulations with (a–c) 109 levels, (d–f) 153 levels, and (g–i) 264 levels, and with the background diffusivity and viscosity components of KPP turned on and off, at different locations as marked in Fig 1 (b) of the main paper are compared to observation (MP) KE spectra. The blue curves represent modeled KE spectra with the KPP background diffusivity and viscosity turned off, while the solid black curves represent modeled KE spectra with the KPP background viscosity set to  $\mathcal{O}(10^{-4}) \text{ m}^2\text{s}^{-1}$  in the momentum equations and KPP background diffusivity set to  $\mathcal{O}(10^{-7}) \text{ m}^2\text{s}^{-1}$  in the scalar equations which are also same as that of the global model. MP KE spectra are shown in black dashed curves in each. Modeled KE spectra are calculated in the depth range of 80–1400m while the MP KE spectra are calculated in the depth range of 85–1384m. The GM76 KE spectral slope of  $-2$  is plotted in each for reference. The legends for all the subplots are in (a).



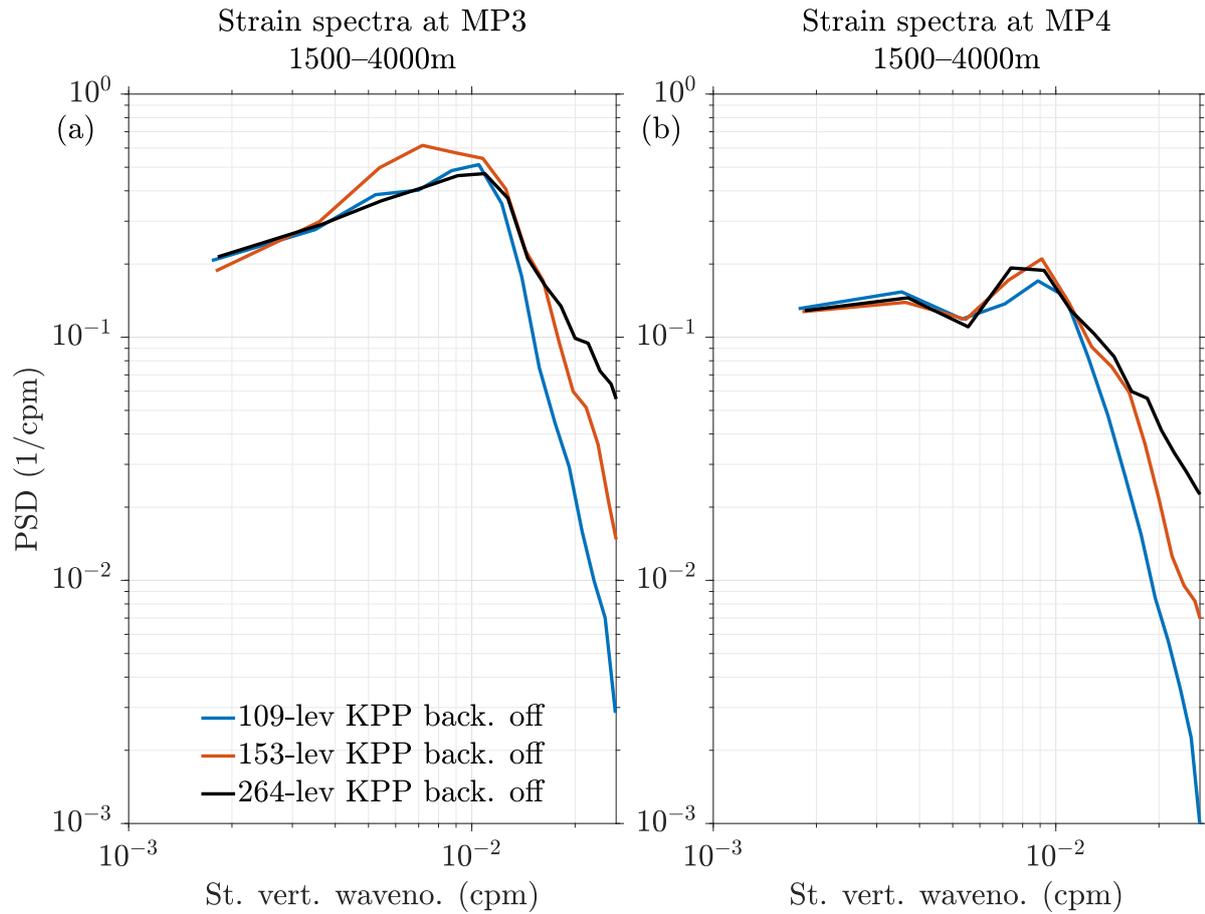
**Figure S4.** Effect of KPP background on the vertical wavenumber spectra of modeled KE at the MP4 location for the highest-resolution (264-level) simulation in the (a) high-pass or supertidal ( $>11.5$  hr), (b) lowpass ( $<11.5$  hr), (c) semidiurnal (11.5–13.5 hr), and (d) near-inertial (90–110% of 23.92 hr) frequency bands. These spectra are calculated in the depth range of 80–1400m. The modeled KE spectra with the KPP background turned off (blue curves) have increased high-wavenumber variance in all the frequency bands when compared to the KE spectra with the KPP background kept on (black curves).



**Figure S5.** Vertical wavenumber spectra of modeled KE in the deep ocean (1500–4000m) demonstrate the improvement of the high-wavenumber internal wave continuum with an increase in model vertical resolution at (a) MP3 and (b) MP4 locations. There are no MP measurements in this depth range to compare the modeled spectra with observational spectra. The GM76 KE spectral slope of  $-2$  is plotted in each for reference. The legends for both subplots are in (a).



**Figure S6.** The ratio of strain  $(N^2 - \overline{N^2})/\overline{N^2}$  from the 264-level simulation in the depth range of 80–1400m with the KPP background turned off and from MP observations at MP1, MP3, and MP4 in different frequency bands as described in Fig S4. The legends for all the subplots are in (a). Extra horizontal lines are drawn at values of unity for reference.



**Figure S7.** Vertical wavenumber spectra of modeled strain in the deep ocean (1500–4000m) demonstrate the improvement of strain variance in the internal wave continuum with an increase in model vertical resolution at (a) MP3 and (b) MP4 locations. All spectra are from simulations with the KPP background turned off. The legends for both subplots are in (a).