Supporting Information for "Ranking IPCC Models Using the Wasserstein Distance"

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Text S1. Wasserstein distance (WD): Background and history

We present herein historical and mathematical information on WD, as well as additional information on the climate models analyzed. We wish to quantify the discrepancies between the output of a climate model and the observed reality by comparing their complete probability distributions and not just some representative quantity, like their variance. One way of doing so is to use the Kullback–Leibler (KL) divergence (Kullback & Leibler, 1951), which is rather widespread in applied statistics. To better explain the difference between the Wasserstein or Kantorovich–Rubinstein distance (Kantorovich, 2006) and the KL divergence, we first list below the axioms associated with the mathematical concept of a metric d. These axioms are inspired by and, of course, satisfied by the usual Euclidean distance.

Given points x, y, z in a topological space X, $x, y, z \in X$, these axioms are

$$d(x,y) = 0 \iff x = y, \tag{1a}$$

$$d(x,y) = d(y,x), \tag{1b}$$

$$d(x,y) \le d(x,z) + d(z,y); \tag{1c}$$

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they are referred to, respectively, as the axiom of identity or indiscernibles; the axiom of symmetry; and the axiom of subadditivity, better known as the triangle inequality. These axioms also imply the nonnegativity or separation condition

$$d(x, y) \geq 0 \quad \text{for all} \quad x, y \in X.$$

A topological space X equipped with such a metric becomes a metric space. Examples well-known in studying partial differential equations of fluid dynamics are so-called Hilbert spaces, which can be seen essentially as infinitely dimensional versions of Euclidean spaces (Halmos, 2017).

Given probability distributions P, Q, R on a metric space X, the KL divergence $D_{KL}(P\|Q)$ for P given Q satisfies neither the symmetry condition (1b) nor the triangle inequality (1c), i.e.

$$D_{KL}(P\|Q) \neq D_{KL}(Q\|P) \quad \text{and, in general,} \tag{2a}$$

$$D_{KL}(R\|P) \leq D_{KL}(Q\|P) + D_{KL}(R\|Q) \quad \text{does not hold.} \tag{2b}$$

The Wasserstein distance (hereafter WD) (Dobrushin, 1970), though, is a true metric and satisfies all three axioms of Eq. (1). It is based on the concept of optimal transport (Villani, 2009) and it allows one to evaluate quantitatively the distance between two distributions: intuitively, the nearer the two distributions of points in phase space, the smaller the effort required to merge the two. WD is also called the "earth mover's distance," since it was originally motivated by minimizing the effort of a platoon having dug a trench of prescribed shape and moving the earth dug up to another, existing trench of a different shape (Monge, 1781).

Using WD, it is possible to estimate the reliability of a model by choosing an appropriate combination of climatic or other physical variables, depending on the goal of the computation. Since an N-dimensional distribution contains much more information than its N one-dimensional marginals, every point in our multidimensional distribution carries information about all the fields at the same time and not just about the product of the marginals.

Text S2. CMIP5 models

The models that participated in CMIP5 are listed in Table S1 below. The three rankings summarized in Fig. 5 of the Main Text are listed here in Tables S2–S4.

References

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Acronym	Model	Center	Country
ACCESS1.0	Australian Community Climate and Earth-System Simulator, version 1.0	Commonwealth Scientific and Industrial Research Organisation – Bureau of Meteorology	Australia
ACCESS1.3	Australian Community Climate and Earth-System Simulator, version 1.3	Commonwealth Scientific and Industrial Research Organisation – Bureau of Meteorology (CSIRO-BOM)	Australia
BCC-CSM1.1	Beijing Climate Center, Climate System Model, version 1.1	Beijing Climate Center (BCC), Chinese Meteorological Administration (CMA)	China
BCC-CSM1.1-m	Beijing Climate Center, Climate System Model, version 1.1, Moderate resolution	Beijing Climate Center (BCC), Chinese Meteorological Administration (CMA)	China
BNU-ESM	Beijing Normal University Earth System Model	Beijing Normal University (BNU)	China
CanESM2*	Second Generation Canadian Earth System Model	Canadian Centre for Climate Modelling and Analysis (CCCma)	Canada
CCSM4*	Community Climate System Model, version 4	National Center for Atmospheric Research (NCAR)	United States of America
CESM1-BGC*	Community Earth System Model, version 1, BioGeoChemistry	National Science Foundation (NSF) – U.S. Department of Energy (DOE) – National Center for Atmospheric Research (NCAR)	United States of America
CESM1-CAM5*	Community Earth System Model, version 1 - Community Atmosphere Model, version 5	National Science Foundation (NSF) – U.S. Department of Energy (DOE) – National Center for Atmospheric Research (NCAR)	United States of America
CMCC-CM	Centro Euro-Mediterraneo per I Cambiamenti Climatici Climate Model	Centro Euro-Mediterraneo per I Cambiamenti Climatici (CMCC)	Italy
CMCC-CMS	Centro Euro-Mediterraneo per I Cambiamenti Climatici Climate Model, Stratosphere version	Centro Euro-Mediterraneo per I Cambiamenti Climatici (CMCC)	Italy
CNRM-CM5	Centre National de Recherches Météorologiques Coupled Global Climate Model, version 5	Centre National de Recherches Météorologiques (CNRM) – Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique (CERFACS)	France
CSIRO-Mk3.6.0*	Commonwealth Scientific and Industrial Research Organisation Mark, version 3.6.0	Commonwealth Scientific and Industrial Research Organisation (CSIRO) – Queensland Climate Change Centre of Excellence (QCCCE)	Australia
EC-EARTH*	European Community Earth-System Model	EC-EARTH Consortium	Europe
GFDL-CM3*	Geophysical Fluid Dynamics Laboratory Climate Model, version 3	National Oceanic and Atmospheric Administration (NOAA) - Geophysical Fluid Dynamics Laboratory (GFDL)	United States of America
GFDL-ESM2G	Geophysical Fluid Dynamics Laboratory Earth System Model, Generalized Ocean Layer Dynamics (GOLD) component	National Oceanic and Atmospheric Administration (NOAA) - Geophysical Fluid Dynamics Laboratory (GFDL)	United States of America
GFDL-ESM2M	Geophysical Fluid Dynamics Laboratory Earth System Model, Modular Ocean Model 4 (MOM4) component	National Oceanic and Atmospheric Administration (NOAA) - Geophysical Fluid Dynamics Laboratory (GFDL)	United States of America
HadGEM2-CC	Hadley Centre Global Environment Model, version 2, Carbon Cycle	Met Office Hadley Centre	United Kingdom
HadGEM2-ES	Hadley Centre Global Environment Model, version 2, Earth System	Met Office Hadley Centre	United Kingdom
INM-CM4	Institute of Numerical Mathematics Coupled Model, version 4.0	Institute of Numerical Mathematics (INM)	Russia
IPSL-CM5A-LR	Institut Pierre-Simon Laplace Coupled Model, version 5A, Low Resolution	Institut Pierre-Simon Laplace (IPSL)	France
IPSL-CM5A-MR	Institut Pierre-Simon Laplace Coupled Model, version 5A, Medium Resolution	Institut Pierre-Simon Laplace (IPSL)	France
IPSL-CM5B-LR	Institut Pierre-Simon Laplace Coupled Model, version 5B, Low Resolution	Institut Pierre-Simon Laplace (IPSL)	France
MIROC5*	Model for Interdisciplinary Research on Climate, version 5	Atmosphere and Ocean Research Institute (AORI)–National Institute for Environmental Studies (NIES)–Japan Agency for Marine-Earth Science and Technology (JAMSTEC)	Japan
MIROC-ESM	Model for Interdisciplinary Research on Climate, Earth System Model	Atmosphere and Ocean Research Institute (AORI)–National Institute for Environmental Studies (NIES)–Japan Agency for Marine-Earth Science and Technology (JAMSTEC)	Japan
MIROC-ESM-CHEM	Model for Interdisciplinary Research on Climate, Earth System Model, atmospheric chemistry coupled version	Atmosphere and Ocean Research Institute (AORI)–National Institute for Environmental Studies (NIES)–Japan Agency for Marine-Earth Science and Technology (JAMSTEC)	Japan
MPI-ESM-LR	Max Planck Institute Earth System Model, Low Resolution	Max Planck Institute for Meteorology (MPI-M)	Germany
MPI-ESM-MR	Max Planck Institute Earth System Model, Medium Resolution	Max Planck Institute for Meteorology (MPI-M)	Germany
MRI-CGCM3	Meteorological Research Institute Coupled Atmosphere–Ocean General Circulation Model, version 3	Meteorological Research Institute (MRI)	Japan
NorESM1-M	Norwegian Earth System Model, version 1, Medium Resolution	Norwegian Climate Centre (NCC)	Norway

Table S1. CMIP5 models used in the paper. The asterisk points out the models not used for sea ice entension tests.

3D WD	Model
0.097	IPSL-CM5A-MR
0.101	MIROC-ESM-CHEM
0.107	MIROC-ESM
0.125	NorESM1-M
0.136	MPI-ESM-LR
0.143	CMCC-CMS
0.157	GFDL-ESM2M
0.158	MPI-ESM-MR
0.162	IPSL-CM5A-LR
0.165	BNU-ESM
0.169	CMCC-CM
0.188	ACCESS1.0
0.188	CNRM-CM5
0.191	IPSL-CM5B-LR
0.192	HadGEM2-ES
0.193	BCC-CSM1.1
0.200	MRI-CGCM3
0.207	HadGEM2-CC
0.223	ACCESS1.3
0.229	INM-CM4
0.235	BCC-CSM1.1-m
0.246	GFDL-ESM2G

Table S2. Ranking of CMIP5 models obtained with the three-dimensional WD.

Average of means	Model
0.881	MIROC-ESM-CHEM
0.978	IPSL-CM5A-LR
0.993	MIROC-ESM
1.030	IPSL-CM5A-MR
1.128	NorESM1-M
1.369	IPSL-CM5B-LR
1.412	BCC-CSM1.1
1.557	BNU-ESM
1.748	CMCC-CM
1.749	BCC-CSM1.1-m
1.785	MRI-CGCM3
1.785	CMCC-CMS
1.893	MPI-ESM-LR
2.120	GFDL-ESM2M
2.224	MPI-ESM-MR
2.335	GFDL-ESM2G
2.508	HadGEM2-CC
2.578	CNRM-CM5
2.657	HadGEM2-ES
2.694	ACCESS1.0
3.163	INM-CM4
3.239	ACCESS1.3

Table S3. Ranking obtained by averaging the three separate mean distances.

Average of the standard deviations	Model
0.160	MPI-ESM-MR
0.186	CMCC-CMS
0.189	MPI-ESM-LR
0.225	CMCC-CM
0.298	CNRM-CM5
0.326	ACCESS1.3
0.360	ACCESS1.0
0.362	IPSL-CM5A-LR
0.366	IPSL-CM5A-MR
0.369	GFDL-ESM2M
0.390	MIROC-ESM
0.391	NorESM1-M
0.406	MIROC-ESM-CHEM
0.434	HadGEM2-CC
0.443	HadGEM2-ES
0.452	IPSL-CM5B-LR
0.455	INM-CM4
0.532	BNU-ESM
0.573	GFDL-ESM2G
0.651	MRI-CGCM3
0.758	BCC-CSM1.1
0.762	BCC-CSM1.1-m

Table S4. Ranking obtained by averaging the three standard deviations.