Supporting Information for "Water vapor spectroscopy and thermodynamics constrain Earth's tropopause temperature"

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Additional Figures

Methods

Isca Framework

For all simulations, we use Isca, a modeling framework that makes it easy to vary between configurations (Vallis et al., 2018). We use Isca configured as a clear-sky general circulation model (GCM) and a clear-sky single column model (SCM). There is no sea ice, land, or topography. The GCM and SCM configurations use the same column-wise physics routines (e.g., radiative transfer, convective adjustment).

In the GCM, we run at T42 resolution with 40 vertical levels, distributed according to $\sigma = \exp[-7(0.25\tilde{z} + 0.75\tilde{z}^7)]$, where $\tilde{z}$ is evenly spaced on the unit interval. This distribution produces levels that are roughly evenly spaced in the troposphere, and spaced more closely in the stratosphere. We use a slab mixed-layer ocean with a standard specified meridional profile of sea surface temperatures (Neale & Hoskins, 2000):

$$T_s(\phi) = \begin{cases} 300\big(1 - \sin^2(3\phi/2)\big)\ \text{K}, & \text{for } -\pi/3 < \phi < \pi/3 \\ 273\ \text{K}, & \text{otherwise}, \end{cases} \tag{1}$$

where ϕ is the latitude.

In the SCM, we run at 80 vertical levels, necessarily omit the dynamical core, and constrain stratospheric water vapor so that it cannot increase with height. We prescribe surface temperature in increments of 10 K by setting the mixed-layer temperature and then setting its depth to 10^9 m.

In both models, we use the simple Betts-Miller convection scheme (Frierson, 2007; O'Gorman & Schneider, 2008), which drives the free troposphere to a prescribed relative humidity of 70%. Large scale condensation is included to prevent supersaturation, following (Frierson et al., 2006), and all condensed water returns immediately to the surface. Boundary layer turbulence is parameterized using a k-profile scheme similar to Troen and Mahrt (1986), and diffusion coefficients are obtained from Monin-Obukhov similarity theory (in the column model, this computation uses a prescribed surface wind of $5\,\mathrm{m\,s^{-1}}$). In the SCM, we set the boundary layer depth to the lifting condensation level. For consistency, we also use this method to determine the boundary layer depth in the GCM.

In both the GCM and the SCM, we compute radiative transfer primarily with RRTM (Mlawer et al., 1997). The incoming solar radiation meridional profile resembles Earth's seasonally-averaged profile with a Second Legendre Polynomial. The surface albedo is set to 0.2. CO_2 and water vapor are the only greenhouse gasses (unless specified otherwise).

In the SCM, we also run experiments with gray radiative transfer configured to resemble the setup of (Frierson et al., 2006), in which water vapor has no effect on radiative fluxes. That is, the gray optical depth is

$$\tau = \tau_0 \left[f_\ell \Big(\frac{p}{p_s}\Big) + (1 - f_\ell) \Big(\frac{p}{p_s}\Big)^4 \right], \tag{2}$$

where $\tau_0 = 6$ is the surface optical depth and $f_\ell = 0.1$ is a constant. See (Frierson et al., 2006) and the Isca documentation (`https://execlim.github.io/Isca/index.html`) for details. Atmospheric shortwave absorption is turned off, the surface albedo is still set to 0.2 and the stellar constant is set to 342.5 Wm^2 unless stated otherwise.

When water vapor is coupled to the gray radiative transfer scheme, our approach resembles (Byrne & O'Gorman, 2013). That is, the optical depth is calculated as a function of specific humidity q (kg kg^{-1}),

$$\frac{d\tau}{d\sigma} = bq, \tag{3}$$

where $b = 1997.9$ and $\sigma = p/p_0$, the pressure normalized by a constant (10^5 Pa). See (Byrne & O'Gorman, 2013; Vallis et al., 2018) for details.

Diagnosing the tropopause

The radiative tropopause is diagnosed as the lowest layer of atmosphere where radiative cooling goes to zero. In the absence of radiative heating from ozone, the radiative cooling profile asymptotes to zero in the upper troposphere and so a threshold of -0.05 K day^{-1} is used for the SCM and -0.2 K day^{-1} for the GCM. To make the diagnostic less sensitive to model's vertical resolution, the vertical profile is linearly interpolated from 40 (GCM) or 80 (SCM) levels to 800.

The lapse rate tropopause is diagnosed as where the lapse rate exceeds -5 K km^1. Again, the vertical profile is linearly interpolated.

Water vapor spectroscopy

We use PyRADS, a validated line-by-line column model (Koll & Cronin, 2018), to plot the spectral line absorption coefficients of water vapor. These data are sourced from the HITRAN 2016 database (Gordon et al., 2017), with a Lorenz line profile assumed for all lines. Data is plotted with 0.1 cm^{-1} spectral resolution.

Table of constants and their values

See Table S1.

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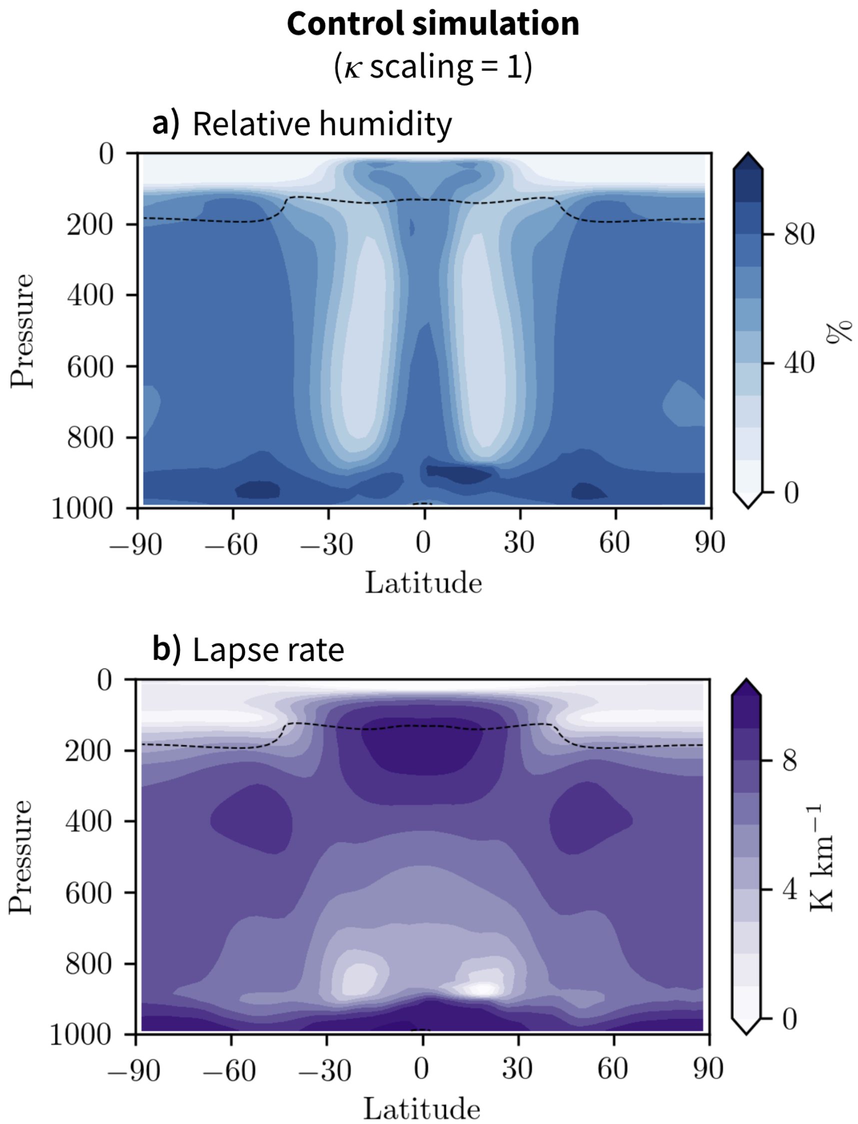

Figure S1. *Zonal-mean profiles from control Isca aquaplanet simulation.* (a) Relative humidity. (b) Lapse rate. The dashed line indicates the radiative tropopause. The globally averaged tropospheric lapse rate is 7 K km^{-1}, defined here as the region between the average lifting condensation level ($\approx$ 950 hPa) and the average tropopause height ($\approx$ 150 hPa).

Table S1. *Definition of symbols used.* See main text for details on computing $\kappa_{\max}$. See Jeevanjee and Fueglistaler (2020) for more details and derivations of many of these quantities.

Symbol	Type	Description	Value/Units
ν	Variable	Wavenumber	cm^{-1}
$\tau_{\mathrm{H_2O}}$	Variable	Optical depth of water vapor at a given wavenumber	—
$\kappa_{\mathrm{H_2O}}$	Variable	Spectroscopic absorption of water vapor at a given wavenumber	$\text{m}^2\ \text{kg}^{-1}$
$\rho_{\mathrm{H_2O}}$	Variable	Density of water vapor	kg m^{-3}
p_{ref}	Constant	Reference atmospheric pressure	500 hPa
T_{em}	Variable	Emission temperature at a given wavenumber	K
T^*	Constant	Characteristic temperature of water vapor	$LR_d\Gamma/(gR_v) \approx 635$ K
T_{ref}	Variable	Characteristic tropospheric temperature	260 K
T_{tp}	Variable	Tropopause temperature	K
M_v^{ref}	Constant	Characteristic column water vapor mass	$T_{\text{ref}}p_v^{\infty}/(\Gamma L) \approx 6 \cdot 10^9$ kg m^{-2}
p_v^{∞}	Constant	Reference value for the saturation vapor pressure	$2.5 \cdot 10^{11}$ Pa
$\kappa_{\max}$	Constant	Maximum absorption of water vapor	≈ 13000 m^2 kg^{-1}
OLR	Variable	Outgoing longwave radiation	Wm^{-2}