

# Capillary Pressure Derived Relative Permeability Relationships

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## Abstract

Surface tension controls all aspects of fluid flow in porous media. Through measurements of surface tension interaction under multiphase conditions, a relative permeability relationship can be determined. Relative permeability is a numerical description of the interplay between two or more fluids and the porous media they flow through. It is a critical parameter for various tools used to characterized subsurface multiphase flow systems, such as numerical simulation for oil and gas development, carbon sequestration, and groundwater contamination remediation. Therefore, it is critical to get a good statistic distribution of relative permeability in the porous media under study.

Empirical relationships for determining relative permeability from capillary pressure are already well established but do not provide the needed flexibility that is required to match laboratory derive relative permeability relationships. By expanding the existing methods for calculating relative permeability from capillary pressure data it is possible to create both two and three-phase relative permeability relationship. Existing laboratory measured relative permeability data along with mercury intrusion capillary (MICP) data coupled with interfacial tension and contact angle measurements were used to determine the efficacy of this approach to relative permeability curve creation. The relative permeability relationships determined with this method were fit to the existing laboratory data to elucidate common fitting parameters that were then used to create relative permeability relationships from MICP data that does not have an associated laboratory measured relative permeability relationship.

## Theory and Derivation

The theory this method is based on is that the total flow rate through a porous media can be approximated by the flow through a bundle of capillary tubes ( 1 ).

$$Q_t = \sum_{i=1}^N Q_i \quad (1)$$

The flow rate through a single capillary tube can be approximated by Poiseuille's Law,

$$\Delta P = \frac{8\mu L Q}{\pi r^4} \quad (2)$$

where  $\mu$  is the fluid viscosity,  $\Delta P$  is the pressure differential across the capillary tube of length  $L$  and radius  $r$  with a fluid viscosity of  $\mu$ . It is assumed that the capillary tube length  $L$  is equivalent to the fluid path length. Darcy's Law for laminar flow of an incompressible fluid through a porous media is,

$$Q = \frac{k \Delta P}{\mu L} \quad (3)$$

where  $k$  is the permeability and  $\Delta P$  is the pressure differential of a fluid with  $\mu$  viscosity across a sample of area  $A$  and length  $L$ . Rearranging ( 2 ) in terms of  $Q$  and then substituting Darcy's Law ( 3 ) for  $Q$  gives the following formula in terms of permeability,

$$k = \frac{\pi r^4 L}{8 A l} \quad (4)$$

The volume of a capillary tube is given as,

$$V = \pi r^2 l \quad (5)$$

where  $V$  is the pore volume. Rearranging ( 5 ) in terms of  $\pi$  and substitute into ( 4 ) to get the volume term, giving the following relationship,

$$k = \frac{r^2 L V}{8 A l^2} \quad (6)$$

Next, the relationship needs to be set in terms of saturation. Saturation is related to volume in a porous media by,

$$S = \frac{V}{\phi A L} \quad (7)$$

where  $\phi$  is the porosity. Rearranging ( 7 ) in terms of volume and substituted into ( 6 ) to give the following relationship,

$$k = \frac{\phi r^2 L^2 S}{8 l^2} \quad (8)$$

Tortuosity ( $\tau$ ) is introduced to account for the fact that in a porous media the fluid flow path length  $l$  is longer than the sample length  $L$  and also assumed to be inversely related to the pore radius by,

$$\tau = \frac{l}{L} = \frac{a}{r^b} \quad (9)$$

where  $a$  and  $b$  are constants defined by the porous media. Substituting ( 9 ) into ( 8 ) gives the following equation in terms of pore radius,

$$k = \frac{\phi r^{2(1+b)} S}{8 a^2} \quad (10)$$

Capillary pressure relationship to pore radius is given by the following formula,

$$P_c = \frac{2\sigma \cos \theta}{r} \quad (11)$$

where  $\sigma$  is the interfacial tension and  $\theta$  is the contact angle of the wetting and non-wetting fluid phases. Rearranging ( 11 ) in terms of pore radius and substituting into ( 10 ) gives the following relationship for a single capillary tube,

$$k = \frac{\phi (2\sigma \cos \theta)^{2(1+b)} S}{8 a^2 P_c^{2(1+b)}} \quad (12)$$

For a bundle of  $N$  capillary tubes  $a$  and  $b$  are assumed constant,

$$k = \frac{\phi (2\sigma \cos \theta)^{2(1+b)}}{8 a^2} \sum_{i=1}^N \frac{\partial S_i}{P_{c_i}^{2(1+b)}} \quad (13)$$

To this point in the derivation we have been following Fatt and Dykstra (1951) derivation. We now modified this relationship by parameterizing the saturation endpoints of the integral from endpoints that span the total saturation range of 0% to 100% to endpoints that span from the residual wetting phase saturation ( $S_{wr}$ ) to the maximum wetting phase saturation ( $S_{max}$ ). If we integrate over the saturation range of  $S_{wr}$  to  $S_{max}$  we get a measure of the permeability of the sample.

$$k = \frac{\phi (2\sigma \cos \theta)^{2(1+b)}}{8 a^2} \int_{S_{wr}}^{S_{max}} \frac{\partial S_w}{P_c^{2(1+b)}} \quad (14)$$

If we integrate over the saturation range of  $S_{min}$  to  $S_w$  we get a measure of the effective permeability.

$$k_e = \frac{\phi (2\sigma \cos \theta)^{2(1+b)}}{8 a^2} \int_{S_{wr}}^{S_w} \frac{\partial S_w}{P_c^{2(1+b)}} \quad (15)$$

Relative permeability  $k_r$  is related to permeability  $k$  and effective permeability  $k_e$  by,

$$k_r = \frac{k_e}{k} \quad (16)$$

Substituting ( 14 ) and ( 15 ) into ( 16 ) gives the following formula for wetting phase relative permeability  $k_{rw}$

$$k_{rw} = \frac{\int_{S_{wr}}^{S_w} \frac{dS_w}{P_c^{2+2b}}}{\int_{S_{wr}}^{S_{max}} \frac{dS_w}{P_c^{2+2b}}} \quad (17)$$

Here, we extend the formula derived above by Fatt and Dyskra (1951) to the non-wetting phase relative permeability  $k_{rn}$ . We change the saturation range in ( 15 ), the effective permeability, to span the interval from the  $S_w$  to the  $S_{max}$ . This represents the change in the non-wetting phase saturation and when substituted into ( 16 ) give the non-wetting phase relative permeability  $k_{rn}$ .

$$k_{rn} = \frac{\int_{S_w}^{S_{max}} \frac{dS_w}{P_c^{2+2b}}}{\int_{S_{wr}}^{S_{max}} \frac{dS_w}{P_c^{2+2b}}} \quad (18)$$

In order to use ( 17 ) and ( 18 ) to generate relative permeability curves from capillary pressure data we use the trapezoidal rule to approximate the integration and make the formula useable for discrete data.

$$k_{rw} = k_w \max * \left( \frac{S_w - S_{wr}}{S_w^{max} - S_{wr}} \right)^c * \left( \frac{S_{wr}}{P_c(S_{wr})^{2+2b} + \frac{S_w}{P_c(S_w^{max})^{2+2b}}} \right) \quad (19)$$

$$k_{rn} = k_n \max * \left( \frac{S_w^{max} - S_w}{S_w^{max} - S_{wr}} \right)^c * \left( \frac{S_w}{P_c(S_w)^{2+2b} + \frac{S_w^{max}}{P_c(S_w^{max})^{2+2b}}} \right) \quad (20)$$

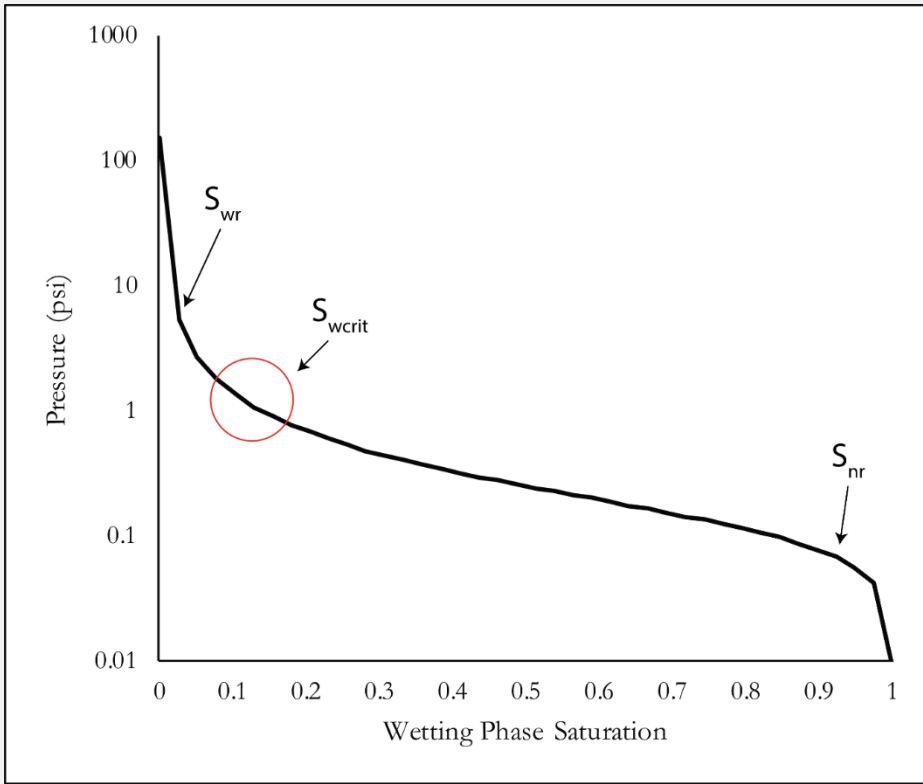
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## Method for determining Relative Permeability from Capillary Pressure

### Two-Phase System

- Determining the saturation endpoints is the first step. Plot the capillary pressure versus wetting phase saturation ( $1-S_{Hg}$ ) data from an MICP or other capillary pressure tests. The inflection points on this graph are used to determine residual phase saturations needed to calculate the relative permeability
  - The non-wetting phase residual saturation is found by observing the inflection point corresponding to the change in the slope from convex to concave that happens during initial non-wetting phased intrusion into the sample (labeled  $S_{nr}$ )
  - The wetting phase saturation is more difficult to determine. This value is normally associated with the point on the graph where it goes to near vertical, indicating no more mercury can be intruded into the very small pores and thus reaching wetting phase residual saturation,  $S_{wr}$ . But, analysis of capillary pressure and relative permeability data from the SWP and in the literature indicates that the wetting phase residual saturation corresponds with the area labeled  $S_{wcrit}$
- The phase permeability needs to be determined next. The wetting phase permeability is calculated using the Purcell method at the  $S_{nr}$  point on the capillary pressure graph. The non-wetting phase permeability is a more difficult to determine. First, the angles of the 1<sup>st</sup> derivative of the capillary pressure data must be calculated for each pressure step. Where that angle equals 45 degrees corresponds to the bulk fluid flowing through the majority of the pore space. Equation 21 is used to calculate phase permeability at this point by applying the appropriate interfacial tension ( $\sigma$ ) and contact angle ( $\theta$ ) parameters for the fluid pair being modeled (CO<sub>2</sub>/brine, CO<sub>2</sub>/oil, oil/water)



$$k_e = \frac{1}{2} (\sigma \cos \theta)^2 * \tau * \phi * \int_{S_{Hg}^{min}}^{S_{Hg}^{max}} \frac{dS_{Hg}}{P_c(S_{Hg})^2} \quad (21)$$

- Next, the relative permeability endpoints will need to be determined. The relative permeability endpoint can be determined by dividing the effective phase permeability by the intrinsic permeability ( $k_{ew}/k$  and  $k_{en}/k$ ). If the effective phase permeability ( $k_e$ ) is larger than the intrinsic permeability ( $k$ ) it is assumed that the maximum phase relative permeability endpoint is 1
- The next step is to calculate each phase's relative permeability curve using our derived formulas ( 19 ) and ( 20 ). Both the 'b' exponent and 'c' exponent are defined independently for the wetting and non-wetting curves
- The relative permeability calculated using equations ( 19 ) and ( 20 ) is then multiplied by the maximum phase relative permeability determined in step 3 to get the 'normalized' phase relative permeability. The wetting phased saturation is then plotted against the 'normalized' phase relative permeability to give the final relative permeability relationship that then can be used in numerical simulators

### Three-Phase System

- In order to modify this method to determine three-phase relative permeability relationships, we assume that a three-phase relationship can be approximated by two fluid pairs, oil/water and gas/oil (or CO<sub>2</sub>/oil) and the use of a combination model, such as the Stone II model
  - The oil/water relative permeability pair is the same as would be determined using the two-phase method outlined above
  - The gas/oil (or CO<sub>2</sub>/oil) relative permeability pair is calculated using a residual wetting phase saturation that is the sum of the wetting and non-wetting phase residual saturations used in the oil/water relative permeability curve. The residual gas saturation is calculated the same as the residual non-wetting phase saturation in the two-phase method

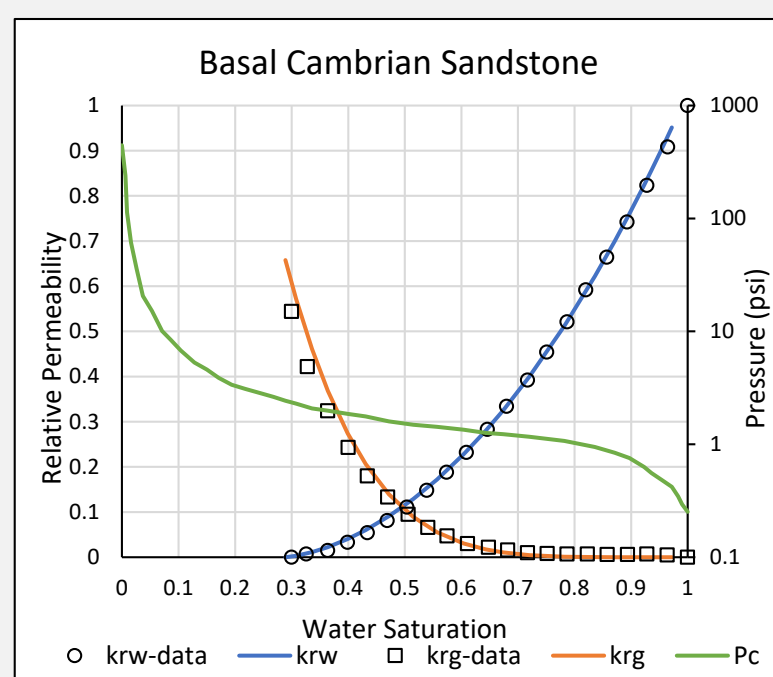
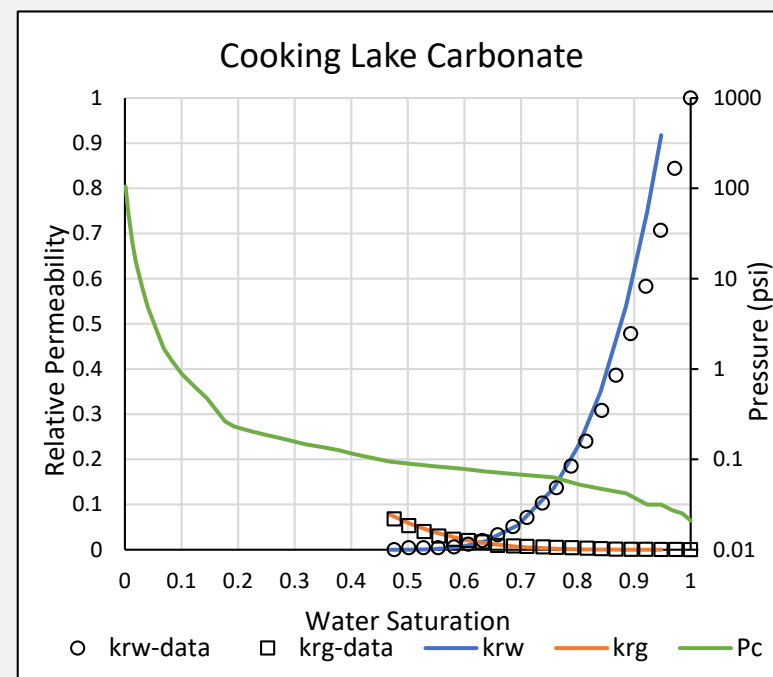
## Fitting Laboratory Data

### Two-phase curves from literature data

To understanding how well the method performed, a number of laboratory data from the literature was studied. The goal was to elucidate common fitting parameters that could be applied to capillary pressure data that lacks a corresponding relative permeability curve. Laboratory data published by Bennion and Bachu (2005, 2006) and Krevor et al. (2012) had both the capillary pressure data and corresponding relative permeability data for a number of sandstone and carbonate formations

Fitting parameters were derived using the sum of least squares minimization algorithm. The wetting phase 'b' value of -1 simplifies ( 19 ), removing the capillary pressure term and reducing the equation to a simple saturation relationship, shown in ( 22 )

When the non-wetting phase 'b' value of 0.5 is substituted into ( 20 ) it indicates that there is an inverse cubic relationship between capillary pressure and saturation



Gas/water pair exponents 'b' and 'c'

Formation Name	Capillary Pressure exponent 'b'		Effective Saturation exponent 'c'	
	b-krn	b-krw	c-krn	c-krw
Basal Cambrian Sandstone (1)	0.5	-1	5.23	1.34
Wabamun - low perm (1)	0.5	-1	5.99	1.22
Ellerslie Sandstone (1)	0.5	-1	2.18	1.89
Viking Sandstone (1)	0.5	-1	2.93	2.93
Cooking Lake Carbonate (1)	0.5	-1	4.55	3.01
Nisku Carbonate (1)	0.5	-1	1.11	2.32
Berea Sandstone (2)	0.5	-1	3.05	3.29
Mt Simon Sandstone (2)	0.5	-1	1.80	5.76
Paaratte Sandstone (2)	0.5	-1	3.52	6.10
Tuscaloosa Sandstone (2)	0.5	-1	5.66	2.50
Likely Fitting Parameters	0.5	-1	3.60	3.03

$$k_{rw} = k_w \max * \left( \frac{S_w - S_{wr}}{S_w^{max} - S_{wr}} \right)^c * \left( \frac{S_{wr} + S_w}{S_{wr} + S_w^{max}} \right) \quad (22)$$

$$k_{rn} = k_n \max * \left( \frac{S_w^{max} - S_w}{S_w^{max} - S_{wr}} \right)^c * \left( \frac{\frac{S_w}{P_c(S_w)^3} + \frac{S_w^{max}}{P_c(S_w^{max})^3}}{\frac{S_{wr}}{P_c(S_{wr})^3} + \frac{S_w^{max}}{P_c(S_w^{max})^3}} \right) \quad (23)$$

### Three-phase curves from SWP data

To understand how the three-phase variation of our method performed we fit the method to laboratory data collected by the SWP. Both relative permeability and capillary pressure were measured on core samples from the same hydraulic unit, allowing the capillary pressure curves to be used in creating the relative permeability curve and then directly compared to the laboratory measured data. From the limited number of samples testes, just like in the two-phase system the 'b' values seem to have the same or similar values across the samples tested while the 'c' values show less of a correlation. ( 24 ) to ( 27 ) illustrate how a common 'b' value simplifies the equation.

Gas/oil pair exponents 'b' and 'c'

Sample ID	Capillary Pressure exponent 'b' (gas/oil)		Effective Saturation exponent 'c' (gas/oil)	
	b-krg	b-krog	c-krg	c-krog
PU 1 Sample 3-7	0.87	-0.5	3.95	1.14
PU 2 Sample 3-7	0.58	-0.5	3.47	1.24
NMT1 Core19	0.5	-0.5	9	2
NMT2 CoreL5	0.20	-2	9.65	0.85
Likely fitting parameters	0.5	-0.5	3.5 or 9	1

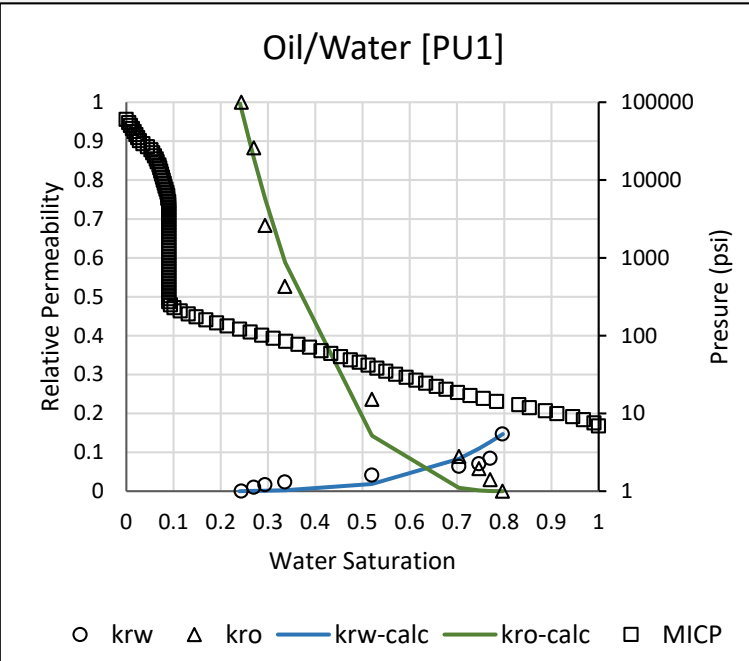
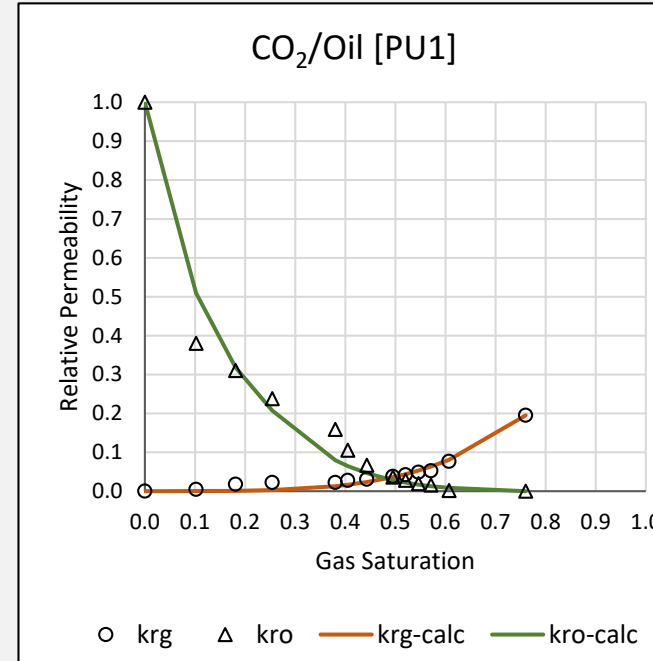
Oil/water pair exponents 'b' and 'c'

Sample ID	Capillary Pressure exponent 'b' (oil/water)		Effective Saturation exponent 'c' (oil/water)	
	b-krw	b-krow	c-krw	c-krow
PU 1 sample 5-1	-0.5	0.84	1	2.80
PU 2 Sample 3-9	-0.5	0.11	0.1	2.86
NMT1 Core19	-1	-0.5	1.18	3.66
NMT2 CoreL5	-0.5	0.1	1	2.82
Likely fitting parameters	-0.5	0.1	1	2.8

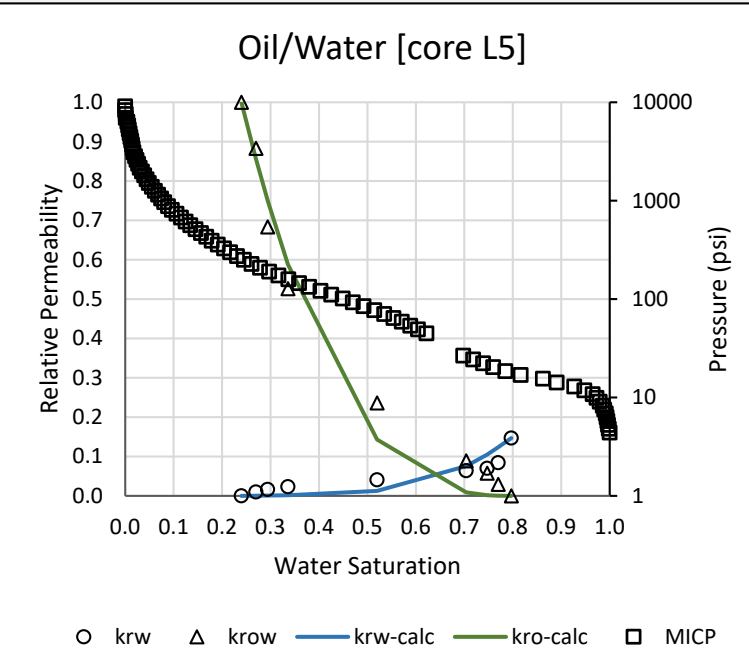
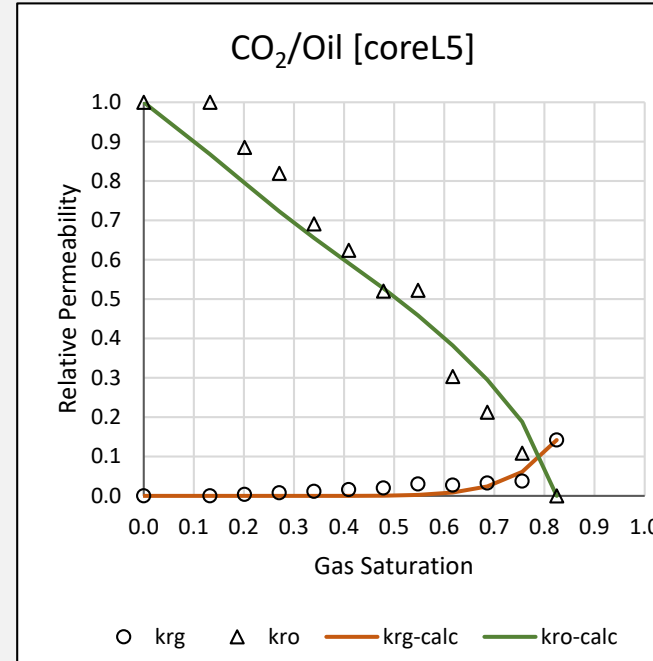
$$k_{rg} = k_g \max * \left( \frac{S_l^{max} - S_l}{S_l^{max} - S_{lr}} \right)^c * \left( \frac{\frac{S_l}{P_c(S_l)^3} + \frac{S_l^{max}}{P_c(S_l^{max})^3}}{\frac{S_{lr}}{P_c(S_{lr})^3} + \frac{S_l^{max}}{P_c(S_l^{max})^3}} \right) \quad (24)$$

$$k_{rog} = k_{og} \max * \left( \frac{S_l - S_{lr}}{S_l^{max} - S_{lr}} \right)^c * \left( \frac{\frac{S_{lr}}{P_c(S_{lr})^3} + \frac{S_l^{max}}{P_c(S_l^{max})^3}}{\frac{S_l}{P_c(S_l)^3} + \frac{S_l^{max}}{P_c(S_l^{max})^3}} \right) \quad (25)$$

Petroleum University Relative Permeability Relationship 1



New Mexico Tech Relative Permeability Relationship 1



$$k_{rw} = k_w \max * \left( \frac{S_w - S_{wr}}{S_w^{max} - S_{wr}} \right)^c * \left( \frac{\frac{S_{wr}}{P_c(S_{wr})^2.2} + \frac{S_w}{P_c(S_w)^2.2}}{\frac{S_{wr}}{P_c(S_{wr})^2.2} + \frac{S_w^{max}}{P_c(S_w^{max})^2.2}} \right) \quad (26)$$

$$k_{row} = k_{ow} \max * \left( \frac{S_w^{max} - S_w}{S_w^{max} - S_{wr}} \right)^c * \left( \frac{\frac{S_w}{P_c(S_w)^2.2} + \frac{S_w^{max}}{P_c(S_w^{max})^2.2}}{\frac{S_{wr}}{P_c(S_{wr})^2.2} + \frac{S_w^{max}}{P_c(S_w^{max})^2.2}} \right) \quad (27)$$

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