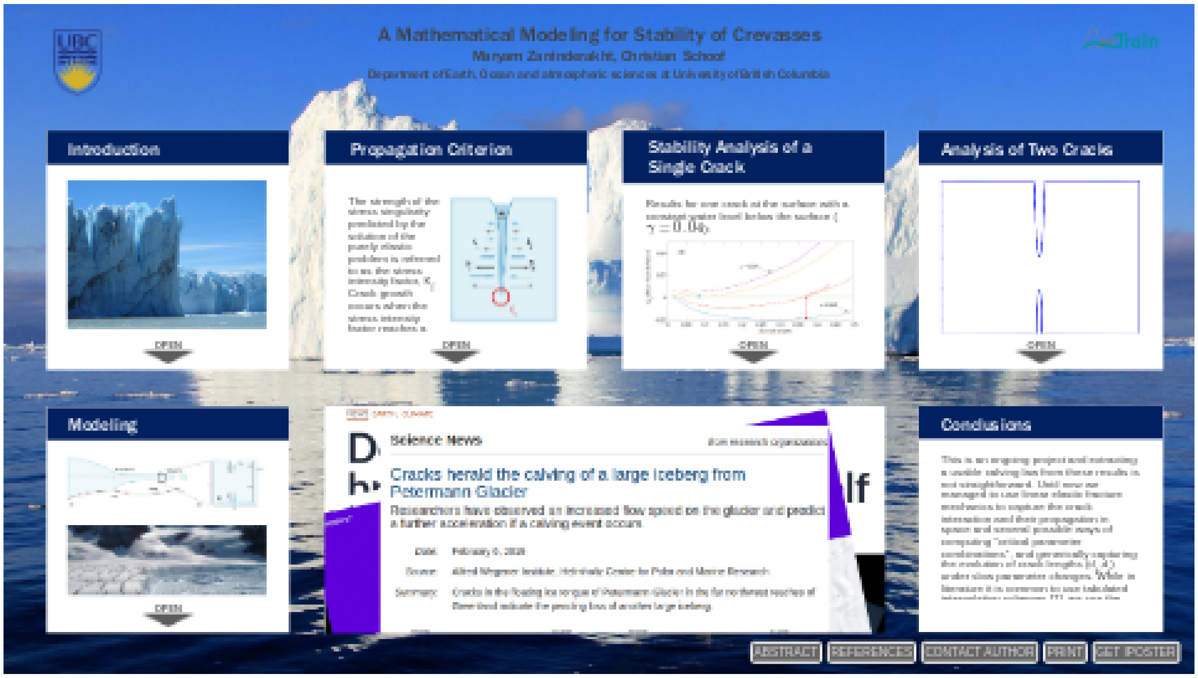
A Mathematical Modeling for Stability of Crevasses



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INTRODUCTION

(https://www.livescience.com)

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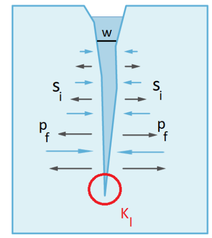
Mass loss from ice sheets can take three basic forms: (1) melting and sublimation at the surface, (2) melting at the base of floating ice sheets; And (3) calving of icebergs from floating ice sheets or grounded calving fronts. The discharge of ice from Greenland and Antarctic ice sheets increased sharply in recent decades [1]. Several ice shelves around the world instead of retreating slowly, suddenly calved dramatically in recent decades. This unsolved calving problem in glaciology, significant ice collapse all around the world, their major effects on climate change and sea-level rise motivated us on working on this project.

PROPAGATION CRITERION



The strength of the stress singularity predicted by the solution of the purely elastic problem is referred to as the stress intensity factor, K_I. Crack growth occurs when the stress intensity factor reaches a critical value, the fracture toughness, K_{Ic}.

$K_I < K_{Ic} \Rightarrow$ crack is stationary.
$K_I = K_{Ic} \Rightarrow$ crack propagates.

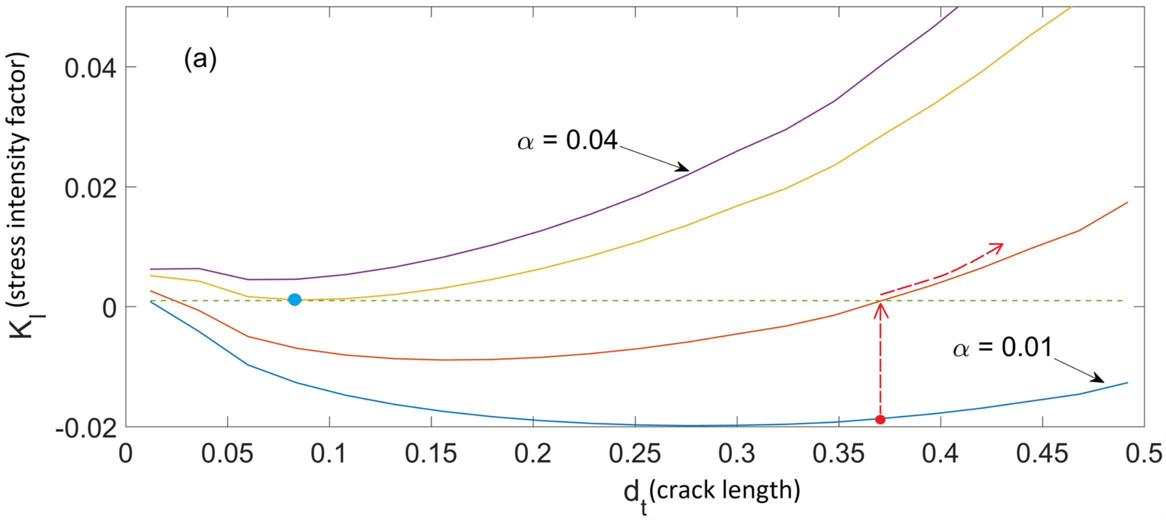
In the case that we aim to model everything as a semi-smooth dynamical system, the propagation criterion can be indicated as the following using analytical based theory be Freund [4].

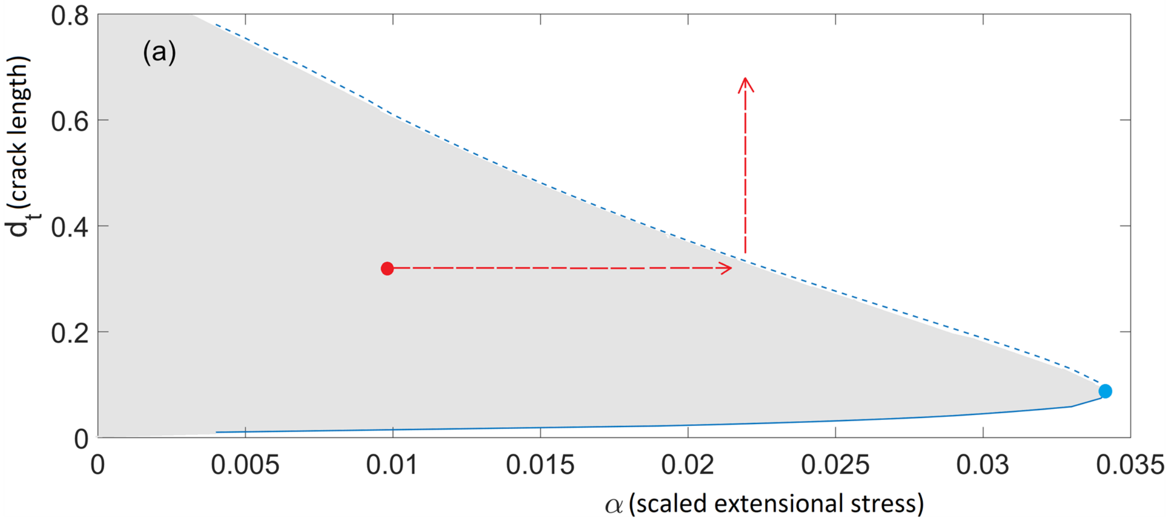
$$\dot{d} = \max(K_{I,stat} - \eta, 0).$$

Here we use the boundary integral method for calculating the stress intensity factor and the displacement discontinuity at the boundary.

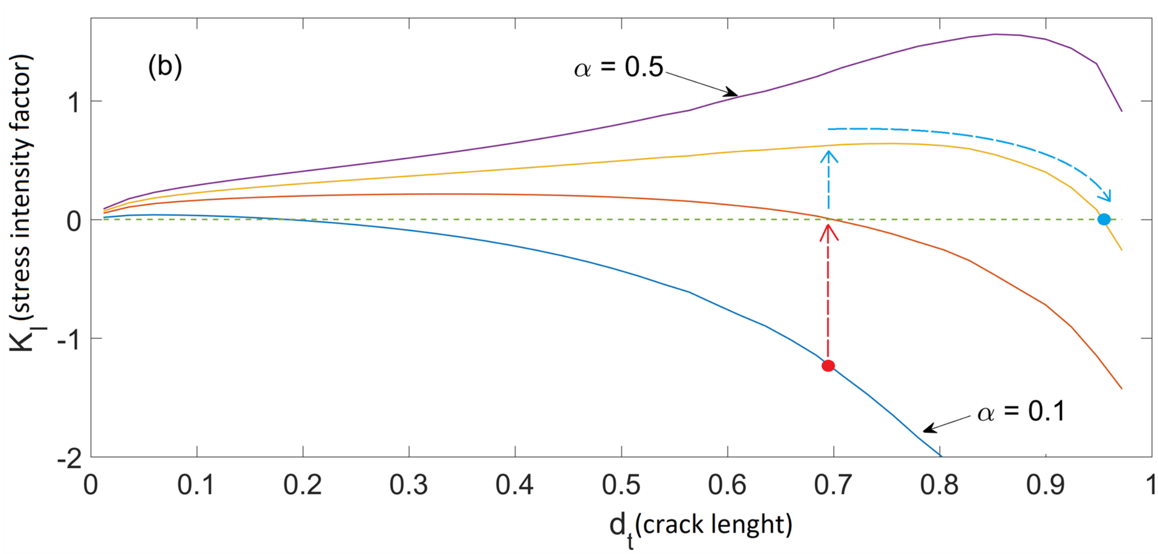
STABILITY ANALYSIS OF A SINGLE CRACK

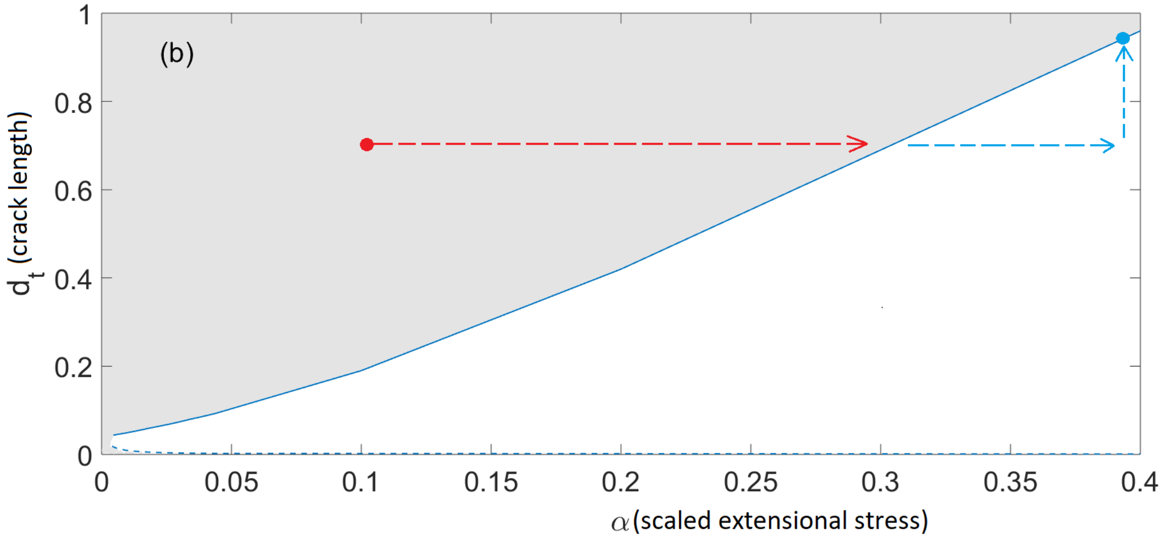
Results for one crack at the surface with a constant water level below the surface ($\gamma = 0.04$):



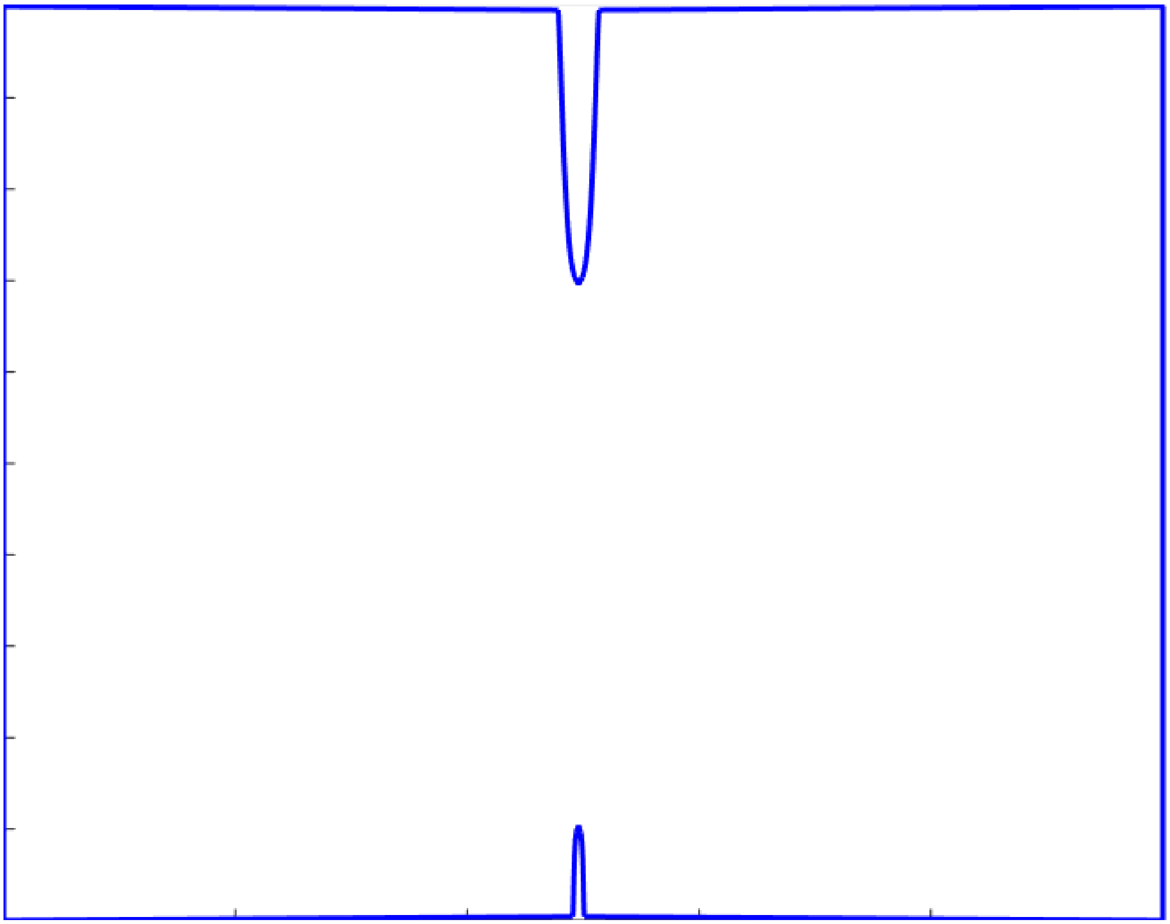


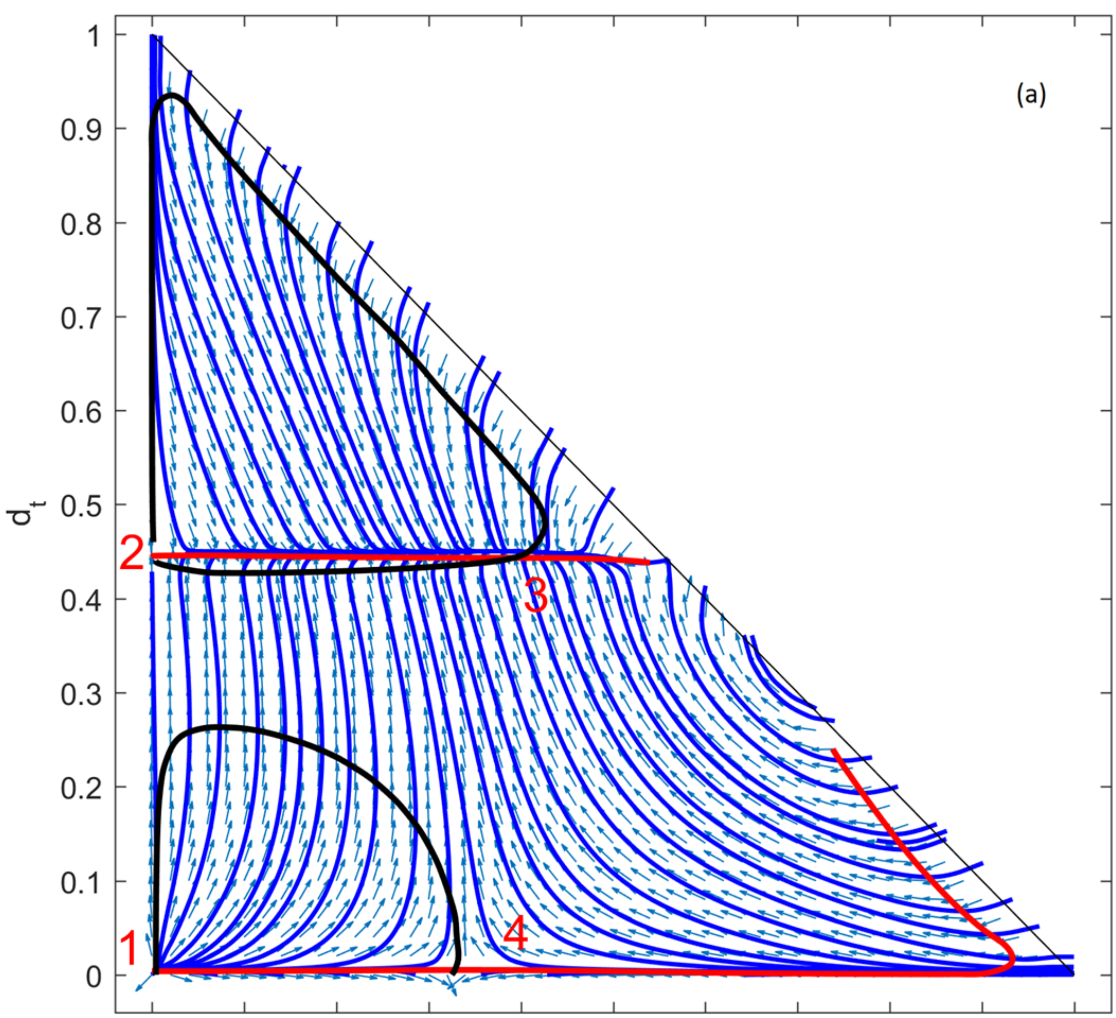
Results for one crack at the surface with constant water volume inside the crack ($\beta = 0.4$):

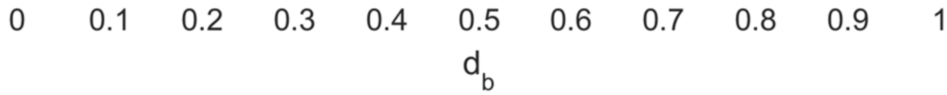


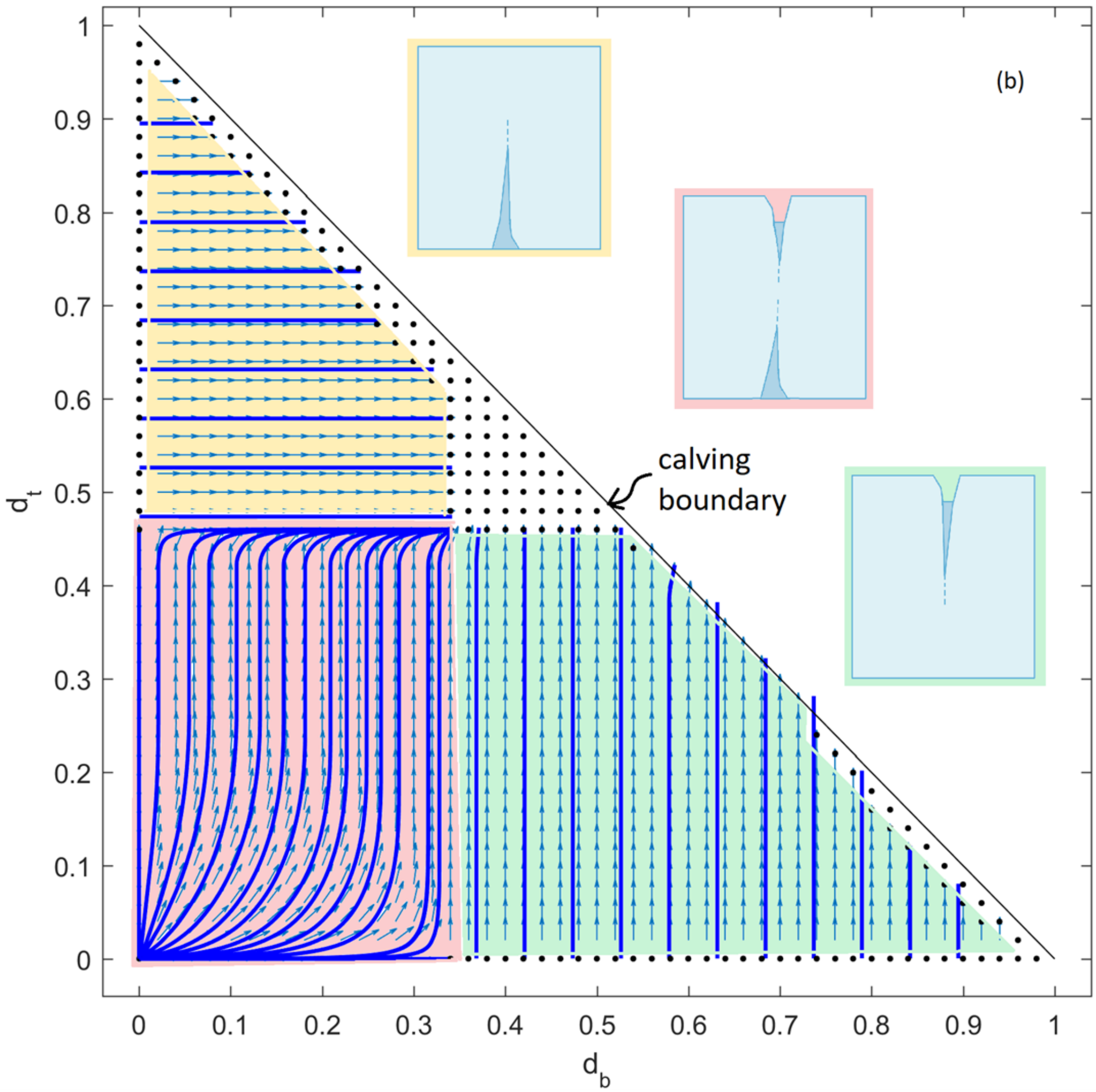

(b)
d_t (crack length)
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
α(scaled extensional stress)

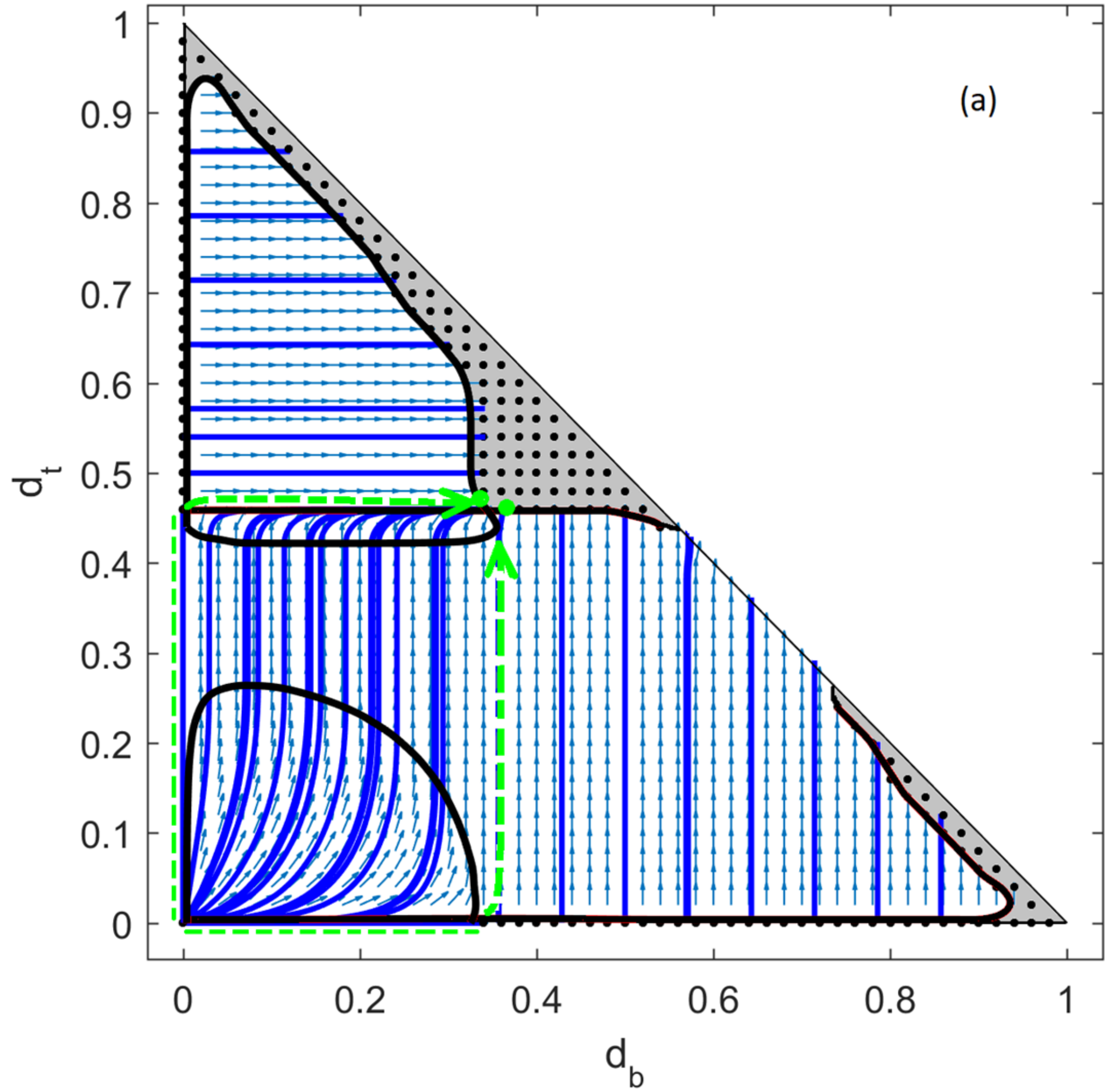
ANALYSIS OF TWO CRACKS

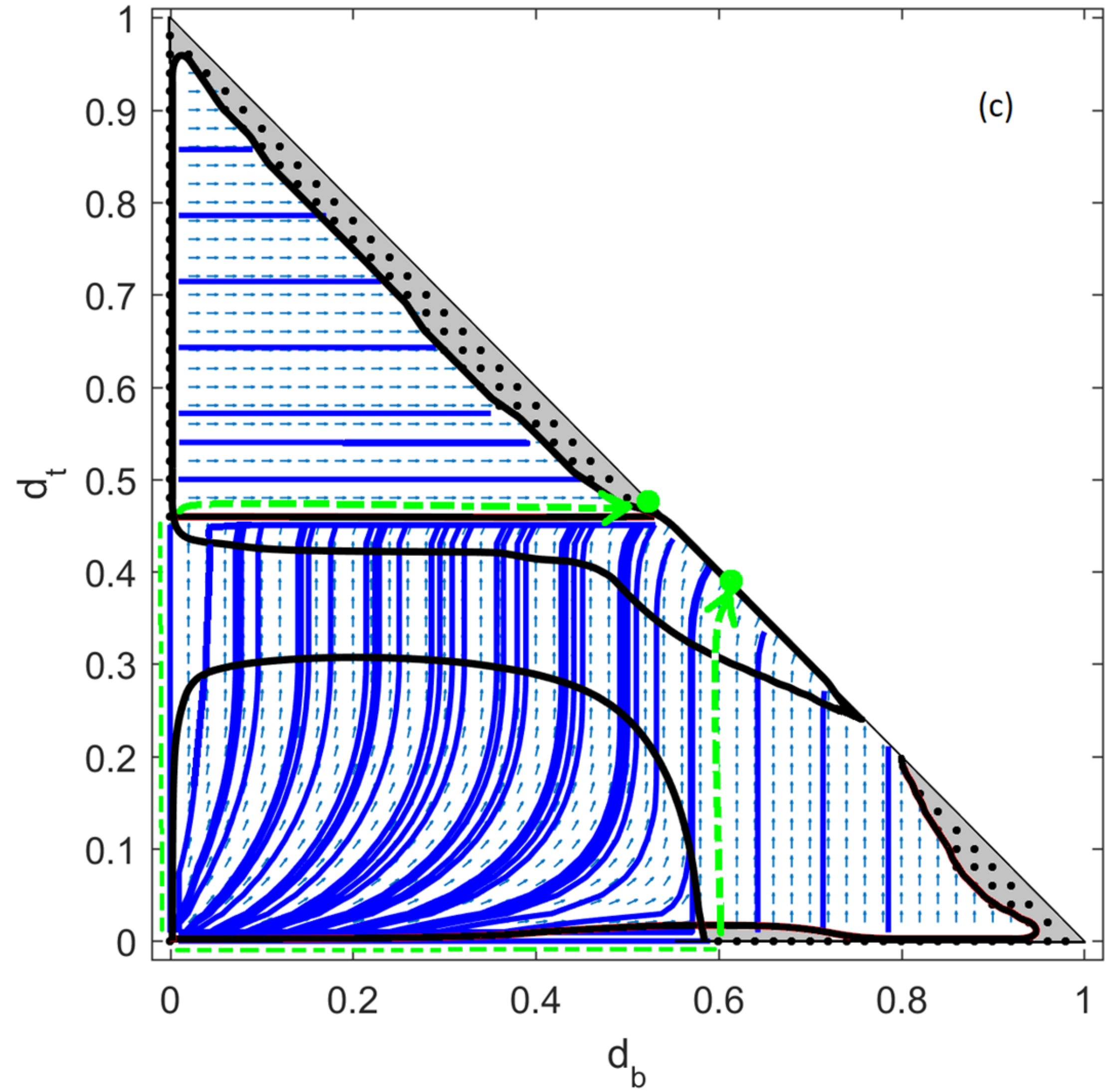



(a)
d_t
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4

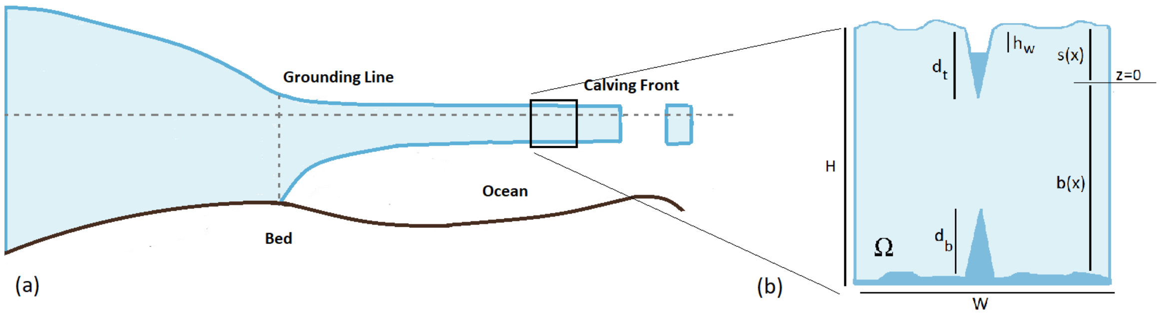

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
d_b


(b)
calving boundary
d_t
d_b
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

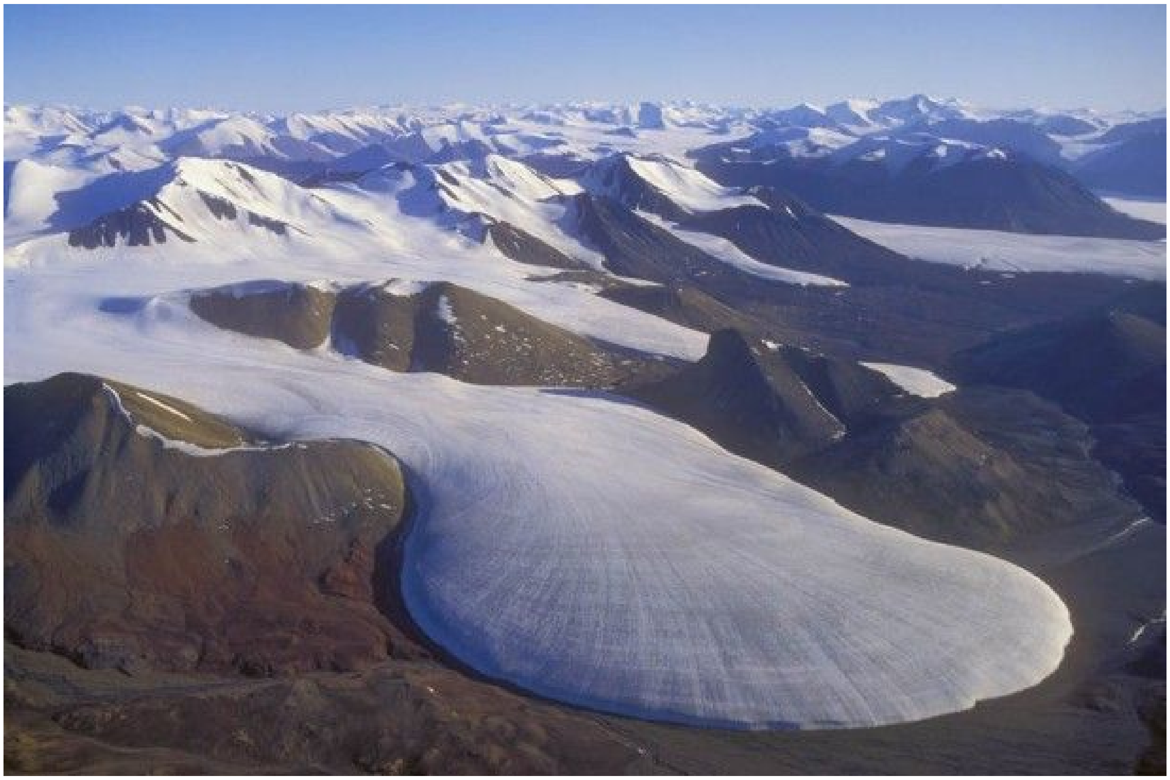

(a)
d_t
d_b
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


(c)
d_t
d_b
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

MODELING



(https://blogs.egu.eu/)

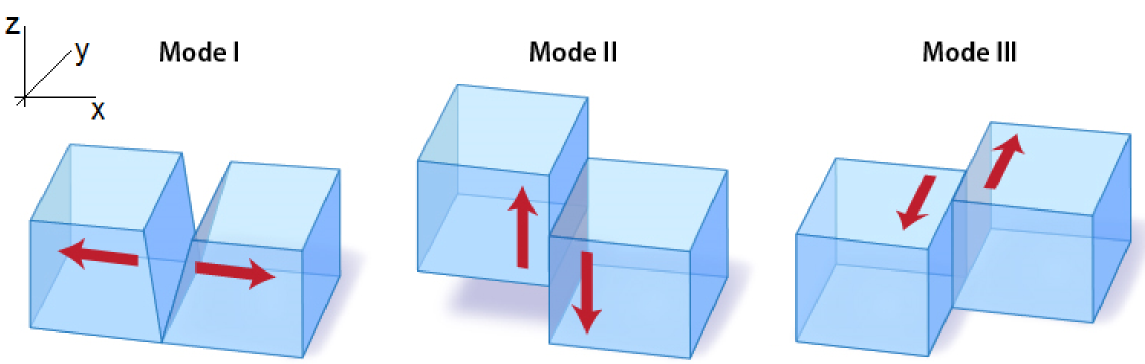


(https://blogs.egu.eu/)

Glaciers are typically modeled as either purely viscous or purely elastic. Ice behaves as a viscoelastic material: over short time scales, ice behaves as a typical elastic solid, with recoverable deformation resulting from an applied load.

A floating ice shelf is defined as the part of a glacier or ice sheet that extended onto the sea. The shelf extends from the grounding line to the calving front. An ice shelf surface is covered with many cracks and crevasses. Crevasses are fractures in the ice due to tensile failure. Based on observations, crevasses appear in near-constant periodic distance from each other. For simplicity, we consider a two dimensional, rectangular domain with two aligned cracks.

The same geometry as we consider here, although in an infinite rather than periodic domain, forms the basis of [2,3,5]. The goal is to determine the conditions under which a single crack can either propagate all the way through the ice or under which the two cracks will meet somewhere in the ice.



(http://williamcolgan.net/blog/?tag=fracture)

Either situation corresponds to the block of ice being fractured through its entire thickness, allowing one side to detach from the other. That is the process of calving. Here we assume a plane strain, quasi-static problem with mode I fracture, ignoring any complications due to the long-term viscoelastic behavior.

We can have two assumptions for water inside the surface crack: limited water supply was locally produced water that is stored in cracks (fixed water volume) or prescribing near-surface hydrological system determining water levels.

Scaling analysis gives us three fundamental groups of parameters that control the crack propagation:

Scaled extensional stress: $\alpha = \frac{R_{xx}}{\rho_i g H}$, Scaled water volume/level: $\beta = \frac{2V_w G}{\rho_i g H^3} / (\gamma = \frac{h_w}{H})$,

Scaled fracture toughness: $\eta = \frac{K_{Ic}}{\rho_i g H^{3/2}}$.

CONCLUSIONS

This is an ongoing project and extracting a usable calving law from these results is not straightforward. Until now we managed to use linear elastic fracture mechanics to capture the crack interaction and their propagation in space and several possible ways of computing "critical parameter combinations", and generically capturing the evolution of crack lengths (d_b,d_t) under slow parameter changes. While in literature it is common to use tabulated interpolation schemes [7], we use the boundary integral method, which allows us to tackle general geometries.

ABSTRACT

Calving is the process of blocks of ice detaching from an ice shelf of grounded calving cliff. Here, we focus on calving that occurs through the propagation of fractures through a floating ice shelf on sufficiently short time scales to allow ice rheology to be treated as elastic. We revisit the linear elastic fracture mechanics models of Weertman and van der Veen, which consider the propagation of cracks into slabs of ice, driven by an applied extensional stress and by water pressure inside the crack, due to sea water and surface melt entering the cracks. We extend their work by considering the interaction between multiple cracks and developing a method that allows us to compute crack propagation in arbitrary domain geometries. We show that the simple case of two aligned cracks, one extending from the ice surface and the other from the base, can be considered as a two-dimensional dynamical system. We are able to show that viable steady crack configurations (where the ice shelf is crevassed without calving) correspond to stable fixed points of that dynamical systems. Calving corresponds to the annihilation of steady states under a parameter change. That can either take the form a bifurcation that happens at specific combinations of forcing parameters, and leads to the abrupt, dynamic propagation of the crack across the remaining unbroken thickness of ice. Alternatively, calving can occur because the two crack tips gradually meet as forcing parameters change. We derive different forms of calving laws, depending on whether crack propagation to full calving is initiated from a previously un-cracked floating slab of ice, or from a previously cracked configuration. For the former, we show that calving laws take the form of a functional relationship between a water storage parameter, extensional stress, ice thickness and fracture toughness. For the latter, we obtain an history-dependent relationship in the form of a steady crack evolution problem that bears abstract similarity with plasticity models. We also discuss how these could be implemented in ice
flow models.

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