

Physics-Informed Data-Driven Seismic Inversion

Recent Progress and Future Opportunities

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Abstract

The goal of *seismic inversion* is to obtain subsurface properties from surface measurements. Seismic images have proven valuable, even crucial, for a variety of applications, including subsurface energy exploration, earthquake early warning, carbon capture and sequestration, estimating pathways of subsurface contaminant transport, *etc.* These subsurface properties (such as wave speed, density, impedance, and reflectivity) influence the transmission of seismic waves through the subsurface media, and well-understood physics models (so-called “forward models”) can be used to predict what surface measurements would be made, for any given subsurface configuration. Seismic inversion is the inverse problem: given actual surface measurements, infer what subsurface configuration would give rise to those measurements. Like most inverse problems, seismic imaging is ill-posed. There is more below the surface than on the surface, and many different subsurface configurations can give rise to the same surface measurements. Because the forward model is itself computationally expensive – the inverse inference is even more so.

But recent advances in algorithms and computing provide an opportunity for remarkable progress in seismic inversion, and efficient solutions to previously infeasible problems have been obtained using data-driven approaches (such as the deep learning methods that were developed primarily for problems in computer vision). The excellent performance of learning-based methods arises from its ability to exploit large amounts of high-quality training data, without the need for hand-designed features. Unlike computer vision, however, seismic inversion is not a data-rich domain. The technical challenges and high cost of acquiring field data prevent the accumulation of large quantities of data (and the commercial value of that data, once acquired) prevents much of the existing data from being widely available. To alleviate the data scarcity issue and improve model generalization, there has been growing interest in combining physics knowledge with machine learning for solving seismic inversion problems.

This review will survey methods for incorporating physics knowledge with machine learning (primarily deep neural networks) to solve computational seismic inversion problems. We will provide a

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structured framework of the existing research in the seismic inversion community, and will identify technical challenges, insights, and trends.

Index Terms

Scientific Machine Learning, Physics Informed Neural Network, Seismic Inversion, Subsurface Characterization, Computational Imaging, Inverse Problems

I. INTRODUCTION

WHILE we can't directly observe the geology of the earth beneath our feet, knowledge of its properties is vital for a variety of applications, including knowing where to build or drill for water or hydrocarbons. From civil infrastructure to energy exploration, characterizing subsurface geology is crucial, but since direct observation of the earth's interior is nearly impossible, inference of unknown subsurface properties primarily relies on indirect and limited geophysical measurements taken at or near the surface. Seismic inversion attempts to reconstruct an image of the subsurface from measurements of natural or artificially produced seismic waves that have traveled through the subsurface medium. A forward model describes how the observations depend on the subsurface map, while the inverse problem is the inference of that map from the observations. The forward model of seismic wave propagation is governed by the wave equations, in the form of a partial differential equation (PDE). A linear approximation can be utilized to simplify the problem, at the cost of loss of accuracy and resolution, leading to travel-time inversion methods [1]. Seismic full-waveform inversion (FWI) addresses the full non-linear problem, providing superior inversion accuracy and resolution [2], but at considerably greater computational cost. Given the large volume of publications in this field, our review focuses on non-linear seismic FWI techniques over other seismic inverse problems (such as travel-time tomography) based on the following considerations:

- **Superior Performance.** FWI leverages extensively more waveform physics than other seismic inversion techniques, resulting in significantly improved imaging results. The technique has been widely applied to various subsurface applications ranging from reservoir-scale characterization [3] to global-scale tomography of Earth's crust and mantle [4].
- **Significant Technical Impact.** FWI faces similar technical challenges to many other ill-posed inverse problems. Our focus here is on how to leverage the underlying governing wave physics (as shown in Fig. 1a), but the techniques discussed here can be not only extendable to other wave imaging problems but also provide potential solutions to a broader context of PDE-governed inverse problems.
- **Active Research.** FWI is one of the most active research topics in subsurface geophysics. A variety of new computational methods have emerged recently (see Fig. 1b for solving the FWI problem).

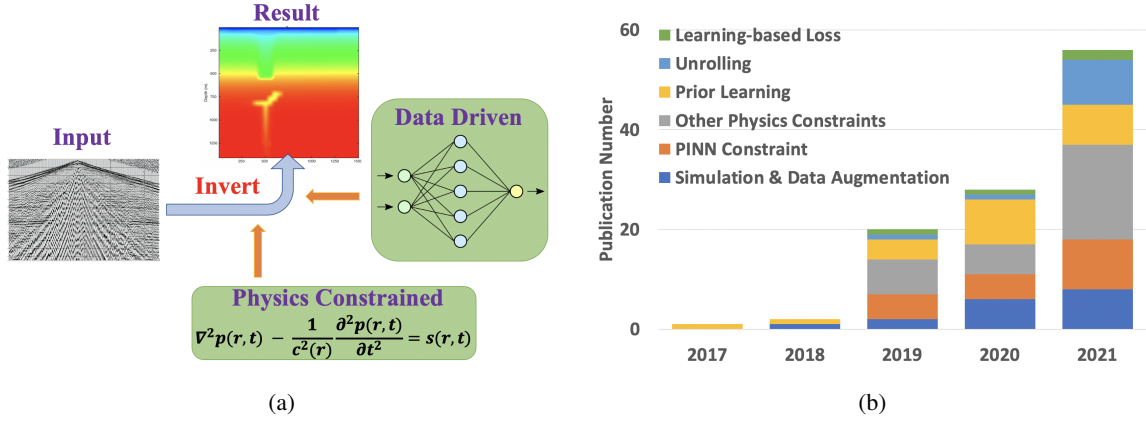


Fig. 1: (a) Physics-informed data-driven seismic imaging. The wave equation is the governing physics. (b) Trends of physics-guided machine learning techniques in computational seismic imaging. Six major technical categories can be identified out of a total 107 papers from 2017 to 2021 collected from the Web of Science and Google Scholar.

This forms an excellent foundation for us to provide a broad and in-depth review of the existing literature.

Seismic full-waveform inversion is a nonlinear inverse problem. Approaches for solving this problem can be categorized into two groups: physics-based methods and machine learning-based methods. Physics-based methods primarily rely on nonlinear optimization techniques that are computationally intensive and highly sensitive to data coverage^{1,2}. Once developed, however, they can be robust and generalize quite naturally to new datasets. In part to alleviate the computational expense of physics-driven methods, data-driven inversion techniques have recently been proposed [5], [6], [7], [8] and reviewed [3]. The data-driven techniques can be extremely efficient once fully trained, but the generalizability of the data-driven approaches is limited by the size and range of the training set. In addition, because data acquisition in subsurface geoscience is always expensive and time-consuming, and because observations can be incomplete and are inevitably contaminated by noise, large training sets are difficult to acquire. Further, because data-driven models learn from data directly without leveraging domain knowledge, those models are prone to unreasonable or unrealistic predictions that do not conform to the physical mechanism. Efforts have been made to combine the physical consistency and generalization ability of physics-based modeling with the computational efficiency of data-driven approaches; these hybrid algorithms

¹Data coverage: the spatial extent to which a subsurface region is covered by a seismic survey.

²We provide a summary of the seismic terminologies used in the review (see Table II).

Reference	Year	Application Domain	Physics	ML	PIML
Virieux and Operto [2]	2009	Exploration Geophysics	✓		
Tromp [4]	2020	Global Geophysics	✓		
Bergen et al. [9]	2019	General Applications in Solid Earth		✓	
Gregory et al. [10]	2020	Computational Imaging and Inverse Problems		✓	
Yu and Ma [11]	2021	General Applications in Solid Earth		✓	
Adler et al. [3]	2021	Computational Seismic Inverse Problems		✓	
Willard et al. [12]	2020	Engineering and Environmental Problems			✓
Karniadakis et al. [13]	2021	Scientific and Engineering Problems			✓

TABLE I: A summary of relevant review articles in the topics of physics-based seismic FWI for exploration geophysics [2] and global geophysics [4], pure ML approaches for general geoscience [9], [11] and computational imaging [10], pure ML-based seismic inverse problems [3], PIML for environmental problems [12], and for broad scientific and engineering inverse problems [13].

are collectively referred to as physics-informed machine learning (PIML). There is ongoing work in the seismic inversion community to develop new models that combine physics-driven methods with machine learning techniques. This review paper will focus on this new trend. The strengths and limitations of these techniques will be discussed based on some numerical examples.

Several review articles relevant to this topic have recently been published. We summarize a number of them in Table I, and categorize them into three groups: physics-based seismic imaging techniques, machine learning methods (ML), and physics-informed machine learning (PIML).

- **Physics-based.** Virieux and Operto [2] provide a thorough overview of physics-based FWI techniques in exploration geophysics, while Tromp [4] gives a different perspective in an overview focusing on applications of FWI for global seismology problems.
- **Machine Learning.** We have found four particularly noteworthy review papers that focus on data-driven methods. Bergen et al. [9] provide a summary of machine learning models for solving general solid Earth geosciences problems ranging from earthquake physics to data visualization, with a focus on existing supervised and unsupervised ML models developed for subsurface applications. Gregory et al. [10] provide an overview on the broad topics of deep learning for computational imaging and inverse problems, concentrating mainly on methodology development rather than on specific imaging problems. In contrast, Yu and Ma [11] explore deep neural networks, and emphasize a broad range of applications in geophysics, including geophysical data processing, interpretation, earthquake detection, and tomography. Adler et al. [3] provide an overview of recent machine learning-based

Terminology	Description in Plain Language
Seismic Data Coverage	The spatial extent to which a subsurface region is covered by a seismic survey.
Velocity Map	A spatial map of the local speed of the seismic wave through the subsurface medium.
Well Logs	Well logs present the detailed record of the geologic information (such as acoustic velocity, electrical resistivity, gamma ray, etc.) penetrated by a borehole.
Reverse Time Migration	Reverse time migration is a seismic processing technique that is able to estimate limited subsurface structural information, such as the locations of layer boundaries.
Prestack Seismic Data	Prestack seismic data is the original raw seismic record.
Post-stacked Seismic Data	Post-stacked seismic data is a processed seismic record that contains seismic waveforms that have been added together from different records to reduce noise and improve overall data quality.

TABLE II: A summary of seismic terminology used in this review.

methods for solving different computational seismic inverse problems including velocity, impedance, reflectivity model building, and seismic bandwidth extension. The focus of their paper is mostly on pure data-driven deep learning approaches with applications to exploration geophysics.

- **PIML.** The literature for physics-informed machine learning is just emerging in geoscience. Neither of the two relevant reviews [14], [13] focuses specifically on computational seismic imaging. Willard et al. [14] discuss the use of physics-informed neural networks (PINN) in engineering and environmental problems. Karniadakis et al. [13] provide a more high-level overview of PINN with applications to inverse problems in different domains such as biophysics, quantum chemistry, material sciences, and molecular simulations.

The remainder of this paper is organized as follows. Section II (Current Research and Development) provides a high-level overview of the physics-based and data-driven computational seismic imaging methods. Section III (Physics-Informed Data-Driven Computational Seismic Imaging) gives a summary of existing hybrid models incorporating physics with machine learning techniques. Section IV (Challenges and Future Research Opportunities) provides a perspective of the existing challenges and some future opportunities. Finally, Section V (Conclusions) summarizes our paper.

II. CURRENT RESEARCH AND DEVELOPMENT

A. Physics-Based Seismic Imaging Techniques

Physics-driven methods infer subsurface properties by directly employing the governing physics and equations. Seismic waves are mechanical perturbations that travel in the medium at a speed governed by

the acoustic/elastic impedance of the material through which they are traveling. In the time domain, the acoustic wave equation is given by

$$\nabla^2 p(\mathbf{r}, t) - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} p(\mathbf{r}, t) = s(\mathbf{r}, t), \quad (1)$$

where t denotes time, $\mathbf{r} = (x, z)$ represents the spatial location in Cartesian coordinates (x is the horizontal direction and z is the depth), ∇^2 is the Laplacian operator, and $c(\mathbf{r})$ is the velocity map (that is, the speed of the wave through the medium at position \mathbf{r}). Here, $p(\mathbf{r}, t)$ is the pressure wavefield, and $s(\mathbf{r}, t)$ is the source term that specifies the location and time history of the source. In the context of this forward model, if we know the velocity map $c(\mathbf{r})$ and the source term $s(\mathbf{r}, t)$, we can use Eq. (1) to solve for the pressure field $p(\mathbf{r}, t)$. Thus, we can write

$$\mathbf{d} = f(\mathbf{m}), \quad (2)$$

to express the deterministic relationship between the model \mathbf{m} , which encapsulates the subsurface velocity map $c(\mathbf{r})$, and the observed data \mathbf{d} , which typically consists of time histories of the pressure wavefield $p(\mathbf{r}, t)$ at a discrete set of locations \mathbf{r} , usually on the surface. The goal of the inverse problem is to infer the subsurface model \mathbf{m} given observed data \mathbf{d} . This can be posed as a minimization problem [15]

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \|\mathbf{d} - f(\mathbf{m})\|_2^2 + \lambda \mathcal{R}(\mathbf{m}) \right\}, \quad (3)$$

where $\|\mathbf{d} - f(\mathbf{m})\|_2^2$ is the data fidelity term with $\|\cdot\|_2$ denoting the ℓ_2 norm, and $\mathcal{R}(\cdot)$ is the regularization term with regularization parameter λ . Solving the minimization in Eq. (3) can be numerically challenging or even infeasible due to the ill-posed nature of the inversion problem (i.e. even small noise levels in the measurements \mathbf{d} can yield large uncertainties in the model \mathbf{m}). For the FWI problem, this can arise from limited data coverage (too few measurements of the pressure wavefield) and in particular from *cycle skipping* (multiple solutions matched to the same data). Many approaches have been proposed to alleviate this problem. These includes advanced regularization [15], dynamic warping techniques [16], incorporation of prior information [17], multiscale inversion approaches [18], and preconditioning techniques [19]. Another issue with current computational methods for solving FWI is the high computational cost. Most of the current techniques are based on first-order or second-order optimization techniques. Both of these types of methods are expensive since they require computation of the gradient of the function Eq. (3), which requires a forward wave propagation, a backward wave propagation, and full-wavefield cross-correlation. A line search strategy, requiring additional steps of forward wave propagation, is also necessary.

B. Data-Driven Seismic Imaging Techniques

We begin this section with the remark that *purely* data-driven techniques are unlikely ever to work for seismic imaging; there just isn't enough data. A lot of wells have been drilled over the last century,

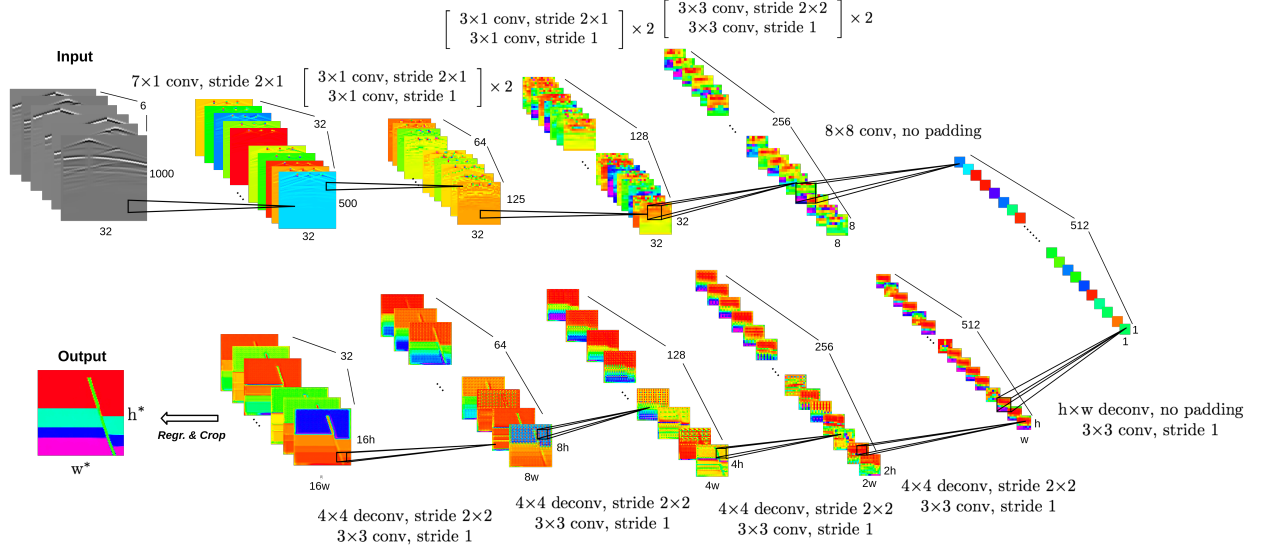


Fig. 2: An illustration of InversionNet, a data-driven seismic waveform inversion solver [6].

and so we have a reasonable idea about what the subsurface is like in general. But for a given site, there is precious little in the way of direct observations of the subsurface. When we speak of data-driven methods being applied to the seismic imaging problem, we are virtually always speaking of simulated data. As with traditional “physics-based” methods described in the previous subsection, we assume we know the governing equation for acoustic waves, and that we can readily simulate the function $f(\cdot)$ described in Eq. (2). Given this, the key idea for data-driven methods is to create many samples of image pairs: of velocity model \mathbf{m} -images along with matched samples of waveform data \mathbf{d} -images. There is some art to the creation of the \mathbf{m} -images – these are chosen to be representative of what we think the subsurface might be like. But once we have a collection of \mathbf{m} -images, we can use $\mathbf{d} = f(\mathbf{m})$ to create the associated \mathbf{d} -images. With all of this simulated data, we can at last apply some data-driven analysis. A large image-to-image deep neural net (as shown in Fig. 2) is used to learn a direct mapping of \mathbf{d} to \mathbf{m} , which can finally be used to map an actual data observation \mathbf{d}_o to an estimate of the subsurface map. Mathematically, this can be posed as

$$\begin{aligned} \widehat{\mathbf{m}}(\mathbf{d}) &= g_{\theta^*}(\mathbf{d}) \\ \text{s.t. } \theta^*(\Phi_s) &= \arg \min_{\theta} \sum_{(\mathbf{m}_i, \mathbf{d}_i) \in \Phi_s} \mathcal{L}(g_{\theta}(\mathbf{d}_i), \mathbf{m}_i), \end{aligned} \quad (4)$$

where: θ represents the trainable weights in the inversion network $g_{\theta}(\cdot)$; $\mathcal{L}(\cdot, \cdot)$ is a loss function; and Φ_s is the training dataset with paired samples of $(\mathbf{m}_i, \mathbf{d}_i)$.

A distinctive feature of this approach is that, in contrast with traditional inversion, it produces a

“whole function” $g_{\theta^*}(\cdot) \approx f^{-1}(\cdot)$, not just a single inversion $\widehat{\mathbf{m}}_{\text{obs}} \approx f^{-1}(\mathbf{d}_{\text{obs}})$. Nonetheless, experience with data-driven methods indicates that they can be trained in substantially less time than it takes for a single traditional inversion [6]; and once trained, the application to a specific observation \mathbf{d}_{obs} is nearly instantaneous. The reason for this efficiency is not fully understood, but some factors that potentially contribute include: the simulations are only of the forward model, and no gradients (or Hessians) are computed; the simulations are restricted to plausible models, so no time is spent (as may be the case for traditional optimizers) in implausible regions of solution space; traditional optimization is inherently serial, whereas the many simulations can be readily parallelized, and neural network architectures naturally lend themselves to efficient massive parallelization. Further, because the function $f^{-1}(\mathbf{d})$ is learned directly and implicit regularization is imposed via training on a large volume of simulations, we do not see the ill-posedness that traditional inversion often exhibits; on the other hand, solutions tend to look like the models \mathbf{m} that they were trained on, so the generalization ability of this approach is limited.

Most recent explorations have focused on data-driven end-to-end reconstruction. For instance, GeoDNN [7] used a fully connected network with 8 layers for FWI. InversionNet [6] introduced an encoder-decoder network with modern convolutional neural network (CNN) features. ModifiedFCN [8] employed a fully connected convolutional network (FCN) design and SeisInvNet [5] enhanced each seismic waveform with auxiliary knowledge from neighborhood traces for better spatial correspondence.

III. PHYSICS-INFORMED DATA-DRIVEN COMPUTATIONAL SEISMIC IMAGING

In this section we discuss a variety of computational techniques that have been developed for incorporating physics in seismic imaging methods.

A. Physics Knowledge

Physics knowledge can be represented in many different ways. What makes seismic imaging problems fundamentally different from those of the computer vision community are the physical laws underlying those problems. These physical laws are “invariant” in the sense that the spatial and temporal dynamics can be characterized by those laws throughout the physical process. Here we provide two different representations of physics knowledge that has been utilized to enhance the machine learning model.

- 1) **Imaging Physics.** The most directly relevant physics is expressed by the governing equations underlying the imaging process. For seismic imaging, this is the partial differential equation in Eq. (1) that describes the propagation of acoustic waves through the subsurface.
- 2) **Physical Properties of the Solution Space.** To the extent that we understand the geophysics of the subsurface, we can constrain the solution space to be consistent with those physical properties,

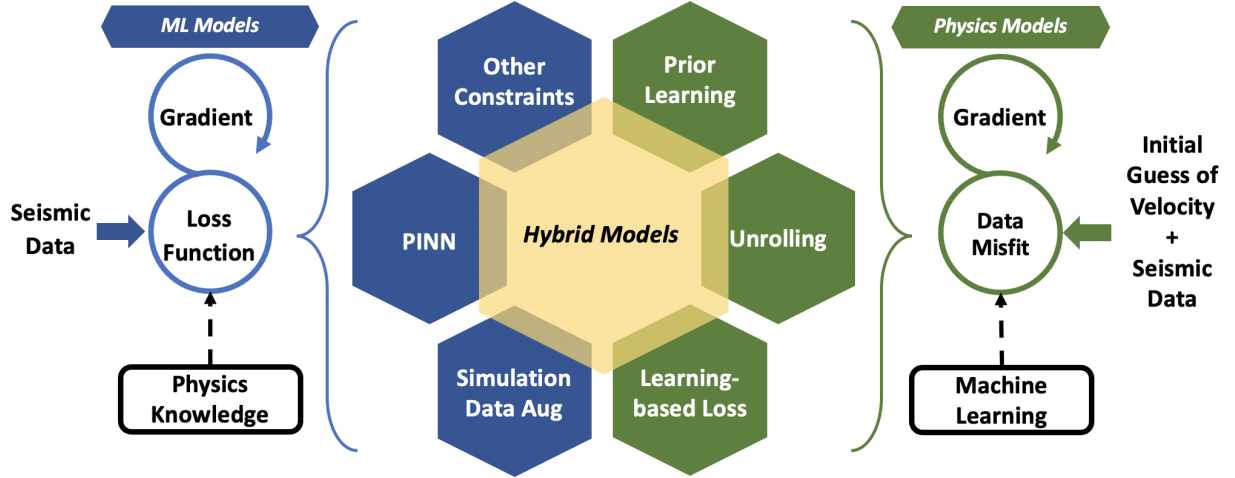


Fig. 3: An illustration of PIML-based seismic imaging techniques, which can be grouped into two categories: those built on ML models (blue hexagons) and those built on physics models (green hexagons).

and we can more effectively choose “plausible” samples from the solution space to train our neural networks. Among these properties are the tendencies for materials to stratify into distinct layers, the existence of faults as distinct geological formations, and the overall variability of densities and sound speeds in subsurface material, etc. We can also make use of dynamical properties of the subsurface phenomena that we are trying to measure. Consider CO_2 sequestration as one example. After injection of super-critical CO_2 into the subsurface, the CO_2 plume will migrate over time, meaning that the spatial spreading of CO_2 should gradually increase over time. If a model can be designed so that the images are constrained to exhibit dynamic spreading (and not, say, compaction) of the CO_2 plume, then this would improve the overall performance [20].

We provide a taxonomy of physics-informed learning-based seismic inversion techniques (see Fig. 3) with two major categories of approaches: one being built on ML models (blue hexagons) and the other built on physics models (green hexagons). There are three groups of techniques within each category. We will discuss all six groups in the following sections.

B. Machine Learning Models Incorporating Physics

In this section, we describe methods identified with blue hexagons in Fig. 3. These models are all fundamentally data-driven (in most cases, they are deep neural networks), but in each case with some aspect of the appropriate physics incorporated. In particular, we will discuss how to embed physics

knowledge through data augmentation techniques, how to constrain the learning model using the governing wave equation, and how to learn underlying solution space distribution to help with the inversion.

1) Simulations and Data Augmentation: As noted previously in Section II-B, we can leverage our knowledge of the imaging physics to simulate large quantities of paired data (m, d) where m is a plausible subsurface velocity map and d is the data that would be observed if m accurately modeled the subsurface. What we did not say in Section II-B, however, is what we meant by “plausible.” or how a large sample of plausible subsurface maps might be generated. In general to do this requires domain knowledge of the subsurface rock physics, and the creation of a large idealized set of subsurface structure maps (to be used as training sets) is commonly known as “velocity building” in the subsurface geophysics community.

While this can be effective, it may not ultimately be scalable, because it often leads to very large training sets – for example, more than 60,000 velocity maps were created for training in [6] – that still may not encompass the full variability of what might be in the subsurface. One approach is to use auxiliary knowledge of the subsurface physics to reduce this variability. Well logs³ from drilled wells has been used to build a large-scale subsurface data set for training a deep learning model [21], and pseudo-wells based on the well statistics (such as porosity, saturation, mineralogy, and thickness) have also been created to train the neural network models [22]. Additionally, generative models (such as the variational auto-encoder and the generative adversarial network) have been leveraged to synthesize more realistic simulations. A variational auto-encoder with physics-informed regularization (shown in Fig. 4a) was designed to synthesize realistic velocity maps [23]. Domain adaptation is another technique that was shown to be effective to overcome the data scarcity issue. A style transfer network (see (Fig. 4b) was developed to convert from natural images to physically meaningful subsurface velocity maps [24].

It is possible, albeit less straightforward, to augment the training set from the seismic data perspective rather than just produce more simulations form a broader variety of velocity maps. A couple of authors have shown the benefits of enhancing supervised training with this kind of data augmentation [25], [26]. Specifically, Gomez et al. [25] developed adaptive data augmentation techniques based on active learning, where physically realistic simulated seismic data are created to enrich the existing training set. Ideas of domain adaptation have also been applied to enrich the quality of seismic data in training set. Alkhalifah et al. [26] construct a high-quality synthetic seismic dataset with more realistic features from the target seismic data (such as noise and artifacts). Their example applications on passive seismic imaging demonstrate the efficacy of their proposed techniques.

³Well logs present a detailed record of the geologic information penetrated by a borehole.

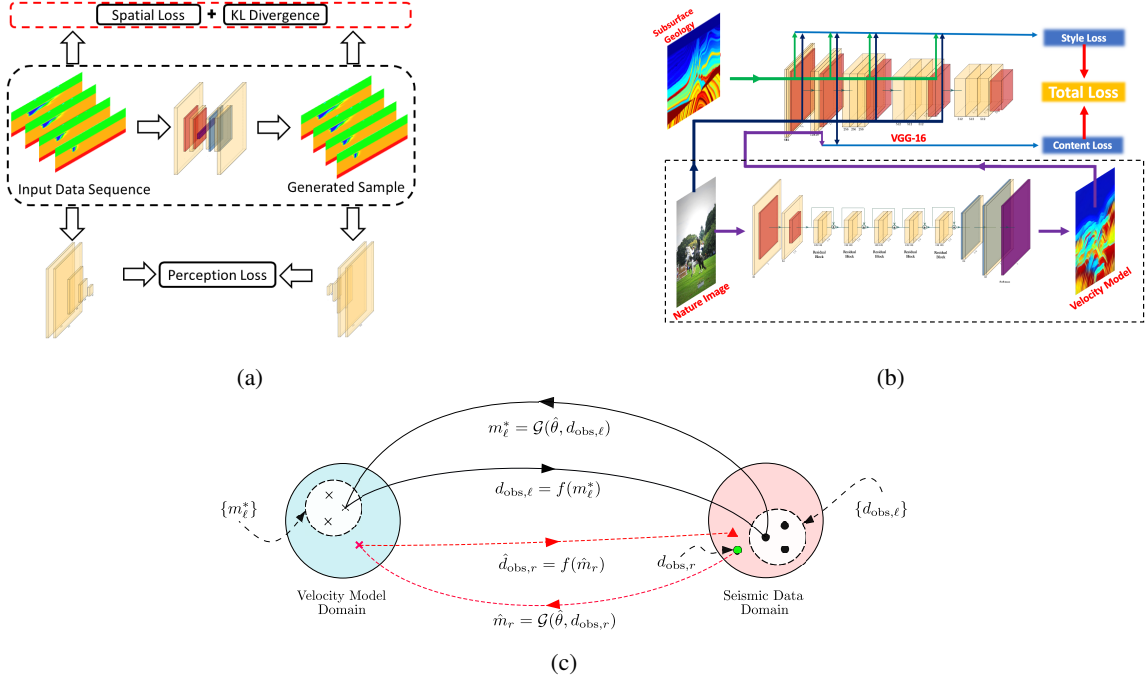


Fig. 4: Schematic illustration of three physics-informed data augmentation techniques to generate physically realistic samples. (a) VAE model with physics-informed loss to synthesize velocity maps [23]. (b) Style transfer model to leverage existing natural images to generate velocity maps [24]. (c) An adaptive active learning method to augment training set with unlabeled seismic data [25].

As a demonstration, we provide a brief overview of the technique developed in [25]. Four steps (illustrated in Fig. 4c) are proposed including:

- 1) Estimate an approximate solver $\mathcal{G}(\hat{\theta}, \mathbf{d}_{\text{obs}})$;
- 2) Generate approximate velocity maps from unlabeled data $\hat{\mathbf{m}}_{\text{obs}} = \mathcal{G}(\hat{\theta}, \mathbf{d}_{\text{obs}})$;
- 3) Create seismic data using forward model $\hat{\mathbf{d}}_{\text{obs}} = f(\hat{\mathbf{m}}_{\text{obs}})$;
- 4) Add new pairs to the original training set.

The augmented dataset plays a key role in model accuracy because it will not only carry useful physics information but also provides examples of velocity maps that are consistent with the target geology feature of interest. Furthermore, the full augmentation process can be applied in an iterative fashion by re-training the approximate solver $\mathcal{G}(\hat{\theta}, \mathbf{d}_{\text{obs}})$ based on the extended training set in order to generate new approximate velocity maps $\hat{\mathbf{m}}_{\text{obs}}$. This approach allows further refinement of the mapping between velocity and seismic subdomains. In Fig. 5, we further provide visualization of the inverted velocity maps using both physics-based FWI and data-driven approaches as proposed in [25] to characterize tiny

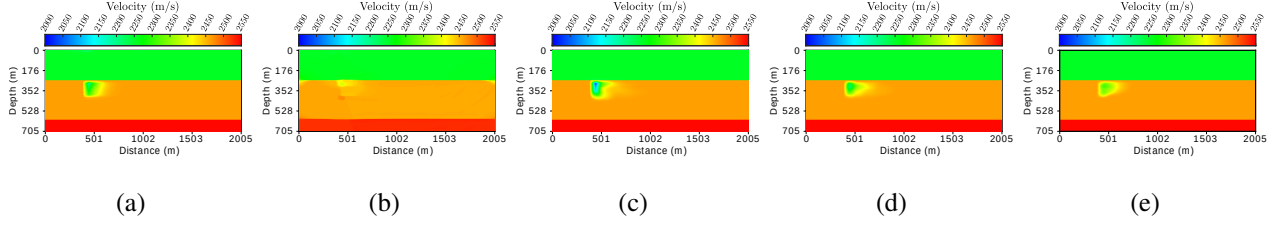


Fig. 5: (a) Ground truth. Inversions and errors obtained using (b) physics-based FWI (0.069, 0.972), data-driven model trained on (c) the *large* and *medium* subsets (0.0620, 0.990), (d) *large* and *augmented medium* subsets (augmented once) (0.0134, 0.993), and (e) *large* and *augmented medium* subsets (augmented twice) (0.0122, 0.994). Error is in the format of (MAE, SSIM).

CO₂ leakage plumes at Kimberlina sequestration site [27]. We observe that the network trained over augmented subsets obtains a more accurate estimate of the plume shape and location.

2) *Physics-Informed Neural Networks*: Physics-informed neural networks (PINNs) [28] are learning methods for solving scientific inverse and forward problems given governing physics PDEs. This technique has been widely adopted in various science and engineering domains. The PINN framework is a popular choice in seismic imaging for incorporating the governing wave equations [29], [30], [31], [32]. In the following, we select and discuss one these techniques, as representative of the group.

Jin et al. [30] developed an Unsupervised Physical-informed Full-Waveform Inversion (UPFWI), which can be posed as

$$\widehat{\mathbf{m}}(\mathbf{d}) = g_{\theta^*}(\mathbf{d}), \text{ where } \theta^*(\Phi_u) = \arg \min_{\theta} \sum_{\mathbf{d}_i \in \Phi_u} \mathcal{L}(f(g_{\theta}(\mathbf{d}_i)), \mathbf{d}_i), \quad (5)$$

where Φ_u represents an unsupervised dataset that contains seismic data \mathbf{d}_i alone. Finite differences were used to approximate the forward modeling of PDE as a differentiable operator (from velocity map to seismic data), and then model its inversion by CNN (from seismic data to velocity map). Hence, the supervised data-driven inversion task in Eq. (4) is transformed into the unsupervised seismic data reconstruction task in Eq. (5), where the PINN plays an essential role. Comparing the network architecture of UPFWI (Fig. 6) to InversionNet (Fig. 2), the major difference is the explicit incorporation of wave equation. It constrains the output of the CNN (the orange network in Fig. 6), when passing through an explicit wave equation, to match the original input (i.e., seismic data). Some additional physical constraints are imposed to further enhance the imaging results: one is to leverage a learning-based perceptual loss function by extracting style and content features of the output seismic data via VGG16 network, and the other is to confine the resulting dimension of the CNN output, which should be the same as that of the

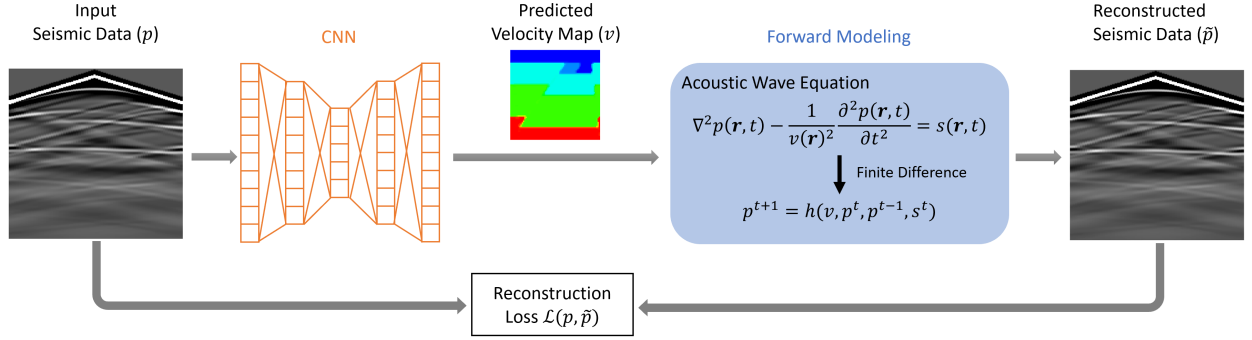


Fig. 6: Schematic illustration of UPFWI structure, which comprises a CNN to learn an inverse mapping and a differentiable operator to approximate the forward modeling of PDE [30].

Supervision	Method	FlatFault			CurvedFault		
		MAE ↓	MSE ↓	SSIM ↑	MAE ↓	MSE ↓	SSIM ↑
Supervised	InversionNet [6]	15.83	2156.00	0.9832	23.77	5285.38	0.9681
	VelocityGAN [33]	16.15	1770.31	0.9857	25.83	5076.79	0.9699
	H-PGNN+ [29]	12.91	1565.02	0.9874	24.19	5139.60	0.9685
Unsupervised	UPFWI	14.60	1146.09	0.9895	23.56	3639.96	0.9756

TABLE III: Quantitative results evaluated in MAE, MSE and SSIM. UPFWI yields better inversion accuracy comparable to that of supervised baselines.

expected velocity maps. The former helps in imaging the reflection layers of the velocity maps while the latter is helpful to some degree to constrain the solution space.

We provide a performance comparison of UPFWI with two pure data-driven methods (i.e., InversionNet [6] and VelocityGAN [33]), and a supervised physics-informed method H-PGNN [29] using two different datasets (FlatFault and CurvedFault [34]). UPFWI performs comparably to all three baseline methods on both FlatFault and CurvedFault. The velocity maps inverted by different methods are shown in Figure 7, which is consistent with the quantitative analysis in Table III.

3) *Other Physics-Informed Constraints:* Another interesting way of incorporating physics is to estimate the velocity maps based on a learned representation of the underlying distribution of the physical parameters. Specifically, the generative adversarial network (GAN) is a popular network architecture that has been used to characterize data distribution. A number of recent works are representative of this

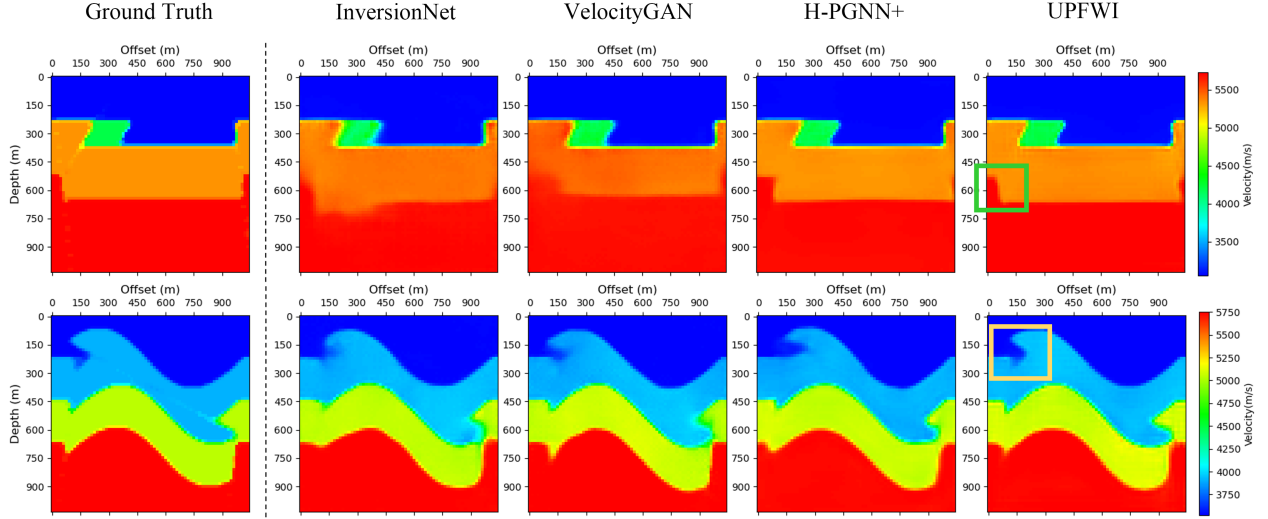


Fig. 7: Comparison of different methods on inverted velocity maps of FlatFault (top) and CurvedFault (bottom). For FlatFault, UPFWI reveals more accurate details at layer boundaries and the slope of the fault in deep region. For CurvedFault, UPFWI reconstructs the geological anomalies on the surface that best match the ground truth.

category [35], [33], [36]. Mathematically, these ideas can be formulated as:

$$\begin{aligned}
 \widehat{\mathbf{m}}(\mathbf{z}^*) &= g_{\theta^*}(\mathbf{z}^*) \\
 \text{s.t. } \mathbf{z}^*(\mathbf{d}) &= \arg \min_{\mathbf{z}} \mathcal{L}(f(g_{\theta^*}(\mathbf{z})), \mathbf{d}) \\
 \theta^*(\Phi_m) &= \arg \min_{\theta} \sum_{\mathbf{m}_i \in \Phi_m} \mathcal{L}_{\text{GAN}}(g_{\theta}(\alpha_i), \mathbf{m}_i),
 \end{aligned} \tag{6}$$

where Φ_m is a training dataset including numerous velocity maps; and \mathbf{z} and α_i are tensors sampled from the normal distribution. The iterative optimization is then performed on \mathbf{z} to draw a velocity map sampled from the adaptive distribution. It is worthwhile mentioning a major difference in learning strategy comparing Eq. (6) with Eq. (5). The optimal weights $\theta^*(\Phi_u)$ in Eq. (5) is obtained based on the dataset Φ_u for a subdomain, whereas the optimal $\theta^*(\mathbf{d})$ in Eq. (6) is optimized per sample \mathbf{d} . After training with sufficient seismic data, Eq. (5) yields a mapping from seismic data to velocity maps that can be directly used for inference without any optimization. In contrast, the optimal weights in Eq. (6) are sample dependent, requiring optimization for any new sample before inference.

C. Physics Models Enhanced by Machine Learning

Three groups of methods are included in this category, as shown in the green hexagons in Fig. 3. These methods are built on physics-based iterative approaches, with machine learning techniques introduced to

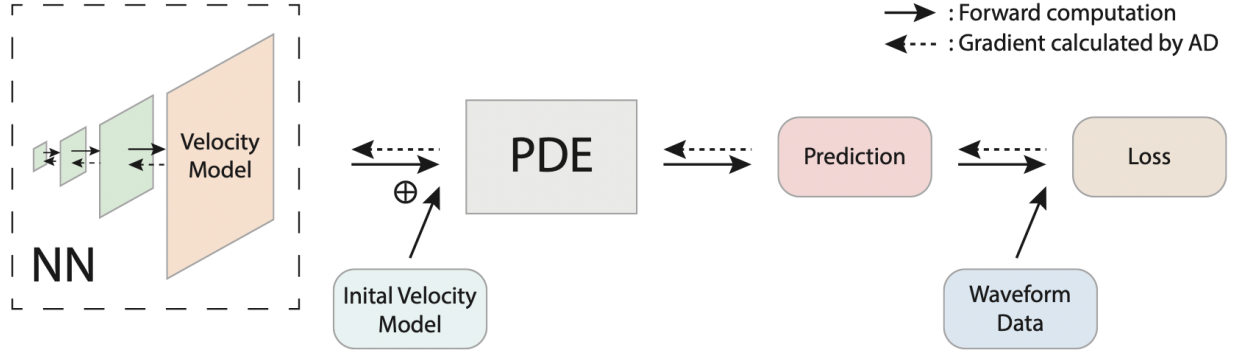


Fig. 8: Schematic illustration of the idea of reparameterization, which combines neural network and PDE of seismic waves. The neural network is used to parameterize the velocity maps while the PDE is to calculate the forward modeling and data misfit term (Image courtesy of [37] and used with permission.)

enhance the computational techniques. Specifically, we will discuss how to learn useful prior knowledge using deep learning models for physics-based methods, how to leverage ML to estimate gradient, and how to learn an effective loss function using ML for physics-based methods.

1) *Prior Knowledge Learning*: A large body of work has demonstrated the importance of prior knowledge in improving the performance of physics-based FWI approaches. This prior knowledge includes the existing parameter distribution [37], [38], [39], [40], use of the low-frequency component for a reasonable initial guess [41], [42], [43], and others. Here our focus is on exploiting the known parameter distribution, for which a number of data-driven methods have been proposed. Those techniques originate from the general idea of deep internal learning that trains deep neural networks on a single training sample. An example would be the deep image prior [44] in which deep learning models are utilized to learn useful prior knowledge via re-parameterization. When applying similar ideas to seismic inversion problems, it has already been shown that a couple of the major benefits are expected including one, the techniques mitigate local minima issue due to the sparse representation of the velocity maps using re-parameterization; two, the re-parameterization can be seen as an implicit regularization, which in turn helps alleviate the ill-posedness of the inversion problem [39]. Schematically, the network structures of those methods usually consist of two components: a neural network and a PDE, which is illustrated in Fig. 8. The neural network is used to re-parameterize the velocity maps while the PDE is to calculate the forward modeling procedure and obtain the data misfit. Mathematically, seismic inversion with re-parameterization can be

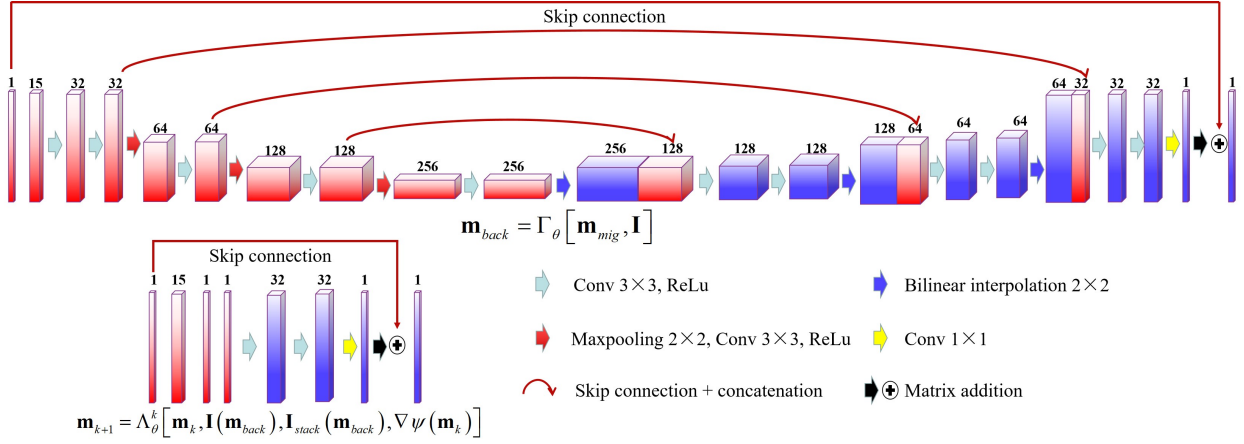


Fig. 9: Schematic illustration of the idea of (Image courtesy of [45] and used with permission.)

formulated as

$$\begin{aligned} \widehat{\mathbf{m}}(\mathbf{d}) &= g_{\theta^*}(\mathbf{d}) \\ \text{s.t. } \theta^*(\mathbf{d}) &= \arg \min_{\theta} \mathcal{L}(f(g_{\theta}(\alpha)), \mathbf{d}), \end{aligned} \quad (7)$$

where α is a random tensor drawn from a normal distribution, i.e., $\alpha \sim \mathcal{N}(0, \sigma^2)$. Different network architectures, $g_{\theta}(\alpha)$, have been explored to study their regularization implicit effect in those existing work including CNN-domain FWI [38], DNN-FWI [39] and FCN-FWI [37]. As shown in Eq. (7), these methods involve iterative optimization, with $\theta^*(\mathbf{d})$ in Eq. (7) being optimized for seismic measurements \mathbf{d} for a particular time and site, and requiring re-training in order to compute the inversion for a different input \mathbf{d} . Unlike those subdomain approaches, the optimal weights $\theta^*(\Phi_u)$ in Eq. (4) are obtained based on the dataset Φ_u , thus one network would be applicable to multiple seismic samples once fully trained. Here the training of the networks, $g_{\theta}(\alpha)$, is usually carried out in two steps. The networks are first pre-trained on an expert initial guess of the solution. After pre-training, the network is used to reparameterize the velocity and the network parameters and is updated iteratively via the computation of the gradient. Comparable results to physics-based inversion methods have been reported in the aforementioned works. There are, however, some technical challenges associated with these methods. One is the heavy reliance on the initial guess of the solution. Eq. (7) requires an *expert* initial guess, which is crucial for optimization and expensive to acquire [39]. An unsatisfactory initial guess would potentially mislead the training and yield a completely wrong inversion. Two is the high computational cost in training, which is due to the nature of the per sample learning strategy. The optimal network parameters, θ in Eq. (7), will not be able to be generalized to other data without a full re-training.

2) *Unrolling-Based Techniques*: The computational techniques solving Eq. (3) are based on iterative methods. The main idea behind this group of methods is to unroll an iterative schemes with a small number of iterations, replacing critical components of the scheme, such as the computation of the gradient of the data fidelity term, with a trainable module. Mathematically, this can be represented as

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \nabla E(\mathbf{m}_k) \approx \Psi_{\theta}^k [\mathbf{m}_k, \nabla E(\mathbf{m}_k)], \quad k = 0, 1, \dots, N-1, \quad (8)$$

where α_k is the line search step size at each iteration; $\nabla E(\mathbf{m}_k)$ is the gradient term; and Ψ_{θ}^k is a neural network with learnable parameters θ . With an appropriately trained network of Ψ_{θ}^k , the gradient of FWI will be obtained efficiently. However, there are two major issues in employing Eq. (8). One is the irreversible loss of some information when calculating the gradient based on the cross-correlation of the forward and backward propagated wavefields. A heuristic interpretation is that the original raw seismic waveform data are mapped from high-dimensional space into a low-dimensional space where the gradient resides in, resulting in unavoidable lost of some information of the observed data. Two is that the accuracy of Eq. (8) depends on a good initial guess of the solution. A variety of approaches have been developed to address these issues [45], [46], [47]. Zhang and Gao [45] developed a two-stage learning strategy described below (illustrated in Fig. 9):

- Stage 1 – a network is designed to estimate background velocity map \mathbf{m}_{back} from input

$$\mathbf{m}_{\text{back}} = \Gamma_{\theta} [\mathbf{m}_{\text{smooth}}, I(\mathbf{m}_{\text{smooth}})], \quad (9)$$

where Γ_{θ} is an encoder-decoder neural network with learnable parameter θ (shown in the upper panel of Fig. 9); \mathbf{m}_{back} is the reconstructed background velocity map; $\mathbf{m}_{\text{smooth}}$ is the smoothed background velocity map; and $I(\cdot)$ is the reverse time migration operator⁴ using prestack seismic data⁵.

- Stage 2 – the high resolution velocity map is updated iteratively from the background velocity map as

$$\mathbf{m}_{k+1} = \Lambda_{\theta}^k [\mathbf{m}_k, I(\mathbf{m}_{\text{back}}), I_{\text{stack}}(\mathbf{m}_{\text{back}}), \nabla R(\mathbf{m}_k)], \quad k = 0, 1, \dots, N-1, \quad (10)$$

where Λ_{θ}^k is a residual neural network with learnable parameter θ (shown in the lower panel of Fig. 9); $I_{\text{stack}}(\cdot)$ is the reverse time migration operator using the post-stacked seismic data⁶; and $\nabla R(\mathbf{m}_k)$ is the gradient of the regularization in Eq. (3).

⁴Reverse time migration is a seismic processing technique that is able to estimate limited subsurface structural information, such as the locations of layer boundaries.

⁵Prestack seismic data is the original raw seismic record.

⁶Post-stacked seismic data is a processed seismic record that contains seismic waveforms that have been added together from different records to reduce noise and improve overall data quality.

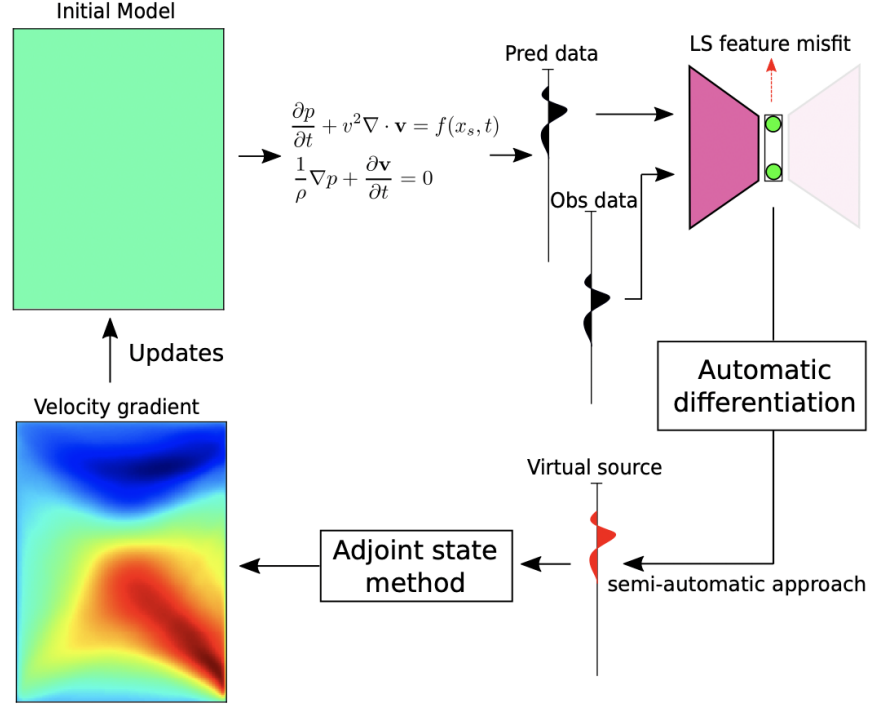


Fig. 10: Schematic illustration of the hybrid machine learning inversion framework (Image courtesy of [48] and used with permission).

An advantage of the two-stage learning strategy (Eqs. (9) and (10)) over the standard unrolling method as in Eq. (8) is that the neural network input and output are in the same space. In addition, more physical information (i.e., reverse time migration images) are used, which not only provides more details to reconstruct the velocity maps, but also alleviates the model reliance on a good initial guess of the solution. As mentioned in [45], the prediction performance of the model heavily relies on the quality and quantity of the samples and labels. Some performance improvements are reported in the literature due to the incorporation of additional physical information, but this is more computationally expensive because of the nature of the iterative procedure.

3) *Learning Loss Functions*: The loss function plays a critical role in physics-based inversion techniques. An inappropriately selected loss function will make the iteration prone to non-convexity/multiple minima problems and over-sensitivity to the initial guess. Traditional methods to tackle this challenge are based on hand-crafted functions, which may succeed in some datasets but fail in others [2]. To overcome this issue, ML approaches have been utilized to learn a data fidelity function [48], [49], [50]. Specifically, a learning-based data fidelity function that measures high-level features has been shown

to be effective in mitigating the local minima issue. Following this, Chen and Saygin [48] develop a multi-scale hybrid machine learning inversion method by defining a data fidelity function that measures the latent space features of the observed and predicted data (see Fig. 10). The velocity maps are updated iteratively by minimizing the learning-based data fidelity term. A different perspective is to use a network to approximate either the data fidelity function or the regularization term. Sun and Alkhalifah [49] employ a meta-learning methodology to learn a robust data fidelity function for FWI. From the same group, Zhang and Alkhalifah [50] trained a deep neural network to first estimate the distribution of facies (the overall characteristics of a rock unit that reflect its origin and differentiate the unit from others around it), which will be used as regularization to constrain the iterative procedure of the FWI. We select the work in [48] to represent this group of approaches due to its promising performance and applicability to real data. Their idea is illustrated in Fig. 10. Rather than using the traditional loss term (i.e., $\|\mathbf{d} - f(\mathbf{m})\|_2^2$ in Eq. (3)), a new loss function is proposed

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \|W_{\theta}(\mathbf{d}) - W_{\theta}(f(\mathbf{m}))\|_2^2 \right\}, \quad (11)$$

where W_{θ} is a neural network with learnable parameter θ used to extract useful latent space features from both real data, \mathbf{d} and predicted data $f(\mathbf{m})$. The gradient of the network can be obtained via automatic differentiation and the adjoint-state method. The proposed learning-based loss function in Eq. (11) is found to be less prone to entrapment in local minima compared to the traditional loss in Eq.(3). Furthermore, the proposed inversion method can recover the subsurface velocity structure in different scales. A low dimensional latent space contains the kinematic information of the seismic waveforms, while large-dimensional latent space features preserve the dynamic information from the waveforms. Leveraging that, in turn, further improves the imaging resolution significantly. Experiments with both synthetic and real data tests have been carried out to demonstrate the performance of the proposed method [48].

D. Open Data and Benchmarks

The availability of an extensive database is vital for machine learning-based inversion techniques. Unlike some data-rich domains (such as computer vision), this is problematic for seismic inversion in two ways. One is that data are often proprietary and hence challenging to share, but the second and more important problem is that acquiring correctly labeled data is extremely difficult or even impossible as the true velocity maps are never known exactly. There have been some recent effort to address this urgent need for more and better open data for the R&D of seismic inversion. The U.S. Department of Energy (DOE) created the Energy Data eXchange (EDX), which is a virtual library and data laboratory built to find, connect, curate, use and re-use data to advance fossil energy and environmental R&D [51].

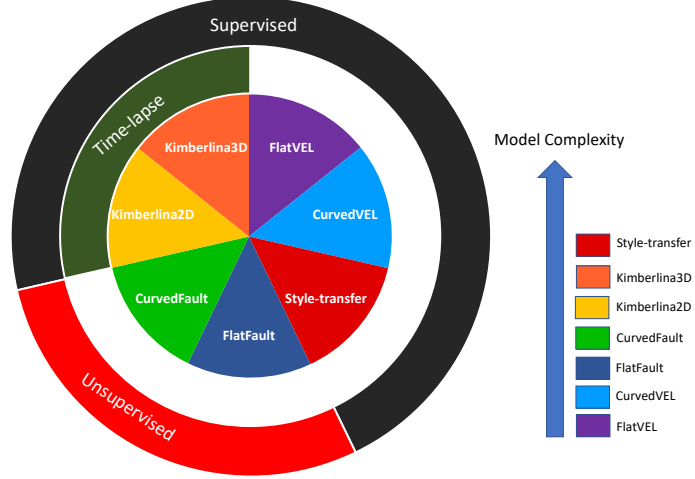


Fig. 11: Overview of the datasets in OpenFWI [34].

EDX supports the entire life cycle of data by offering secure, private collaborative workspaces for ongoing research projects until they mature and become catalogued, curated, and published. Recently, Los Alamos National Laboratory created and released OpenFWI, a collection of large-scale open-source datasets, particularly suited for benchmarking seismic inversion algorithms [34]. The purpose of OpenFWI is to facilitate diversified, rigorous, and reproducible research on data-driven seismic inversion. As shown in Fig. 11, OpenFWI includes datasets of multiple scales, encompasses diverse domains/applications, and covers various levels of model complexity. Along with the dataset, an extensive study on performance benchmarks using both physics-based inversion and data-driven techniques are provided.

IV. CHALLENGES AND FUTURE RESEARCH OPPORTUNITIES

A summary of the inversion techniques mentioned in this review is provided in Table IV. This collection of papers is by no means exhaustive, but it does represent some of the major R&D efforts in the past five years. While physics-informed data-driven techniques have already shown great potential in addressing some of the existing issues in seismic inversion (such as high computational cost, non-unique solutions, etc), new challenges have been encountered during the development of those techniques. Here we briefly discuss some of these, and propose potential future research topics.

- 1) **Generalization and Model Robustness.** These two topics are critical to any machine learning problems, but especially to seismic imaging. Generalization formally refers to the issue of performance on out-of-sample data (data that is not in the training set), but for seismic imaging, the concern

Method	Learning Strategy		Supervision		Physics Knowledge				
	Data-Driven	Hybrid	Supervised	Unsupervised	Augmentation	Governing Eq.	Priors	Unrolling	Loss
InversionNet [6]	✓		✓						
Araya-Polo et al. [7]	✓		✓						
ModifiedFCN [8]	✓		✓						
Li et al. [5]	✓		✓						
Renan et al. [25]		✓	✓		✓	✓			
Yang et al. [23]		✓	✓		✓		✓		
Feng et al. [24]		✓	✓		✓		✓		
UPFWI [30]		✓		✓		✓			✓
VelocityGAN [33]		✓	✓				✓		
H-PGNN+ [29]		✓	✓			✓			
NNFWI [37]		✓		✓		✓			
He & Wang [39]		✓		✓		✓			
Wu & McMechan [38]		✓		✓		✓			
Mosser et al. [35]		✓	✓				✓		
Richardson et al. [36]		✓	✓				✓		
Zhang & Gao [45]		✓	✓					✓	
Zhang et al. [47]		✓	✓					✓	
Chen & Saygin [48]		✓		✓					✓
Zhang & Alkhalifah [50]		✓		✓					✓
Sun & Alkhalifah [49]		✓		✓					✓

TABLE IV: Summary of machine learning based seismic inversion techniques addressed in this paper.

extends to data that is drawn “out of distribution,” or from a drifted distribution. Robustness refers to the performance degradation of a model with perturbed noisy test data. Incorporation of physics seeks to enhance both the generalization and the robustness, though it is difficult to explicitly demonstrate the effectiveness of the models on field data, which are usually contaminated with a significant amount of noise and artifacts. We remark that Deng et al. [52] used theoretical arguments to provide a rigorous derivation of worst-case error bounds in seismic imaging.

- 2) **Scientific “Sim2Real.”** Given that data are scarce in subsurface geoscience, full-physics simulations provide some compensation for this lack of data. How to bridge the “knowledge gap” (the gap between physical simulations and real applications) by utilizing machine learning and the underlying physics is a scientifically important question that needs to be carefully addressed. A similar challenge also exists in other scientific disciplines (such as robotics), where the notion of *Sim2Real* has been proposed to transfer knowledge learned in simulation to the real data [53]. That

leads to two potential solutions: one is the pure model-based simulators and the other is machine learning methods (such as generative models). The former usually simplifies the complex physics resulting in an unavoidable reality gap between the simulation and real data. The latter may be used as a compensating tool to learn underlying physics directly out of data. We are beginning to see new research and development has been carried out in both areas. However, the feasibility of these approaches on real applications with field data needs to be validated.

- 3) **Penalties and Constraints.** For most of the methods in Table IV, the physics knowledge is embedded in the loss function as soft constraints via penalties such as regularization terms. Mathematically, these problems can be generally posed as

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \{ \mathcal{L}(g_{\boldsymbol{\theta}}(\mathbf{m}), \mathbf{d}) + \lambda \mathcal{R}(f(\mathbf{d}), \mathbf{m}) \} , \quad (12)$$

where \mathbf{m} and \mathbf{d} are the inputs and predictions, respectively; $g_{\boldsymbol{\theta}}(\cdot)$ represents the mapping relationship fitted by the network model with learnable parameters $\boldsymbol{\theta}$; $f(\cdot)$ is the mathematical representation of the domain knowledge (such as the governing PDEs); and λ is the regularization parameter connecting the loss term $\mathcal{L}(\cdot)$ and the regularization term $\mathcal{R}(\cdot)$. With soft constraint as posed in Eq. (12), the physics knowledge can be only weakly enforced in the prediction results, meaning that they can not ensure the predictions do not violate the underlying physics constraints. To overcome this limitation of soft constrained approaches, different optimization techniques are recently proposed to incorporate physics knowledge as hard constraints, which ensures a given governing equation is strictly satisfied in a given domain [54].

- 4) **Uncertainty Quantification.** Considering issues of generalization, model robustness, and information gap between synthetic and field data, it is important to quantitatively evaluate the model performance on both inversion accuracy and the associated confidence. Uncertainty quantification (UQ) has been widely used as a tool to achieve this. There has been some recent work studying data-driven seismic inversion with UQ to provide insights into the various UQ induced by different perturbations including measurement noise and model parameters [37].

V. CONCLUSIONS

The problem of seismic full-waveform inversion, first formulated more than 35 years ago [55], continues to be a major area of research in subsurface geoscience. From characterizing fossil-fuel reservoirs in subsurface energy exploration to imaging global subduction zones in earthquake seismology, there is a demand for algorithms that are *faster*, *more accurate*, and *scalable* to very large datasets. As demonstrated by the research efforts highlighted in this review, scientific machine learning is showing great potential

on all of these fronts. Serious challenges remain – direct subsurface measurements are difficult and necessarily sparse; generalizing learned solutions to regimes outside their training is inherently fraught with uncertainty; network robustness when handling inevitable measurement noise and artifacts – but we believe that there are ample opportunities for physics-informed machine learning to enable exciting progress in this science-rich and data-starved field.

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