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Multiplicative noise and intermittency in bedload sediment transport

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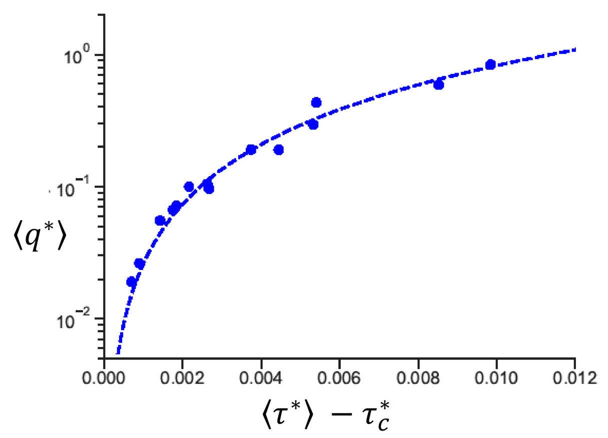
Key Points

- Sediment transport near the threshold of motion is intermittent; it comes in short, intense bursts. This makes it difficult to measure the average sediment flux and define the threshold of motion itself.
- We use bifurcation theory and the concept of multiplicative noise to understand and describe the intermittency.
- Applying this to a set of flume experiments[1], we find a new way of measuring the critical shear stress, τ_c^*, and a way of predicting when intermittency will dominate sediment transport.

Bedload Transport

- Want to predict mean (dimensionless) volume flux, $\langle q^* \rangle$, given a mean shear stress, $\langle \tau^* \rangle$ at the bed.

Himachal Pradesh, India



Experimental results [1]

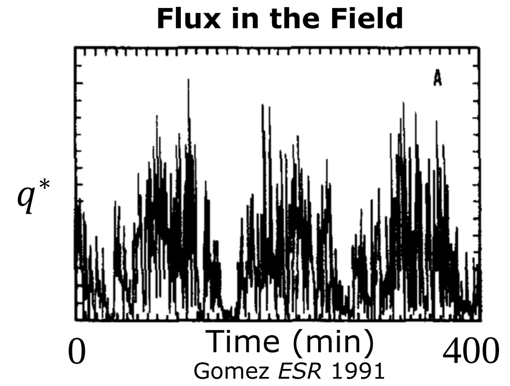
- For example, most common and successful version is the 3/2 law:

$$\langle q^* \rangle \propto (\langle \tau^* \rangle - \tau_c^*)\left(\sqrt{\langle \tau^* \rangle} - \sqrt{\tau_c^*}\right)$$

Intermittency

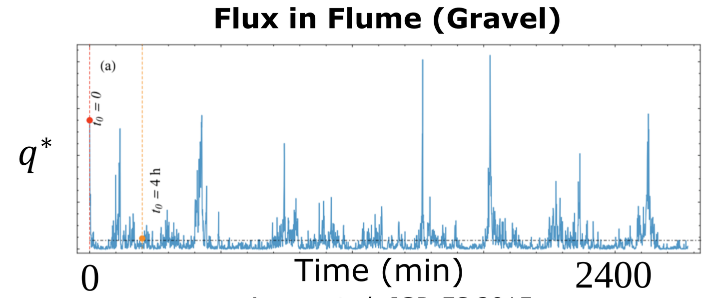
- Experimental and field measurements show the presence of intermittency at low transport stages.
- Implications include:
  - Can't measure $\langle q^* \rangle$ accurately over short intervals.
  - Hard to define τ_c^*
- Models of average bedload flux don't account for intermittency.
  - Intermittency can cause *breakdown* of average laws.

Flux in the Field



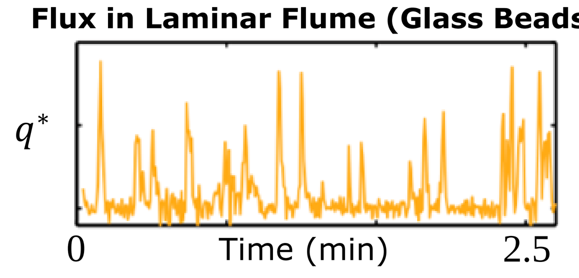
Gomez *ESR* 1991

Flux in Flume (Gravel)



Ancey *et al.* *JGR:ES* 2015

Flux in Laminar Flume (Glass Beads)



Houssais *et al.* *Nat. Comm.* 2016

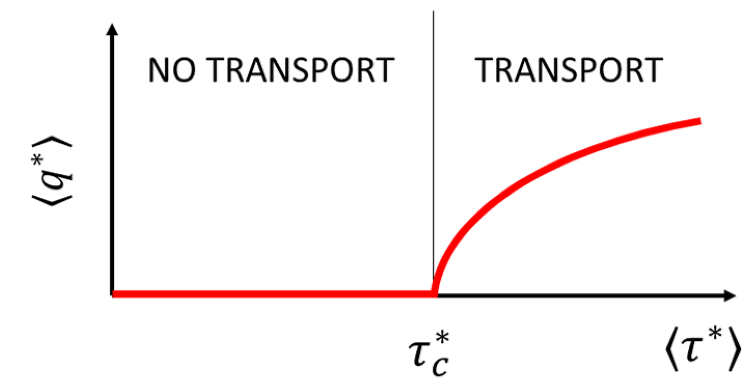
Bifurcations and Multiplicative Noise

Bedload transport as a bifurcation:

- For $\tau^* < \tau_c^*$, no transport: $\langle q^* \rangle = 0$
- For $\tau^* \geq \tau_c^*$, transport: $\langle q^* \rangle > 0$

Close to the threshold, $q^* \ll 1$, and:

$$\frac{dq^*}{dt} = g(q^*, \langle \tau^* \rangle, \tau_c^*, \ldots) \approx (\langle \tau^* \rangle - \tau_c^*)q^* - NL(q^*)$$

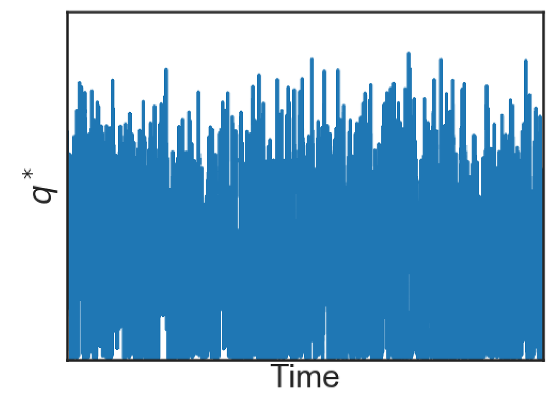


Bifurcation diagram schematic for bedload transport

However, sediment transport is noisy (turbulence, collisions...). How do we include the noise?

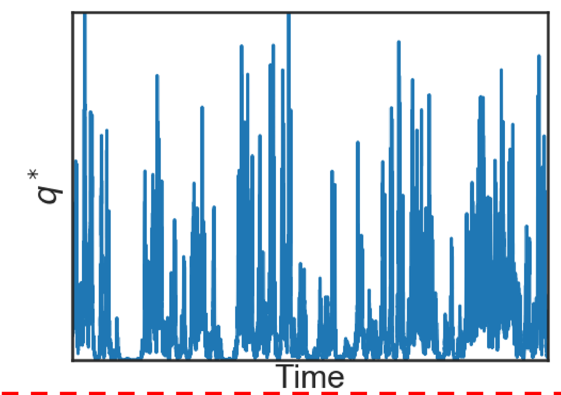
<u>Additive Noise</u>

$$\frac{dq^*}{dt} \approx (\langle \tau^* \rangle - \tau_c^*)q^* - NL(q^*) + Noise$$



<u>Multiplicative Noise</u>

$$\frac{dq^*}{dt} \approx (\langle \tau^* \rangle - \tau_c^* + Noise)q^* - NL(q^*)$$



Multiplicative noise causes what's called on-off intermittency [2] when $\langle \tau^* \rangle \to \tau_c^*$. Statistical predictions [3]:

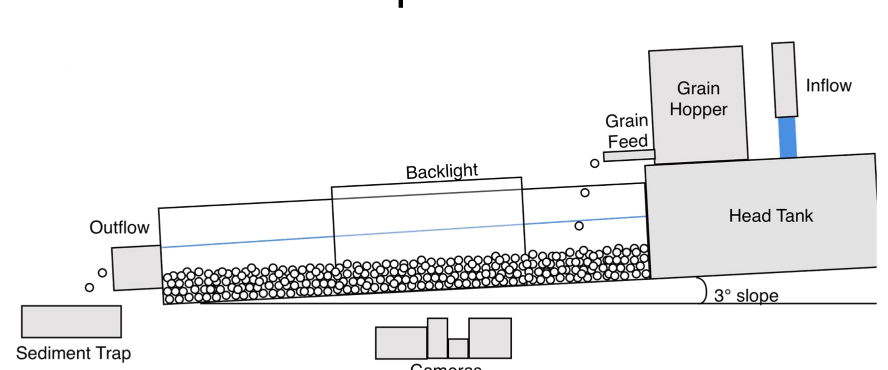
a) Intermittent if $\langle \tau^* \rangle - \tau_c^* < S$, and S = autocorrelation of the noise.

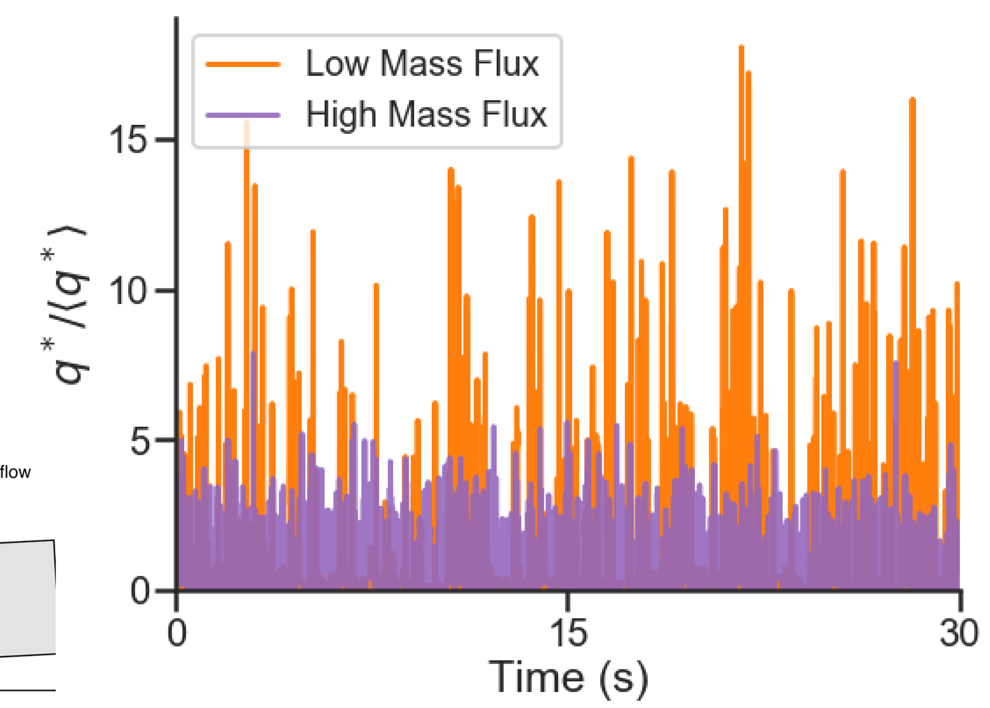
b) Waiting time between large events is power law with exponent -3/2

c) Distribution of possible mass flux also power law, with exponent -1

Laboratory Flume Experiments

- Performed at SFU in the Venditti lab.
- 5mm glass beads, ~10mm-wide flume.
- Constant water flux, varied mass flux input.





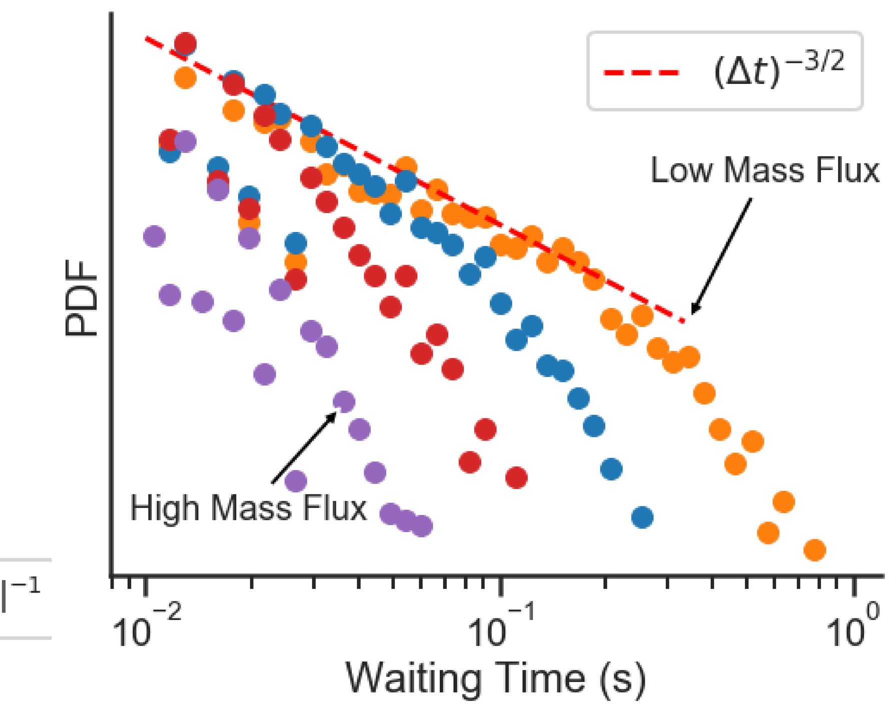
Experimental Time Series [1]

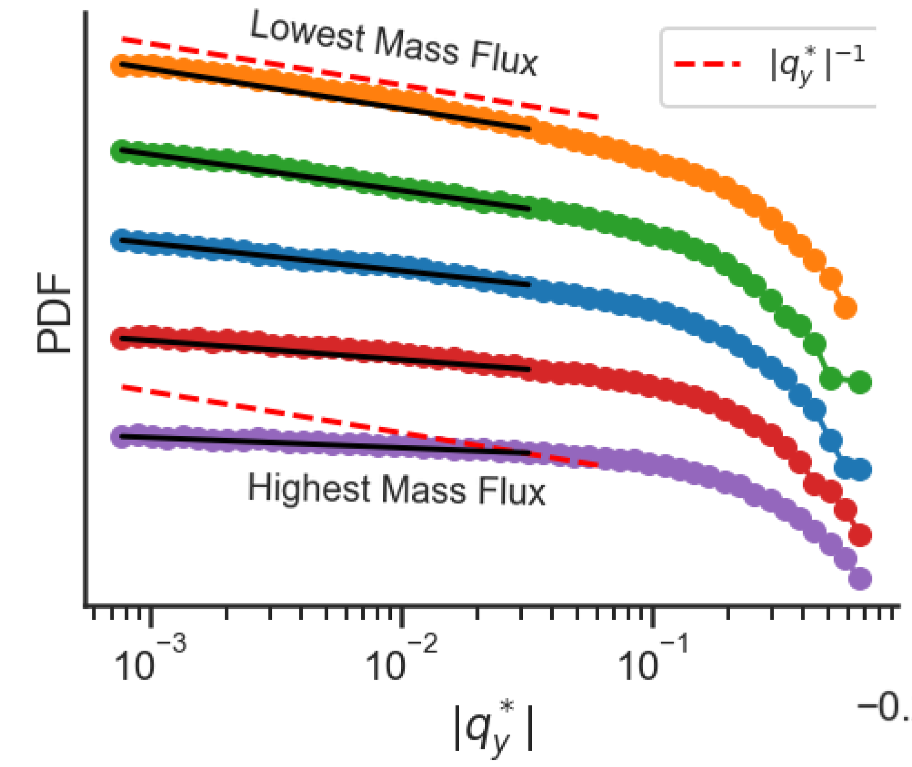
Testing Predictions

- Prediction:

$$\mathrm{PDF}_{q_0^*}(\Delta t) \propto (\Delta t)^{-\frac{3}{2}}$$

- Cutoff depends on threshold proximity

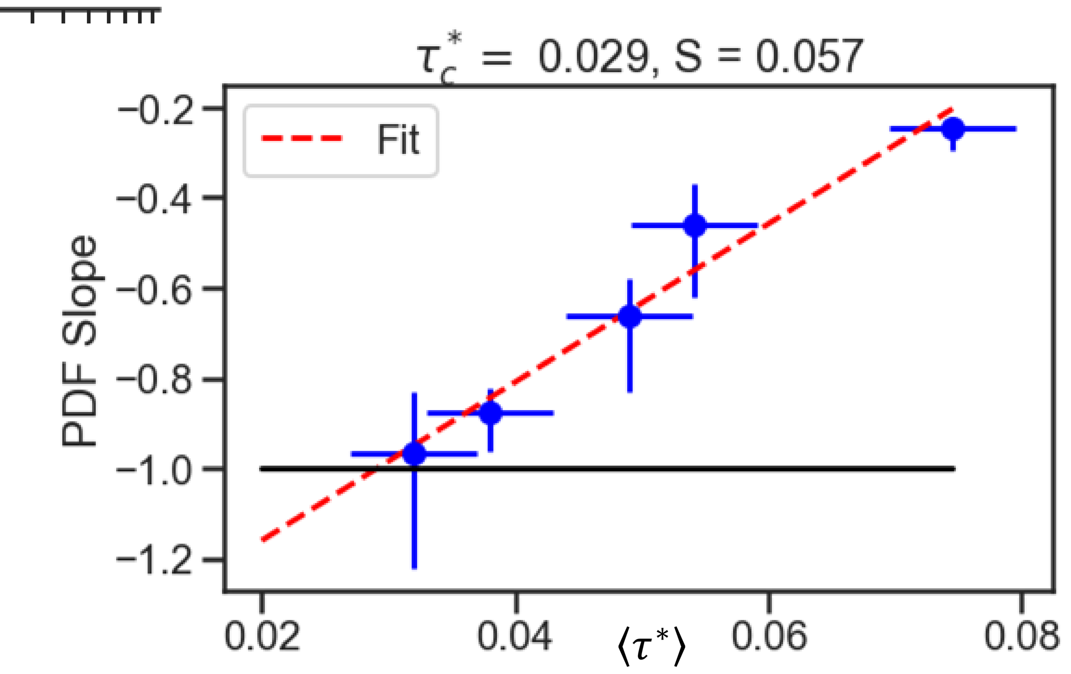




- Prediction:

$$\mathrm{PDF}(q^*) \propto (q^*)^{\frac{\langle \tau^* \rangle - \tau_c^*}{S} - 1}$$

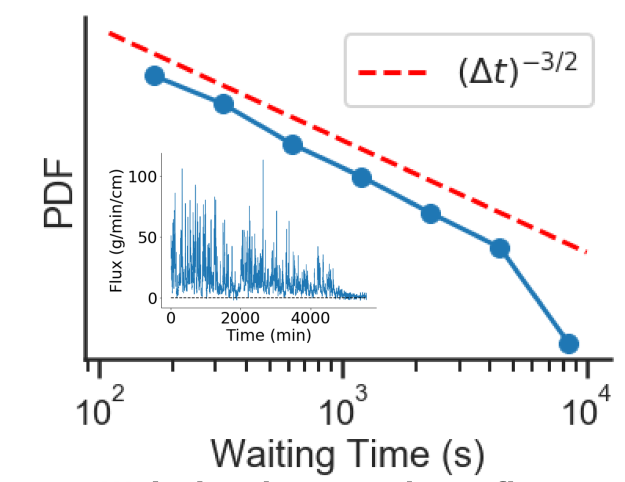
- Slope measures threshold proximity



- Calculated τ_c^* in a novel way.
- Extracted property of noise, S.

Implications and Future Work

- Want to confirm with other systems
  - Flume with natural grains (in progress)
  - Field data (preliminary, see figure)
- Future questions/studies:
  - How does S vary between systems?
  - Motivate simple ODE with physics?



Wait time between large flux, Nelson *et al.* *JGR* (114) 2009

References

[1] Eric Deal *et al.* EP41B-2650: Observing the role of grain shape on bedload transport in paired flume experiments and numerical simulations. AGU Fall Meeting, 2018.
[2] Heagy, J. F., Platt, N., & Hammel, S. M. *Phys. Rev. E,* 49(2):1140-1150, 1994.
[3] Aumaître, S., Pétrélis, F., & Mallick, K. *Phys. Rev. Lett.* 95:064101, 2005