

Chameleonic noise in GPS position time series

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Key points:

- Position offsets attenuate most of the power spectrum at long periods and severely impact velocity estimates and their uncertainties.
- Due to estimated position offsets, the best-fitting noise color changes from a pinkish power-law to a white Gauss-Markov process.
- Position offsets severely reduce the chances of detecting random walk noise and long-period Earth deformation signals in long GPS series.

Abstract

It has been standard practice for about two decades to compute GPS-based station velocity uncertainties using the apparent noise statistics of the non-linear position residuals rather than assume white noise (WN) behavior. The latter choice would yield unrealistic velocity uncertainties. The most common noise types used are power-law, usually close to flicker noise (FN), over most frequencies mixed with WN at the shortest periods. The complicating impact of offsets in the position time series, mostly caused by equipment changes or tectonic events, has not been fully appreciated. These are far less benign than recently suggested. In addition to contributing a pseudo-random walk noise (RW) component to the velocity errors, estimating offset parameters changes the apparent noise color towards whiter, to the point that FN can no longer be considered the model that objectively best describes the observed GPS position spectra. Spectral power is effectively drained by offsets at periods longer than roughly the mean span between them. This consequently promotes a Gauss-Markov process as the preferred noise model and, importantly, obscures the presence of RW and long-period Earth deformation in the series. Both effects can lead to potentially under-estimated velocity uncertainties. The full value of decadal-long GPS time series for geodynamical applications is thereby greatly eroded by recurring offsets, especially when they occur quasi-regularly. In addition, contrary to common assumption, the noise color is generally not fixed with time, but clearly becomes whiter in more recent data. The origin of the colored noise and its whitening over time remain elusive.

1. Introduction and motivation

This paper addresses the central problem of GPS time series analysis: the separation of what is considered signal, i.e., the systematic variations, from what is considered to be noise, i.e., the random variations of usually unknown origin. A functional model is commonly fit to a GPS position time series to represent the signal, consisting of a linear trend, periodic variations, and irregularly timed position offsets as needed. It was realized in the 1990s that weighted least-squares fitting for the functional model with formal error propagation yields highly optimistic uncertainties for geodetic parameters. Then the discovery that the GPS-based errors are correlated both spatially (Wdowinski et al., 1997) and temporally (Zhang et al., 1997) led to more robust methods to assess and quantify the levels and types of noise in time series of GPS station positions (Mao et al., 1999). The main goal was to obtain more reliable uncertainties for GPS-based velocities, which are needed for many geodynamical studies such as measuring intra-plate tectonic stability and correcting vertical land motion from tide gauge records. A general consensus quickly emerged that GPS time series errors combine mostly white noise (WN) at shorter periods, typically less than a month, with flicker noise (FN) or similar power-law noise (PL) over longer spans, from monthly up to decadal periods (Mao et al., 1999; Zhang et al., 1997), although some level of background random walk (RW) error cannot be excluded. Consequently, velocity uncertainties can be under-estimated by as much as an order of magnitude if WN alone is assumed (Mao et al., 1999).

Development quickly followed of several mathematical tools to enable users to evaluate models of alternative noise types and amplitudes in observed GPS time series (Amiri-Simkooei et al., 2007; Bos et al., 2013; Langbein, 2004; Williams, 2008), which have been widely adopted for published noise analyses. One of the most common mathematical approaches found in the literature is the maximum likelihood estimator (MLE) which is well adapted to fit any noise model or combination of noise models to univariate time series. Very recently, this approach has been extended to analyze spatial common-mode noise models from regional GPS networks (Dmitrieva et al., 2015).

In this study, we analyze the impact the functional model, particularly position offsets, has on the apparent color of the noise model fitted to the position time series. Two very recent papers have raised questions about the orthodox interpretation of the long time series that are increasingly available nowadays. He et al. (2019) conclude that GPS noise does not flatten at low frequencies while assuming that the noise properties are stable over time, which we will reiterate below is not justified, in part due to the effect of occasional position offsets. Meanwhile, Wang and Herring (2019) attempt to show that the impact of position offsets is less serious than previously thought. These authors base their conclusions on serious misrepresentations. It is the objective of this contribution to clarify these points and to offer new perspectives on the information obtained from the colored noise in GPS position time series.

2. Background

2.1 Basics of time series computation

Most studies of GPS noise start with some particular set of time series with little or no regard for the process that was used to generate them or how alternative results might compare. It is, however, instructive to first consider the processing steps that produce observed GPS time series as this affects their noise content. “GPS” in the following can increasingly be regarded generically as “GNSS”, but in actual fact, for the long time series considered here, GPS observations have been used exclusively.

Three types of GPS data processing can be distinguished that produce time series of station positions: 1) full global network solutions free of any a priori over-constraints; 2) precise point positioning (PPP) of several sorts; and 3) regional network solutions. The International GNSS Service (IGS) and its Analysis Centers (ACs) are the prime examples of global network providers. Their occasional IGS reprocessing campaigns generate the GNSS inputs for the International Terrestrial Reference Frame (ITRF; Altamimi et al. 2016) and for many PPP solutions. PPP allows a single GNSS station to be positioned by assuming a priori satellite orbits and clocks from some given global solution provider (Zumberge et al., 1997). This is an efficient and autonomous method to densify a global solution that is usually limited in size by CPU constraints. The quality of PPP solutions is poorer when integer phase ambiguities are not fixed, but improve notably, especially in the east component, when double-differenced or one-way ambiguities are resolved, which is the usual case today. Regional networks are processed for many local or national applications requiring dense coverage, and can yield highly precise differential positions due to the natural rejection of common-mode errors.

The assignment of absolute geocentric coordinates to estimated station positions depends on the type of processing used. The IGS global network positions are realized, after combining independent AC solutions, by aligning each unconstrained daily frame via no-net-rotation (NNR) to the current linear ITRF coordinates using a "core" subset of well-distributed and reliable reference stations, typically numbering between about 40 and 80 (Rebischung et al., 2016b). This alignment process unavoidably redistributes non-linear displacements, both genuinely physical as well as technique artifacts, among the reference stations to all other network members (Collilieux et al., 2012). By including as many reference stations as possible, as uniformly spaced as possible, the net alignment-induced scatter should be much less than if only a few fiducials were used. Ray et al. (2017) showed that the long-term weighted root-mean-square (WRMS) of this rotational scatter is about 25 to 30 μs , equivalent to about 0.8 to 0.9 mm of equatorial motion, which corresponds to the estimated accuracy of daily IGS polar motion measurements. This same level of scatter also matches the observed WRMS error floor for daily IGS horizontal GPS positions after removing periodic variations, actually, about 0.9 and 0.8 mm for North and East components, respectively. Individual stations often display scatter that is considerably larger than these floors due to a variety of local effects. Ray et al. (2017) also found a WRMS error floor for non-periodic GPS station heights of 3.0 mm, which is commensurate with the usual ratio of GPS vertical to horizontal errors. The spectral qualities of the net frame alignment scatter were not studied by Ray et al. (2017) except to describe them as broadband and with more power at longer periods, i.e., reddish noise.

At a minimum, time series of PPP estimates inherit the net noise properties of their reference global network solution, and its ITRF alignment, that have been fixed in the processing. In fact, contrary to assertions by Zumberge et al. (1997), the actual performance of a PPP solution must always be degraded somewhat by loss of the full solution covariance information and by any potential deviation from identical modeling applied in the global network versus the PPP data reductions. (See further points in section 4.1).

Regional network solutions are particularly effective in estimating very precise horizontal positions and velocities by innately suppressing spatially coherent errors and by being generally less sensitive to large-scale effects, e.g., orbit errors. But they do not provide reliable absolute geocentric heights for the same reason and long-term alignments of regional frames to ITRF are subject to significant biases (Legrand et al., 2010).

2.2 Analysis effects in GPS noise

There have been few quantitative comparisons between the GPS noise characteristics from different IGS ACs or from different types of computation strategy (global, PPP, regional). Using the AC-specific station residuals from continuously updated operational IGS combined products might be instructive to some extent, but the interpretation is confused by comparison to the weighted AC mean. To address this concern, Rebischung et al. (2016a) formed individual long-term solutions for each AC in the second IGS reprocessing campaign (i.e., Repro2) and examined the station residuals. There was a large variation among ACs in the number of stations processed, but the quality of solutions was generally rather similar. The background non-periodic power is well described, except for JPL, as high-frequency WN plus a PL with spectral index near -1 for North and East and near -0.8 for up. The JPL behavior differs in having less WN, which is undetectable for heights, and having smaller PL spectral indices of -0.72 to -0.74 for North and East and -0.52 for up.

On the surface this result seems to imply that WN is largely a product of data analysis and not derived from GPS observational noise. But that conclusion could be undermined by the facts that the JPL Kalman filter processor might simply absorb WN into its parameter estimates or that the JPL use of data arcs that overlap by 3 hr with adjoining days before and after could exert a strong smoothing effect. EMR/NRCan is the only other IGS AC using the JPL Kalman filter but they fit strict 24-hr arcs and have noise properties similar to the other non-JPL ACs, which supports the second possibility of implicit smoothing.

The attenuation of WN is at least partly produced by the implicit smoothing effect of including 3 hr overlaps from the previous and the following daily observations. This is the case in the JPL processing, but it is less clear how any difference of the JPL analysis scheme can reduce their spectral indices by ~ 0.3 . After all, the geophysical processes that affect GPS data and positions, such as surface pressure loading, contribute effects that have distinctly red spectral variations, at least up to periods of a year or longer. One guess is that the critical element might involve JPL's estimation of satellite solar radiation pressure (SRP) variations as 3D stochastic offsets rather than as harmonic parameters, as most ACs do, but EMR also uses 3D stochastic SRP estimation.

Concerning position offsets, Williams (2003) showed that the presence of undetected and unmodeled position offsets in GPS time series contributes a RW noise component. He also evaluated the impact of adding offset parameters to the functional model, for various background noise types. Griffiths and Ray (2016) quantified the magnitude of the RW-like noise due to adding position offsets in an ITRF-type global network solution. Under these circumstances and because some station hardware changes cause discontinuities while others do not, different ACs are expected to introduce varying amounts of RW noise when they use incomplete metadata for the station equipment changes. This effect is probably not dominant among AC noise differences, however.

2.3 Assumptions of noise analysis and the evolution of colored noise

In constructing analytical methods to evaluate GPS positional noise, a number of assumptions must be made, some of them by implication only. Basic is the natural notion that the total observed GPS noise is a superposition of different physical processes each with its own spectral characteristics. Some processes might be so slight, compared to others, that their noise effects cannot be detected directly or the data span that would be needed is very long, e.g., RW. Under these circumstances, it is not generally possible to invert a GPS spectrum to infer its constituent noise components. Instead, forward modeling of a range of possible noise types and amplitudes is usually performed and tested by best-fitting against the observations, which is of course non-unique and prone to subjectivity.

Another key assumption almost always made is that the mix of noise characteristics for any given time series is constant over time, despite observations of Williams et al. (2004) and Santamaría-Gómez et al. (2011) to the contrary. Time series, especially long ones, often display non-stationary noise properties that are evident to the eye. For examples, please view the JPL PPP website at <https://sideshow.jpl.nasa.gov/post/series.html> and check plots for these long-running stations: ALIC, CAS1, CHUM, COCO, DAV1, GENO, KIRO, KIRU, MAS1, MATE, MEDI, NRC1, ONSA, PIN1, RAMO, SFER, TOW2, UCLU, ZIMM. There has been an overall trend towards improved GPS tracking hardware and more robust ground networks on the one hand, but degradations and gradual failures at individual stations can happen at any time. This is why visual inspection of the series, as done in this study, is of uttermost importance to support the conclusions of a given analysis. The JPL plots also illustrate the pervasiveness of position offsets in any GPS solution, which average about 0.9 per station per decade in the IGS Repro2 global analysis (Rebischung et al., 2016b), the dataset used by Griffiths and Ray (2016).

3. Data and methods

In this study we used residual daily GPS position time series from the JPL online service (<https://sideshow.jpl.nasa.gov/post/series.html>; accessed in May 2019). These series are estimated using the PPP technique with phase ambiguity resolution and JPL final orbits and clocks products. The daily PPP frames were NNR-aligned to the IGS14 realization of ITRF2014 (Rebischung et al., 2016b) using transformation parameters estimated at the same time as the orbit/clock products in a full global network solution. JPL data reduction models and procedures were updated following Repro2 (IGS Mail 7637, 5 June 2018) and applied in the network solution and in the PPP processing, including antenna calibrations consistent with the IGS14 frame. This latest JPL PPP reprocessing ensures the temporal coherence of the historic series with their current operational products. The residual series were obtained after removing linear trends, seasonal variations, position offsets, and outliers. JPL uses an automatic offset detector that Gazeaux et al. (2013) ranked as the second-best automatic solution in their study.

From the initial set of 2567 stations available, we retained 137 stations having at least 20 years of data since 1999, less than 10% of missing points, and gaps not longer than 5 months. These 137 stations were screened for quality using SARI (Santamaría-Gómez, 2019) in which we removed several outliers most probably caused by snow/ice on the antennas. In addition, we added position offsets for 8 stations that were visually identified and judged to be significant against the colored noise content in the series. After this check, we rejected 8 more stations having unusual residual distributions or strong changes in their variability, leaving 129 long, high-quality stations for this study (see all the information in the supplemental material). We observed a very irregular temporal sampling for some of these stations with, occasionally, multiple points estimated on the same day separated by only a few hours. This is probably caused by JPL merging their global network and PPP estimates for the same station. To avoid this, we removed the daily estimates with epochs before 6 h UTC or after 18 h UTC. We also note there are more than 3 offsets per station per decade on average among the selected 129 stations, which is particularly large compared to the value for the IGS Repro2 series.

We acknowledge the JPL PPP series have distinct noise characteristics compared to other GPS position series, but they provide the longest homogeneously reprocessed residual series to date, before the next IGS reprocessing campaign, and for an extensive number of global stations. Judging from IGS combination reports (see weekly orbit and frame combination summaries at <https://lists.igs.org/pipermail/igsreport/>), the JPL analysis quality is outstanding also. This dataset is

therefore the best available for addressing several points concerning the noise content and the impact of position offsets.

In the following, we used the Lomb-Scargle periodogram (Scargle, 1982) to represent the error power spectra and the Create and Analyze Time Series (CATS) software (Williams, 2008), which implements MLE, for the estimation of the noise power spectra. Here, we define, on one hand, the noise power spectrum as the power spectrum that is obtained from a noise analysis (e.g., CATS) that is conveniently used to assess the formal errors of a fitted model. On the other hand, we define the error power spectrum as the power spectrum that is actually observed in residual GPS series. In other words, the error spectrum represents the spectral distribution of the variance that is not explained by the reduction of the raw GPS observations nor by the fitted model to the series, and it includes random and systematic variations, both physical and spurious. Generally speaking, the noise spectrum is obtained by fitting the error spectrum with a particular stochastic process or mix of processes. Ideally, the noise spectrum and the error spectrum should be very similar, but we show in the next section that this is not always the case and that a distinction between them needs to be recognized. Since JPL did not fit the draconitic harmonics in the PPP series (see Fig. 1), we removed up to the tenth harmonic of the 1.04 yr draconitic period before estimating the noise spectrum with CATS.

4. Results

4.1 Noise types in the JPL PPP series

The stacked error spectra of the 129 residual JPL series are shown in Fig. 1. Overall, the error spectra have a crowbar shape that could be described by a power-law process up to periods of ~ 5 years. The slope of the power-law is clearly lower than FN, especially in the vertical component. Beyond 5 years, the power spectra decay significantly in all the components. At the shortest periods, we observe that WN is not visible in any coordinate component. The PL clearly dominates the stacked spectra even at high frequencies of at least 100 cycles-per-year (cpy) or more. To quantify an upper bound of the WN amplitude, we assume the PL is actually dominated by FN, which is likely for the horizontal components, but not so much for the vertical component as we discuss later on. Considering a

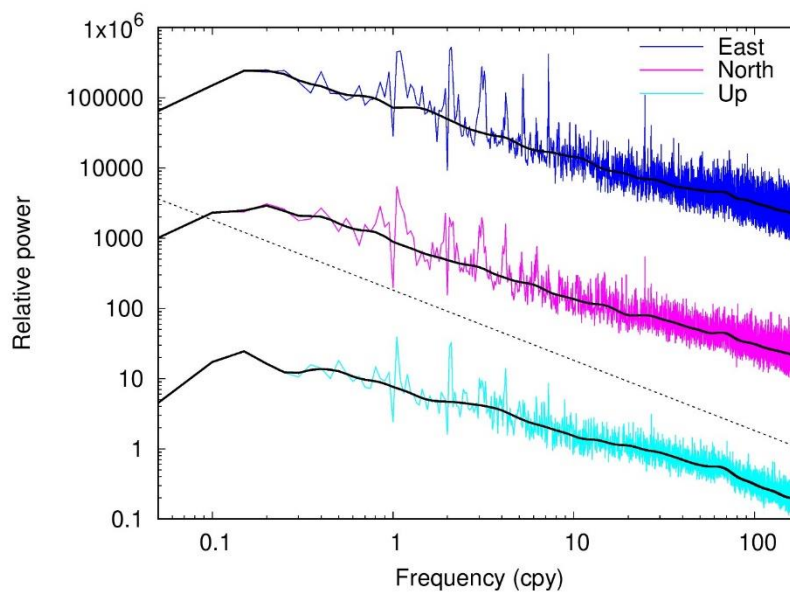


Figure 1. Stacked error spectra of the 129 selected JPL series for the three coordinate components. The spectra are shifted for the sake of visibility. The thick solid black lines represent the smoothed spectra. The thin dotted black line represents a pure flicker noise process.

median FN amplitude for the daily series of $4 \text{ mm yr}^{-1/4}$ for the horizontal and $11 \text{ mm yr}^{-1/4}$ for the vertical component, and a cross-over period from FN to WN of less than 4 days, we bound the maximum white noise level to be 0.4 and 1.1 mm for the horizontal and vertical components, respectively. This is much lower than the WN reported for the other AC solutions of the last IGS Repro2 (Reischung et al., 2016a) and reflects the smoothing inherent in the JPL global processing used for their PPP series. In addition, Ray et al. (2013) and Amiri-Simkooei et al. (2017) have found spurious ~ 5 d signals in the preceding JPL PPP time series that are not present in their corresponding global solutions, the source of which was unknown. The ~ 5 -day signals reported in those past studies are not visible in this latest JPL solution, but nevertheless we still note a bump around 5.5 days (~ 65 cpy) common to all the components and of unknown origin, but possibly related to the overlapped $3 + 24 + 3$ hr data arcs used by JPL. The power spectrum decays faster for periods shorter than 5.5 days in the vertical component only.

Because white noise obviously does not play a part in the background error spectrum, in the following noise analysis we average the daily series into weekly series in order to speed up the processing without losing information on the colored noise content.

Using CATS, we estimated the type and amplitude of the colored noise that best describes the error spectra of the individual series. To assess the impact of offsets in the noise analysis, we assume the background noise content of the series is constant and can be described by a unique noise model. We will demonstrate later that this hypothesis, commonly applied in most noise analyses, is not valid. At least we can assess how well a single model describes the background noise because all our series are available for the same period of time. Four noise models were tested: FN, PL, flicker plus random walk noise (FNRW) and a generalized Gauss-Markov noise model (GM), which includes a PL process at high frequency and a WN process at low frequency (Langbein, 2004). In order to discriminate the goodness of fit for each model, we used the estimated maximum likelihood plus an empirical threshold computed by Santamaría-Gómez et al. (2011) that accounts for the different degrees of freedom of each noise model. We verified the validity of these empirical thresholds using ad hoc FN synthetic series.

The distribution of the best fitted noise model per component (labelled “Full series”), the noise model parameters and its degrees of freedom are given in Table 1. The FNRW noise model (2 degrees of freedom) is not included in Table 1 because it was not selected as the preferred noise model in any of the series. These results corroborate our qualitative description of the stacked error spectrum in Fig. 1, i.e., the GM model dominates the other noise models and it is clearly the best model explaining the crowbar shape of the stacked full-series error spectrum. We observe that the spectral index of the GM process is significantly closer to FN than RW, consistent with the error spectra shown in Fig. 1 and indicating that the First-Order GM noise process used by Wang and Herring (2019), which combines WN at long periods with RW at shorter periods, may not be the best model to describe the noise in most of the series.

Figure S1 in the Supporting Information shows the distribution of the 129 stations used in this study together with their preferred noise model for each component. Most of the stations are concentrated in Europe, Australasia and, especially, Western USA. To explore to what extent the stacked error spectra of Fig. 1 could be biased by the geographic distribution of the 129 sites, we computed two additional smoothed stacked spectra corresponding to the stations located in Western USA (64) and Europe (36). Figure S2 in the Supporting Information shows that while the spectra from the regional subsets are more variable due to the smaller number of sites, their general shape is consistent with the one from Fig. 1.

Table 1. Percentage of the best fitted noise model per component. The numbers in parenthesis after the noise model names in the first column provide the degrees of freedom. The numbers after the different percentages indicate the 5%/50%/95% values of relevant noise model parameters for the selected fits (σ for variance in mm, k for spectral index and β for cross-over period in years). Note that the variance corresponds to weekly sampled series.

		East	North	Up
FN (1)	Full series	16% $\sigma=2.0/2.3/2.9$	7% $\sigma=1.9/2.2/2.6$	2% $\sigma=6.5/6.7/6.7$
	Segments	51% $\sigma=1.9/2.1/2.7$	37% $\sigma=1.8/2.3/3.3$	21% $\sigma=5.7/6.8/11.1$
PL (2)	Full series	9% $\sigma=2.0/2.3/3.1$ $k=-0.62/-0.83/-1.13$	9% $\sigma=1.8/2.1/4.0$ $k=-0.48/-0.74/-0.87$	21% $\sigma=5.1/6.4/8.3$ $k=-0.61/-0.69/-0.82$
	Segments	27% $\sigma=1.6/2.1/2.8$ $k=-0.57/-0.76/-1.16$	25% $\sigma=1.3/2.0/3.3$ $k=-0.45/-0.72/-1.28$	64% $\sigma=4.4/5.7/7.9$ $k=-0.52/-0.70/-0.82$
GM (3)	Full series	75% $k=-0.92/-1.20/-2.26$ $\beta=0.2/1.1/2.2$	84% $k=-0.98/-1.29/-2.09$ $\beta=0.3/0.8/1.7$	77% $k=-0.82/-1.05/-1.51$ $\beta=0.3/0.8/1.7$
	Segments	22% $k=-1.10/-1.51/-3.36$ $\beta=0.2/0.5/2.1$	38% $k=-1.14/-1.49/-2.38$ $\beta=0.3/0.6/1.2$	15% $k=-0.97/-1.25/-2.18$ $\beta=0.2/0.4/0.9$

4.2 Loss of low-frequency power

Although the GM model is by far the best fitting amongst those tested over the full data spans, the power spectrum of this noise model does not explain the sharp loss of power at long periods observed in the error spectra of Figure 1 (Langbein, 2004). It is known that the trend estimation absorbs part of the colored noise at the longest period and that the estimated colored noise from the residual series is biased low for short series (Langbein & Johnson, 1997). Here we argue that a significant amount of the low-frequency power spectrum is also absorbed by the estimated position offsets.

To demonstrate this, we first reran the noise analysis, but using this time the continuous segments of the series between each pair of estimated offsets. From our initial 129 series of 20 years, we extract 163 segments longer than 5 years with a median length of almost 9 years. Table 1 shows the results from this second run (labelled "Segments"). The dominant noise model for the East component is now FN. For the North component, it is equally distributed between FN and GM, closely followed by PL with a median spectral index of -0.7 . For the vertical component, the dominant noise model is clearly PL with a median spectral index of -0.7 . We also observe that GM, while not being the preferred noise model in any component anymore, still fits better the error spectrum of a quarter of the series on average. The FNRW was still not selected in any of the series. Considering the three components together, the preferred noise model is PL with a spectral index lower than FN. We note however that it is difficult to reach solid conclusions about the general noise model that best describes the error spectrum when the series do not have the same length nor cover the same period of time. This exercise at least confirms that the GM model is apparently promoted by the effect of estimated position offsets in long discontinuous series.

Comparing the ratio of the selected GM models between the “Full series” and the “Segments” runs in Table 1, one would be tempted to state that the effect of offsets is to triple the chances to select a GM model on average for any component. However, this increase should be considered the upper bound effect. The true effect is very likely smaller because by extracting segments of the original series and comparing their noise content to the original 20-year series we are repeating the common flaw of comparing what is not comparable, i.e. comparing the noise content between series having different length. A more rigorous approach that could be applied, especially when comparing noise levels from different sets of series, is to use continuous segments of the same length and period, at the expense of reducing the number of exploitable series. Since that was not possible in this case, we carried out another test to demonstrate the impact of the offsets in the error spectrum by creating synthetic series with the same sampling as our 129 original series and different amplitudes of FN from Table 1. These series were then fitted with a linear trend and the same offsets as in the original JPL series. Figure 2 (black curves) shows how the FN power is absorbed at long periods slightly by the trend and mostly by the offsets. The simulated FN power spectrum starts flattening around a period of 3 years and the power starts dropping around a period of 7 years, which lies close to the median offset separation of the dataset of 8.2 years. The decay follows the crowbar shape of the real error spectra reasonably well. This indicates that, very likely, the error spectra of the GPS series actually would not decay or flatten if the series were continuous.

4.3 Detectability of RW noise

The loss of power at long periods caused by estimating offsets hinders the analysis of the error sources that may contribute in that part of the spectrum, for instance, the RW noise. In addition to a RW contribution due to remaining offsets in the series (Williams, 2003), it has been suggested that the instability of the antenna monument introduces some level of RW process noise in all series (Johnson & Agnew, 1995). For the best geodetic-class monuments, the amplitude of RW is expected

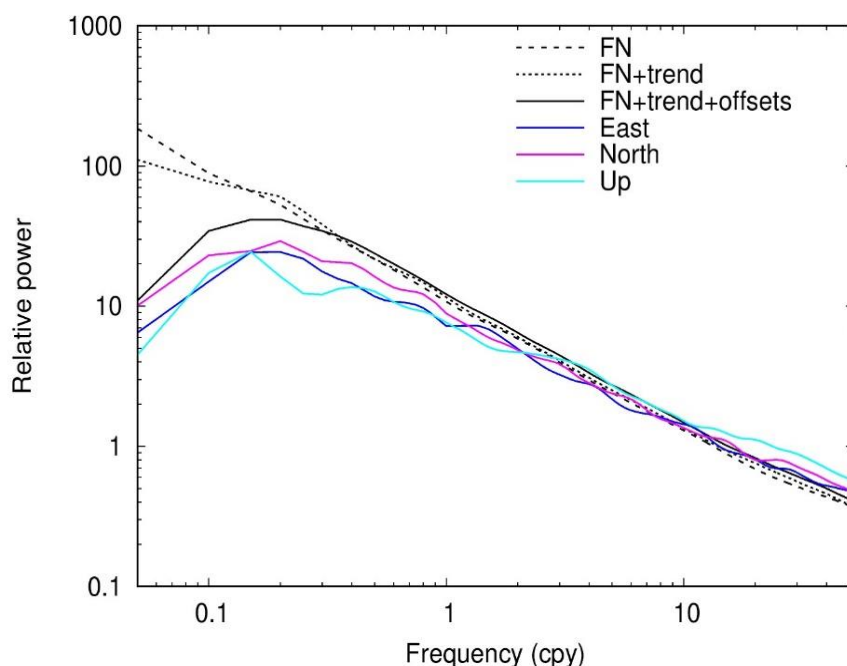


Figure 2. Comparison of simulated (in black color) and real (in colors) smoothed error power spectra. The real spectra for East, North and Up are the same as in Figure 1. The simulated spectra includes flicker noise (dashed), flicker noise with trend estimation (dotted) and flicker noise with trend and offsets estimation (solid).

to be several times smaller than the amplitude of FN and, therefore, it can only be detected in very long series or when the spatially-correlated FN is reduced, for instance in regional solutions and in short-baseline solutions (Dmitrieva et al., 2015; Hill et al., 2009; King & Williams, 2009; Langbein & Svarc, 2019). In our dataset of 129 stations of 20 years length we did not detect RW being significantly present in any of the series. Recent studies using network or PPP global solutions also report that RW does not play a significant role in the error spectrum (e.g., He et al. 2019). The low proportion of detected RW in global solutions has been attributed to the dominance of FN and the series not being long enough (Williams et al., 2004), to which we should add the estimation of velocities and the position offsets.

Since the estimated position offsets absorb significant amounts of power in the band of the error spectrum where we expect RW to emerge, our chances to detect RW in long series are reduced. A lower probability to detect RW implies that the amount of RW potentially being hidden in the series could be quite large relative to the amount of FN. To quantify this, we ran a new noise analysis using synthetic series of 20 years having FN+RW in different proportions and with offsets being estimated at different intervals. Figure 3 shows that the success rate of detecting RW over a general PL is heavily affected by the amplitude ratio FN/RW, by the velocity estimate and in smaller amount, but still significant, by the length of the continuous series without offsets. The impact of the offsets is particularly important when the FN clearly dominates over the RW. As an example, for a series having an estimated offset every 6 years, which is a common scenario, the daily series could accommodate RW amplitude equal to 1/6 of the FN amplitude, i.e., typically $\sim 0.7 \text{ mm yr}^{-2}$ for the horizontal components and almost 2 mm yr^{-2} for the vertical component, and yet, we will have less than 10 % chance to detect this RW component. Lower RW amplitudes have been reported in past studies (King & Williams, 2009), which implies even less chances to detect it. For the same scenario, the success rate more than doubles if the offsets were not present in the series and, ideally, tops around 45 % if the velocity was not estimated (Fig. 3).

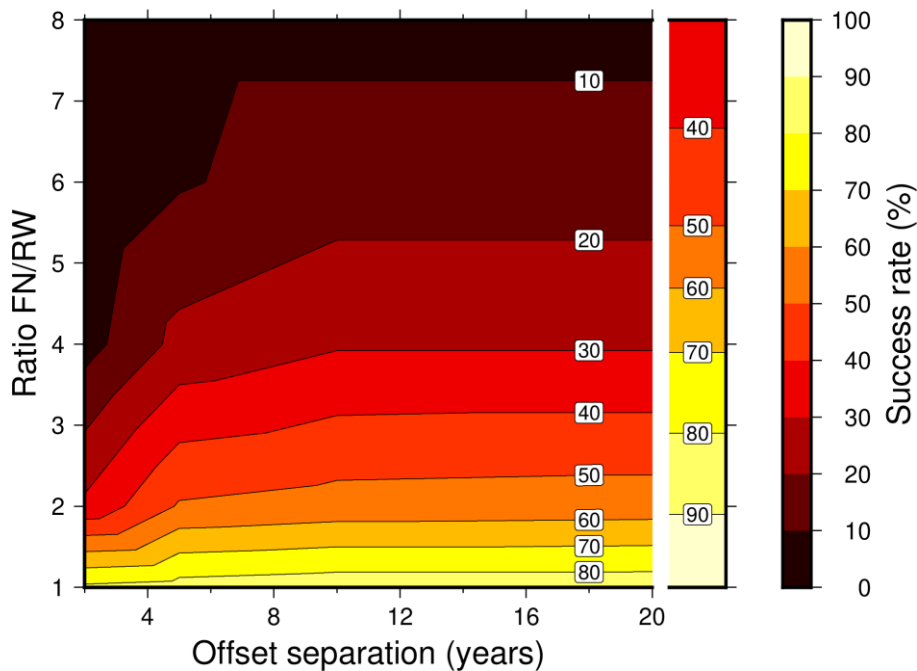


Figure 3. Success rate of random walk detection in a FN+RW model against a PL model as a function of the amplitude ratio between flicker noise and random walk noise (y-axis) and as a function of the maximum length of the series without offsets (x-axis). The separated box to the right of the main image represents the success rate when the velocity and the offsets are not removed from the series. The impact of removing the velocity is determined as the difference between the rightmost values of the figure and the separated box on the right of the figure.

4.4 Impact of frame alignment on error spectra

It is clear that the functional model can severely alter the error power spectrum. Therefore, a natural question arises as to what would be the effect of the frame alignment on the colored noise of the series. The frame alignment is part of the functional model in any network solution and is also implicitly applied to any PPP solution via the fixed orbit and clock products from the network solution. To answer this question, we used 20 years of weekly IGS SINEX files starting in 1999. We modified the SINEX files by extracting the IGS14 core stations and replacing their weekly station coordinates by their linearized positions, extracted from the mean position and velocity of the IGS long-term frame, plus a synthetic FN series for each station. The idea is to quantify how much the FN series change after the frame alignment process.

The variance-covariance of the weekly network solutions was left intact in the SINEX files. By doing this, we assume the spatial correlation of position errors and its changes through time are independent of the temporal correlation of each individual station. This hypothesis was necessary as it is not practically possible to synthesize a covariance matrix that accounts for FN in both the temporal and spatial domains at the same time in a least-squares network solution (Benoist et al., 2020). In addition, this assumption is commonly applied in most studies when the colored noise content in a given series is estimated with no regard for its spatial correlation.

We performed two cumulated solutions using these modified SINEX files with and without the estimation of the transformation parameters of the frame alignment. In addition, we examined the impact of including/excluding a weekly scale offset and also the impact of the full/diagonal covariance matrix of the SINEX files, i.e., the spatial correlation. Our results indicate that neither the frame alignment nor the consideration of the spatial correlation have a significant impact on the retrieved FN series. The velocity estimation absorbs a small amount of the FN variance at the longest period, as expected, but the estimation of the transformation parameters does not alter significantly the background error spectrum of the residual series after the alignment (Fig. S3). This result also validates the hypothesis that the temporal and spatial correlations can be considered independently when analyzing the noise content of individual series. On a series-by-series basis, the FN amplitude changes typically less than 0.1 mm for both horizontal and vertical components, even if the added FN amplitude was ~3 times larger in the vertical component. The FN change is marginally larger in the vertical component if the scale parameter is estimated. For most of the stations, the FN amplitude increases in the horizontal component and decreases in the vertical component.

4.5 Evolution of noise types over time

Concerning changes of the colored noise content with time, Williams et al. (2004) and Santamaría-Gómez et al. (2011) showed that the amplitude of the PL content in the series is generally getting smaller with time. Bos et al. (2010) also questioned the hypothesis that noise properties are constant over time. In that case, the estimated noise content of a given series will depend on the period of time chosen. This dependency can void the conclusions from separate studies about the average noise level in a particular set of series if they do not cover the same time period. It can also have a significant impact on comparisons of noise levels between different sets of series (Amiri-Simkooei et al., 2017; Langbein & Svarc, 2019).

Here, we revisit this issue by using our selection of 129 series from the last JPL PPP solution having the same length, minimum gaps, homogeneous good-quality data from 1999 onwards, and almost twice as long as those in Santamaría-Gómez et al. (2011). We split the original 20-year series into 4 segments of 5 years each and ran separate noise analyses for each segment including the same four

noise models (Fig. 4). We observe that not only the PL amplitude reduces, but the noise color itself changes as well throughout the last 20 years of GPS observations (see Table 2). The proportion of FN and GM is falling in favor of a PL with a spectral index becoming whiter (Table 2). Part of the apparent change of the noise color is undoubtedly caused by the irregular distribution of the estimated offsets in the series (Fig. 4d). Offsets are more frequent in the first half of the series than in the second, particularly before 2005. For instance, in 2002 and 2004, around 60% of the stations had at least one estimated offset. After 2005, the number of removed offsets stabilizes. If the noise amplitude and type were to be constant, we should expect the noise to get redder if less offsets were removed in the second half of the series, but we observe the contrary. This result confirms earlier findings that colored noise is becoming whiter for newer observations, but in a more general framework, i.e., considering also changes in the preferred noise model from FN towards PL. The whitening of the noise color can also be seen in the reduction of the median formal velocity uncertainties obtained from the fitted noise models in each segment and component, i.e., not considering the contribution of offsets to the uncertainty (Fig. 4d). We also note that in the segmented series, the GM model is much less dominant than the FN and PL models and also tends to disappear in the newest part of the series. This is contrary to the analysis of the full 20-year series discussed earlier and confirms again that the GM model is favored by the power absorption of the estimated offsets at long periods.

Table 2. Percentage of selected PL and FN models, with the median noise amplitude in mm (σ) and spectral index (k), for the four segments of 5 year length as in Figure 4. Values in red are computed from less than 30 series.

		East			North			Up		
		%	σ	k	%	σ	k	%	σ	k
PL	1999-2004	19	2.0	-0.73	12	2.4	-0.71	42	5.7	-0.65
	2004-2009	20	2.2	-0.74	15	2.6	-0.77	55	5.3	-0.65
	2009-2014	36	1.5	-0.70	20	1.8	-0.68	54	5.3	-0.65
	2014-2019	36	1.6	-0.67	45	1.7	-0.68	74	5.0	-0.63
FN	1999-2004	67	2.5	-1	76	2.9	-1	39	8.2	-1
	2004-2009	59	2.1	-1	47	2.3	-1	31	6.5	-1
	2009-2014	45	2.1	-1	56	2.3	-1	35	6.6	-1
	2014-2019	56	2.0	-1	48	2.1	-1	21	7.1	-1

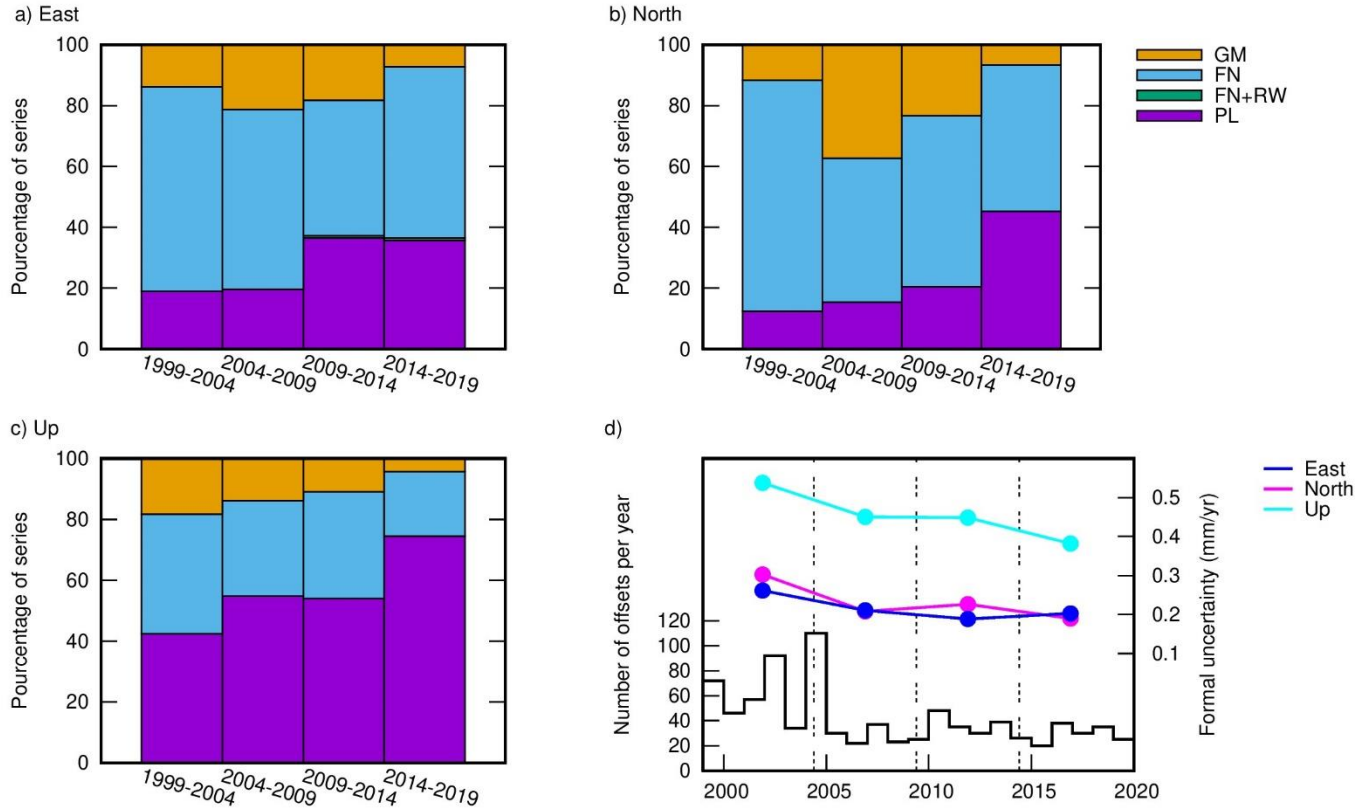


Figure 4. Distribution of the preferred noise model in the JPL series for 4 periods of 5 years each (x-axis) for the East (upper left), North (upper right) and Vertical components (lower left). Lower right: distribution of offsets per year and the median formal uncertainty of velocities for each period and component.

5. Discussion

5.1 Impact of offsets on the noise color

It is clear that offsets should be, first, avoided as much as possible and, second, removed from the series in order to get the least biased estimate of the velocity. The main limitation for reaching the second goal is that offsets are usually mixed and hidden by other features in the series, including colored noise. Manual offset detection by expert analysts generally outperforms automatic algorithms which tend to over-fit the series with spurious offsets (Gazeaux et al., 2013). Despite this finding, with the increasing number of series available and also their length, automatic offset detection is being used nowadays by many scientists as a purely practical matter and it is very likely that this trend will continue.

Leaving aside the question whether the removed offsets are correctly timed or not, we demonstrate for the first time how the offset estimation absorbs most of the low-frequency colored noise in the series, independent of both the series length and the number of offsets, but depending on the separation between the offsets. This translates into a different shape of the error spectrum and changes estimates of the colored noise content and hence velocity uncertainty. The impact is most dramatic in the case of a series with regularly spaced offsets every N years. In that case, the colored noise content of the series becomes unobservable beyond a period of approximately N years. For an irregular distribution of offsets, the fit of a noise model will provide, at best, a biased lower noise amplitude and, at worst, it will indicate that the best fitted model is much whiter, for instance a PL

model with spectral index closer to zero or even a GM model where the spectral index is zero at the longest periods.

In their recent study, He et al. (2019) analyzed 110 series of at least 12 years length to investigate whether the power spectrum flattens at the longest periods. They found that for 90 % of the series, the noise could be described by a GM process. This figure dropped to 5 % of the horizontal and 13 % of the vertical series when excluding the GM fits having a cross-over period between the Gaussian and the Markov-like processes smaller than 1 year. From the JPL series in our study, we find that around 80 % of the horizontal and 77 % of the vertical series are better described by a GM process, falling to 32 % for the North, 42 % for the East and 32 % for the vertical component if we exclude the GM fits having a cross-over period smaller than 1 year. These results are inconsistent with those from He et al. (2019). To choose the best-fit model, He et al. (2019) used a modified Bayesian Information Criterion penalty function that was added to the logarithm of the maximum likelihood estimate. This penalty function is slightly less restrictive than the empirical log-likelihood thresholds that we used from Santamaría-Gómez et al. (2011), i.e., in a synthetic FN series, the penalty function would allow the GM to be selected about 2 % more of the time than our empirical threshold, which would not explain these differences.

We may instead explain the difference of our results with those from He et al. (2019) due to a combination of the following reasons. First, they considered series of different lengths and only 20 out of their 110 stations had the maximum length of 20 years, compared to all 129 in our study. If we consider that the GM model was mainly retained in the longest 20 stations, then their GM ratios would rise to a maximum of 28 % and 72 %, closer to our results. Second, it is very likely that their series had a different set of estimated offsets than those in our series. Since almost all of their fitted GM models had a cross-over period shorter than a year, it would indicate that the power in many of their error spectra was decaying at shorter periods than our spectra, i.e., their series may have been fitted by more offsets than ours. Third, they used a different software for the noise content analysis that provides slightly lower spectral indices for short series (Bos et al., 2013). It is unknown to us if their software would also provide shorter cross-over periods for the GM model. Fourth, they analyzed PPP series from an older version of the same software with, importantly, obsolete orbits and clock products consistent with the IGS 1st reprocessing campaign. Those products may possess higher levels of FN (Amiri-Simkooei et al., 2017).

We also note that, from their method description, He et al. (2019) did not consider the spurious draconitic signal and its harmonics, with periods located between 1 year and 1 month, when fitting the noise model to the series. As with the velocity and the offsets, any parameter in the functional model could have an impact on the noise model fit to the residual series. If the draconitic oscillations are stationary and one removes them, then the error power spectrum will drop below the background noise at the removed lines. Otherwise, the error power spectrum at those lines will be higher than the background noise. In both cases, the background noise at those specific lines will be affected by the choices made in the functional model. Note that the same situation happens with the seasonal oscillations. We favored including all the systematic oscillations in the functional model, in this case up to the 10th harmonic of the 1.04 cpy frequency. Alternatively, one could restore the background noise power for the removed periods, but this requires a reliable hypothesis of the background power since the true noise power is not separable from the power of the systematic oscillations. Nevertheless, it would be expected that the draconitic oscillations, if left intact in the series as in He et al. (2019), would slightly bias the fit towards a GM model. We confirm that if the draconitic oscillations were not removed from our series, the GM model would be selected in ~4 % more of the series, which renders our results even less consistent with He et al. (2019). We also

observe that the power corresponding to some of the draconitic harmonics is not completely removed in our stacked error spectrum (see Fig. S4 in the Supporting Information), which indicates that the draconitic amplitude/phase may change over time or that the actual period may be different among the stations as shown by Santamaría-Gómez et al. (2011). For instance, Santamaría-Gómez et al. (2016) showed that for the station AZU1, the fourth draconitic oscillation changed its amplitude and phase when the antenna was replaced, indicating that part of the draconitic variance is due to local near-field multipath effects. This observation precludes the stationarity of the draconitic oscillations at least for those stations where the multipath environment may have changed.

When the estimated noise level is biased low or the best-fitted model is whiter than it should be, the resulting formal velocity uncertainty will be biased low. This is the main reason to reject the GM process when the cross-over period is too small (He et al., 2019; Santamaría-Gómez et al., 2011). However, when designating the noise model that best describes long residual GPS series, it is clear that we cannot consider a long-memory process like FN, as in most past publications, but rather a GM process. However, this statement does not mean that the actual noise of the series is dominated by a GM process, but instead that the true noise color has changed and it is not observable anymore due to the fitted functional model, particularly the position offsets.

In addition, since the estimated offsets change the amplitude and even the noise color, they could negate the conclusions drawn from the comparison of noise levels between different solutions (Amiri-Simkooei et al., 2017) or different monument types (Langbein & Svarc, 2019). In order to obtain comparable noise levels between different series, we recommend using similar sampling, including the series length and gaps, and also, if possible, the same set of offsets. The latter may not be possible if an offset is generated by incorrect metadata during the reduction of the GPS observations or if the series belong to different instruments with different antenna monuments and tracking histories.

We foresee that, if the current frequency of position offsets does not change or even increases in the future, as the series get longer the error power spectrum will continue to drop at long periods. This will prevent us from obtaining further information on the interannual to decadal error band compared to the information we already have today. Like a chameleon, we will never see the true noise color of the GPS position errors at such long periods. A consequence of this is that it will be necessary to use a heuristic noise model, not supported by the observations, in order to obtain less biased formal velocity uncertainties. Somehow, we are already seeing this scenario today when the best fitting model is a GM process, but it is rejected because of the low formal uncertainty it implies. A more conservative approach would be to fix the noise model to FN and also adding a relatively small amount of RW, compared to the estimated FN, to compensate for both the unobserved monument instability and also the reduction of the estimated FN amplitude by the offsets.

This recommendation is subjective and would depend on the analyst choice, but the fact that offsets are potentially hiding random walk noise in the series is of extreme concern for research using GPS velocities applied to tectonics, reference frames, sea-level change and GIA. Beyond the impact on velocities, the power spectrum drop caused by the estimated offsets seriously limits our ability to address relevant scientific questions such as the expected periodic Earth deformation at long periods recently modelled by Ding et al. (2020). A signal with a period of ~5.9 years lies very close to the period where the power spectrum is mostly flat and starts decaying in the best and longest JPL series available today. Whether this signal exists or not, we have not been able to find any sign of a significant spectral peak emerging out of the noise in this band in any coordinate component (Fig. 1).

The lack of observability of the noise spectrum at long periods also blocks our ability to unambiguously answer whether it naturally flattens at some long period or still continues unabated, contrary to the conclusions by He et al. (2019). This question, that has puzzled geodesists for the last 20 years or more, could provide invaluable information on the process generating the colored noise of the GPS series. Due to the estimated offsets, it will probably remain unanswered, therefore preventing scientists from exploiting even more precise GPS velocities.

5.2 Impact of offsets on velocities

Beyond the impact on the estimated noise level and color, the offsets also have a significant impact on the estimated velocity. Indeed, there exists a threshold on the offset size below which correcting the offset may produce a larger velocity error than if the offset was left uncorrected (Gazeaux et al., 2013; Griffiths & Ray, 2016). Williams (2003) demonstrated that an offset in the middle of the series introduces the largest velocity error compared to a different location in the series. If the offset is located near the beginning or end of the series, the velocity error being added becomes negligible compared to the velocity error without the offset. A similar scenario results if more than one offset is added very close to the beginning or end of the series, provided the length of the series before or after the offsets does not change significantly. In other words, the velocity error obtained from the fitted functional model is inversely proportional to the length of the continuous series between the offsets and not proportional to the number of offsets in the series. Certainly, when the offset is in the middle, the longest continuous segment of the series is the shortest of any other offset location possible, and therefore the velocity error is the largest possible.

In the case of a series free of offsets for N years, adding a new offset at the end will barely affect the present velocity error. However, after adding an offset, as the series length continues to grow, the velocity error will decrease much slower than if the offset had not been added. The maximum impact of the added offset on the velocity error will occur approximately when the series reaches a length of twice the current length ($2N$ years), or slightly earlier in case the series is dominated by FN according to Williams (2003). The actual velocity error added by the offset will be indirectly proportional to the length N . From the epoch of maximum impact around $2N$, as the series gets longer, the velocity error will decrease until the impact of the added offset vanishes more than 4 or 5 times N years in the future. Therefore, when planning a configuration change that would likely introduce a new offset into a series, the station manager should consider the increased velocity error that will result in the long-term, especially when the series is approximately twice the current length. For series having several offsets, the velocity error will accumulate the effects of the different offsets. If the introduced offsets are periodic and mostly equally spaced, then the resulting velocity error will decay very slowly, simulating a random walk process, but considering the separation between the offsets and not the series length (see Fig. 5 left panel), similar to the findings by Griffiths and Ray (2016). In the most extreme case of having an offset every year, as can occur in a GPS campaign, and assuming a common FN of 6 mm, the offsets will introduce a velocity error of $\sim 0.6 \text{ mm yr}^{-1}$ after 20 years of measurements (not shown in Fig. 5). It will be necessary to collect 80 years of measurements to reduce the velocity error down to 0.3 mm yr^{-1} , i.e. the error obtained from typical offset rates in continuous 20-years long series (Griffiths & Ray, 2016). This velocity error does not include the impact of seasonal terms in the series, which would aggravate the velocity error as discussed by Wang and Herring (2019). This velocity error is mostly accounted for by the functional model and is independent of the formal velocity error provided by the fitted noise model that best describes the

residual series, which will reduce as the number of offsets increases (Fig. 5 left panel) and most probably will not include RW, as we demonstrated in section 4.3.

When the offsets are equally spaced in the series, as typically in simulation studies, both the separation between the offsets and number of offsets provide equivalent assessments of the velocity errors created by offsets, though their representation is still different and may lead to confusion. Figure 5 shows different metrics for the velocity errors from zero-trend synthetic series having decreasing numbers of estimated offsets. This figure shows that in the case of considering the total number of equally-spaced offsets (right panel), the velocity error increases linearly. However, in the case of considering the equivalent separation between equally-spaced offsets (left panel) the velocity error increases similarly to a RW process generated exclusively by the functional model being fitted. We note that the same velocity errors are shown in both panels of Fig. 5, only the independent variable in the X axis of the panels is different. The functions used to predict the increase of the velocity error with offsets are different depending on the chosen independent variable, i.e., quadratic or linear. This reconciles the apparently contradictory results from Wang and Herring (2019), who considered the total number of offsets in the series, with those obtained earlier by Griffiths and Ray (2016), who systematically split the available series in half and considered the rate of offset occurrence, i.e., the inverse of their separation.

Another major difference between the findings by Griffiths and Ray (2016) and those by Wang and Herring (2019) is that while the former discussed true velocity errors, i.e., the scatter of velocity estimates compared to a reference value, the latter discussed formal velocity errors, i.e. the estimate of the velocity error from the law of propagation of variance. To explain their difference, Wang and Herring (2019) emphasize the importance of considering colored noise against WN when estimating the formal velocity error and its change with offsets. They created synthetic series with different types of noise processes, some of which, for instance WN or RW series, are not reasonable examples of actual GPS series. They concluded that, depending on the noise type of the series, the impact of offsets on the formal velocity errors can be quite different. Indeed, the impact of colored noise in the formal velocity errors is established since the late 1990s (Zhang et al., 1997), together with the fact that the GPS error spectra is generally consistent with a power-law noise spectrum close to FN. In their study, Griffiths and Ray (2016) employed actual IGS series, which contain FN, to observe how the true velocity error changes by varying only the rate of artificially inserted position offsets. When estimating the impact of offsets on the true velocity errors, the type and amplitude of the colored

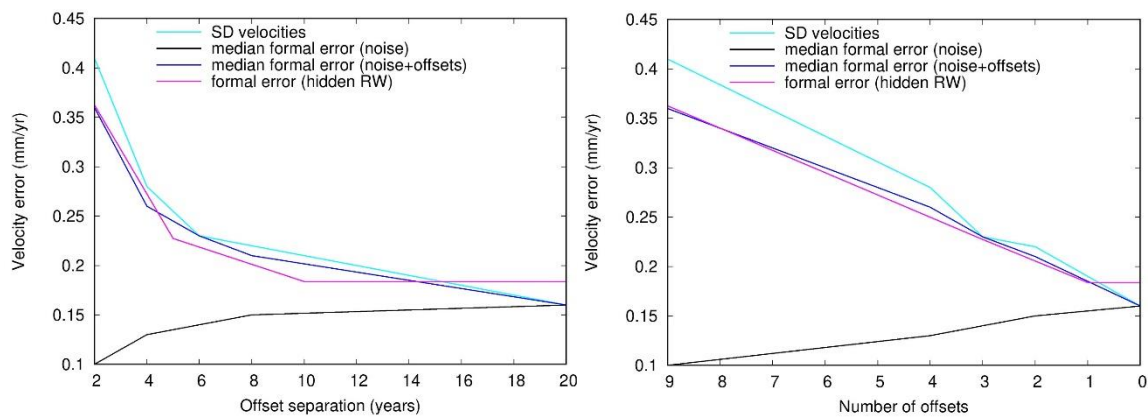


Figure 5 Velocity errors from 1000 synthetic series of 20 years having $6 \text{ mm yr}^{-1/4}$ of FN as a function of the separation between offsets of 2, 4, 6, 8, 10 and 20 years (left), and their equivalent number of total offsets in the series, i.e., 9, 4, 3, 2, 1 and 0 offsets (right). Formal velocity uncertainties are given for a fitted FN model only (noise), the FN model plus the functional model (noise+offsets) and the potentially hidden RW considering a 10 % detection threshold (hidden RW). The dispersion of the estimated velocities (SD velocities) represents the true velocity error.

noise used to estimate the formal velocity error has a limited impact, provided the noise type is consistent with FN or a smaller (whiter) spectral index. This fact is not observed if the noise in the series is consistent with RW, which, we insist, is not a realistic noise model to describe real GPS series. In other words, provided a real series with a given colored noise between white and flicker, the true velocity error induced by offsets when fitting the trend will not change significantly whether we consider the series as WN or FN in the covariance matrix of the least-squares fit. It is in fact the estimate of the formal velocity error that changes depending on the assumed temporal correlation. Ideally, when the correct noise model is used, both the true and formal rate errors will provide the same numbers, i.e., the formal velocity error depends on the chosen noise model while the true velocity error does not. This is a major difference between both studies, and we argue that is more informative providing true velocity errors, as in Griffiths and Ray (2016), than considering changes of the formal velocity error that depend on the chosen noise model, as in Wang and Herring (2019). In addition, in their analysis with the simulated series, Wang and Herring (2019) estimated the impact of offsets on the formal velocity errors without re-estimating the apparent change of the noise type and its parameter values after adding the offsets; they used the same covariance matrix as the one used for creating the synthetic series in the first place. Therefore, their estimated formal velocity errors do not account for the fact that the error spectrum at long periods is severely reduced by the added offsets as we have demonstrated in this study. For instance, we showed in Fig. 5 that, while the scatter of the estimated velocities, i.e., the true velocity error, increases with the number of offsets, the contribution of the colored noise to the formal velocity error decreases because the fitted noise model is progressively whiter. This means that the findings by Wang and Herring, with respect to the synthetic series, are not describing the complete impact offsets have on GPS velocities as we do in our study.

By adding equally-spaced artificial offsets into real IGS position series, Griffiths and Ray (2016) found that the velocity error due to the actual rate of offsets removed from the series, i.e., their average span separation, is at the level of 0.09 and 0.34 mm yr⁻¹ for the horizontal and vertical components, respectively. These values correspond to the average IGS rate of offset occurrence at that time. In our study, using synthetic FN series that reasonably match the JPL PPP series (Fig. 2), we find velocity errors at the level of 0.08 and 0.21 mm yr⁻¹ due to the offsets that JPL removed from the series. These values were obtained from the scatter of the fitted trends with and without the JPL offsets while also fitting the parameters of a PL process. If we consider WN instead of PL, as Griffiths and Ray (2016) did, the true velocity errors change by less than 0.02 mm yr⁻¹, supporting the quantification of velocity errors by Griffiths and Ray (2016). Our numbers are smaller most probably due to the fact that colored noise in the JPL series is lower than in the IGS series, and also due to the fact that we considered synthetic series with real irregularly-distributed offsets, while Griffiths and Ray (2016) considered real series with artificial equally-spaced offsets, which tends to increase the error.

Also importantly for velocity estimates is that, due to the removed trend and offsets, it is very difficult to say how much RW is truly contributing to the series variance. At most, given the separation between offsets in the series, we provided in Fig. 3 the probability that the RW does not exceed a given threshold. Inversely, by setting a detection threshold, we can assess the maximum amount of RW that could be present in the series. This is relevant because if RW noise were to be included in the error budget, one would obtain a more conservative formal uncertainty for the GPS velocities. For example, if we assume the upper limit of the hidden RW corresponds to a 10 % detection probability, from Fig. 3 we obtain a ratio FN/RW between 7.3 (RW ~14 % of FN amplitude) and 3.7 (RW ~27 % of FN) for a separation between offsets of 20 and 2 years, respectively. Considering the FN amplitude for weekly series shown in Table 1, the amount of potentially hidden RW translates into formal velocity errors (pink curve in Fig. 5) much higher than those obtained from

the estimated noise content (black curve in Fig. 5) and very similar to those caused by the offsets in the functional model (blue curve in Fig. 5). However, if the real amount of hidden RW is larger than the 10 % threshold we have used, which still has some probability to occur on a series-by-series basis, then the true velocity errors would be larger than those provided by both the fitted functional and noise models together, potentially resulting in a biased velocity estimate. At the end, the estimated formal uncertainty of the velocity becomes a subjective choice, i.e., depending on how conservative velocity uncertainties one prefers to obtain, a different amount of RW noise could be assumed. Despite RW noise having been detected in several regional or very-short baseline solutions, most of the studies using global GPS solutions have chosen to not consider RW noise in the velocity uncertainty budget, certainly because it has not been detected as a main contributor of the error spectrum. A remarkably exception is Langbein (2012) who proposed using a noise model composed of FN and RW noises instead of the typical PL noise model to obtain a more conservative velocity error budget. We argue that, with the state-of-the-art GPS series in our study, we cannot rule out the possibility that the RW noise is indeed affecting the velocity estimates even if we were not able to detect it. The GPS velocity errors of a global solution would be much larger if RW noise were to be considered, especially in the vertical component where the larger series variability allows for a larger RW noise to pass undetected. For example, the long-term stability of the terrestrial reference frame (Altamimi et al., 2016), assessing recent sea-level changes (Santamaría-Gómez et al., 2017) or our understanding of the Earth rheology and the glacial history (Schumacher et al., 2018) rely on accurate GPS velocity errors.

In light of the broad impact that offsets have on the GPS velocities as shown for the first time in this study, our recommendation is that if the station manager wants to change an already running GPS antenna, for instance to augment the tracking capabilities of the station, the antenna change should be made immediately after an unplanned offset occurred, e.g., an earthquake that displaced the antenna. Otherwise, we recommend installing a new station nearby to benefit from the already existing infrastructure like access, security, power, communications, etc. As a general rule, a functioning antenna must never be touched for the interest of science, contrary to the advice of Wang and Herring (2019).

5.3 Origin of colored noise

The underlying physical origin of colored GPS noise and its change in time is still an open question. Santamaría-Gómez et al. (2011, 2013) investigated the impact of the evolving tracking network, the evolving satellite constellation and the evolving rate of fixed phase ambiguities. None of these effects could explain the observed decrease in colored noise amplitude with time. Here we demonstrate that the frame alignment process cannot explain either the observed colored noise or its change with time.

Using synthetic series with different proportions of FN, we were unable to explain the stacked error spectra resulting from the JPL residual series. The real series exhibit spectra that are significantly lower in power at the longest periods even if we account for the damping of the colored noise caused by the estimation of position offsets. The mismatch between the real stacked spectra and the simulated FN spectrum is more evident for the vertical component than for the horizontal (Fig. 2). Spectral indices of the horizontal components have been reported to be closer to flicker noise than the vertical component (Reischung et al., 2016b; Santamaría-Gómez et al., 2011; Williams et al., 2004). Although surface load mass variations can introduce significant colored noise in the vertical component (Santamaría-Gómez & Mémin, 2015), the fact that flicker noise is visible more clearly in the horizontal component argues against the hypothesis of flicker noise being mainly created by surface mass load variations, which are significantly more important on the vertical component than

in the horizontal. Another concurrent indication is the fact that the colored noise content is getting whiter with time even if non-tidal loading corrections are not currently included in the conventional corrections applied to the series and their effects are most likely not decaying with time.

What causes a spectral index that is whiter than flicker is as intriguing as what causes the FN itself. This situation can only be explained by a combination of different error sources, i.e., we are looking for a process that dampens FN in the vertical component while having minimal impact on the horizontal components. We demonstrated that the frame alignment process certainly contributes in the same direction, but its magnitude is rather negligible. Actually, surface mass load variations are a good candidate for such a process. Santamaría-Gómez and Mémin (2015) showed that the combination of modeled atmospheric, oceanic and hydrologic load variations results in colored noise that can be described by a PL with a spectral index varying spatially from WN to RW (see map of figure S1 in Santamaría-Gómez and Mémin (2015)). The median spectral index of the modeled load variations for the vertical component at the 129 JPL series used in this study is -0.7 , i.e. significantly lower than a flicker process. In addition, the median amplitude of the colored noise obtained from the modeled surface loads at the 129 stations and scaled to weekly sampling is ~ 4 mm. This equals two-thirds of the observed PL amplitude of ~ 6 mm in the residual weekly vertical JPL series, which implies that the surface mass load variations add noise to the series, mostly on the vertical component, but the noise added is less colored than FN. One may conclude that the surface mass loads, contrary to causing the FN observed in the series, probably contribute to hide it, making the vertical series appear whiter than the horizontal components.

Conclusions

Offsets in GPS position time series are one of the biggest limitations concerning the precision of the estimated GPS velocities. Yet, the best GPS series are populated with offsets, many of which are related to equipment testing and upgrading which are avoidable. The use of algorithms for automatic offset detection may even increase the number of spurious offsets that are included in the model fitted to the GPS series. Here, we address for the first time the complete impact that estimated position offsets have on the GPS velocities, their uncertainty and the colored noise of the residual series.

We have shown how the estimated offsets absorb most of the series variability at periods longer than their time spacing. The result is that at interannual to decadal periods, the noise power spectra of the GPS position time series changes color and becomes uninterpretable as a quantitative measure of the contributing noise processes. This severely affects our ability to objectively provide a noise model that fully describes the temporal correlation of the GPS series. Any colored noise model fitted to GPS series having offsets will be biased low, i.e. towards WN, and the preferred noise model may even be wrongly selected, e.g. a Gauss-Markov process instead of a flicker or power-law process. As a result, the estimated formal velocity uncertainty will be biased low. If one is interested in obtaining conservative formal velocity uncertainties, it is necessary to subjectively choose a different noise model that is not clearly supported by the observations, for example a FN model topped with a relatively small amount of RW.

The subjectivity to choose a more accurately representative noise model also concerns the amount of RW that the analyst is willing to add to the velocity uncertainty budget. On most past studies using global network or PPP solutions, the RW has never been included in the velocity error budget because it has not been detected over other noise types. We show how the inclusion of offsets in the series renders RW detection even more difficult in very long series that include RW. We conclude

that the common assumption that RW does not contribute to the velocity error is questionable, especially considering that RW has been detected in regional or short-baseline solutions with greater sensitivity due to spatial filtering. The addition of a small quantity of RW, even if not observed directly, will significantly raise the GPS velocity uncertainty and especially in long series with frequent offsets where the amount of RW that could be allocated in the error budget is larger.

The unobservability of the noise power spectra at long periods caused by the estimated position offsets also has severe consequences for our ability to observe and discover long-period Earth deformation signals. A periodic Earth deformation signal with a period of ~5.9 years has been suggested in recent literature. This period is exactly located where the error spectrum is blurred by the estimated position offsets in the best and longest JPL series to date used in this study. Despite recent claims for the detection of this signal, we were unable to find any sign of a significant signal emerging out of the noise in this band in any coordinate component. The origin of colored noise in GPS position time series and its change with time is still an open question. We have demonstrated how the frame alignment process, that is inherent to the construction of any GPS series, does not play a significant role. Surface mass loads could at least partly explain why the noise is whiter in the vertical component than in the horizontal, but they cannot explain why the noise color is becoming whiter in the most recent data. In addition, the offsets already present in the GPS series are preventing us from observing the properties of the colored noise at long periods. If offsets continue to appear in the series at the current rate, it will represent a major limitation for our understanding of the GPS error spectrum reducing the value of longer series in the future.

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