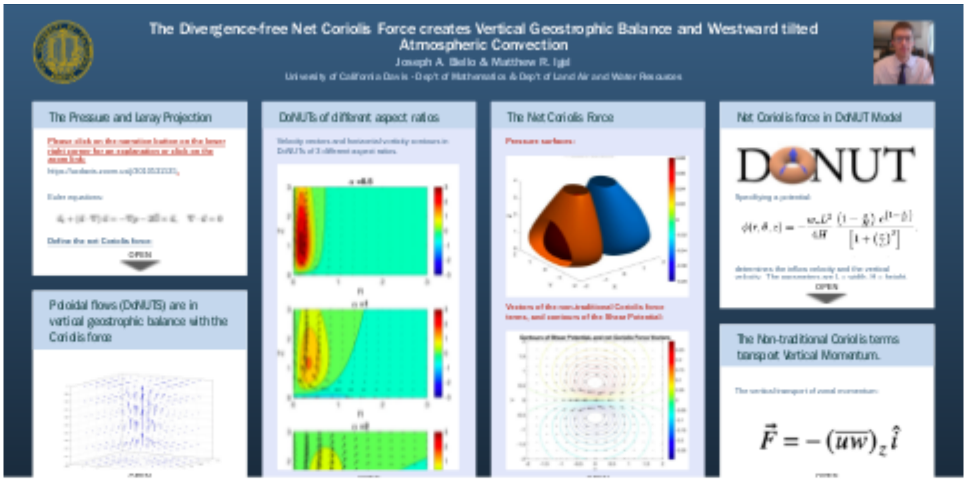
The Divergence-free Net Coriolis Force creates Vertical Geostrophic Balance and Westward tilted Atmospheric Convection



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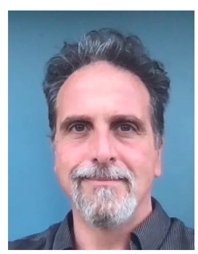


PRESENTED AT:

THE PRESSURE AND LERAY PROJECTION

Please click on the narration button on the lower right corner for an explanation or click on the zoom link:

https://ucdavis.zoom.us/j/3010531535.

Euler equations:

$$\vec{u}_t + (\vec{u} \cdot \nabla)\,\vec{u} = -\nabla p - 2\vec{\Omega} \times \vec{u}, \quad \nabla \cdot \vec{u} = 0$$

Define the net Coriolis force:

$$\vec{F} = -\nabla p - 2\vec{\Omega} \times \vec{u},$$

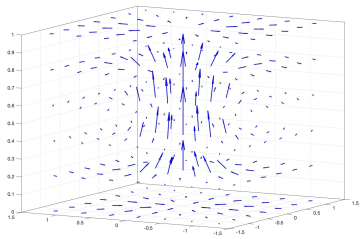
Where

$$\nabla \cdot \vec{F} = 0 \quad \vec{F} \cdot \hat{n} = 0 \quad \text{on rigid boundaries,}$$

The equation for the pressure becomes:

$$\nabla^2 p = 2\vec{\Omega} \cdot \vec{\omega},$$
$$\hat{k} \cdot \nabla p = 2\left(\vec{u} \times \vec{\Omega}\right) \cdot \hat{k}, \quad \text{on} \quad z = 0.$$

POLOIDAL FLOWS (DONUTS) ARE IN VERTICAL GEOSTROPHIC BALANCE WITH THE CORIOLIS FORCE



The **Helmholtiz decomposition** splits any vector field into a poloidal and horizontal part:

$$\vec{u} = \left(-\phi_x \hat{i} - \phi_y \hat{j} + w\hat{k}\right) + \left(-\psi_y \hat{i} + \psi_x \hat{j}\right)$$

where the potential of the poloidal flow is uniquely determined by the vertical velocity:

$$\phi_{xx} + \phi_{yy} = w_z$$

$$\hat{n} \cdot \left(\phi_x \hat{i} + \phi_y \hat{j}\right) \to 0 \quad \text{as} \quad \sqrt{x^2 + y^2} \to \infty,$$

The crucial fact that we build on is that the vorticity of a poloidal flow is completely in the horizontal plane:

$$\vec{\omega} = \left[\left(w_y + \phi_{yz}\right)\hat{i} - \left(w_x + \phi_{xz}\right)\hat{j}\right]$$

From Igel & Biello 2020, we realized that the pressure gradient of a poloidal flow balances the vertical component of the total Coroilis force in the DoNUT Model. In fact, this is a general result for any poloidal flow.

Mathematical theorem:

Let d be some constant direction, then if the component of the vorticity in the d direction does not vary in the direction of rotation,

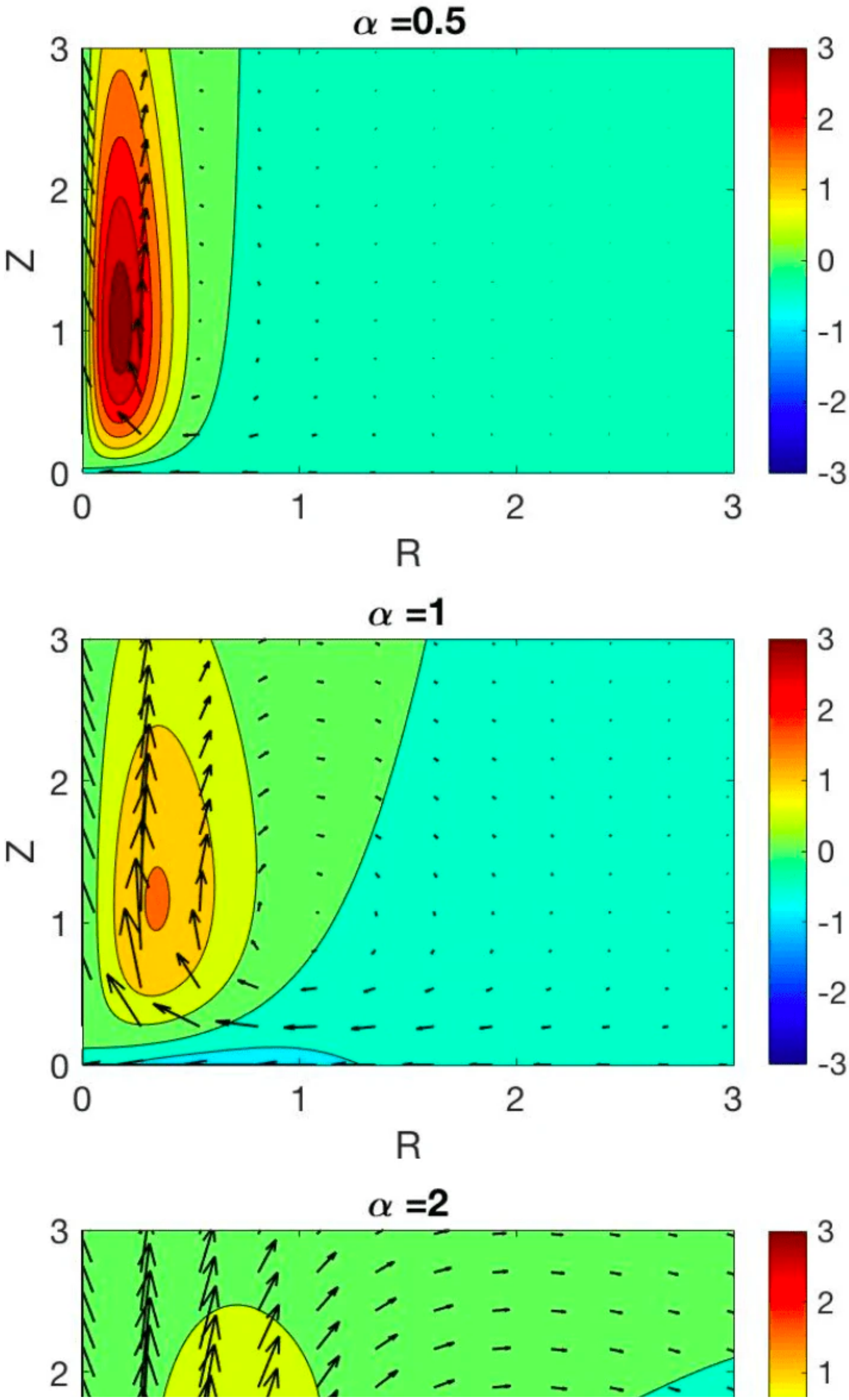
$$2\vec{\Omega} \cdot \nabla \left[\hat{d} \cdot \vec{\omega}\right] = 0.$$

then the net Coroilis force in that direction is zero.

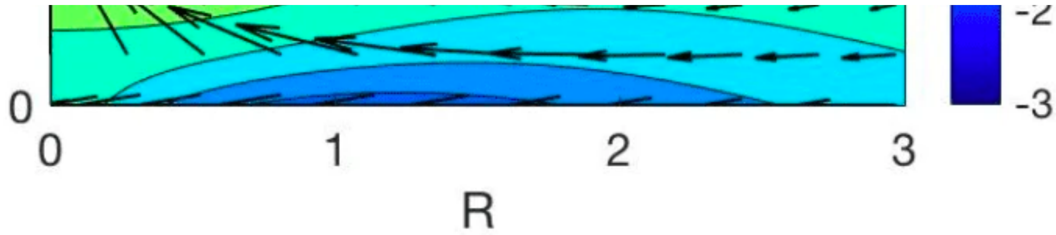
For a poloidal flow, the vertical component of vorticity is zero, so too is the vertical component of the net Coroilis force.

DONUTS OF DIFFERENT ASPECT RATIOS

Velocity vectors and horizontal vorticity contours in DoNUTs of 3 different aspect ratios.

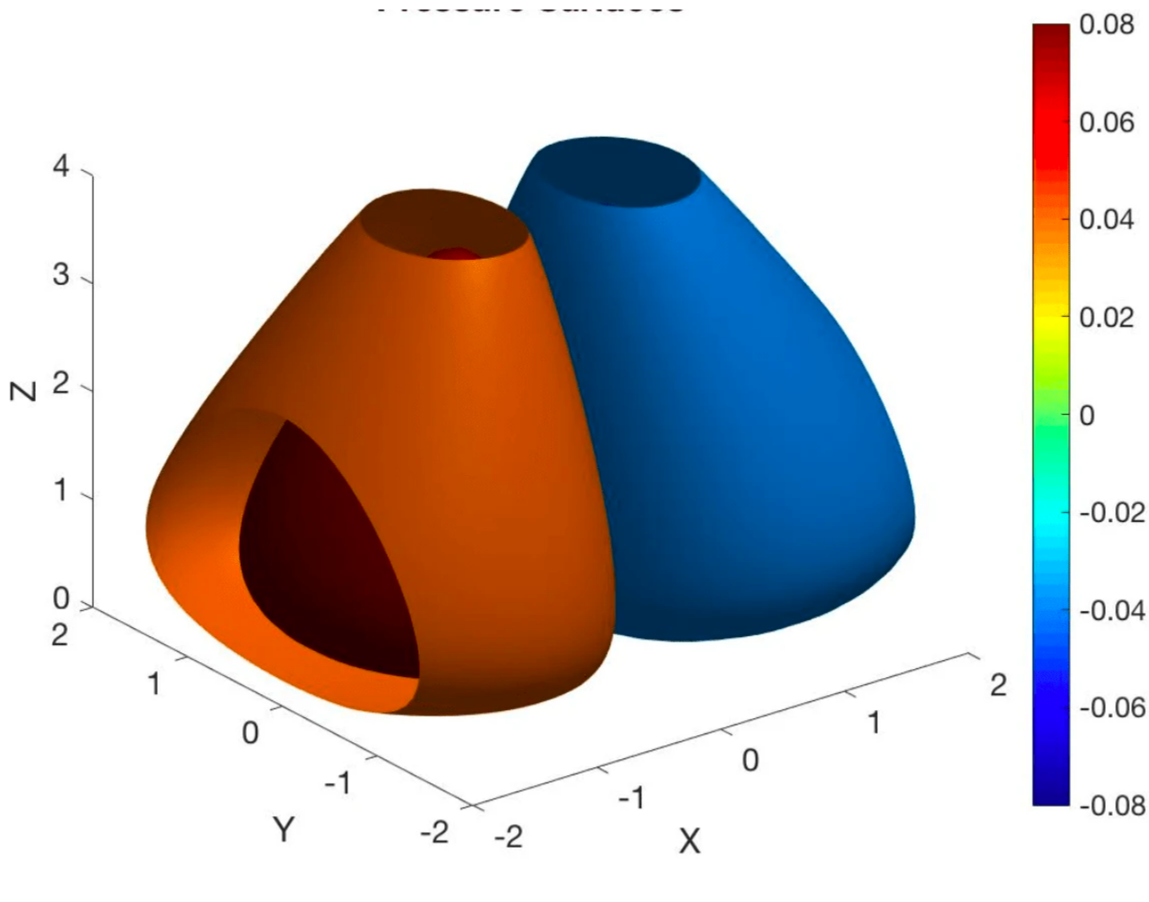


α =0.5
Z
R
α =1
Z
R
α =2

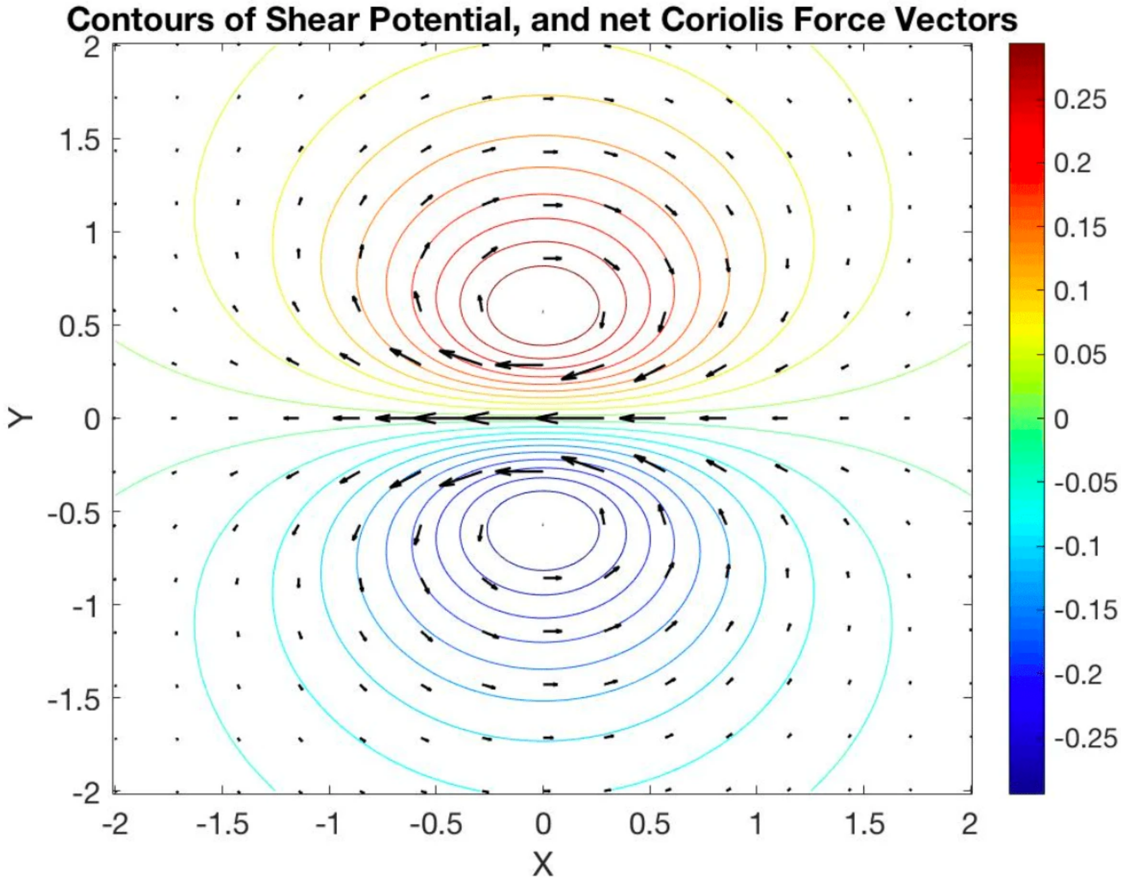

-2
-3
0
0
1
2
3
R

THE NET CORIOLIS FORCE

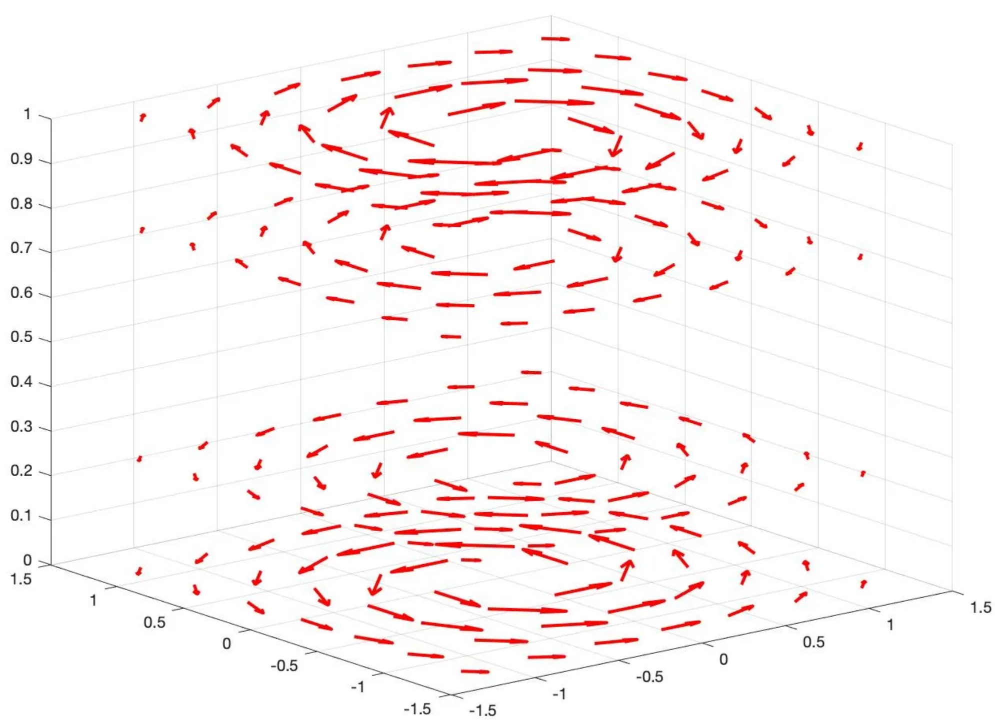
Pressure surfaces:



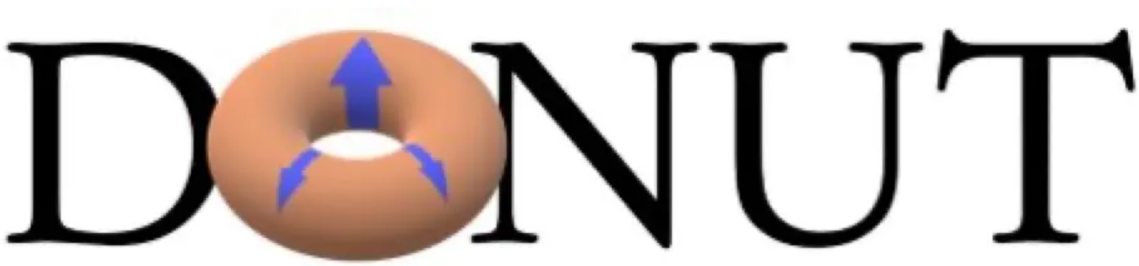
Vectors of the non-traditional Coriolis force terms, and contours of the Shear Potential:



Vectors of the traditional Coriolis force terms:



NET CORIOLIS FORCE IN DONUT MODEL



Specifiying a potential:

$$\phi(r,\theta,z) = -\frac{w_* L^2}{4H}\,\frac{\left(1-\frac{z}{H}\right)\,e^{\left(1-\frac{z}{H}\right)}}{\left[1+\left(\frac{r}{L}\right)^2\right]}.$$

determines the inflow velocity and the vertical velocity. The parameters are L = width, H = height, and w* = maximum vertical velocity of DoNUT convection.

Using Lambda to denote the latitude, we can easily compute the pressure is the vertical integral of the zonal velocity

$$p = -2\Omega_0\,\cos(\lambda)\,\int_z^\infty u(x,y,z')\,dz';$$

One more quantity to define, the integral of the potential:

$$\Phi = \int_z^\infty \phi(x,y,z')\,dz'.$$

Net Traditional Coriolis Force:

$$\vec{F}_{\mathrm{TCT}} = 2\Omega_0 \sin(\lambda) \left[-\phi_y \, \hat{i} + \phi_x \, \hat{j}\right]$$

Net Non-Traditional Coriolis Force:

$$\begin{aligned}\vec{F}_{\mathrm{NCT}} &= 2\Omega_0 \cos(\lambda) \left[\Phi_{yy} \, \hat{i} - \Phi_{xy} \, \hat{j}\right] \\ &= -2\Omega_0 \cos(\lambda) \, \nabla^{\perp} \Phi_y.\end{aligned}$$

The NCT is in the horizontal plane, and force lines are tangent to contours of constant

Φ_y

We call this the **Shear Potential.**

THE NON-TRADITIONAL CORIOLIS TERMS TRANSPORT VERTICAL MOMENTUM.

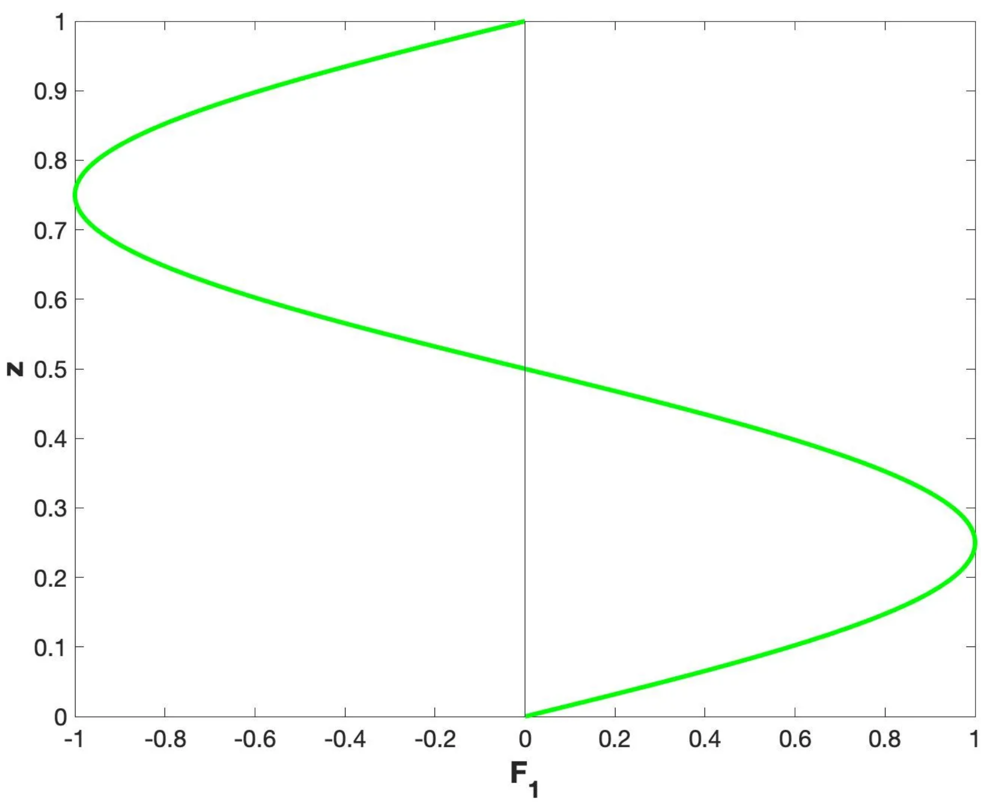
The vertical transport of zonal momentum:

$$\vec{F} = -\left(\overline{uw}\right)_z \hat{i}$$

can be computed explicity from the DoNUT model

$$\bar{F}_1 = \frac{f}{H}\left(\frac{\Omega}{D}\right) w_*^2 \sin\left(\frac{2z}{H}\right)$$

and acts to force a vertical shear of zonal momentum



ABSTRACT

There has been increasing realization that the non-traditional Coriolis force terms may have a significant effect on convective circulation and organization in the tropics.

In this talk, we introduce the concept of the net Coriolis force of a fluid dynamical flow. In order to compute the net Coriolis force, we assume incompressible flow - and therefore divergence free net Corolis force. Through the Leray projection we are able to construct, both, the pressure needed to maintain an incompressible force, and the net Coriolis force, itself.

We explore the effects of the net Coriolis force on basic flows by decomposing the velocity field using the Helmholtz decomposition, and we describe poloidal and horizontal flows separately. We then compute the net Coriolis force associated with the Traditional Corolis terms (proportional to the sine of latitude) and, separately, the net Non-Traditional Coriolis terms (proportional to the cosine of latitude).

We show that all poloidal circulations - which are flows which lack a vertical component of vorticity - are in vertical gesotrophic balance. Therefore, the pressure induced by such flows is simply computed without the need to invert a Laplacian.

Using the Dynamics of Non-rotating Updraft Tori (DoNUT), which is a polodial circulation framework introduced by Igel & Biello (2020) to describe the full kinematic circulation of atmospheric convection, we show that the net non-traditional Coriolis force has zero component in the vertical direction, is westward in the regions of upward flow, and recirculates eastward poleward of the upward flow. The resulting circulations lead to vertical/westward oriented momentum flux from the resulting Reynold's stress terms. We will conclude by discussing implications of these circulations for tropical convective organization.

The research that this poster extends the work of Igel & Biello 2020 J.A.S. 77 (https://journals.ametsoc.org/view/journals/atsc/77/12/JAS-D-20-0024.1.xml), and is based on is submitted to QJRMS, Biello & Igel.