A Fourier-series modeling approach to develop corrections to drag in orbit

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Once upon a drag-coefficient

- Parameter governing atmosphere-satellite interaction
- Models used in orbit-determination

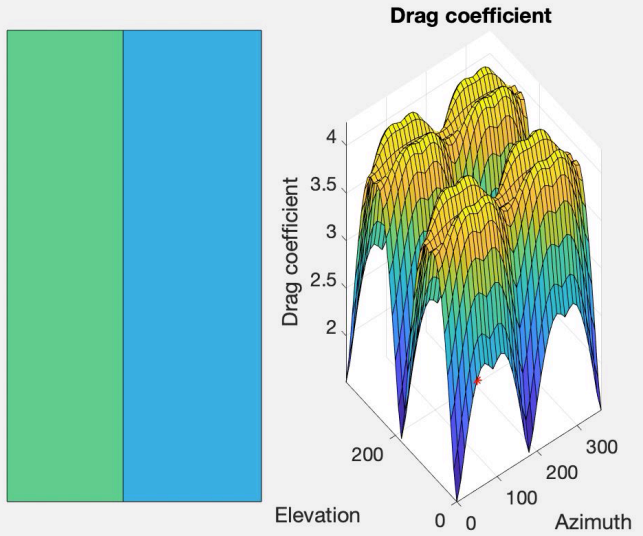
$$a_{drag} = -\frac{1}{2}\rho C_d \frac{A_{ref}}{m} v_r^2 \hat{u}$$

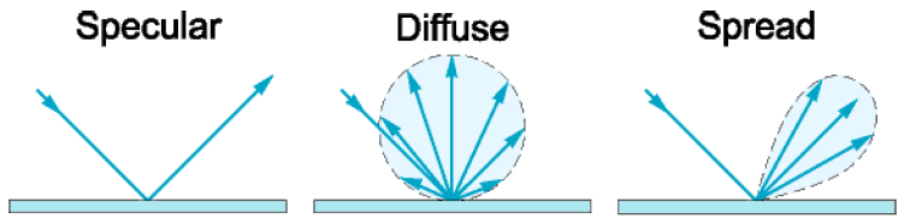
- Analytical approach
- Numerical approach
- Machine-learning

Physics-based

- Cannonball model

Estimation-based





Specular, diffuse, and spread reflection from a surface.

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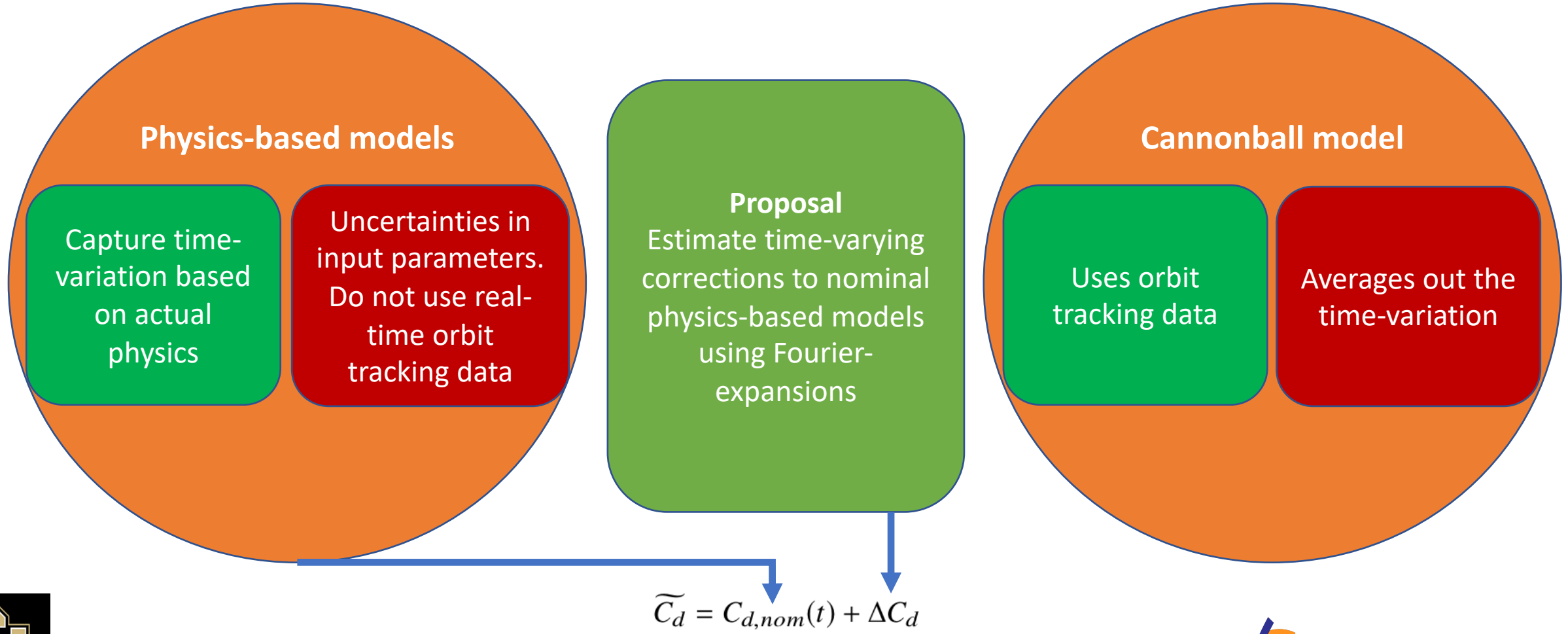
A Fourier-expansion based approach



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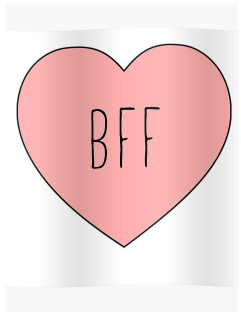
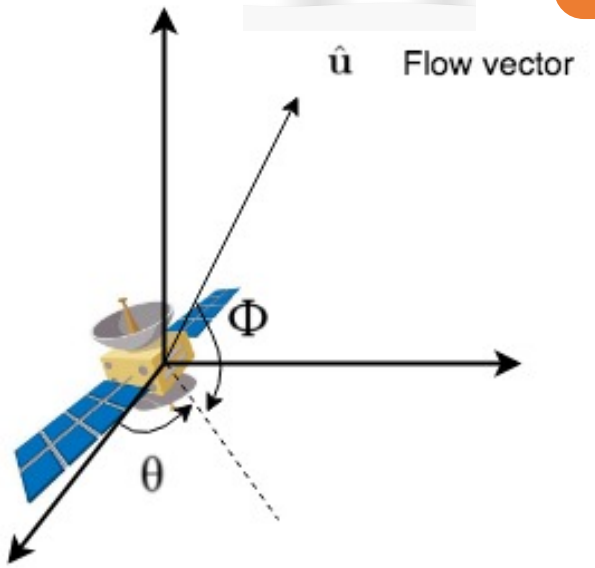
Fourier what?



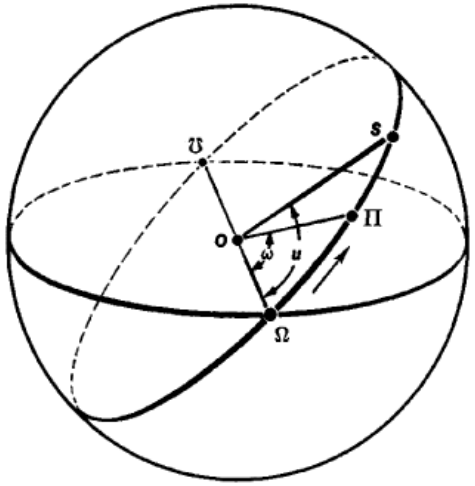


Body-fixed Fourier (BFF) model

- Fourier expansions around orientation of velocity vector in body frame
- Captures variations due to attitude

Orbit-fixed Fourier (OFF) model

- Fourier series expansion around the argument of latitude of the satellite
- Captures variations due to ambient parameters in orbit



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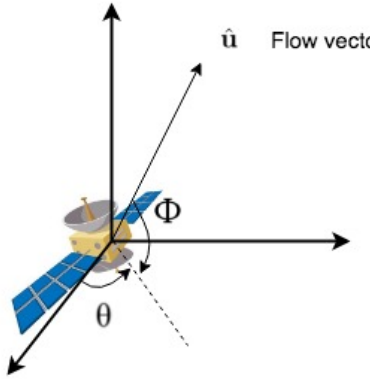
Body-orbit models

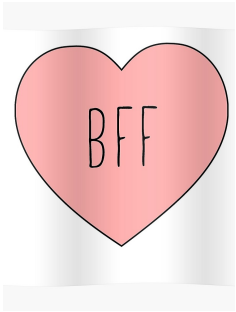
Body-orbit double Fourier (BODF) model

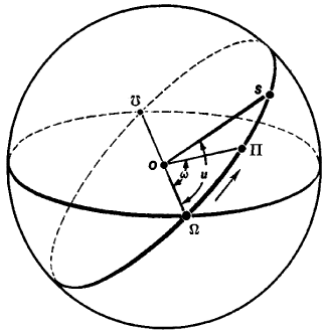
- Capture variations due to both attitude and ambient parameters

Body-orbit summation (BOS) model

- Ignore the cross-coupled terms







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Some results

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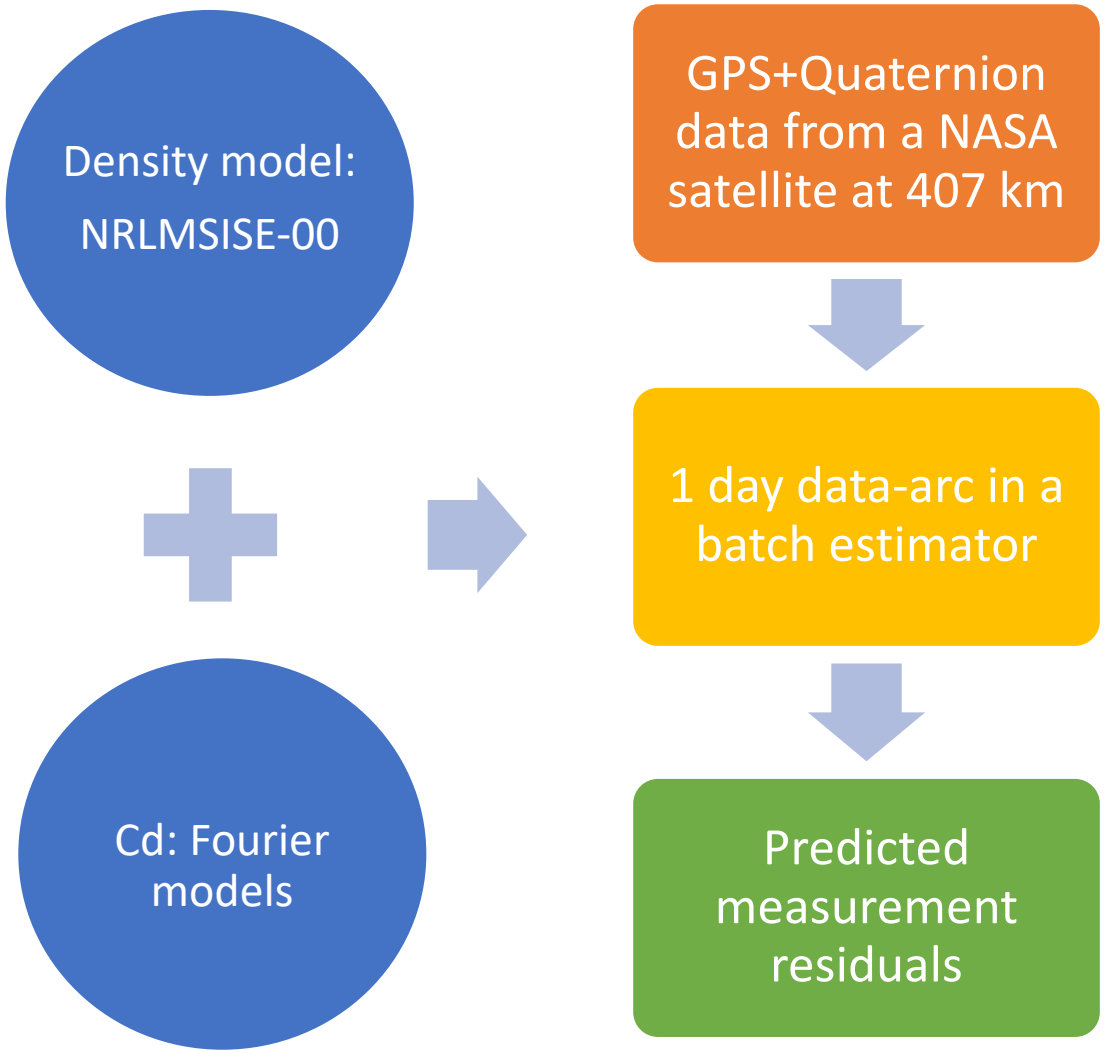
Application to a NASA satellite



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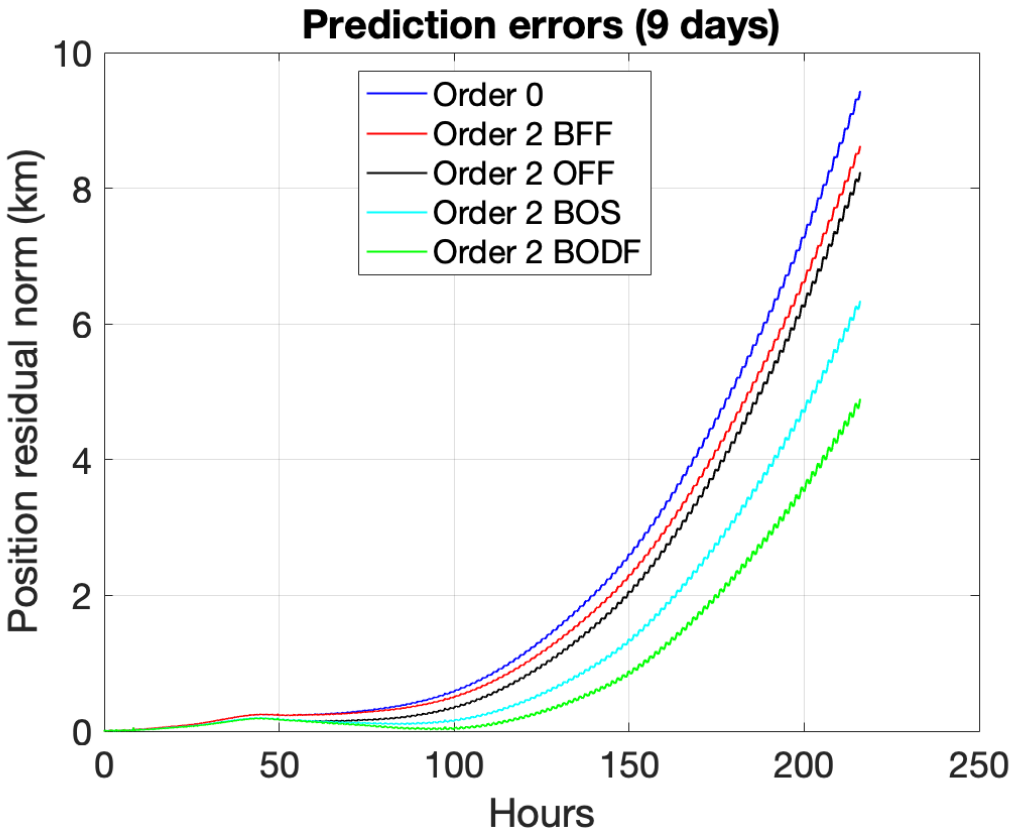
Application to a NASA satellite

- Data: Jun 29- Jul 6, 2017
- BODF reduces error by ~ 50 % over cannonball

- Order 0: Cannonball
  - BFF: Body-fixed, OFF: Orbit-fixed, BODF: Body-orbit, BOS: Body-orbit summation



Jun 29-Jul 6, 2017: Attitude maneuver

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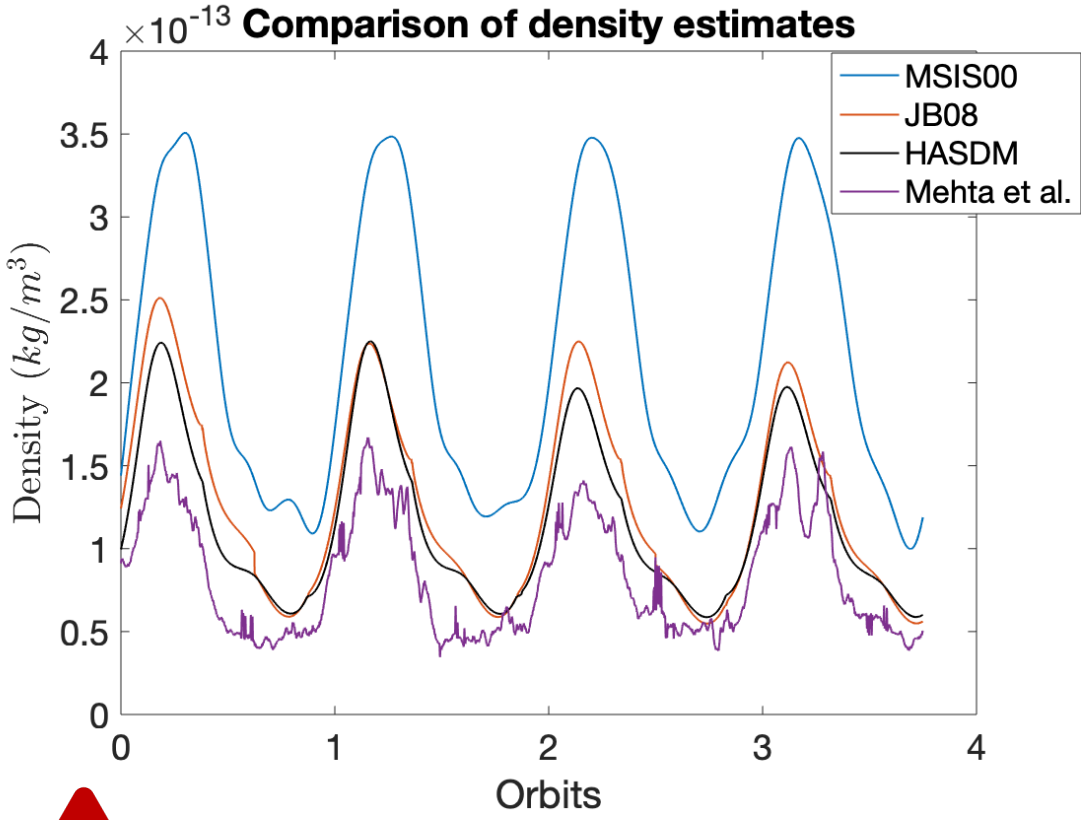
Application to GRACE

- Day 82-87, 2007, quiet geomagnetic conditions
- Densities used in estimator
  - NRLMSISE-00
  - JB08
  - HASDM
  - Estimates from Mehta et al.[1]
- Drag coefficient model:
  - Nominal drag coefficients from Mehta et al.
  - Fourier model to estimate corrections

Highly correlated
Order of truncation dependent on density accuracy

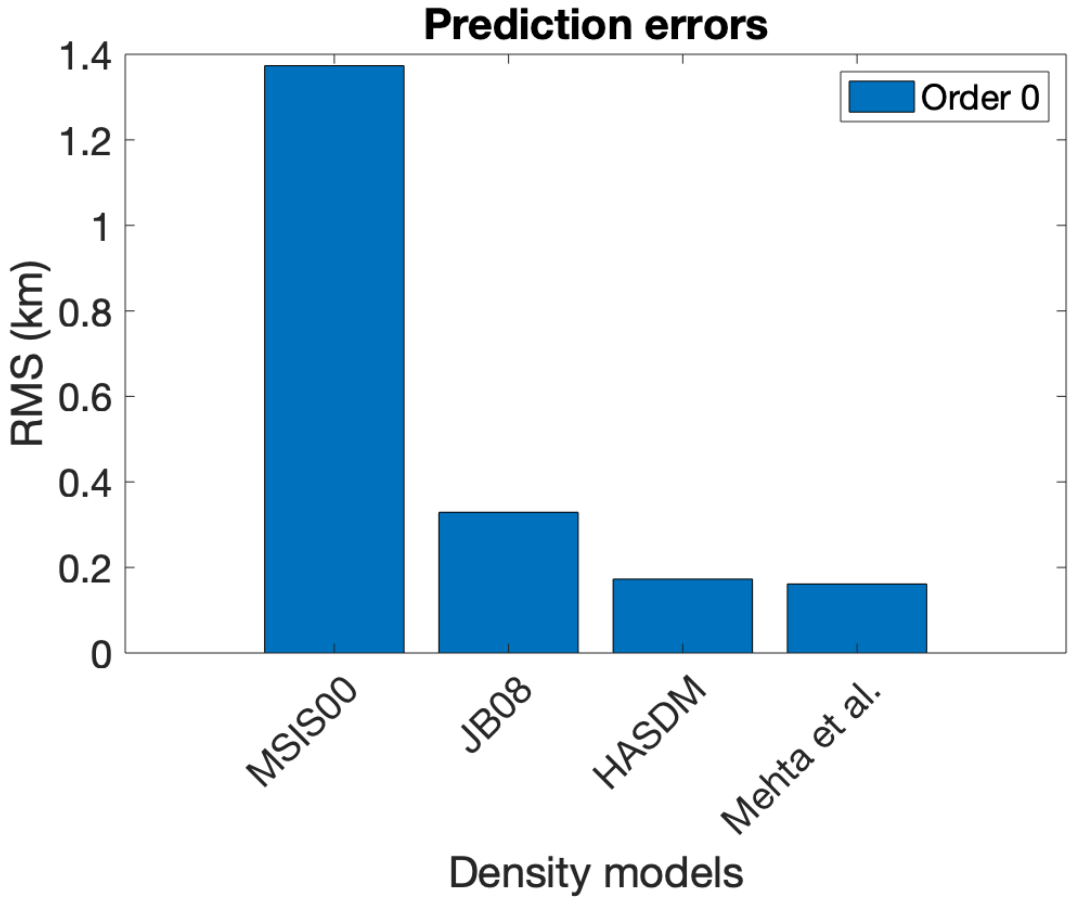


[1] Mehta et al., New density estimates derived using accelerometers on board the CHAMP and GRACE satellites, *Space Weather,* DOI:10.1002/2016SW001562

Application to GRACE

Order 0 Cd (Cannonball model)

- MSIS00 > JB08>HASDM> Mehta et al.



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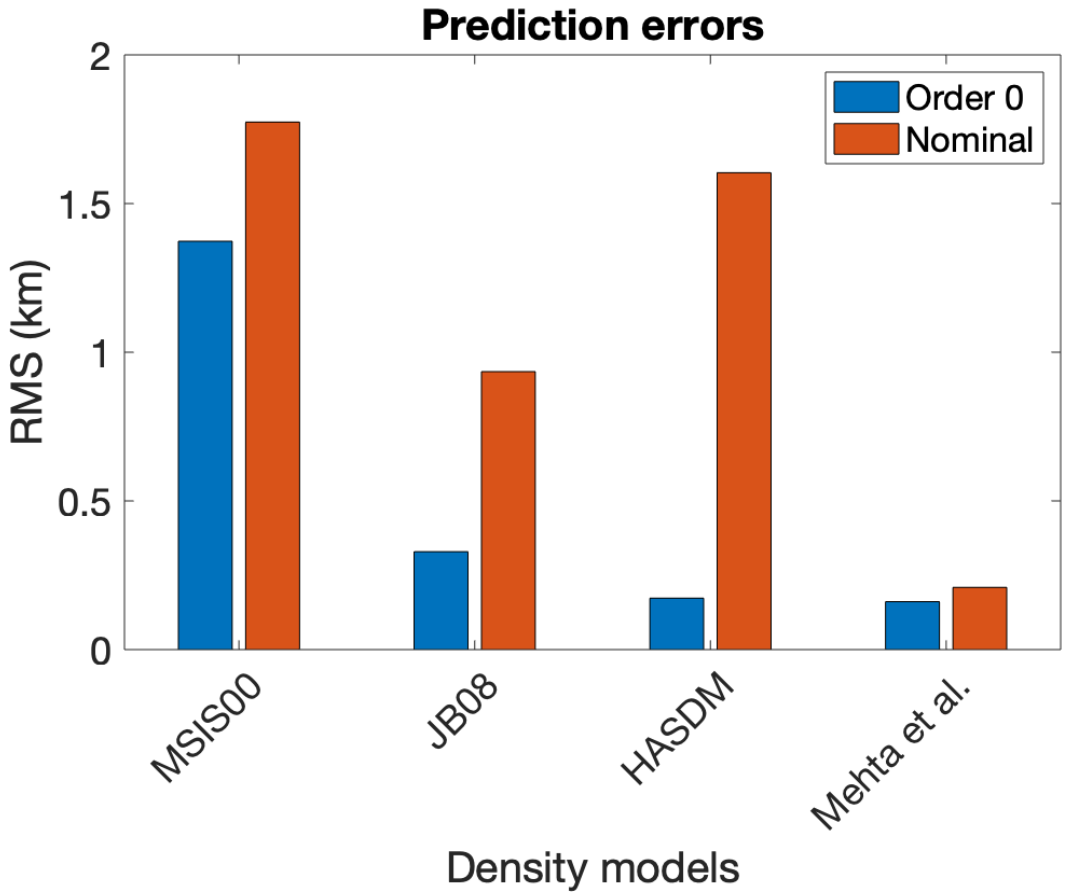
Application to GRACE

Nominal Cd (Mehta et al.)

- Increase in prediction errors



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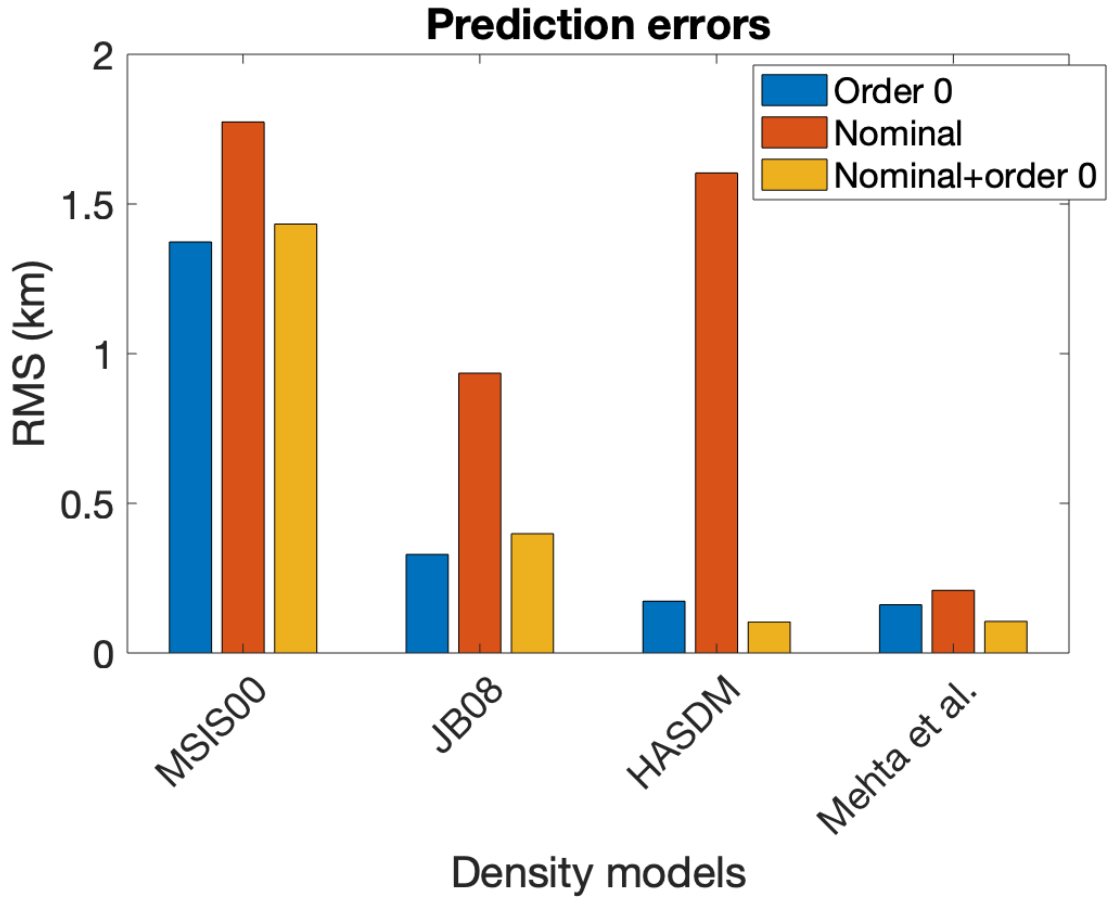
Application to GRACE

Nominal + Cannonball

- Improved performance for HASDM and Mehta et al.



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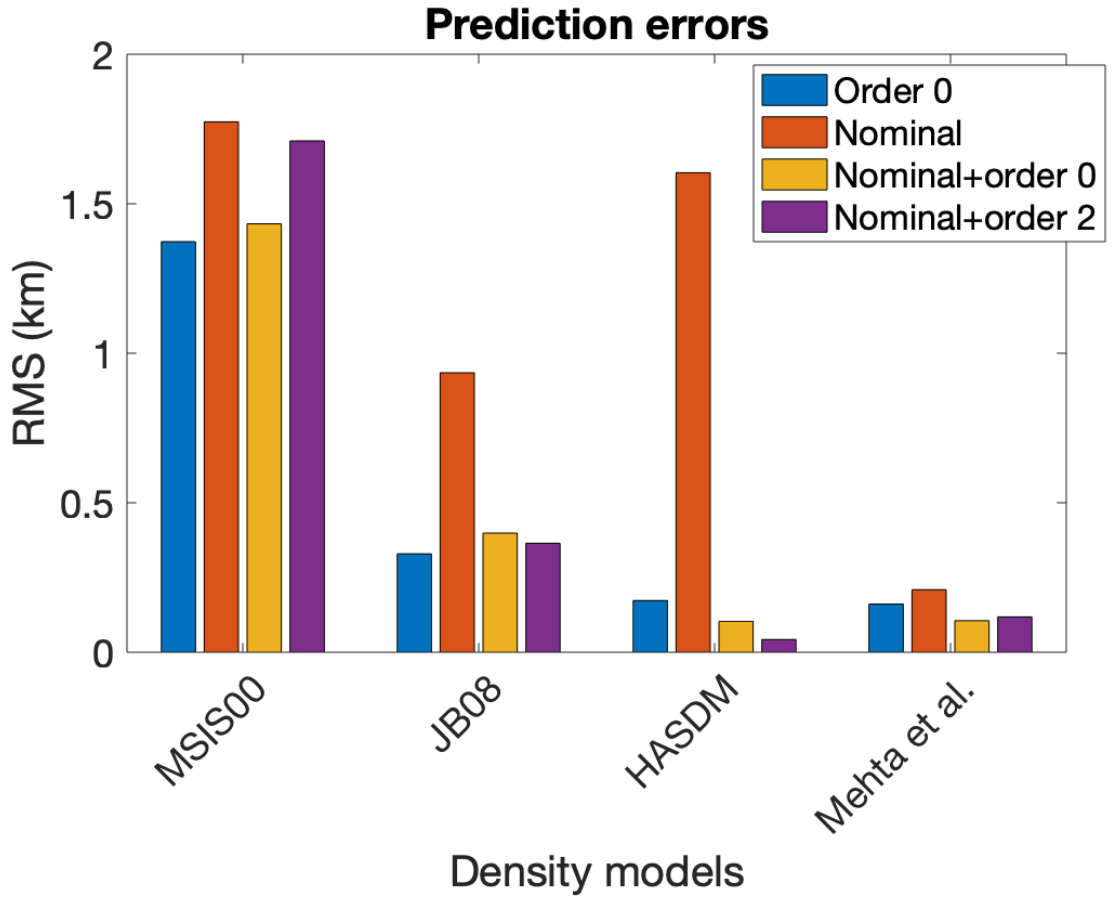
Application to GRACE

Nominal + Order 2 OFF

- Improved performance for HASDM

Possibilities of improving MSIS00 results?



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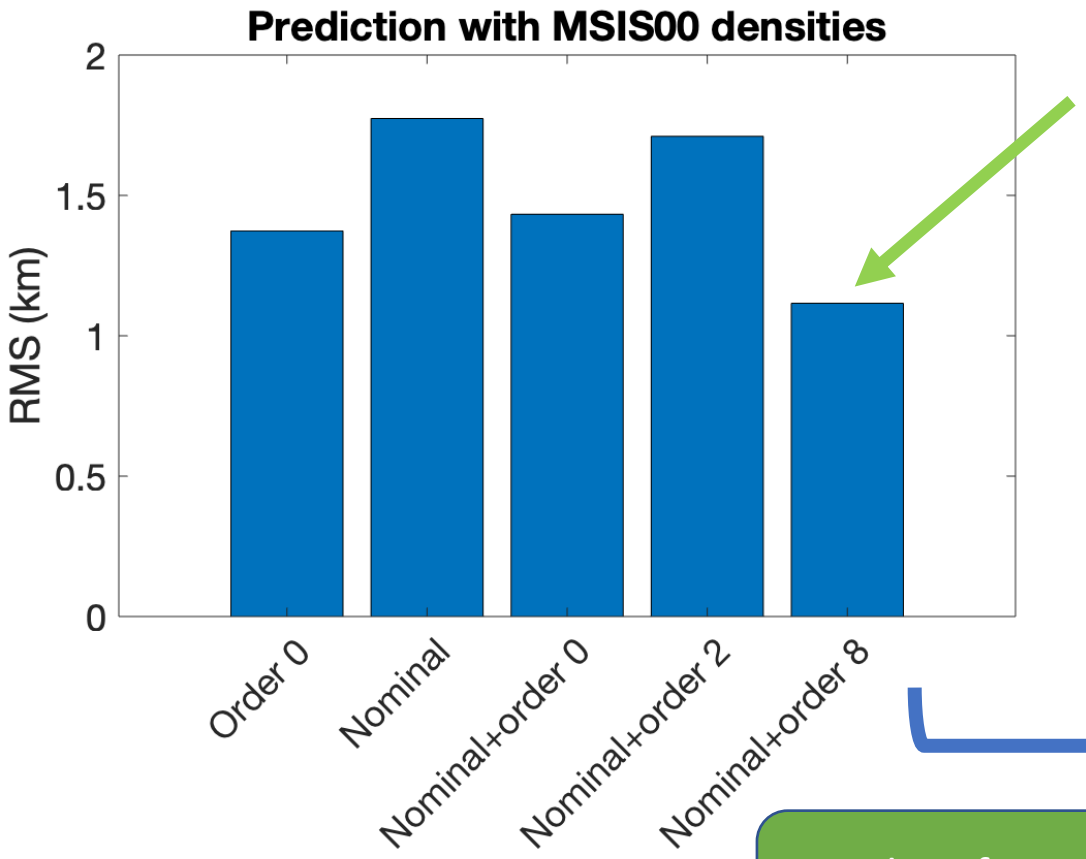
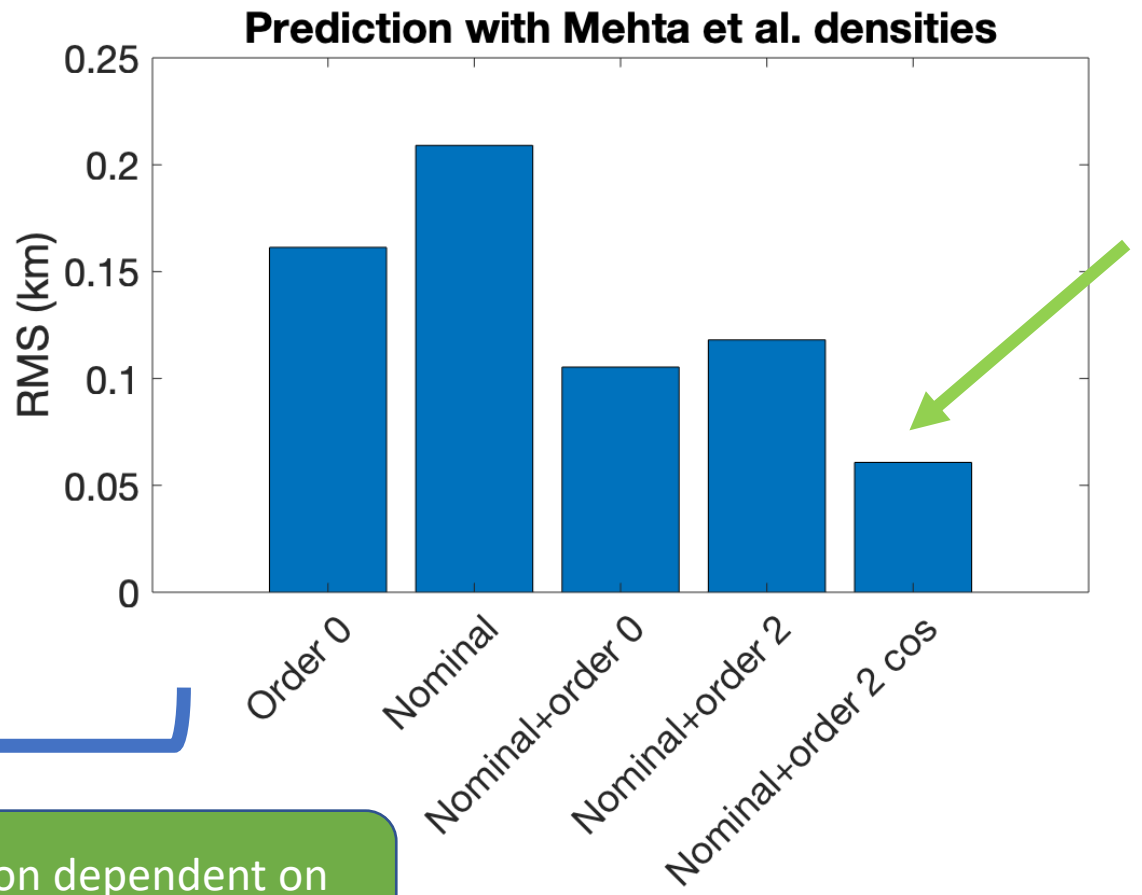
Application to GRACE





Order of truncation dependent on density accuracy

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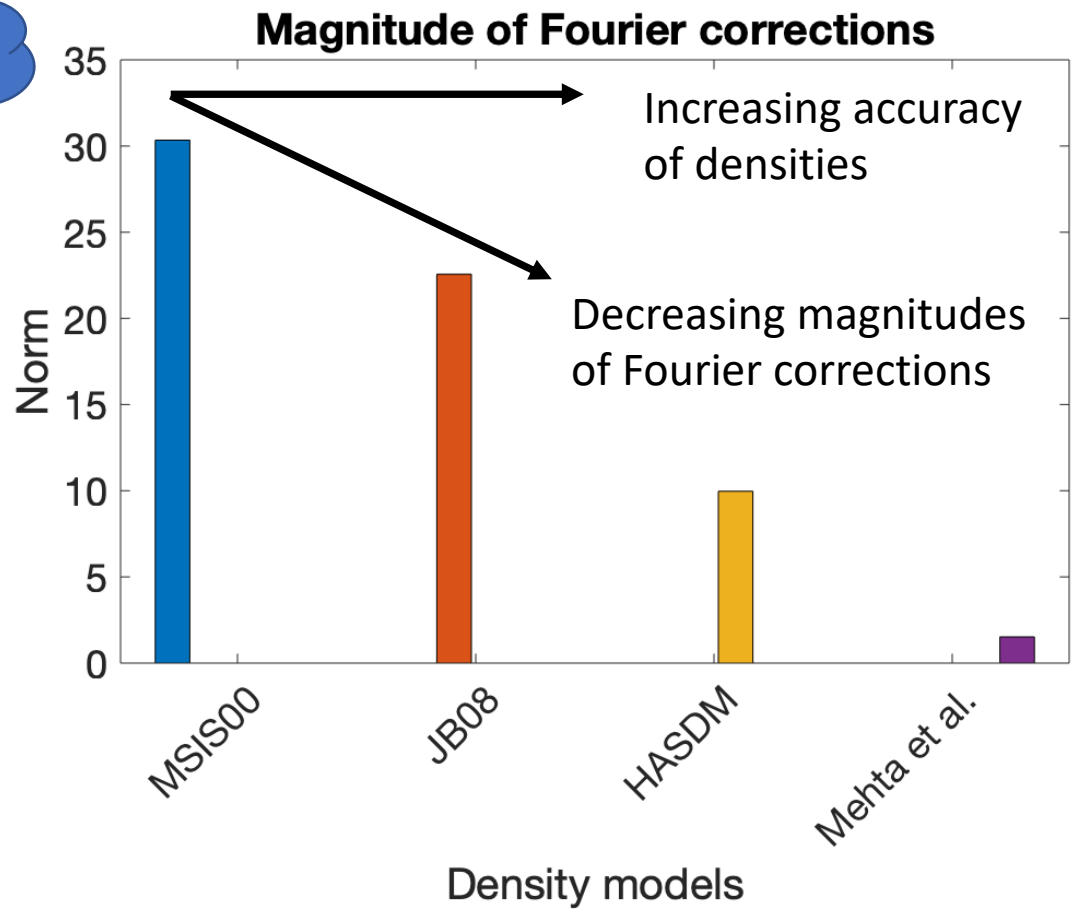
Application to GRACE

Bar charts

Magnitude of Fourier corrections

- Significant implications as a validation tool for models



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Take home points

Fourier expansions can provide periodic corrections to nominal models

Coefficient magnitudes: Validation of Cd and densities

Performance highly dependent on density models

Future work →

Time-varying Fourier coefficients: Markov process

Evaluation for geomagnetically active conditions

Further details: Ray et al., A drag coefficient modeling approach using spatial and temporal Fourier expansions for orbit determination, *Journal of Astronautical Sciences,* Doi: 10.1007/s40295-019-00200-4

Additional slides

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Filter implementation

Initial estimates

- GSIM with a macro-model
- Numerically integrate to find Fourier coefficients
- Truncate at chosen order

Initial covariance

- Monte-Carlo simulation
- Calculate mean and standard deviations

Estimate

- Correlations with mismodeled dynamics: Atmospheric density, truncated geopotential, time-varying SRP

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Body-orbit models

Body-orbit double Fourier (BODF) model

- Capture variations due to both attitude and ambient parameters

$$C_d = \sum_{n=0}^{\infty} (\bar{A}_n \cos n\psi + \bar{B}_n \sin n\psi).$$

$$C_d = \sum_{n=0}^{\infty} (\bar{A}_n(u) \cos n\psi + \bar{B}_n(u) \sin n\psi).$$

Body-orbit summation (BOS) model

- Ignore the cross-coupled terms

$$C_d = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (\bar{A}_{mn} \cos mu \cos n\psi + \bar{B}_{mn} \sin mu \cos n\psi + \bar{C}_{mn} \cos mu \sin n\psi + \bar{D}_{mn} \sin mu \sin n\psi)$$

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