# Wavelet methods for detecting slow slip events in GNSS recordings

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## 1. Slow slip events

- Duration: Several days to several years. Recurrence time: Months to years.
- Generate much weaker seismic waves than ordinary earthquakes.
- Observed in many subduction zones and some transform fault zones (Beroza and Ide, 2011 [2] ; Audet and Kim, 2016 [1]).
- In some places, slow slip events are spatially and temporally correlated with tectonic tremor events (Episodic Tremor and Slip in Cascadia and Nankai).

**We explore whether the wavelet decomposition of GNSS time series can be used to detect slow slip events, which would be useful for systems where tremor is not abundant or detected.**

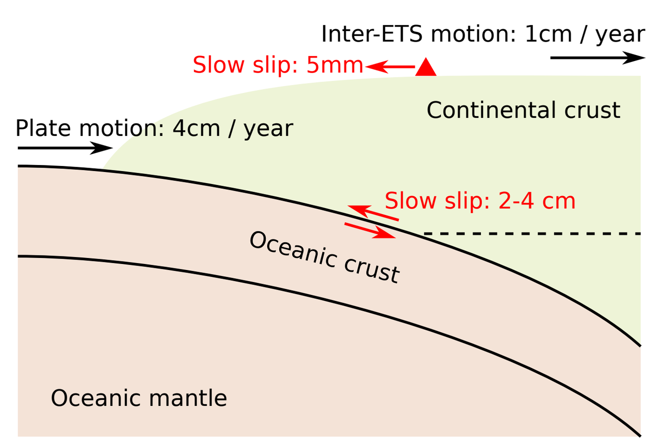



Figure 1: Slow slip event in northern Cascadia.

## 2. Wavelet decomposition

### Maximal Overlap Discrete Wavelet Transform (MODWT)

$X_t$ (length $N$) $\rightarrow$ $J$ wavelet vectors $W_j$ $(j = 1, ..., J)$ of length $N$ $\rightarrow$ J details $D_j$ $(j = 1, ..., J)$ of length $N$ and one smooth $S_J$ of length $N$. Each wavelet vector $W_j$ and each detail $D_j$ is associated with changes on time scale $\tau_j = dt 2^{j-1}$, where $dt$ is the time step of the time series, and corresponds to a bandpass filter of the original GPS time series $X_t$ between frequencies $\frac{1}{dt 2^{j+1}}$ and $\frac{1}{dt 2^{j}}$.

$J$ = Level of the wavelet decomposition (Percival and Walden, 2000 [4]).

**Multiresolution analysis (MRA)**: $X = \sum_{j=1}^{J} D_j + S_J$.

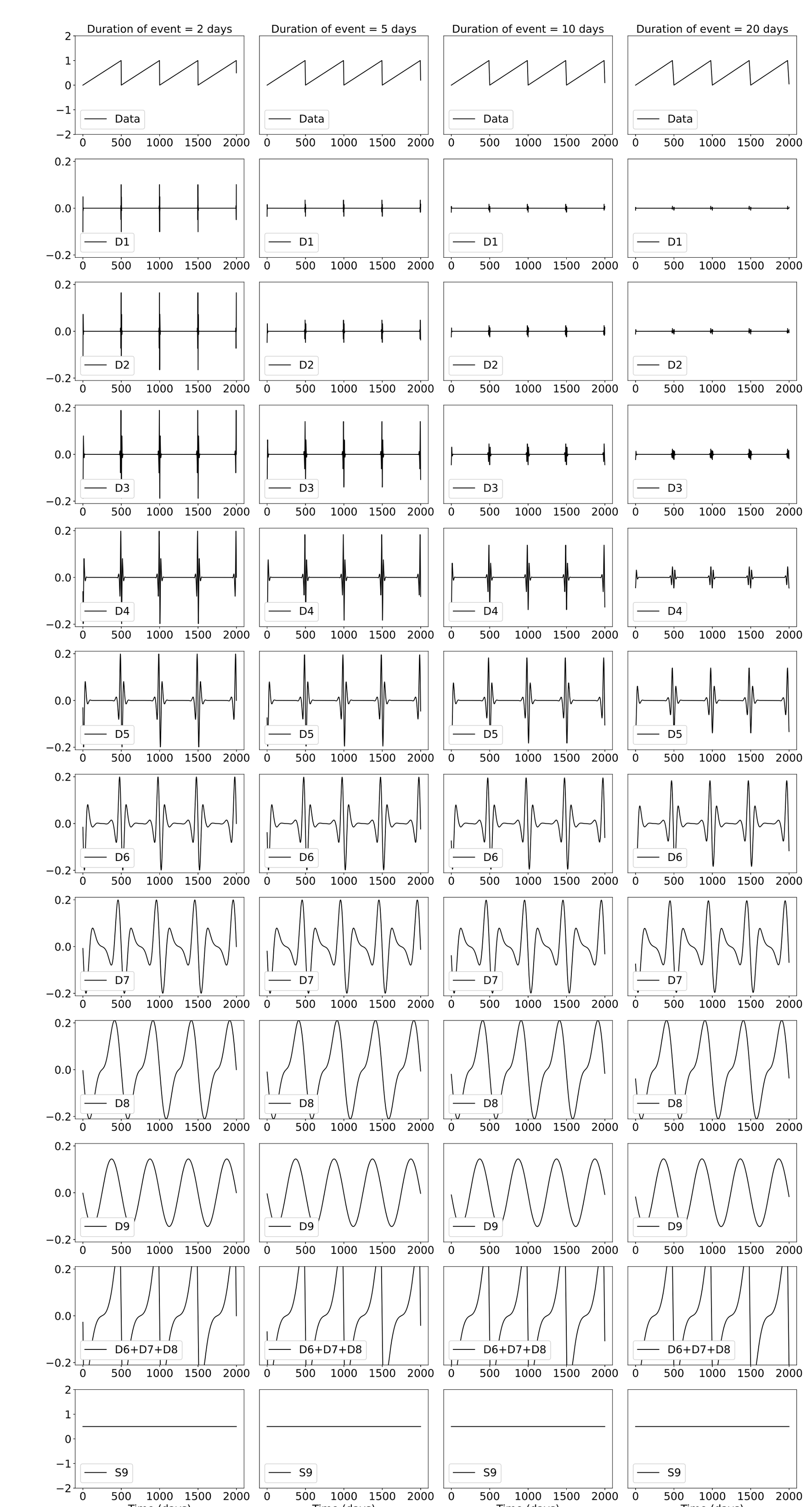


Figure 2: Demonstration of a wavelet decomposition for a synthetic dataset. A synthetic time series is created (top row) with steps of period 500 days, and transient durations of 2 days (left), 5 days, 10 days, and 20 days (right). The resulting details and smooths are shown in increasing level. The amplitude of the synthetic time series is normalized to 1, and the details and smooths show the relative amplitude.

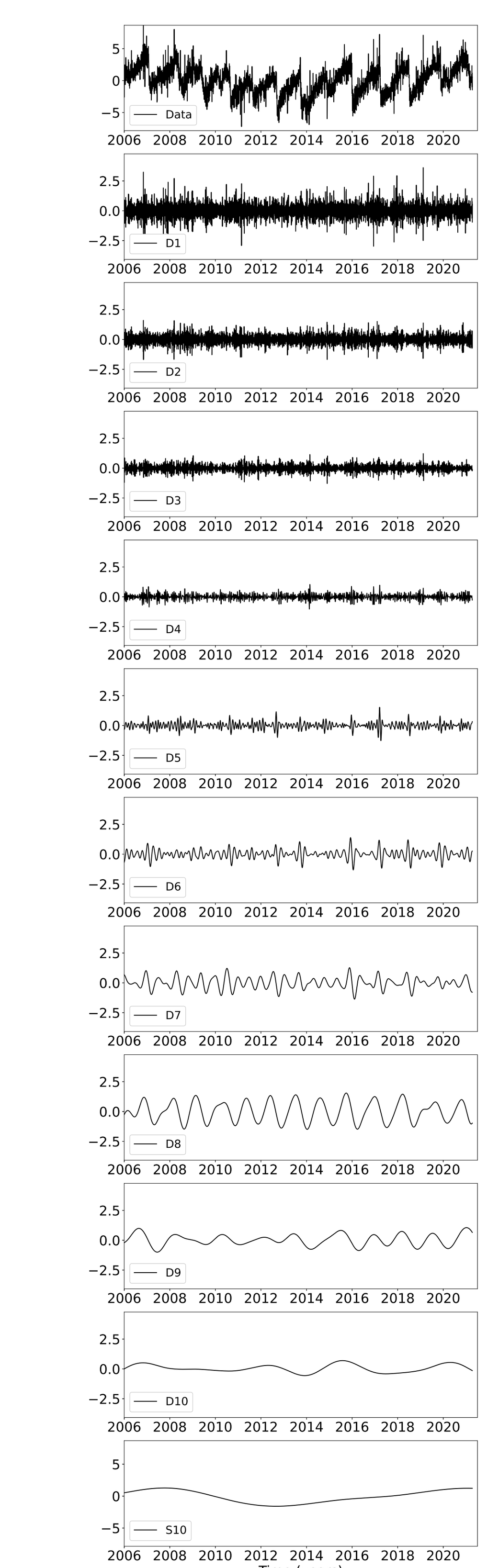


Figure 3: Top: Longitudinal displacement recorded at GPS station PGC5. The resulting details and smooth of the wavelet decomposition are shown in increasing level.

## 3. Slow slip detection and comparison with tremor

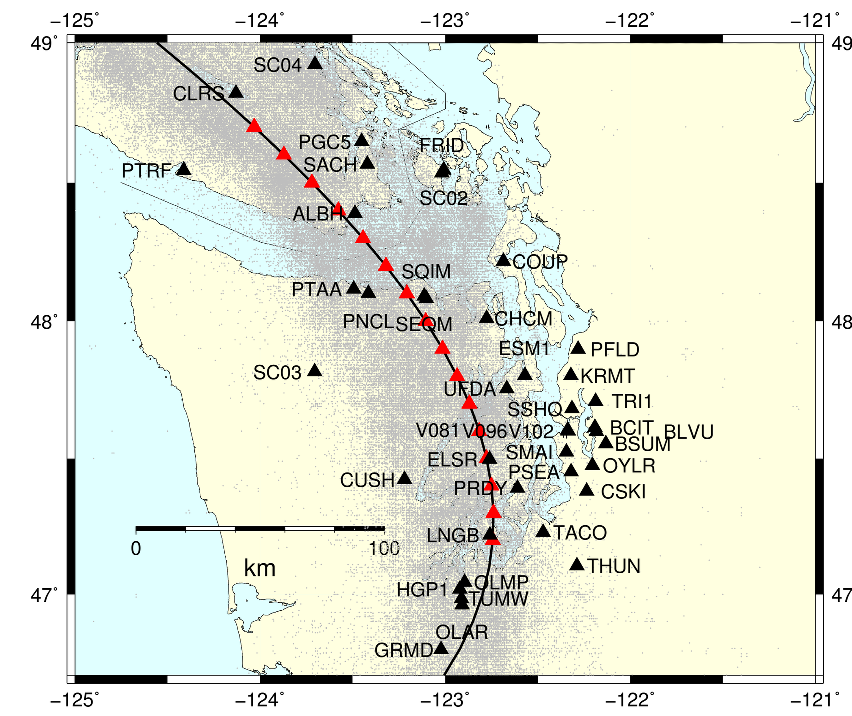


Figure 4: GPS stations used in this study (black triangles). The black line represents the 40 km depth contour of the plate boundary model by Preston *et al.* (2003 [5]). The red triangles are the locations where we stack the GPS data. The small grey dots are all the tremor locations from the PNSN catalog.

**Signal processing for the GPS data**

- Take points along the 40km depth contour of the plate boundary.
- Take all GPS stations in a 50 kilometers radius.
- Compute the MODWT for all the stations
- Stack each detail over all the stations.
- Sum the 6th, 7th and 8th levels details together (corresponds to periods 64 to 512 days).

**Signal processing for the tremor data**

- Compute the cumulative number of tremor in a 50 kilometers radius.
- Remove the trend.
- Compute the MODWT.
- Multiply by -1 for easier comparison.
- Sum the 6th, 7th and 8th levels details together.

- Red rectangles = Sum of details $> D_{cut}$ $\rightarrow$ Replace by 1
- Blue rectangles = Sum of details $< -D_{cut}$ $\rightarrow$ Replace by -1
- $-D_{cut} <$ Sum of details $< D_{cut}$ $\rightarrow$ Replace by 0

- True positive (TP) = Value equal to 1 for both GPS and tremor / Value equal to -1 for both GPS and tremor
- False positive (FP) = Value of GPS equal to 1 but value of tremor equal to 0 / Values of GPS and tremor have opposite signs
- True negative (TN) = Value equal to 0 for both GPS and tremor
- False negative (FN) = Value of GPS equal to 0 but value of tremor equal to 1 or -1

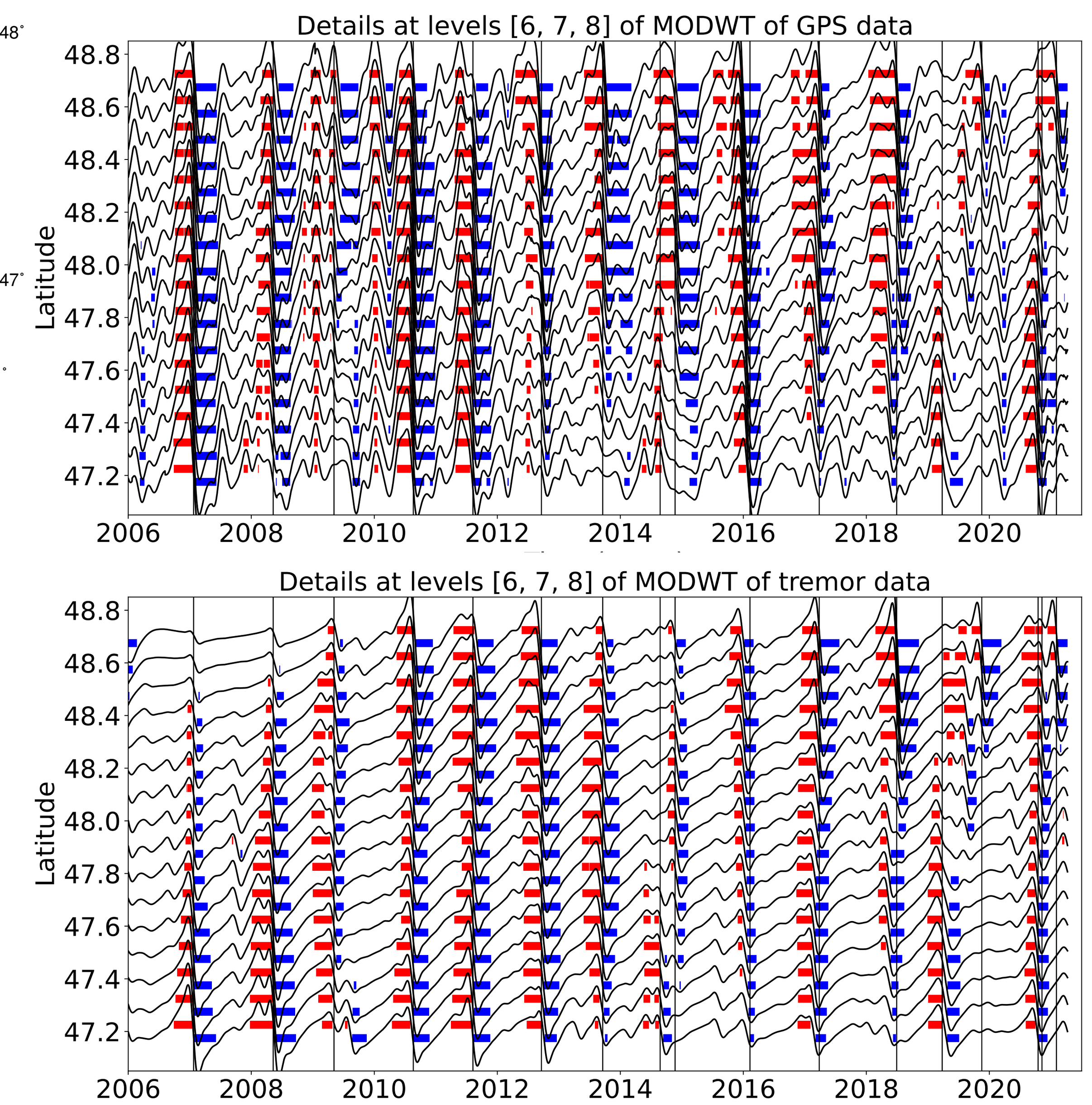



Figure 5: Top: Sum of the stacked 6th, 7th and 8th levels details of the wavelet decomposition of the displacement over all the GPS stations located in a 50 km radius of a given point, for the 16 red triangles indicated in Figure 4. Bottom: Sum of the 6th, 7th and 8th levels details multiplied by -1 of the cumulative tremor count in a 50 km radius of a given point for the same 16 locations. The black lines represent the timings of the main ETS events. We mark by a red rectangle every time where the amplitude is higher than a threshold of 0.8 (for the GPS) or 0.01 (for the tremor). We mark by a blue rectangle every time where the amplitude is lower than minus the threshold.

The receiver operating characteristic curve (ROC curve) is widely used for binary classification problems in statistics and machine learning.

$$\text{sensitivity} = \frac{TP}{TP + FN} = \text{True positive rate}$$

$$\text{specificity} = \frac{TN}{TN + FP} = 1 - \text{False positive rate}$$

- Vary the value of $D_{cut}$ for both the GPS and the tremor data.
- Compute the corresponding sensitivity and specificity and add to ROC curve.
- Choose the values of the threshold that maximize the sensitivity while keeping the specificity large enough.

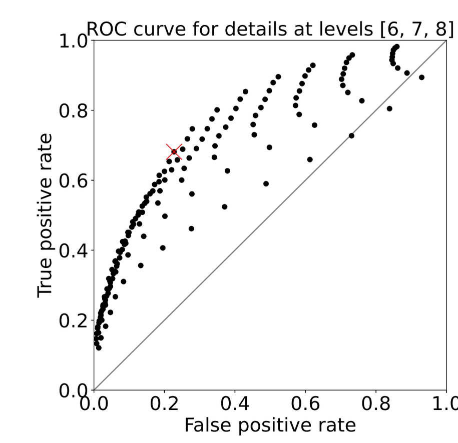


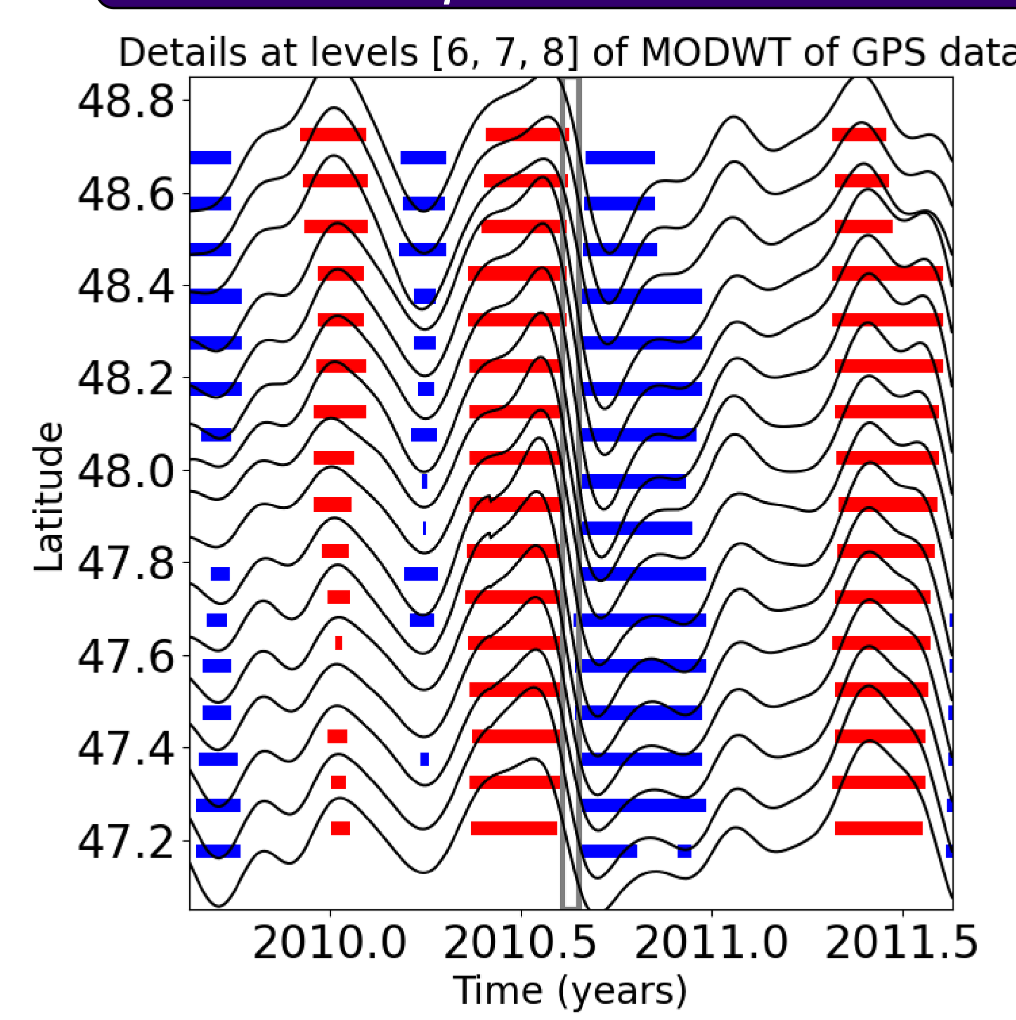

Figure 6: ROC curve for the sum of the 6th, 7th, and 8th level details of the wavelet decomposition. Each black dot represents the true positive rate of event detections and the false positive rate of event detections for a given pair of thresholds (for the GPS and for the tremor). The red cross marks the true positive rate and the false positive rate obtained with the thresholds used to make Figure 5.

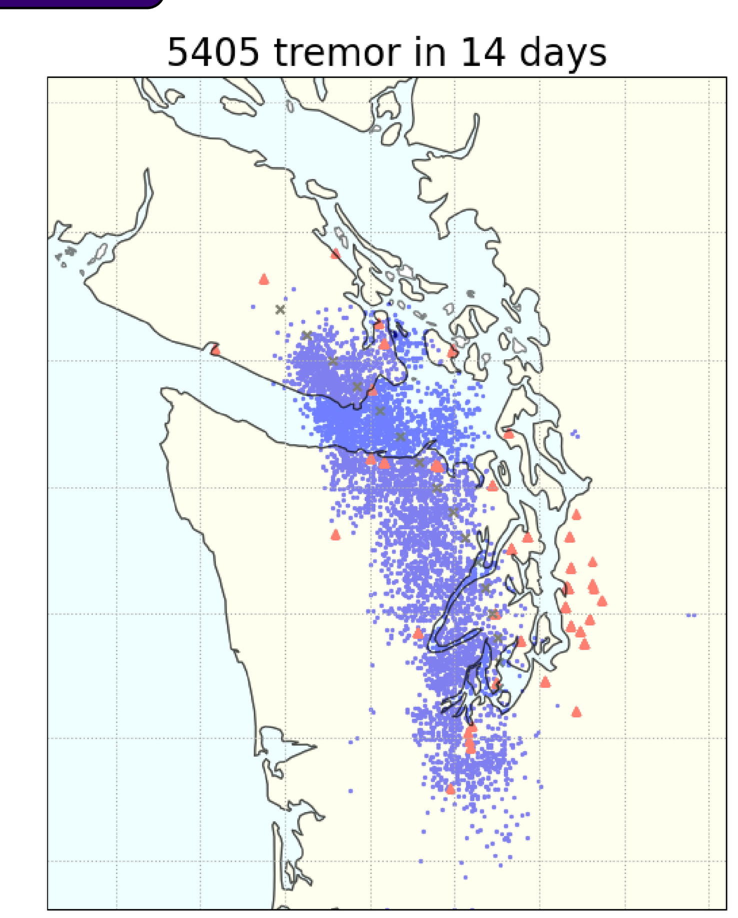
## 4. An example: the 2010 slow slip event

Left panel: Same as top panel of Figure 5, but zoomed over a shorter time window.

Right panel: Location of the tremor occurring during a 14 days period centered on the middle of the time window from the left panel (marked as a grey rectangle).

An animated version of this figure is available on the online version of the poster.




## 5. Summary

- In many subduction zones, tectonic tremor is observed in relation to slow slip $\rightarrow$ Tremor can be used as a proxy to study slow slip events where surface deformation is hidden in GNSS noise.
- In some subduction zones, no clear relationship between tremor and slow slip occurrence $\rightarrow$ This method cannot be applied.
- DWT and MODWT: Analyze time series simultaneously in the time and the frequency domain.
- Project objective: Use wavelet methods to analyze GNSS time series and seismic recordings of slow slip events in northern Cascadia.
- Detrended GNSS data $\rightarrow$ MODWT transform $\rightarrow$ Stack wavelet details over several nearby GNSS stations.
- Cumulative number of tremor in the vicinity of the GNSS stations $\rightarrow$ Detrend $\rightarrow$ MODWT transform.

**Findings**

- Timing of waveforms corresponds to the timing of slow slip events.
- Slow slip event = Positive peak followed by a negative peak.
- Good correlation between slow slip events detected with only GNSS data, and slow slip events detected with only tremor data for northern Cascadia.

## 6. References

[1] P. Audet and Y. Kim. Teleseismic constraints on the geological environment of deep episodic slow earthquakes in subduction zone forearcs: A review. *Tectonophysics*, 670:1–15, 2016.

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[6] P. Wessel and W. H. F. Smith. Free software helps map and display data. *EOS Trans. AGU*, 72:441, 1991.

## 7. Acknowledgements

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The GPS recordings used for this analysis can be downloaded from the PANGA website [3] http://www.panga.cwu.edu/. The Python scripts used to analyze the data and make the figures can be found on the first author's Github account https://github.com/ArianeDucellier/slowslip. Figure 4 was created using GMT [6]. This work was funded by the grant from the National Science Foundation EAR-1358512. A.D. would like to thank Professor Donald Percival for introducing her to wavelet methods during his excellent class on Wavelets: Data Analysis, Algorithms and Theory taught at University of Washington.