Supporting Information for "Slide-hold-slide protocols and frictional healing in a simulated granular fault gouge"

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1. Background of the Discrete Element Method (DEM) used in this study

For two spheres $\{i, j\}$ in contact with each other that have the positions $\{\boldsymbol{r}_i, \boldsymbol{r}_j\}$, and diameters d_i and d_j, the normal ($F_{n_{ij}}$) and tangential ($F_{t_{ij}}$) forces on particle i in its interaction with particle j can be calculated from the following equations:

$$\boldsymbol{F}_{n_{ij}} = \sqrt{\delta}_{ij}\sqrt{\frac{d_i d_j}{2(d_i + d_j)}}(k_n \delta_{ij} \boldsymbol{n}_{ij} - m_{eff}\gamma_n \boldsymbol{v}_{n_{ij}}) \tag{1}$$

$$\boldsymbol{F}_{t_{ij}} = \sqrt{\delta}_{ij}\sqrt{\frac{d_i d_j}{2(d_i + d_j)}}(-k_t \boldsymbol{u}_{t_{ij}} - m_{eff}\gamma_t \boldsymbol{v}_{t_{ij}}) \tag{2}$$

in which k_n and k_t are the normal and tangential stiffness, and are defined as $k_n = (2/3)E/(1-\nu^2)$ and $k_t = 2E/(1+\nu)(2-\nu)$ (Mindlin, 1949). In the relations for the normal and tangential stiffnesses, E and ν are the Young's modulus and Poisson's ratio, respectively, and $m_{eff} = m_i m_j/(m_i + m_j)$ is defined as the effective mass of the two interacting spheres that have masses m_i and m_j. The relative normal and tangential velocities, $\boldsymbol{v}_{n_{ij}}$ and $\boldsymbol{v}_{t_{ij}}$, of the grains used in Eqs. 1 and 2 are defined as:

$$\boldsymbol{v}_{n_{ij}} = (\boldsymbol{v}_{ij} \cdot \boldsymbol{n}_{ij})\boldsymbol{n}_{ij} \tag{3}$$

$$\boldsymbol{v}_{t_{ij}} = \boldsymbol{v}_{ij} - \boldsymbol{v}_{n_{ij}} - \tfrac{1}{2}(\boldsymbol{\omega}_i + \boldsymbol{\omega}_j) \times \boldsymbol{r}_{ij} \tag{4}$$

in which $\{\boldsymbol{v}_i, \boldsymbol{v}_j\}$ are the linear, and $\{\boldsymbol{\omega}_i, \boldsymbol{\omega}_j\}$ are angular components of grain velocities, and $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\boldsymbol{n}_{ij} = \boldsymbol{r}_{ij}/r_{ij}$, with $r_{ij} = |\boldsymbol{r}_{ij}|$, and $\boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j$. Additionally, δ_{ij} is the normal compression of the grain and is defined as

$$\delta_{ij} = \tfrac{1}{2}(d_i + d_j) - r_{ij} \tag{5}$$

In Eqs. 1 and 2, the parameters γ_n and γ_t are the normal and tangential damping (viscoelastic) constants of the grain-grain interaction, respectively; For the choices of these two damping constants, we use the default LAMMPS option where $\gamma_t = 0.5\gamma_n$ (it has been shown that the choices of the ratio have little impact on the rheology of granular materials in the dense and quasi-static regime of shearing of hard particles we explore in this work (Ferdowsi & Rubin, 2020; Gaume et al., 2011; da Cruz et al., 2005; Silbert et al., 2001)). In the granular module of LAMMPS, the damping is implemented as a spring and dashpot in parallel for both the normal and tangential contacts.

Having defined the equations for contact forces and torques on each particle, i, we solve the Newton's second law to find the translational and rotational accelerations of particles located in a gravitational field $\boldsymbol{g}$,

$$\boldsymbol{F}_i^{tot} = m_i\boldsymbol{g} + \sum_j \left(\boldsymbol{F}_{n_{ij}} + \boldsymbol{F}_{t_{ij}}\right) \tag{6}$$

$$\boldsymbol{\tau}_i^{tot} = -\frac{1}{2}\sum_j \boldsymbol{F}_{t_{ij}} \times \boldsymbol{r}_{ij} \tag{7}$$

Slip occurs at grain contacts when the local shear stress exceeds the specified (constant) local grain-grain friction coefficient, μ_g. The value of μ_g determines the upper limit of the tangential force between two grains from the Coulomb criterion $F_t \leq \mu_g F_n$. This tangential force grows according to the non-linear Hertz-Mindlin contact law up to the point where $F_t/F_n = \mu_g$. After this point, the tangential force is held at $F_t = \mu_g F_n$ until the point that due to rearrangement of grains either $F_t \leq \mu_g F_n$ or the contact between grains is lost. While the model solves the

Newton's second law for each particle, it does not take into account wave propagation inside the grains.

Energy loss at contacts in the granular model is characterized by the "restitution coefficient", which potentially varies from 0 (complete energy loss) to 1 (zero loss). At the low sliding speeds of interest the adopted value of "restitution coefficient" appears to have very little influence on the macroscopic behavior of the system (Gaume et al., 2011; da Cruz et al., 2005; Silbert et al., 2001). The values of restitution coefficients, ϵ_n and ϵ_t for the normal and tangential directions respectively, are controlled by the choices of the damping coefficients $\gamma_{n,t}$ and contact stiffness $k_{n,t}$. For Hertzian contact law at the grain-grain scale, the restitution coefficient in the normal direction is obtained from the equation of relative motion of two spheres in contact:

$$\ddot{\delta} + \frac{E\sqrt{2d_{eff}}}{3m_{eff}(1-\nu^2)}\left(\delta^{3/2} + \frac{3}{2}A\sqrt{\delta}\dot{\delta}\right) = 0 \tag{8}$$

with the the initial condition $\dot{\delta}(0) = \boldsymbol{v}_n$ and $\delta(0) = 0$. Further, the variable A is defined as $A = \frac{1}{3}\frac{(3\gamma_t-\gamma_n)^2}{(3\gamma_t+2\gamma_n)}\left(\frac{(1-\nu^2)(1-2\nu)}{E\nu^2}\right)$, and $d_{eff} = d_i d_j/(d_i + d_j)$ is the effective diameter for spheres of diameters d_i and d_j. From solving this equation, the normal component of the coefficient of restitution is defined as the ratio of normal velocity of grains at the end of the collision, defined as $\dot{\delta}(t_{col})$, to the initial normal impact velocity of the grains: $\epsilon_n = \dot{\delta}(t_{col})/\dot{\delta}(0)$. Solving the same equation also gives the collision time t_{col} for given choices of the physical properties of grains and initial velocities that two grains collide. The restitution coefficient in the tangential direction can be obtained from a similar procedure but with implementing tangential damping coefficient (Brilliantov et al., 1996). The time step of our simulations is defined as $\Delta t = t_{col}/100$, with t_{col} evaluated here with the assumption of an impact velocity $\dot{\delta}(0)$ of 25 m/s (to be on the safe side for the choice of the simulation time-step and solve the equations of motions accurately; grain-grain impact velocities are highly unlikely to achieve 25 m/s at a given time-step in the quasi-static simulations reported in this work). The time-step criteria $\Delta t = t_{col}/50$ is based on previous values used and recommended by Silbert et al. (2001). The majority of the simulations in this study were performed with a very high restitution coefficient of $\epsilon_n = 0.98$, such that the system is damped minimally. However, we also have run a series of slide-hold simulations with a much lower restitution coefficient of $\epsilon_n = 0.3$. It can be argued that damping introduces some time-dependence at the contact scale. However, we have previously tested the transient behavior of the model considered here using the restitution coefficients that varied from nearly zero (complete damping) to nearly 1 (no damping) and found that the choice of the restitution coefficient exerted no significant influence on the system's behavior in the slow-sliding regime that we have been interested in exploring here and in the previous work (Ferdowsi & Rubin, 2020). With the very high choice of the restitution coefficient ($\epsilon_n = 0.98$), we refer to the model as practically having no time-dependence at the contact scale. Further information about the granular module of LAMMPS can be found in the LAMMPS manual and several references (Zhang & Makse, 2005; Silbert et al., 2001; Brilliantov et al., 1996). For more details about the implementation of the model in this manuscript, and a complete list of the governing dimensionless variables, we refer the reader to the "Computational Model" section and Appendix A of Ferdowsi and Rubin (2020). All details of the present model, except for the values of pulling spring stiffness or unless otherwise specified in the following, are identical to the "default" model of our previous paper.

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Table S1. DEM simulation parameters. If in some limited simulations, different parameter values are used, they are explicitly mentioned in the text.

Parameter	Value
Grain density, ρ	2500 [kg/m^3]
Young's modulus, E	50 [GPa]
Poisson ratio, ν	0.3
Grain-grain friction coefficient, μ_g	0.5
Confining pressure, σ_n	5
Coefficient of restitution, ϵ_n	0.98
Time step, Δt	2×10^{-8} [s]

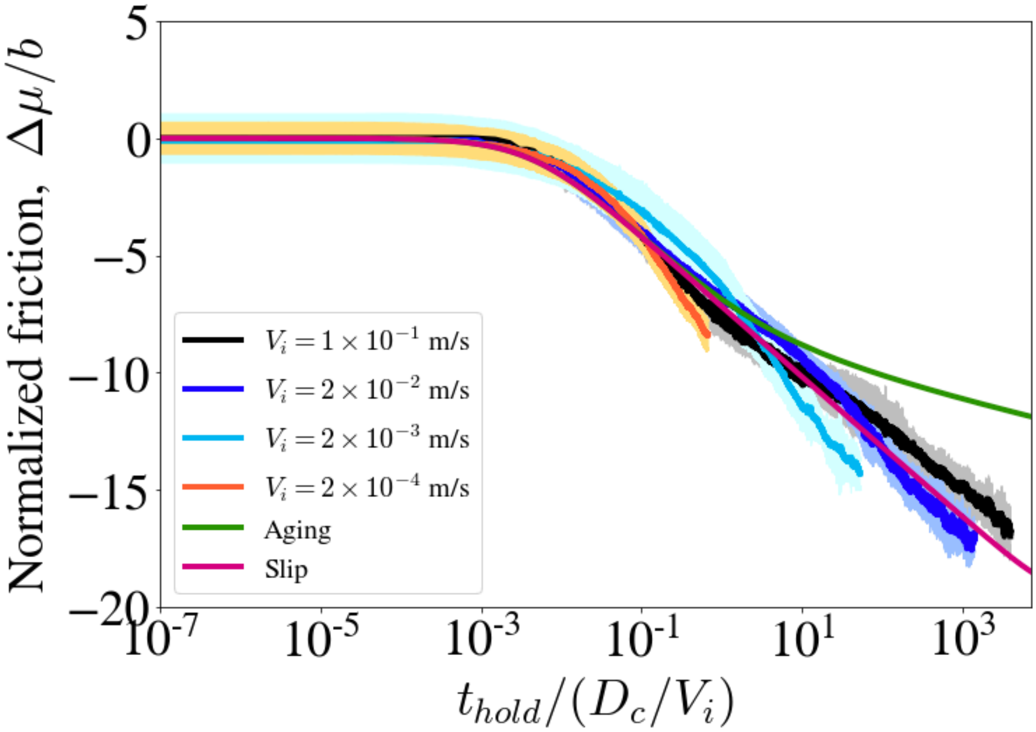

Figure S1. The variation of friction coefficient in slide-hold simulations with prior sliding velocities V_i of 2×10^{-4}, 2×10^{-3}, 2×10^{-2}, and 10^{-1} m/s. All simulations are run with system stiffness $\bar{k}_d \approx 425$ at the confining stress 5 MPa. The lines show the mean behavior of 8 realizations for each system, and the width of the shades regions around each line shows the 2-sigma deviations. The pink and green lines in panels (a) & (b) further show the predictions of the Slip and Aging laws, respectively, using the RSF parameters ($D_c = 0.0053$ m, $a = 0.0247$, $b = 0.0178$) determined independently from Slip-law fits to velocity-step tests performed on the same model (Ferdowsi and Rubin, 2020).

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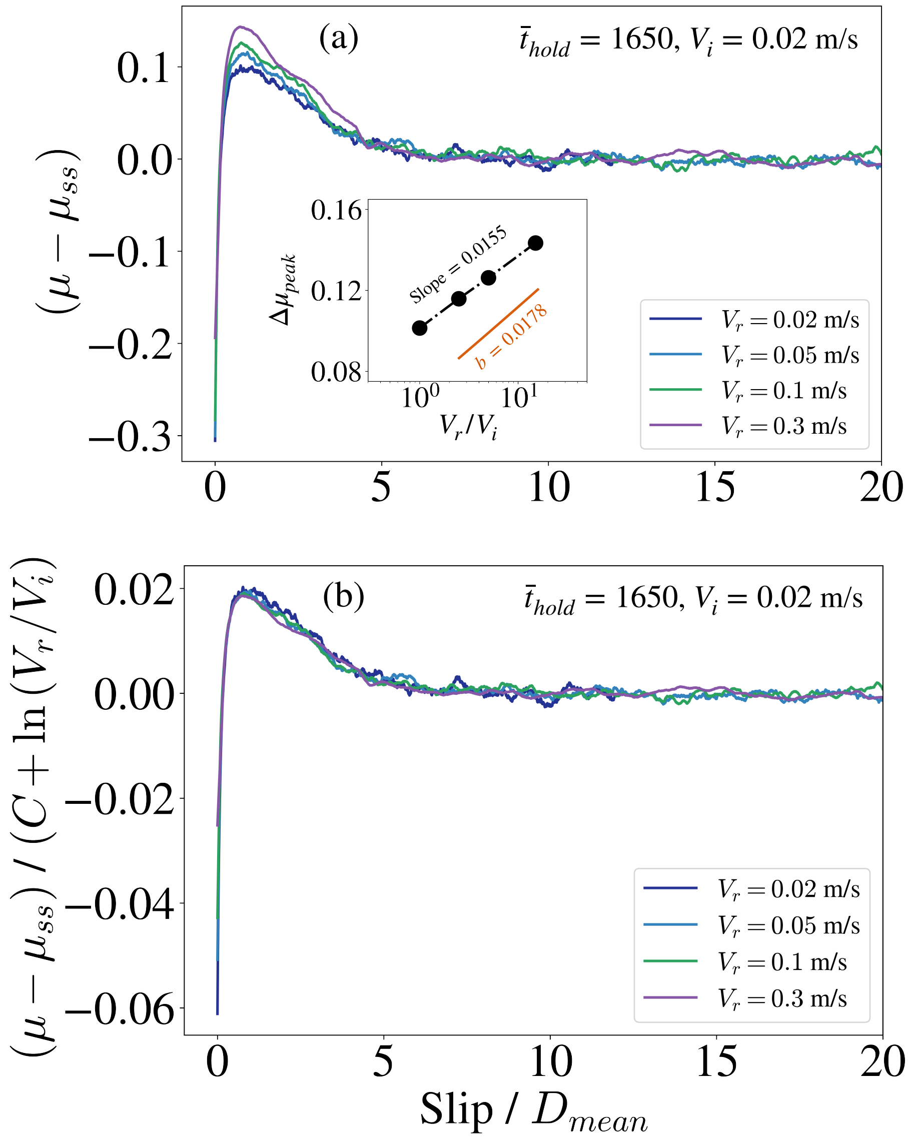

Figure S2. The variation of (a) friction $(\mu - \mu_{ss})$ versus slip distance (Slip / D_c), and (b) normalized friction $(\mu - \mu_{ss})/(C + \ln(V_r/V_i))$ versus slip distance (Slip / D_c), during reslide portion of slide-hold-slide simulations for normalized hold time $\bar{t}_{hold} \approx 1650$, with the initial sliding velocity, $V_i = 0.02$ m/s, and different reslide velocities, $V_r = 0.05$ m/s, 0.1, and 0.3 m/s. The value of $C \sim 5$ is chosen empirically. The inset in panel (a) shows the variation of peak friction $(\mu - \mu_{ss})_{peak}$ versus the ratio of reslide to initial velocity, V_r/V_i. All simulations are run with system stiffness $\bar{k}_d \approx 425$ at the confining stress 5 MPa.

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