

MODELING COMPLEX FLUID FLOW IN ROUGH-FRACTURES: A LUBRICATION-BASED APPROACH

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Session: H35R. Non-linearity in Subsurface Flow and Transport: Modeling, Experiments, and Applications

1 FLOW OF SHEAR-THINNING FLUIDS IN GEOLOGICAL FRACTURES

The **hydraulic behavior of geological formations** is mainly governed by the **fractures connectivity** and **permeability**, Fracture heterogeneity strongly affects flow and transport, with fluid rheology playing an important role, often oversimplified.

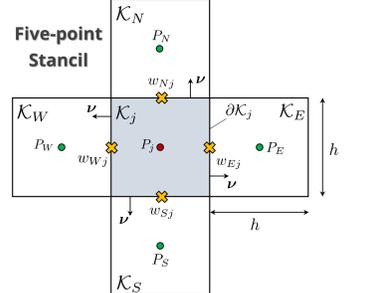
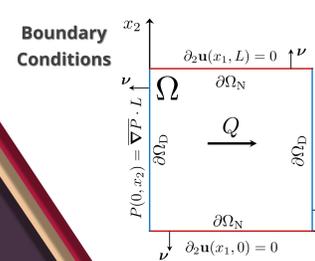
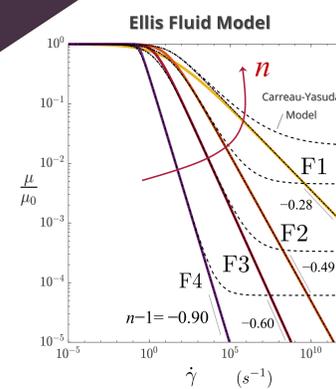
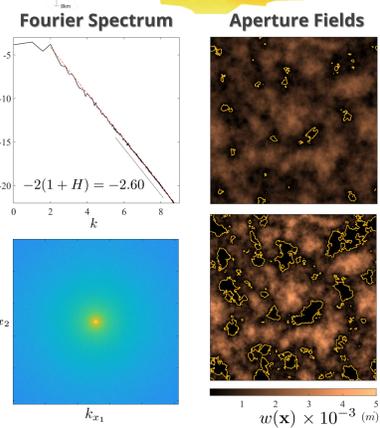
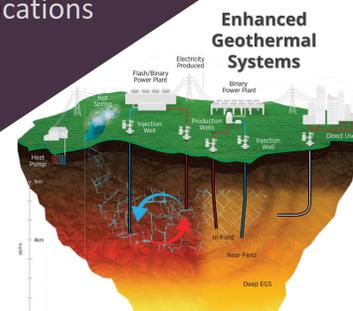
- Typically, unconventional and deep geothermal reservoirs present both **low porosity** and **low permeability**
- Operations in gas shale or hot rock require **hydraulic stimulation** to enhance productivity and become cost effective

Fluids involved in subsurface industrial activities present a **shear-thinning behaviour** at **continuum scale**, due to their complex microstructure.

- muds
- foams
- water-based suspensions

Subsurface industrial activities:

- Enhanced Oil Recovery (EOR)
- Enhanced Geothermal Systems (EGS)
- Carbon sequestration

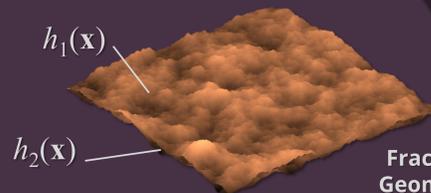


2 NUMERICAL MODELING

2.1 Synthetic fracture generator

A synthetic aperture field is estimated by mating two **isotropic self-affine surfaces** $h_i(\mathbf{x})$

$$w(\mathbf{x}) = h_1(\mathbf{x}) - h_2(\mathbf{x}) + \langle w \rangle$$



Fracture Geometry

$\sigma_w / \langle w \rangle$	1.0
$\langle w \rangle$	10^{-3} m
H	0.8
L_c	0.1 m
L	0.4 m

A rough surface can be generated as a 2D white noise and introducing spatial correlations: multiplying the modulus of the Fourier transform by the modulus of the wave numbers $|k| = (k_{x_1}^2 + k_{x_2}^2)^{1/2}$ to the power $-1-H$

$$|h(\mathbf{k})| \rightarrow |k|^{-1-H} |h(\mathbf{k})|$$

2.2 Fluid Rheology: Ellis model

The Ellis rheology is a **three-parameter model**

$$\eta = \frac{\mu_0}{1 + \left(\frac{\tau_{xz}}{\tau_{1/2}} \right)^{1/n-1}} \mu_0$$

low-shear rate viscosity ($\eta \rightarrow \mu_0$ for $\dot{\gamma} \rightarrow 0$)
shear-thinning index n
characteristic shear stress: $\eta(\tau_{1/2}) = \mu_0/2$

Fluids	n (-)	μ_0 (Pa·s)	$\tau_{1/2}$ (Pa)
F1 CMC at 0.3 wt%	0.72	0.05	4.07
F2 CMC at 0.5 wt%	0.51	0.22	2.50
F3 CMC at 1.0 wt%	0.40	2.99	5.14
F4 VES	0.10	49.00	1.07

2.3 Generalized Non-linear Reynolds Equation

Lubrication theory holds ($\nabla w \ll 1$ and $Re \ll 1$)

$$-\nabla \cdot \left[\frac{w(\mathbf{x})^3}{12\mu_0} + \frac{n}{(2n+1)} \left(\frac{1}{2^{1+n} \mu_0^n \tau_{1/2}^{1-n}} \right) w(\mathbf{x})^{\frac{2n+1}{n}} |\nabla P|^{\frac{1}{n}-1} \right] \nabla P = 0$$

Numerical modeling via finite volume method:

- The non-linear system of equations is solved via **inexact Newton-Krylov** method
- Variable-fill-in Cholesky preconditioned conjugate gradient** \rightarrow linear problem
- A **parameter continuation strategy** is adopted to handle strongly non-linear cases

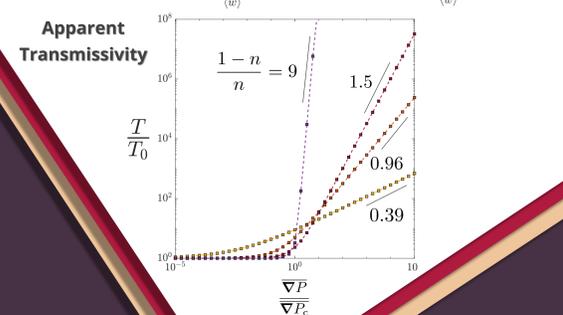
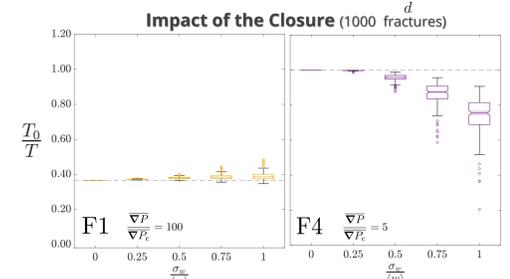
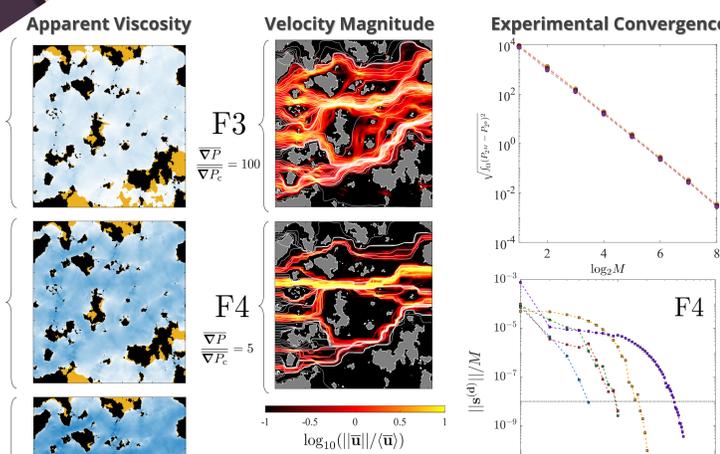
3 RESULTS

3.1 Experimental Convergence

- Starting from a 2x2 aperture field (level mesh 0), we estimate the error at different mesh levels. The **convergence of this sequence of errors** together with the **scheme consistency** implies the **convergence to the true solution** of the problem.
- The adopted strategy aims at **reducing the pre-asymptotic phase of the Newton method**, thus engaging efficient quadratic convergence as quickly as possible.

3.2 Impact of Rheology

- The fluid shear-thinning behaviour **promote flow localization**, with parts of the fractures characterized by high velocities (channels) and others where the flow is almost stagnant.
- Flow mainly occurs in channels of low apparent viscosity and high velocity**, resulting from flow localization.
- Shear-thinning behaviour reduces the impact of the closure**, increasing the apparent transmissivity.



4 CONCLUSIONS AND FUTURE PERSPECTIVES

4.1 Conclusions

- Transmissivity attenuation due to fracture closure is mitigated by the shear-thinning rheology.
- High $\overline{\nabla P}$ and low n increases shear-thinning behaviour, favouring flow localization
- The smaller the fracture length, the higher the dispersions of the velocities
- Shear-thinning behaviour enhances fracture transmissivity, leading to non-darcian flow regime for sufficiently high pressure gradients

4.2 Future Perspectives

- Comparison with full **3-D CFD simulations** to investigate the **limits of the lubrication approximation**.
- Implement a **transport solver** to study the **impact of the shear-thinning rheology on breakthrough curves**.

Méheust, Y., Schmittbuhl, J. (2000) Flow enhancement of rough fracture. *Geophys. Res. Lett.*, 27(18), 2989-2992. doi: 10.1029/1888g100464

Lenci, A., Méheust, Y., Putti, M., Di Federico, V. (2021) Monte Carlo Simulations of Shear-thinning Flow in Geological Fractures, *Water Resour. Res.* (preprint)