Supporting Information for "The 2021 Pacific Northwest heat wave and associated blocking: meteorology and the role of an upstream cyclone as a diabatic source of wave activity"

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Introduction This document describes some technical aspects of local wave activity (LWA) diagnostic. The formalism itself is laid out in the Supporting Information of Huang and Nakamura (2017, hereafter S17), and that document is still a good starting point for the reader unfamiliar with the diagnostic procedure. The present document recaps some of the basics but mostly highlights applications specific to ERA5 (Hersbach et al., 2020). We will also describe the formulation of a one-dimensional model to reconstruct LWA with modified forcing (Fig. 5 of main text).

Computing local wave activity (LWA) from ERA5 reanalysis data

The LWA diagnostic used in the present (and most previous) work is built on the quasi-geostrophic theory, and as such, the key quantity is quasigeostrophic potential vorticity (QGPV). It is well known that some of the assumptions for the QG theory do not hold accurately on a sphere even in the extratropics (e.g. deviation of the Coriolis parameter from a constant must be on the order of the Rossby number), and we do not attempt to fill those gaps. We will keep the Coriolis parameter f a full function of latitude and use full vorticity instead of geostrophic vorticity to define an 'approximate' QGPV. In practical terms, we use the horizontal velocity and (potential) temperature from reanalysis data (u, v, θ) to compute QGPV as

$$q(\lambda, \phi, z, t) = f + \frac{1}{a \cos\phi}\left(\frac{\partial v}{\partial \lambda} - \frac{\partial (u \cos\phi)}{\partial \phi}\right) + f e^{z/H} \frac{\partial}{\partial z}\left(\frac{e^{-z/H}(\theta - \tilde{\theta})}{\partial \tilde{\theta}/dz}\right). \tag{1}$$

In the above, (λ, ϕ, z, t) denote longitude, latitude, pressure pseudoheight and time, respectively, $z = -H \ln(p/1000\text{hPa})$ (p is pressure and $H = 7$ km is assumed), $f = 2\Omega \sin\phi$, $a = 6378$ km and $\Omega = 7.29\times10^{-5}$ rad s^{-1} are the radius and rotation rate of the planet. $\tilde{\theta}(z, t)$ is hemispheric mean potential temperature. It is computed from instantaneous values, but its time dependence is generally very weak, consistent with the QG theory. To evaluate Eq. (1) we first vertically interpolate (u, v, θ) from the 37 pressure levels of ERA5 onto uniformly spaced z-surfaces — in our case with a 500 m interval — up to $z = 48$ km. (We use this coordinate to maintain sufficient details in the stratosphere, but for the column budget this is not necessary, since the stratosphere does not contribute much to the density weighted vertical average — one may well formulate Eq. (1) with the pressure coordinate in the vertical.) We then use finite difference to evaluate Eq. (1) [S17 Eq. (5)].

LWA is defined as

$$A(\lambda, \phi, z, t) = -\frac{a}{\cos\phi} \int_0^{\Delta\phi} q_e(\lambda, \phi + \phi', z, t) \cos(\phi + \phi') d\phi', \tag{2}$$

$$q_e(\lambda, \phi + \phi', z, t) = q(\lambda, \phi + \phi', z, t) - q_{\mathrm{REF}}(\phi, z, t). \tag{3}$$

To evaluate Eq. (2) one must first evaluate Eq. (3), and to evaluate Eq. (3) one must evaluate q_{REF}. We compute q_{REF} by zonalizing the instantaneous QGPV field through an area-preserving map. The easiest way to do this is to first evaluate Eq. (1) for both hemispheres and create a global map of QGPV for each z-surface. With $1^\circ \times 1^\circ$ horizontal resolution, there are 360×180 grid points, indexed by (i, j). At each z and t, all grid points (i, j) are sorted according to equally spaced 181 values of q between the minimum and maximum values on that level $[Q_n = (n-1)\Delta Q,\ 1 \leq n \leq 181,\ \Delta Q = (\max(q_{ij}) - \min(q_{ij}))/180]$. Because of f, typically the minimum value of $q(\lambda, \phi)$ is found near the South Pole, and the maximum value is found near the North Pole. We then compute the area of the region in which $q_{ij} \geq Q_n$ $[\equiv A_n(Q_n)]$ by conditional box counting, weighting each grid with a fractional area $a^2 \cos\phi_j (\Delta\lambda)^2$ $(\Delta\lambda = \pi/180)$. The area $A_n(Q_n)$ is then mapped to equivalent latitude with the formula

$$\phi_n(Q_n) = \sin^{-1}\left(1 - \frac{A_n}{2\pi a^2}\right), \tag{4}$$

which effectively associates the minimum QGPV with the South Pole and the maximum QGPV with the North Pole. Finally by inverting this one-to-one relationship between latitude and QGPV, one obtains $q_{\mathrm{REF}}(\phi)$ at given z and t.

Note that q_e in Eq. (3) must be reevaluated for different ϕ, because q_e is q relative to a value of q_{REF} *at a certain latitude* ϕ. Since q_{REF} increases with latitude, $q_e < 0$ where the QGPV contour $q(\lambda, \phi + \phi') = q_{\mathrm{REF}}(\phi)$ is displaced northward from ϕ (i.e. $\phi' > 0$), and $q_e > 0$ where it is displaced southward ($\phi' < 0$). Either way Eq. (2) is positive. Care must be taken where $\Delta\phi$ is multivalued due to an overturned or cutoff QGPV contour. To take care of this situation automatically, the line integral in Eq. (2) is evaluated at each longitude by scanning the entire latitudes from the South Pole to the North Pole, collecting all contribution from ($q_e < 0,\ \phi' > 0$) and ($q_e > 0,\ \phi' < 0$) (see Fig. 1 of Huang & Nakamura, 2016).

Evaluating the column LWA budget with ERA5 reanalysis data

The column budget of LWA reads

$$\frac{\partial}{\partial t}\langle A\rangle\cos\phi = \underbrace{-\frac{1}{a\cos\phi}\frac{\partial\langle F_\lambda\rangle}{\partial\lambda}}_{\text{(I)}} \underbrace{-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi'}\langle F_{\phi'}\cos(\phi+\phi')\rangle}_{\text{(II)}} \underbrace{+\frac{f\cos\phi}{H}\left(\frac{v_e\theta_e}{\partial\tilde{\theta}/\partial z}\right)_{z=0}}_{\text{(III)}} \underbrace{+\langle\dot{A}\rangle\cos\phi}_{\text{(IV)}}, \tag{5}$$

$$\langle F_\lambda\rangle = \langle u_{\mathrm{REF}}A\cos\phi\rangle - a\left\langle\int_0^{\Delta\phi} u_e q_e\cos(\phi+\phi')d\phi'\right\rangle + \frac{\cos\phi}{2}\left\langle v_e^2 - u_e^2 - \frac{R}{H}\frac{e^{-\kappa z/H}\theta_e^2}{\partial\tilde{\theta}/\partial z}\right\rangle, \tag{6}$$

$$\langle F_{\phi'}\rangle = -\langle u_e v_e\cos(\phi+\phi')\rangle. \tag{7}$$

Note that the RHS terms in Eq. (5) are evaluated at $\phi' = 0$. In the above the angle bracket denotes density-weighted vertical average, R is gas constant and $\kappa = R/c_p$ (c_p is specific heat at constant pressure). The contribution of the last term of Eq. (6) to Term (I), plus Terms (II) and (III) constitutes the vertical average of Eliassen-Palm flux divergence, or equivalently, $\cos\phi\langle v_e q_e\rangle$ via Taylor's identity. Note also

$$u_e(\lambda, \phi+\phi', z, t) = u(\lambda, \phi+\phi', z, t) - u_{\mathrm{REF}}(\phi, z, t), \tag{8}$$

$$v_e(\lambda, \phi + \phi', z, t) = v(\lambda, \phi + \phi', z, t), \tag{9}$$

$$\theta_e(\lambda, \phi + \phi', z, t) = \theta(\lambda, \phi + \phi', z, t) - \theta_{\mathrm{REF}}(\phi, z, t), \tag{10}$$

where $(u_{\mathrm{REF}}, \theta_{\mathrm{REF}})$ are the reference-state zonal wind and potential temperature, and they must be inverted (hemispherically) from $q_{\mathrm{REF}}(\phi, z, t)$. $(u_{\mathrm{REF}}, \theta_{\mathrm{REF}})$ is related to q_{REF} through

$$q_{\mathrm{REF}}(\mu, z, t) = 2\Omega\mu - \frac{1}{a}\frac{\partial}{\partial\mu}\left(u_{\mathrm{REF}}\cos\phi\right) + 2\Omega\mu e^{z/H}\frac{\partial}{\partial z}\left(\frac{e^{-z/H}(\theta_{\mathrm{REF}} - \tilde{\theta})}{\partial\tilde{\theta}/\partial z}\right), \tag{11}$$

where $\mu \equiv \sin\phi$. Using thermal wind balance

$$2\Omega\mu\frac{\partial}{\partial z}\left(u_{\mathrm{REF}}\cos\phi\right) = -\frac{R(1-\mu^2)e^{-\kappa z/H}}{Ha}\frac{\partial\theta_{\mathrm{REF}}}{\partial\mu}, \tag{12}$$

Eq. (11) may be transformed into an elliptic equation for $u_{\mathrm{REF}}\cos\phi$

$$\frac{\partial}{\partial\mu}\left[\frac{1}{2\Omega\mu}\frac{\partial(u_{\mathrm{REF}}\cos\phi)}{\partial\mu}\right] + \frac{2\Omega Ha^2\mu}{R(1-\mu^2)}e^{z/H}\frac{\partial}{\partial z}\left[e^{(\kappa-1)z/H}\frac{\partial(u_{\mathrm{REF}}\cos\phi)/\partial z}{\partial\tilde{\theta}/\partial z}\right] = -a\frac{\partial}{\partial\mu}\left(\frac{q_{\mathrm{REF}}}{2\Omega\mu}\right). \tag{13}$$

We solve Eq. (13) hemispherically on a uniform grid in (μ, z) for $z \in [0, 48]$ km, $\mu \in [0.0872, 1]$. We have also solved Eq. (13) with uniform ϕ and obtained a virtually identical result. Note we set the southernmost boundary of the domain at $\phi = 5°$N to avoid the difficulty with a vanishing μ at the equator. Avoiding the equator proves particularly important for high-resolution data to ensure the convergence of inversion algorithm. The boundary conditions are:

$$u_{\mathrm{REF}}\cos\phi = 0 \quad \text{at} \quad \mu = 1 \quad \text{and} \quad z = 0 \tag{14}$$

$$2\Omega\mu\frac{\partial}{\partial z}\left(u_{\mathrm{REF}}\cos\phi\right) = -\frac{R(1-\mu^2)e^{-\kappa z/H}}{Ha}\frac{\partial[\theta]}{\partial\mu} \quad \text{at} \quad z = 48 \text{ km} \tag{15}$$

$$u_{\mathrm{REF}}\cos\phi = (K - 2\pi\Omega a^2\cos^2\phi)/2\pi a \quad \text{at} \quad \mu = 0.0872\ (\phi = 5°\text{N}). \tag{16}$$

In the above, $[\theta]$ denotes the zonal-mean potential temperature at $z = 48$ km, and $K(z,t)$ is Kelvin's circulation at 5°N equivalent latitude. K is evaluated as the surface integral of absolute vorticity over the domain where QGPV is greater than q_{REF} at 5°N. Since Kelvin's circulation around the QGPV contour is nearly conservative, this boundary condition does not introduce spurious eddy forcing. We have tested different boundary conditions at 5°N and found that they do not affect u_{REF} in the extratropics very much. The no-slip boundary condition at the lower boundary is chosen because u_{REF} represents an eddy-free reference state: a nonzero surface wind requires eddy momentum flux in the presence of surface friction and incompatible with the notion of eddy-free state (Nakamura & Solomon, 2010).

Once we obtain u_{REF}, we reconstruct θ_{REF} using the thermal wind balance [Eq. (12)]. At each altitude a constant value is added to θ_{REF} so that its hemispheric mean matches $\tilde{\theta}(z,t)$. After obtaining $(u_{\mathrm{REF}}, \theta_{\mathrm{REF}})$ we can compute (u_e, θ_e) from Eqs. (8) and (10) and finally evaluate the terms in Eq. (5). We approximate the density weighted vertical average $\langle \cdots \rangle$ as

$$\langle (\cdots) \rangle = \frac{\int (\cdots) e^{-z/H} dz}{H} \approx \frac{\sum_{k=2}^{96} (\cdots)_k e^{-z_k/H} \Delta z}{\sum_{k=2}^{96} e^{-z_k/H} \Delta z} = \frac{\sum_{k=2}^{96} (\cdots)_k e^{-z_k/H} \Delta z}{6.745\mathrm{km}}, \quad \Delta z = 500 \text{ m}. \tag{17}$$

Note that $H = 7$ km in the denominator is replaced by 6.745 km due to discretization. With this approximation, Term (III) in Eq. (5) consists of contributions from k = 1 and 2 ($z = 0$ and 500 m) due to the form of the vertical discretization of QGPV [S17 Eq. (5)]:

$$\frac{f \cos\phi}{H} \left(\frac{v_e \theta_e}{\partial \tilde{\theta} / \partial z} \right)_{z=0} \approx \frac{2\Omega \sin\phi_j \cos\phi_j}{6.745\mathrm{km}} \left(\frac{e^{-z_2/H} v_{eij2} \theta_{eij2}}{\tilde{\theta}_3 - \tilde{\theta}_1} + \frac{v_{eij1} \theta_{eij1}}{2(\tilde{\theta}_2 - \tilde{\theta}_1)} \right) \Delta z. \tag{18}$$

In S17 we conducted several different methods to evaluate the lowest level temperatures in reanalysis and found the result to be insensitive to the chosen methods except for regions with high topography.

Formulation of one-dimensional model for reconstructing LWA with modified forcing

To evaluate the impact of upstream forcing on the downstream block formation, we first evaluate the terms in Eq. (5) using data at a given latitude ϕ for a given period. We can then estimate the zonal transport velocity for column LWA $C(\lambda, t)$ and forcing coefficient $\gamma(\lambda, t)$ from the observed $\langle F_\lambda \rangle$, $\langle A \rangle$, $\langle \dot{A} \rangle$, using the following relationships:

$$\langle F_\lambda \rangle = C\langle A \rangle \cos\phi, \quad \langle \dot{A} \rangle \cos\phi = \gamma \langle A \rangle \cos\phi. \tag{19}$$

With Eq. (19), Eq. (5) may be rewritten as

$$\frac{\partial}{\partial t}\langle A \rangle \cos\phi = -\frac{1}{a\cos\phi}\frac{\partial\left(C\langle A \rangle \cos\phi\right)}{\partial\lambda} + \text{Term (II)}+\text{Term (III)} + \gamma\langle A \rangle \cos\phi. \tag{20}$$

Now we perturb γ to $\gamma + \Delta\gamma$, and as a result $\langle A \rangle$ is also perturbed to $\langle A \rangle + \Delta\langle A \rangle$, but for simplicity we assume that C, Term (II) and Term (III) do not change. (We found little correlation between C and $\langle A \rangle$ during 20-26 June at 49°N.) The above equation is then modified to

$$\begin{aligned}\frac{\partial}{\partial t}\left(\langle A \rangle + \Delta\langle A \rangle\right)\cos\phi = &-\frac{1}{a\cos\phi}\frac{\partial\left[C\left(\langle A \rangle + \Delta\langle A \rangle\right)\cos\phi\right]}{\partial\lambda} \\ &+\text{Term (II)}+\text{Term (III)} + \left(\gamma + \Delta\gamma\right)\left(\langle A \rangle + \Delta\langle A \rangle\right)\cos\phi.\end{aligned} \tag{21}$$

Subtracting Eq. (20) from Eq. (21) and dividing by $\cos\phi$,

$$\frac{\partial}{\partial t}\Delta\langle A \rangle = -\frac{1}{a\cos\phi}\frac{\partial\left(C\Delta\langle A \rangle\right)}{\partial\lambda} + \left(\gamma + \Delta\gamma\right)\Delta\langle A \rangle + \langle A \rangle\Delta\gamma. \tag{22}$$

We solve Eq. (22) for $\Delta\langle A\rangle$ with the knowledge of C, γ, $\Delta\gamma$ and $\langle A\rangle$. In practice, we interpolate these four parameters onto a smaller time interval of 10 minutes, and add a small second-order diffusion with a diffusion coefficient of 2×10^5 $\mathrm{m^2s^{-1}}$ to keep the solution smooth.

References

Hersbach, H., Bell, B., Berrisford, P., Hirahara, S., Horanyi, A., Munoz-Sabater, J., ... Thepaut, J. N. (2020). The era5 global reanalysis. *Quarterly Journal of the Royal Meteorological Society*, *146*(730), 1999–2049. doi: 10.1002/qj.3803

Huang, C. S. Y., & Nakamura, N. (2016). Local finite-amplitude wave activity as a diagnostic of anomalous weather events. *Journal of the Atmospheric Sciences*, *73*(1), 211–229. doi: 10.1175/JAS-D-15-0194.1

Huang, C. S. Y., & Nakamura, N. (2017). Local wave activity budgets of the wintertime northern hemisphere: Implication for the pacific and atlantic storm tracks. *Geophysical Research Letters*, *44*, 5673–5682. doi: 10.1002/2017GL073760

Nakamura, N., & Solomon, A. (2010). Finite-amplitude wave activity and mean flow adjustments in the atmospheric general circulation. part i: Quasigeostrophic theory and analysis. *Journal of the Atmospheric Sciences*, *67*, 3967-3983. doi: 10.1175/2010jas3503.1