

1 **Statistical and Machine Learning Methods Applied to**
2 **the Prediction of Different Tropical Rainfall Types**

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8 **Key Points:**

- 9 • The occurrence and intensity of deep convective, stratiform, and shallow convec-
10 tive rain can be predicted by each method to varying degrees.
11 • The generalized linear model (random forest) predicts rain (no-rain) occurrence
12 best, the neural network produces best intensity statistics.
13 • None of the methods can predict the extreme tail (top 1%) of the rain rate dis-
14 tributions, although random forest comes closest.

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Abstract

We explore the use of three advanced statistical and machine learning methods (a generalized linear model, random forest, and neural network) to predict the occurrence and rain rate distribution of three tropical rain types (deep convective, stratiform, and shallow convective) observed by the radar onboard the GPM satellite over the West Pacific at three-hourly, 0.5-degree resolution. Temperature and moisture profiles from MERRA-2 were used as predictors. All three methods perform reasonably well at predicting the occurrence and rain rate distribution of each rain type. However, none of the methods obviously distinguish themselves from one another and each method still has issues with predicting rain too often and not fully capturing the high end of the rain rate distributions, both of which are common problems in climate models.

Plain Language Summary

Predicting rain from just large-scale environmental variables remains a challenging problem for climate models and it is unclear how well numerical methods of any kind can predict the true characteristics of rainfall without smaller (storm) scale information. The goal of this study was to explore the ability of multiple statistical and machine learning methods to predict rain occurrence and intensity over the tropical Pacific Ocean using satellite rain observations and large-scale environmental profiles of temperature and moisture. We also separated the rain into different types because of their varying kinematic and thermodynamic structures that might respond to the large-scale environment in different ways. Our expectation was that the machine learning methods (i.e., the neural network and random forest) would outperform the statistical model because of their more flexible structures, especially in predicting the highly skewed distribution of rain rates for each rain type. However, this was not the case for a standard neural network while the random forest produced a modest improvement over the statistical model. A possible moral of this story is that machine learning tools must be carefully assessed and are not necessarily applicable to solving all big data problems.

1 Introduction

Rainfall is fundamental to water resources, agriculture, and ecosystems and can cause massive damage in the form of too little or too much rain. However, rainfall can vary strongly in space and time making it hard to measure and even harder to predict. The rain rate distribution of most global climate models (GCMs) is far different than observed, with too much weak rain and not enough heavy rain (e.g., Dai, 2006; Stephens et al., 2010; Fiedler et al., 2020), which hinders predictions of extreme events. The goal of this study is to analyze the ability of advanced statistical and machine learning techniques to predict the occurrence and rain rate distribution of tropical rainfall using environmental temperature and humidity profiles as predictors. An eventual goal would be to determine if these techniques could be implemented in GCM predictions of short-term climate phenomena like El Niño, and perhaps even long-term climate change.

Most of the global rain falls in the tropics and warm season mid-latitudes and over half of this rain comes from large, organized rain systems (Nesbitt, Cifelli, & Rutledge, 2006; R. S. Schumacher & Rasmussen, 2020). These systems are much larger than the individual convective cells targeted by most conventional GCM convective parameterizations and contain elements of deep convection and stratiform rain (Houze, 1997; C. Schumacher & Houze, 2003a). An example of this kind of organized convective system is shown in Figure 1. Shallow convective rain is another type of rainfall that is ubiquitous over the tropical ocean and occurs regularly over some continental locations (C. Schumacher & Houze, 2003b; Funk, Schumacher, & Awaka, 2013).

63 As discussed by Mapes et al. (2006), these rain types form the building blocks of
 64 larger convective systems ranging from mesoscale convective systems (with scales on the
 65 order of 100 km and 12 h) to the Madden-Julian Oscillation (with scales on the order
 66 of 1000 km and many weeks), so predicting each of these rain types is important to stud-
 67 ies of weather and climate. However, the ability of GCMs to simulate these building blocks
 68 and their interactions remains a challenge. For example, while deep convective rain is
 69 produced by GCMs via a convective parameterization, most GCMs produce shallow con-
 70 vection in their boundary layer parameterization, which is run separately from the con-
 71 vective parameterization. In addition, GCM convective parameterizations do not typ-
 72 ically account for stratiform (or mesoscale) rain processes and the rest of a GCM's rain-
 73 fall is produced explicitly at the grid scale as large-scale rain. It is important to note that
 74 large-scale rain in a GCM does not represent the observed stratiform building block dis-
 75 cussed above.

76 Weather radar has the unique capability to view the 3-dimensional structure of pre-
 77 cipitating storms (e.g., Figure 1), which can be used to determine the occurrence and
 78 evolution of the three tropical rainfall building blocks. Thus, this study utilizes space-
 79 borne radar observations separated into deep convective, stratiform, and shallow con-
 80 vective rain to assess the predictive capability of advanced statistical and machine learn-
 81 ing methods.

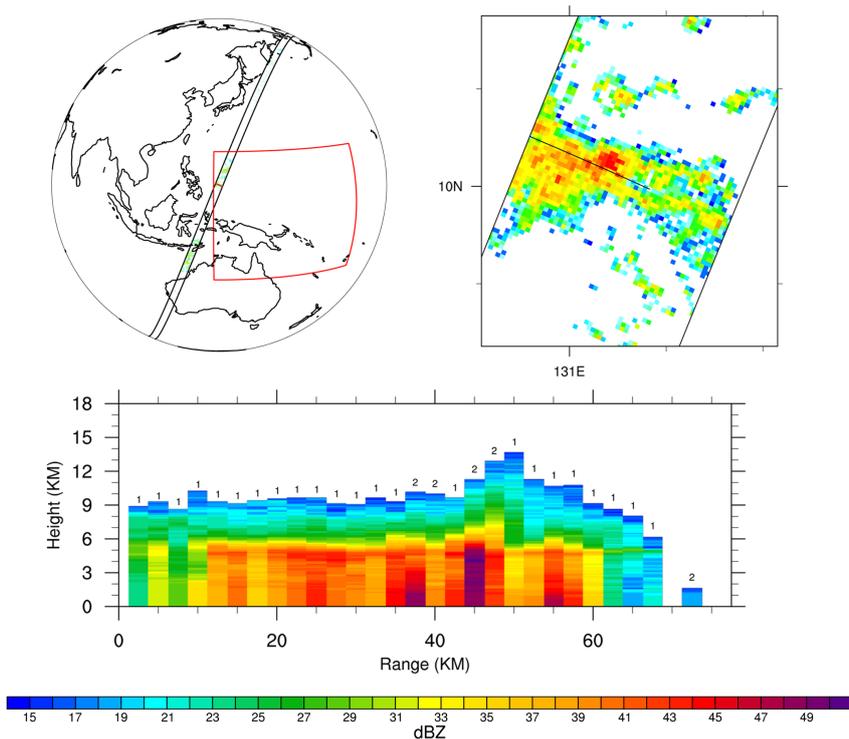


Figure 1. GPM DPR reflectivity observations at 01 UTC on 4 February 2017. The red box indicates the bounds of the study area over the West Pacific. The horizontal cross section is at 2 km AMSL and the vertical cross section is taken along the black line. Stratiform profiles are labeled as 1, convective profiles are labeled as 2. The far right cell in the vertical cross section would be considered shallow convection because its top is below the 0 degree Celsius level (typically about 5 km in the tropics).

82 There are currently a number of efforts to use data science to improve the repre-
 83 sentation of subgrid processes in climate models. Since there is often very limited amount
 84 of data available for unresolved processes, especially in situ measurements, many of these
 85 efforts apply machine learning to conventional model parameterizations or a large en-
 86 semble of higher resolution simulations (Brenowitz & Bretherton, 2018; O’Gorman & Dwyer,
 87 2018; Rasp, Pritchard, & Gentine, 2018). Training on conventional parameterizations
 88 can improve computational efficiency, but does not address the physical deficiencies. The
 89 higher resolution simulations also have their own built-in assumptions about a different
 90 set of smaller scale unresolved processes. Yang et al. (2019) considered a data-centric
 91 approach, using a large satellite rainfall data set and reanalysis fields to show that a gen-
 92 eralized linear model (GLM) can do well at predicting the occurrence of rain in the trop-
 93 ics, but it failed at capturing the tail of the rain rate distributions. This is mainly due
 94 to the restriction of parametric probability distributions used for rain rate. Although dis-
 95 tributions such as Gamma, log-normal, or Weibull are commonly used for rain rate due
 96 to their shape of density curves with long tails (e.g., Yang et al. (2019) used a Gamma
 97 distribution), they are often not flexible enough to capture the heaviest rain rates. This
 98 study builds on Yang et al. (2019) by applying two machine learning techniques, i.e., a
 99 random forest (RF) and deep feedforward neural network (NN), to a similar data set to
 100 determine how well these methods compare to one another and the GLM in predicting
 101 rain occurrence and capturing the high rain rate end of the distribution for multiple rain
 102 types.

103 2 Statistical and Machine Learning Methods

104 2.1 Generalized Linear Model

105 GLMs (McCullagh & Nelder, 1989) are a popular class of statistical models used
 106 to predict a response variable whose mean is assumed to be some parametric function
 107 of covariates. It is a more general modeling framework than multiple linear regression
 108 in that response variables may not follow a Gaussian distribution. Furthermore, unlike
 109 multiple linear regression models, which often use the least squares method for model
 110 fitting, GLMs are fitted using a maximum likelihood estimation (MLE) method. The MLE
 111 method utilizes the distribution function of the response, thus giving generally better
 112 statistical properties of estimators than the least squares method. A GLM does not nec-
 113 essarily assume a direct linear relationship between the response and covariates, and of-
 114 ten their nonlinear relationship is introduced by a *link* function. For instance, a common
 115 log-link function assumes that the log transformed mean of the response can be written
 116 as a linear combination of covariates. Widely used examples for distributions and link
 117 functions for GLMs include *logistic regression* (a Bernoulli distribution for the response
 118 and log link), *loglinear regression* (a Poisson distribution for the response and log link),
 119 and *Poisson regression* (a Poisson distribution for the response and log link).

In this work, we adopt the two-step modeling procedure used in Yang et al. (2019).
 Two separate GLMs, a logistic regression and a Gamma regression, are employed to deal
 with rain occurrence and rain amount, respectively. At a given time, let $p(\mathbf{s})$ denote the
 probability of rain at a grid point \mathbf{s} . Then the rain event is assumed to follow a Bernoulli
 distribution with

$$\log\left\{\frac{p(\mathbf{s})}{1-p(\mathbf{s})}\right\} = \beta_0 + \beta_1 z_1(\mathbf{s}) + \cdots + \beta_p z_p(\mathbf{s}), \quad (1)$$

where $z_i(\mathbf{s})$ denotes predictors (i.e. covariates) at the grid point \mathbf{s} . If $y(\mathbf{s})$ denotes the
 rain amount at \mathbf{s} , we assume that y follows a Gamma distribution with

$$\log[\mathbb{E}\{y(\mathbf{s})\}] = \eta_0 + \eta_1 z_1(\mathbf{s}) + \cdots + \eta_p z_p(\mathbf{s}). \quad (2)$$

120 For both models, parameters, including the coefficients β_i and η_i in (1) and (2), are es-
 121 timated using the MLE method. We fit the GLM models using data aggregated over space

122 and time altogether, similar to Yang et al. (2019). Although models (1) and (2) do not
 123 have explicit temporal structure in them, the temporal structure of the covariates effec-
 124 tively account for that of the responses, and it did not seem necessary to add more tem-
 125 poral terms in (1) or (2).

126 Statistical inference on the estimated parameters, including the significance of co-
 127 efficients, is made possible by using GLMs, and the estimated coefficients are readily in-
 128 terpretable. On the other hand, a possible drawback of the approach outlined above is
 129 the linearity assumption given in (1) and (2), as well as the distribution assumption on
 130 rain amount. In particular, the Gamma distribution may be too restrictive to account
 131 for some heavy rain events (Yang et al., 2019). Other commonly used distributions such
 132 as log-normal and Weibull distributions have similar problems, due to their particular
 133 parametric forms and restrictions. In view of the potentially restrictive nature of GLMs,
 134 we explore two popular machine learning methods, RF and artificial NNs, which oper-
 135 ate under much weaker (i.e., non-linear) assumptions compared to GLMs. RF and NNs
 136 offer the most competitive predictive performances in many applications, and are now
 137 standard tools for machine learning.

138 2.2 Random Forest

139 Random forest (Breiman, 2001) is an ensemble learning method that makes pre-
 140 dictions based on multiple decision trees. A random *forest* is built upon these many de-
 141 cision *trees*. A decision tree is a simple model that predicts the label associated with a
 142 sample by a series of splitting rules. An example decision tree is shown in Figure 2, where
 143 a tree is used to determine if a binary response Y is 1 or 0. The root node has a split-
 144 ting condition: “ $X_1 > 0$?” If the observation fulfills this condition, it will be passed to
 145 the next condition: “ $X_2 < 10$?” Otherwise, the tree predicts $Y = 0$. The procedure
 146 is applied recursively until the tree reaches a prediction of Y . For the construction of a
 147 decision tree, we refer the readers to Breiman (2001). In the above example, the under-
 148 lying goal is classification, where the response is categorical. Decision trees can also be
 149 modified to handle a regression problem, where the response is quantitative.

150 The core idea of ensemble methods like RF is to combine weak predictive models
 151 to achieve strong predictive performance. A RF is usually trained with two “random”
 152 ideas. The first is bagging – for each tree, the training set is formed by resampling from
 153 the original data set with replacement. The second is feature randomness – each tree in
 154 a RF is trained with a random subset of features. Bagging lowers variance while feature
 155 randomization reduces the dependence across trees. They are beneficial to ensemble learn-
 156 ing. The prediction of the RF is obtained by a majority vote over the predictions of the
 157 individual trees.

158 Similar to the GLM analysis, a two-step modeling procedure was implemented for
 159 RF in our work. Namely, we trained an RF model on rain occurrence and another RF
 160 model on rain amount. For both models, we used the default setting of the “random-
 161 Forest” function from the R package “randomForest”, except that we restricted the num-
 162 ber of decision trees to 100 when predicting rain amount in order to alleviate the com-
 163 putational burden. As opposed to GLM, RF is a nonparametric method and can pro-
 164 duce a highly nonlinear regression function. On the other hand, it is significantly more
 165 difficult to interpret the results of the RF model, although RF provides a measure of vari-
 166 able importance. In practice, one might also examine individual classification trees within
 167 the random forest to understand the results.

168 2.3 Neural Network

169 In recent years, artificial NNs (especially those with deep architecture) have be-
 170 come one of the most prominent models for complicated functions. A NN is based on

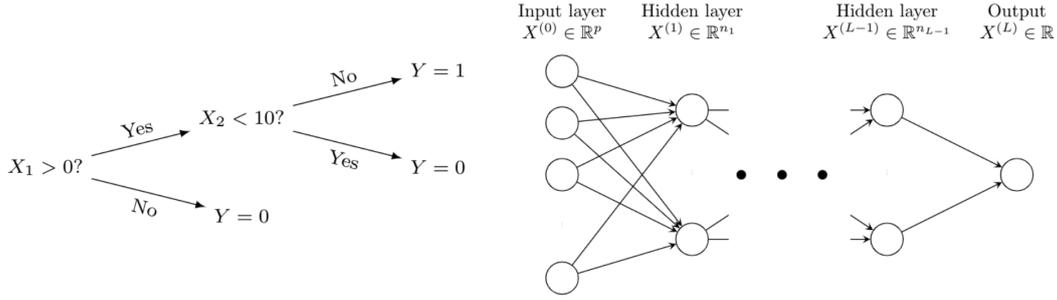


Figure 2. Illustrations for decision tree (left) and deep feedforward neural network (right).

171 a collection of connected nodes. Different ways to connect the nodes result in different
 172 NN architectures, such as fully connected (Hsu et al., 1990), sparsely connected (Ardakani
 173 et al., 2016), convolutional (Lo et al., 1995), and recurrent (Mikolov et al., 2010). Nodes
 174 are typically organized into layers, which can be classified as input, hidden and output.
 175 Networks with multiple hidden layers are said to have deep architectures, and are referred
 176 to as deep NNs. Deep architectures are commonly used nowadays, due to their strong
 177 empirical performance in many areas.

In our analysis, we adopt a deep feedforward NN in which consecutive layers are fully connected (Svozil et al., 1997; Schmidhuber, 2015) because it is one of the most standard forms of deep NN. Figure 2 depicts an example. We use $X^{(l)} \in \mathbb{R}^{n_l}$ to represent the nodes at layer l , where n_l is the number of nodes at layer l . Take $X^{(0)}$ as the input and $X^{(L)}$ as the output. The hidden and output layers are generated as follows. Let $x_k^{(l)}$ be the node k of layer l , where $l = 1, \dots, L$ and $k = 1, \dots, n_l$. Then

$$x_k^{(l)} = \sigma_k^{(l)}(b_k^{(l)} + \sum_{i=1}^{n_{l-1}} w_{i,k}^{(l)} x_i^{(l-1)}),$$

178 where $\sigma_k^{(l)}$ is the activation function, and $b_k^{(l)}$ and $w_{i,k}^{(l)}$ are parameters to be trained by
 179 the data. For simplicity, it is common to use the same activations within the same layer:
 180 $\sigma^{(l)} := \sigma_k^{(l)}$, for $k = 1, \dots, n_l$.

181 Similar to the previous two models (GLM and RF), we adopted the two-step ap-
 182 proach for the NN analysis. More specifically, we trained one NN to perform the binary
 183 classification on rain occurrence and another NN using training samples with positive
 184 rain values only to predict the rain amount. We considered different number of layers
 185 for NN. More specifically, we considered $L = 2, 3, \dots, 10$. Note that $n_0 = 80$ and $n_L =$
 186 1 for all L since they are representing the input size and the output size. For any exist-
 187 ing hidden layer, the number of nodes are set as follows: $n_1 = 40$, $n_2 = 20$, $n_3 = \dots =$
 188 $n_{L-2} = 6$ and $n_{L-1} = 3$. For instance, for $L = 1$, there is only one hidden layer and
 189 so only n_1 is relevant. For $l = 1, \dots, L-1$, the corresponding activation functions $\sigma_k^{(l)}$
 190 were chosen as the rectified linear unit (ReLU) functions ($\sigma(x) = \max(0, x)$). The ac-
 191 tivation function for the output layer had to be chosen based on the response type, i.e.,
 192 classification or regression. We used $\sigma^{(L)}(x) = 1/(1 + \exp(-x))$ for the classification,
 193 while we used the exponential function for the regression since the response is positive.
 194 As for the estimation of the NN, we adopted mean square error as the loss function and
 195 trained the network via the popular algorithm Adam (Kingma & Ba, 2014).

196 To prevent over-fitting, we also adopted the dropout procedure, which is a com-
 197 mon regularization method for training deep neural networks (Baldi & Sadowski, 2013;
 198 Gal et al., 2017). In the dropout procedure, neurons are stochastically dropped out dur-
 199 ing the training at each layer. In our implementation, the dropout rate was set to be the

200 same at every layer and three possible values 0, 0.2, 0.5 were considered. Both the dropout
 201 rate and the number of layers, L , were regarded as the hyper-parameters and were cho-
 202 sen via a validation procedure — we randomly separated 20% of the training data as the
 203 validation set to select the best combination of dropout rate and number of layers.

204 **3 Training and Test Data**

205 We used two years of observations from the NASA Global Precipitation Measure-
 206 ment (GPM; Hou et al., 2014) dual-frequency precipitation radar (DPR) to calculate rain
 207 occurrence and rain rates, which were the predictands of the study. The full year of 2017
 208 was used for training and the full year of 2018 was used for testing. The rain type clas-
 209 sifications (i.e., deep convective, stratiform, and shallow convective; Funk et al., 2013)
 210 and associated rain rates were retrieved from 2ADPR v6 files. Figure 1 shows an exam-
 211 ple orbit from the GPM radar with all three rain types present. We regridded the DPR
 212 orbital rain observations, which are made at a 5-km footprint scale over a 245-km swath,
 213 to 0.5-degree horizontal resolution and 3-hourly temporal resolution. The predictors for
 214 the study were temperature and humidity fields at 40 pressure levels from the MERRA-
 215 2 reanalysis (Rienecker et al., 2011) for 2017 and 2018. The MERRA-2 data was regrid-
 216 ded to a similar horizontal and temporal resolution as the DPR data and points were
 217 only analyzed if a DPR orbit occurred in a grid during the 3-hour period. We limited
 218 our domain to the tropical West Pacific ($130^{\circ}\text{E} - 180^{\circ}\text{E}$, $20^{\circ}\text{S} - 20^{\circ}\text{N}$; Figure 1), but
 219 found similar results in the tropical East Pacific (not shown). Overall, we had 569,596
 220 training samples and 572,968 test samples.

221 The training and test data are generally similar to the observational data sets used
 222 in Yang et al. (2019). However, we used rain observations from the GPM DPR instead
 223 of the Tropical Rainfall Measuring Mission (TRMM) precipitation radar (PR) because
 224 of the DPR’s higher sensitivity to weaker rain rates and thus better shallow convective
 225 rain retrievals (Hamada & Takayabu, 2016). We also used a slightly higher time reso-
 226 lution (3 hours vs 6 hours) to better isolate environment-rain relationships and we used
 227 all times of day instead of just 0-6 UTC to capture the full range of diurnal conditions.
 228 Finally, we only used temperature and humidity as predictors because they accounted
 229 for the majority of the predictive performance by the GLM in Yang et al. (2019), who
 230 also tested other environmental variables such as horizontal wind profiles and surface fluxes.
 231 We further utilized the full temperature and humidity profiles rather than just the first
 232 three empirical orthogonal functions so that the machine learning techniques had more
 233 flexibility in determining the vertical relationship of the predictors to the surface rain
 234 rate.

235 **4 Prediction Results**

236 **4.1 Rain occurrence**

237 When solving for occurrence, we treat grids with extremely small rain amounts as
 238 no-rain cases to avoid retrievals from the radar likely associated with clutter or noise.
 239 For each rain type, we selected a rain rate cutoff that accounts for less than 1% of the
 240 total rain amount in the training data. The cutoff values are 0.056, 0.0395, and 0.0087
 241 mm/hr for deep convective, stratiform, and shallow convective rain, respectively. As will
 242 be illustrated in the next section, the three rain types produce different ranges of rain
 243 rate intensity, which is why separate cutoff values are needed for each rain type.

244 Rain does not occur often at the time and space scales being considered in this study
 245 (i.e., 3 hourly and 0.5 degrees), so there are many more no-rain cases than rain cases.
 246 To deal with this severely imbalanced classification problem, we created a “balanced”
 247 training data set by using a random under-sampling procedure. That is, we randomly
 248 sample the no-rain cases until we have the same number of no-rain and rain samples in

Table 1. [Table updated] The top four rows describe the performance of the occurrence predictions for each rain type by each method. The values in each column are the fraction of the total cases that fall into each prediction category and sum to one, while bold values are the highest correct predictions. The bottom two rows quantify the accuracy of the the rain rate (mm/hr) prediction in terms of root mean square error (RMSE) and mean absolute error (MAE), with bold values representing the smallest errors among the three methods.

	Deep convective			Stratiform			Shallow convective		
	GLM	RF	NN	GLM	RF	NN	GLM	RF	NN
True Negative	0.485	0.568	0.536	0.474	0.529	0.502	0.325	0.415	0.323
False Negative	0.036	0.054	0.054	0.052	0.069	0.076	0.084	0.137	0.106
True Positive	0.122	0.103	0.103	0.188	0.171	0.164	0.267	0.214	0.245
False Positive	0.357	0.275	0.387	0.286	0.231	0.306	0.324	0.234	0.325
RMSE	0.758	0.975	0.749	0.624	0.730	0.619	0.095	0.105	0.094
MAE	0.405	0.504	0.385	0.295	0.367	0.275	0.058	0.062	0.059

249 our training data set. Note that we classify rain/no-rain cases for each rain type sepa-
 250 rately.

251 The top four rows of Table 1 show how well the three statistical and machine learn-
 252 ing methods described in section 2 predict no-rain and rain cases for each rain type. The
 253 actual time the GPM radar observed each rain type over the West Pacific is indicated
 254 by adding the false negative and true positive values (i.e., about 16%, 24%, and 35% for
 255 deep convective, stratiform, and shallow convective rain, respectively). All three meth-
 256 ods do a reasonable job at distinguishing truly raining cases, with GLM slightly outper-
 257 forming the other two methods. However, all methods suffer from a relatively high false
 258 positive rate (i.e., predicting rain too often), which is a persistent problem in most cli-
 259 mate models as well (Fiedler et al., 2020). While GLM had the best true positive pre-
 260 dictions, it had the worst true negative predictions (i.e., predicting no rain when no rain
 261 is observed). RF had the best true negative prediction and NN fell between the two other
 262 techniques.

263 4.2 Rain rate distributions

264 We next apply the statistical and machine learning methods to predict the rain rate
 265 distribution of the three rain types. Figure 3 compares the prediction of each method
 266 to the “True” distribution observed by the GPM DPR. Note that the GPM-observed 99.9%
 267 rain rate varies by rain type with values of 14, 10, and 1.1 mm/hr for deep convective,
 268 stratiform and shallow convective rain, respectively. Even though shallow convective rain
 269 has the highest occurrence it has by far the smallest rain amounts over the 0.5-degree
 270 grid because it doesn’t cover much of the grid and is composed of more lightly raining
 271 cells. Stratiform rain is also normally less intense than deep convective rain on a pixel-
 272 by-pixel basis but because it tends to cover more area than deep convective cells, strat-
 273 iform rain amounts approach deep convective values over the 0.5-degree grid.

274 Figure 3 shows that all three methods (indicated by different green lines) tend to
 275 underestimate weaker rainrates (i.e., around the 50% quantile or first tick mark) in the
 276 deep convective and stratiform distributions, shifting to overestimations around the 90%
 277 quantile (or second tick mark). Between the 90 and 99% quantiles, there is a rapid drop
 278 off in prediction counts compared to the true distribution with NN and GLM showing
 279 the most rapid decrease. RF is the only technique to produce predictions past the 99.9%
 280 quantile for deep convective rain, the category associated with the most extreme rain amounts.

281 All methods do better predicting the shallow convective rain rate distribution with the
 282 drop-off in counts not occurring until after the 99% quantile.

283 To provide context on how the observed and predicted rain rate distributions in
 284 Figure 3 compare to standard GCM output, we obtained a year of data from the NCAR
 285 Community Atmospheric Model, version 5 (CAM5; Neale et al., 2013). We use model
 286 output for 2003 instead of 2018 because it was readily available. While there may be small
 287 year-to-year variations in the rain rate distributions over the West Pacific, we do not ex-
 288 pect them to be large, especially since neither 2003 or 2018 experienced strong El Niño
 289 or La Niña events. The original rain rate data had a 25×25 km grid resolution so we
 290 aggregated rain rates to a 0.5×0.5 degree grid resolution to match our analysis. Hourly
 291 total precipitation (PRECT) and convective (PRECC) precipitation rates were also ag-
 292 gregated into 3 hourly rain rates. We use PRECC to represent deep convective rain and
 293 the difference between PRECT and PRECC (PRECT-PRECC) to represent stratiform
 294 rain. GCMs do not typically calculate a separate shallow convective rain rate, but there
 295 are only small differences between the GPM convective deep rain rate distribution com-
 296 pared to when we combine the observed deep and shallow convective rain rate distribu-
 297 tions (i.e., deep convective rain dominates the convective rain rate distribution in the
 298 West Pacific).

299 As seen in the top panel of Figure 3, CAM5 (indicated by the dashed yellow line)
 300 does not provide a good density estimation for deep convective rain (and is, in fact, close
 301 to the GLM and NN distributions). Recent work has shown that a stochastic version of
 302 the Zhang-McFarlane convective parameterization used in CAM5 can improve the deep
 303 convective rain rate distribution (e.g., Wang et al., 2021), but stochastic techniques are
 304 still not regularly implemented in standard GCM runs. CAM5 appears to characterize
 305 the stratiform rain distribution well (Figure 3, middle panel), although large-scale rain
 306 from GCMs and stratiform rain from radar are not considered to be produced the same
 307 way (e.g., Dai, 2006), so caution must be taken in this comparison.

308 To further assess predicted rain amounts using GLM, RF, and NN, we calculated
 309 the following metrics to measure the performance of the techniques:

- 310 1. Root mean squared error (RMSE) = $\sqrt{\sum_{i=1}^N (\hat{y}_i - y_i)^2 / N}$ and
- 311 2. Mean absolute error (MAE) = $\sum_{i=1}^N |\hat{y}_i - y_i| / N$,

312 where y_i is the observed rain amount for the i -th sample, and \hat{y}_i is the predicted rain
 313 amount for the i -th sample, for $i = 1, \dots, N$. Here samples are aggregated over space
 314 and time, and thus there are a total of N samples for each rain type. Note that MAE
 315 is in general less sensitive to large values compared to RMSE. Table 1 shows that RF
 316 has the highest (and thus worst) RMSE and MAE among the three techniques for each
 317 rain type. NN usually provides the smallest errors among the three methods, and GLM
 318 usually performs only slightly worse than NN.

319 5 Conclusions

320 There is strong motivation to use “big data” to parameterize unresolved processes
 321 in GCMs, such as rainfall production. While training and testing data can come from
 322 higher resolution models, we chose to use a multi-year data set of rain observations from
 323 satellite radar along with temperature and humidity fields derived from a model constrained
 324 by observations (i.e., reanalysis). There are also a number of advanced statistical and
 325 machine learning techniques with which to analyze the available data. We chose a rep-
 326 resentative set that ranged in ease of implementation and interpretability: a generalized
 327 linear model, random forest, and neural network.

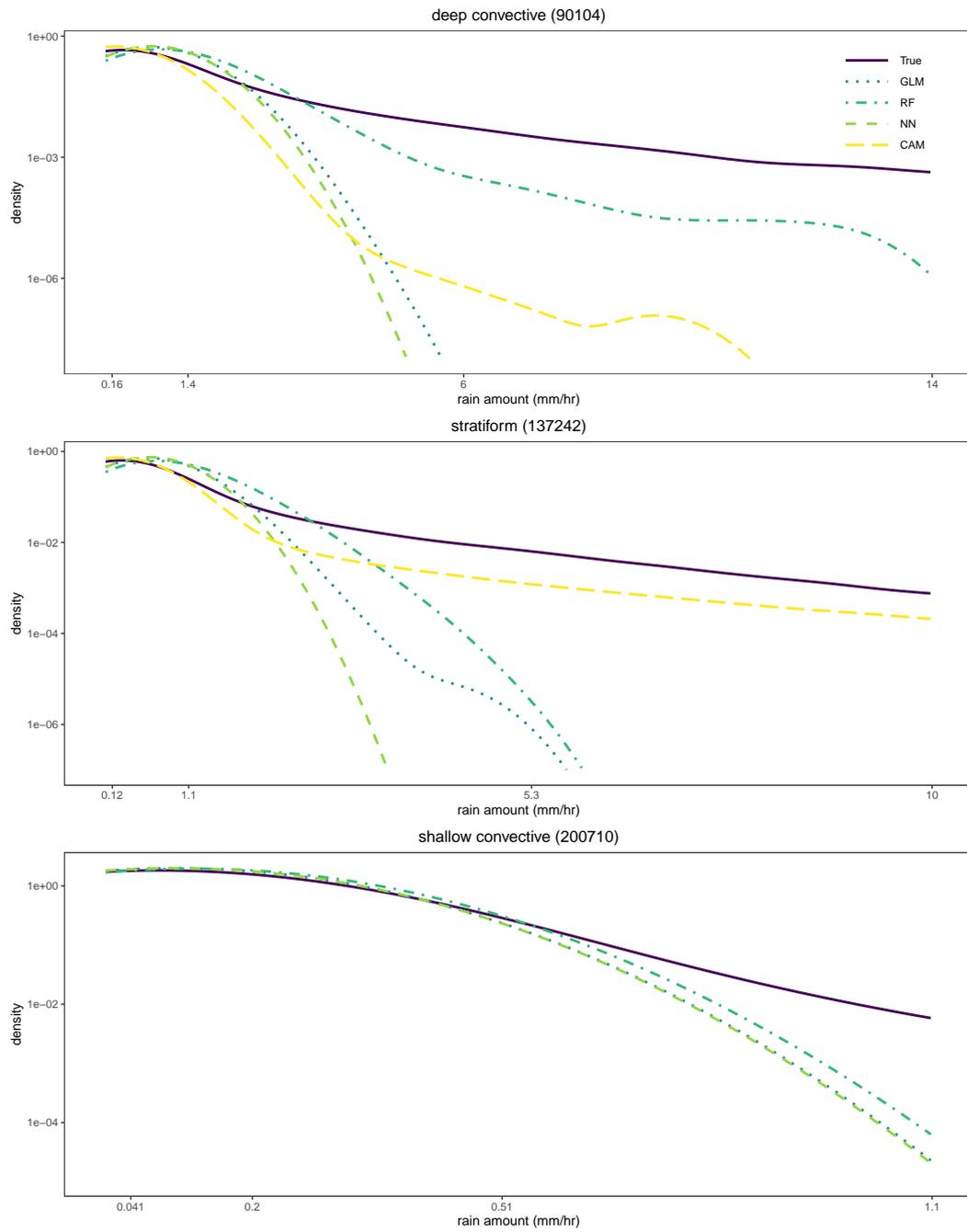


Figure 3. [Figure updated] GPM-observed and model-predicted rain rate distributions for deep convective, stratiform, and shallow convective rain in the base-10 log scale. Values in parentheses are the total cases in the testing data that rain. Values on the x-axis for the three plots are the 50, 90, 99, and 99.9% quantiles of the rain rate distribution, respectively.

328 All three methods performed reasonably well in predicting the occurrence of each
 329 of the three tropical building block rain types: deep convective, stratiform, and shallow
 330 convective. However, each method still predicted rain too often. Due to the high com-
 331 plexity of the model structure, regularization is usually needed for NN. With the dropout
 332 regularization, NN performed similarly to GLM in predicting the rain rate distributions
 333 of each rain type, while RF was more flexible in modeling the true response. However,
 334 the very highest rain rates were still underpredicted by all methods. Future work will
 335 assess the ability of each method to capture the temporal and spatial distribution of ob-
 336 served tropical rainfall, with the ultimate goal of implementing the best overall technique
 337 in a GCM to improve the representation of convection.

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