

**Terrestrial Approach:**  
Compute or predict lava flow length and width from basic flow properties.

**Planetary Approach:**  
Compute or predict basic flow properties from lava flow length and width.

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To date, actively flowing lava has only been observed on Earth and on Jupiter's moon Io. This lack of observation means that for the vast majority of volcanic systems in the Solar System, solidified lava-flow morphologies are used to infer important information about eruption and emplacement parameters. These include: lava supply rate, lava composition, lava rheology, and determination of laminar or turbulent emplacement regimes. Commonly used models that relate simple lava flow morphologic properties (e.g., width, thickness, length) to emplacement characteristics are based on assumptions that are readily misinterpreted. For example, the simplifying assumption of fully turbulent lava flow allows for a thermally mixed flow interior, but ignores the lava properties that naturally work to suppress full turbulence (such as thermal boundary layers encasing active lava flows, and a temperature-dependent lava rheology). However, full turbulence in silicate lava flows erupted into environments that have temperatures lower than the lava solidification temperature requires a rare combination of characteristics. We model Bingham Plastic, Newtonian, and Herschel-Bulkley fluids in rectangular channels, tubes, and sheets with computational fluid dynamics (COMSOL) software to obtain flow solutions and general flow rate equations and compare them to field measurements of volcanic velocity and flow rates. We present these as more realistic alternatives to older simpler rate-from-morphology models. We find that several lava rheology properties work together to delay the onset of turbulence as compared to isothermal Newtonian materials, and that while turbulent lavas flows certainly exist, they are not as prevalent as the published literature might indicate. Results obtained from models that assume full turbulence in silicate flows on the terrestrial planets should therefore be interpreted cautiously.

### Hulme model: (Isothermal laminar Bingham Approximation)

$$Y_R = 2\rho g W_L \sin^2 \theta$$

→ This approximation (e.g. Hulme, 1976) relies on problematic assumptions and should be replaced with an exact or empirical solution (e.g. Skelland 1967, Deane and Sakimoto, 1997; Burger et al. 2015; and others).

$\gamma_B$  = Bingham yield stress  
 $\rho$  = flow density  
 $g$  = acceleration of gravity  
 $W_L$  = Levee width  
 $\theta$  = slope

**Jeffries Equation: (Isothermal laminar Newtonian Approximation)**

$$u = \frac{h^2 g \rho \sin \theta}{B \eta}$$

→ This approximation (Jeffries, 1925) should be replaced with the appropriate exact solution. See, for example, White (2006).

- u = flow velocity
- h = flow depth
- g = acceleration of gravity
- $\rho$  = density
- B = geometry parameter
- $\eta$  = fluid viscosity

## Isothermal laminar Bingham Flow Examples

➔ **Numerous exact and empirical solutions:**  
Skelland, 1967, circular tube, parallel plates  
Burger et al (2015), and many others: rectangular channel  
Deane and Sakimoto (1997) Parabolic Channel  
...etc...

**LESS COMPLEX** ➡ **MORE COMPLEX**

The diagram illustrates the three stages of boundary layer development from left to right:

- Laminar:** Shows smooth, parallel streamlines with a clear velocity profile (parabolic shape) indicated by arrows of varying lengths.
- Transitional:** Shows the onset of mixing and irregularities in the flow, with some streamlines beginning to wobble.
- Turbulent:** Shows fully developed, chaotic mixing with eddies and a flatter velocity profile (logarithmic shape) indicated by arrows.

The figure consists of three separate graphs, each with 'Stress' on the vertical axis and 'Shear Rate' on the horizontal axis.

- Newtonian:** The graph shows a straight line passing through the origin (0,0), indicating a linear relationship between stress and shear rate.
- Bingham Plastic:** The graph shows a straight line with a positive slope that intersects the vertical axis at a positive value, representing a yield stress.
- Herschel-Bulkley:** The graph shows a curve that starts at a positive value on the vertical axis and increases with a decreasing slope (concave down).

The figure consists of three panels, each showing a graph of Height vs. Flow Velocity and a corresponding temperature profile below it.

- Left Panel:** The graph shows a parabolic flow velocity profile. Below it, a red 'X' is placed over the text "Independent Solutions". The temperature profile is a simple vertical gradient bar.
- Middle Panel:** The graph shows a parabolic flow velocity profile with a blue double-headed arrow indicating a change in the velocity profile. Below it, the text "Partial Rheology temperature dependence" is shown. The temperature profile is a vertical gradient bar with a red 'X' over it.
- Right Panel:** The graph shows a parabolic flow velocity profile with a blue double-headed arrow indicating a change in the velocity profile. Below it, the text "All rheology parameters temperature dependent" is shown. The temperature profile is a vertical gradient bar with a red 'X' over it.

Stress

Shear Rate or  $dV/dy$  or rate of strain

Shear-Thickening w/ Yield Stress

Bingham Plastic

Hershel-Bulkley

Newtonian

Shear Thinning (i.e. Power Law)

**For: Observed Flow Dimensions**  
**Model: Coupled Rheology and**  
**Flow Rate**

**On Earth, rheology is often further constrained with geochemistry or geothermometry**

All computational models are done with COMSOL Multiphysics 5.4 (2018) using the Computational fluid dynamics (CFD) module, the Heat Transfer Module, and (where applicable) the Material Library Module. See [www.comsol.com](http://www.comsol.com) for a more complete description of the computational capabilities of each module and the base multiphysics package.

- parabolic velocity profile
- expected turbulence transition

- **more flow in the boundary layers**
- **slower center velocity**
- **delayed turbulence transition**

3D visualization of the velocity field in a rectangular duct. The color scale ranges from 0.00 (blue) to 0.25 (red). The velocity is highest in the center of the duct and decreases towards the walls, showing a parabolic profile.

- less boundary layer flow
- faster center velocity
- delayed turbulence transition

3D visualization of the electric field distribution in a rectangular waveguide. The field is concentrated in the center, with a color scale on the right ranging from 0 to 1.0.

- substantial fast center plug
- thin boundary layer
- delayed turbulence transition

- Planetary volcanology approach inverts the terrestrial approach:
  - Planetary ... often predicting flow properties from flow dimensions and shape rather than flow dimensions from flow properties.

- **Planetary flows do not have geochemistry or geothermometry constraints that may be available for terrestrial flows. Ambient conditions must be considered.**

- **Computational approaches modeling flows with changed ambient conditions for planetary flows are expected to yield improved results compared to exporting vintage terrestrial approximations. Computational approaches can yield empirical equations appropriate for specific model/planet conditions.**

## References

[illegible]

- The Hulme and Jeffries approximation approaches for estimating planetary flow properties should be retired, since they are demonstrably unreliable, and we have vastly improved analytic AND computational approaches that yield more tightly constrained and consistent results.

- We do not yet adequately understand the effects of temperature-dependent rheology, composition, and ambient conditions on terrestrial or planetary flows. So:

—Inferring composition variations from flow morphology is fraught with pitfalls

**—Inferring laminar or turbulent emplacement from flow morphology and multiple simplifying assumptions has significant potential for incorrect results.**

- With current computational tools, we can construct semi-empirical relationships as well as self-contained model applications that are specific to flow conditions and planetary conditions

—...increasing our understanding of planetary flow properties AND general lava flow processes under different ambient conditions..