Dynamically-connected eddy mixing coefficients: Estimates from multi-tracer method

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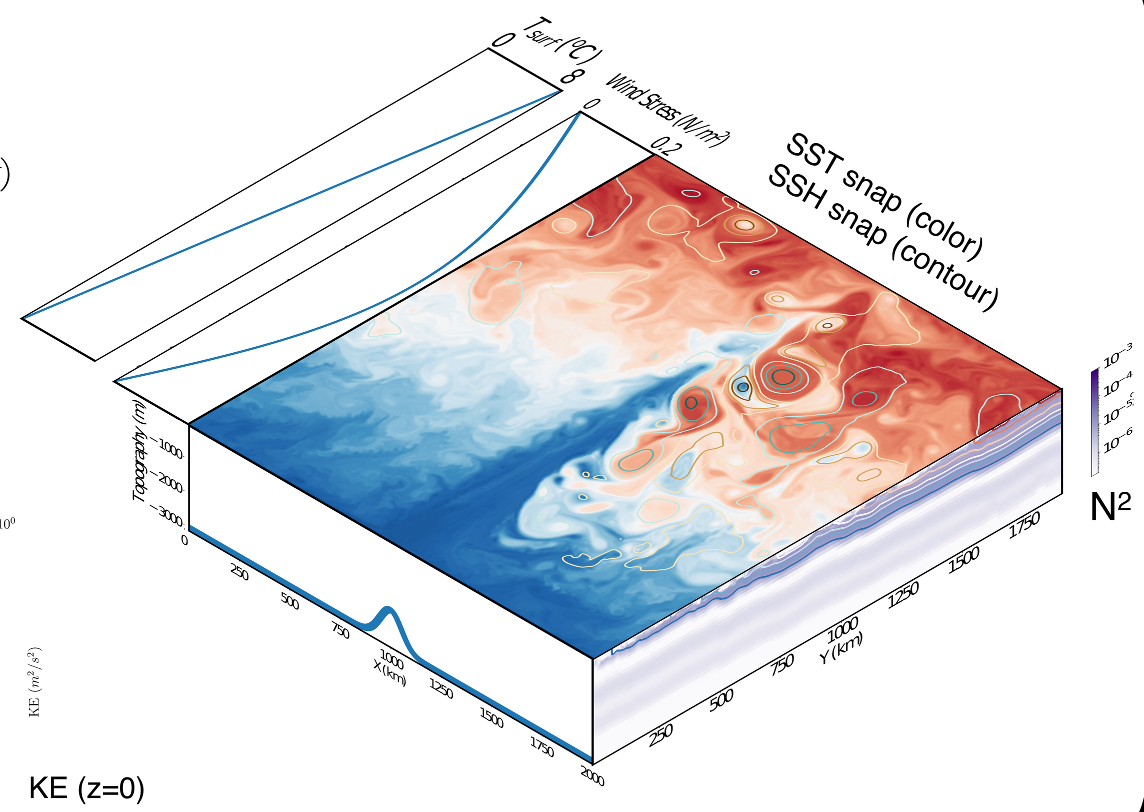
Introduction

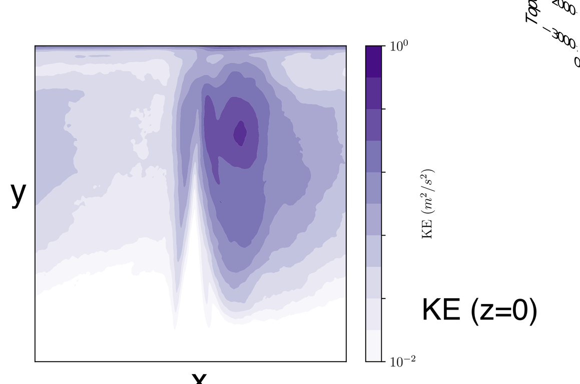
The Gent-McWilliams (GM) and Redi eddy parametrizations are essential features to ocean climate models. GM helps to maintain stratification, balancing the steepening of isopycnals by Ekman forcing and convection, with a relaxation that dissipates potential energy adiabatically. The Redi parametrization represents unresolved isopycnal mixing of tracers, while keeping diabatic mixing small. Due to its direct impact on the simulated circulation, research has focused more on theories for the GM than Redi coefficient, the latter typically being set equal to the former without justification.When tuned to values of O(500) m^2/s, GM-based simulations are able to reproduce observed ocean stratification. By contrast, observational estimates of along-isopycnal mesoscale diffusivity (the Redi part) are typically an order of magnitude larger. Setting the Redi coefficient to the too-small GM value results in serious errors in biogeochemical tracers like oxygen and carbon (Gnanadesikan et al. 2013, 2015).

Here we investigate this idea using a high-resolution MITgcm simulation of an idealized channel with a topographic ridge. Ten independent passive tracers are advected by the flow, and linear restored to target states that effectively maintain mean gradients in each direction, at each point. The eddy fluxes and mean gradients of these tracers are used to solve an inverse problem, resulting in an estimate of an eddy diffusivity matrix. Notably, the mean is defined as a combined time-average and spatial coarse-graining, resulting in a 3D estimate. GM and Redi diffusivities are extracted from the diffusivity, and are shown to be consistent with a relationship expected from QG theory.

Experimental setup

- 2000^2km x 3km
- Resolution: 5km x 40 levs
- Channel, no-slip sides
- SST restored to linear T(y)
- Constant windstress
- No salinity, linear EOS
- Quadratic drag
- LLC4320 params
- Leith dissipation
- 150 year spin-up





Using tracers to construct diffusivity matrix

N tracers $c_j(x,y,z,t)$, j = 1:N, each advected by nondivergent, 3D flow $\mathbf{v}(x,y,z,t)$, obey Reynold's averaged advection

$$\partial_t \bar{c}_j + \bar{\mathbf{v}} \cdot \nabla \bar{c}_j = -\nabla \cdot (\overline{\mathbf{v}' c_j'}) \equiv \nabla \cdot (\mathbf{K} \nabla \bar{c}_j)$$

K is a 3 x 3 diffusivity matrix. Measurements of fluxes and mean gradients provides an over-determined matrix problem for **K**:

$$\mathbf{K}\, \nabla \bar{c}_j = -\overline{\mathbf{v}' c_j'}$$

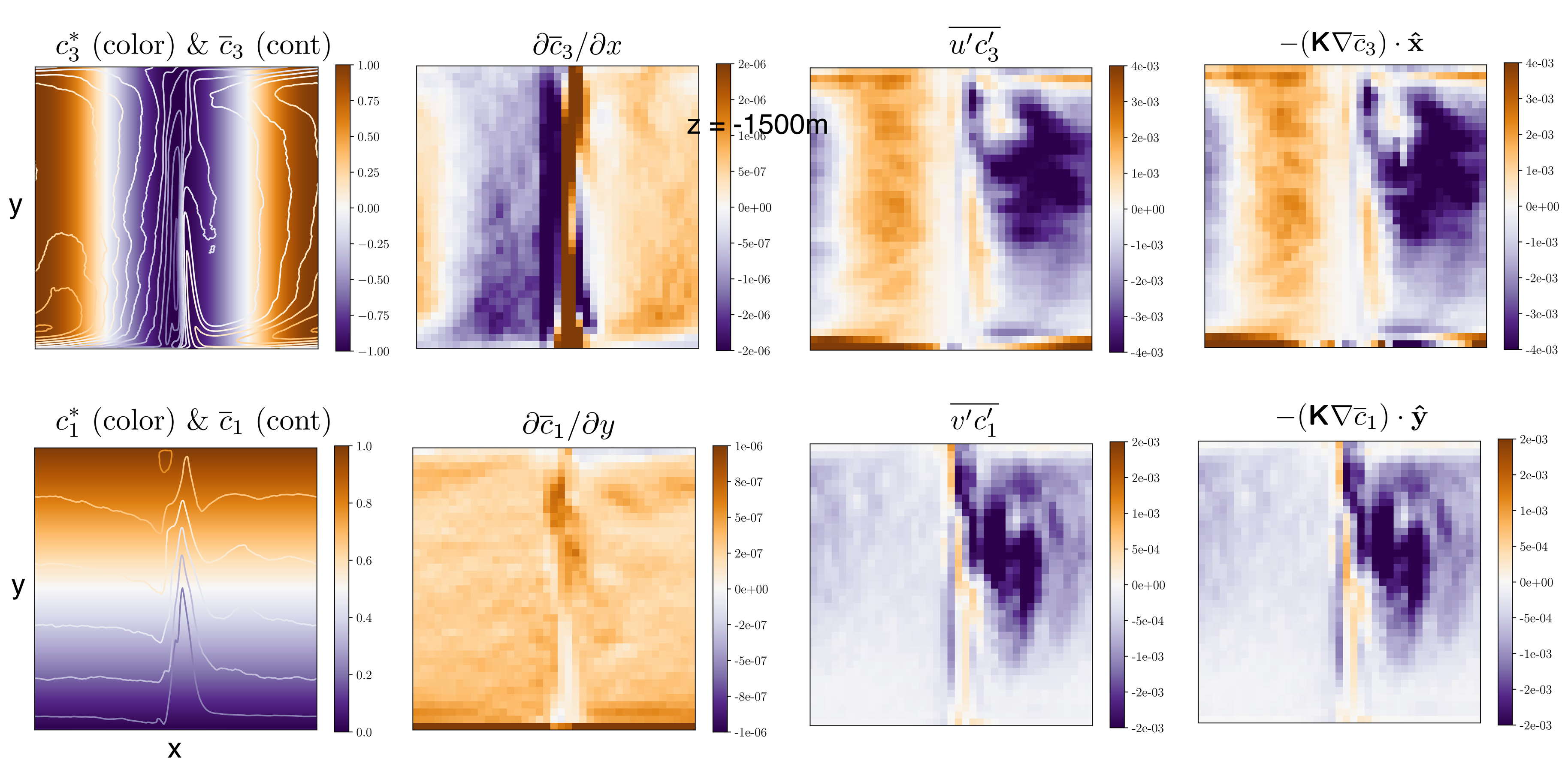
where the gradients and fluxes are arranged as **3 x N** matrices. If non-parallel mean tracer gradients can be maintained, then least-squares provides an optimal solution (Plumb & Mahlman 1987; Bachman, Fox-Kemper & Bryan 2015).

We used N=10 tracers, run for 50 years, restored to a target fields with RHS term $-\tau^{-1}(c_j - c_j^*)$ with τ = 6 years (many methods and timescales were tried). Target fields:

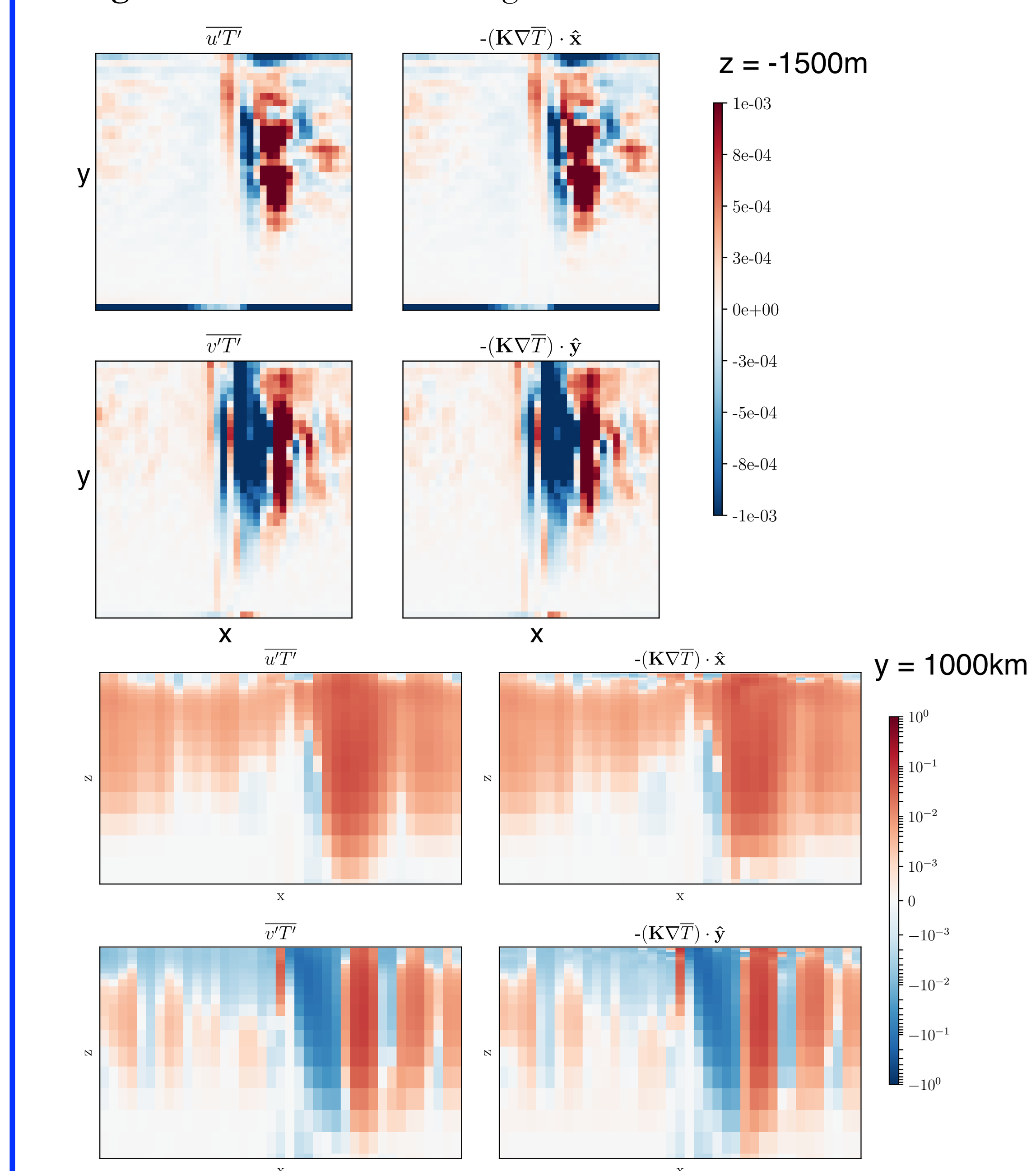
$$c_1^* = y/L \quad c_2^* = -z/H \quad c_3^* = \cos(2\pi x/L) \quad c_4^* = \sin(2\pi x/L) \quad c_5^* = \sin(4\pi x/L)$$
$$c_6^* = \sin(\pi y/L) \quad c_7^* = \cos(2\pi y/L) \quad c_8^* = \sin(2\pi y/L) \quad c_9^* = \cos(\pi z/H) \quad c_{10}^* = \sin(\pi z/H)$$

Average: Full time average + lateral spatial coarse-graining over 50km boxes.

Example test: c_3 and c_1, which have target fields (col 1) with dominant x and y gradients, resp. Weak mean gradients are retained (col 2). Eddy fluxes in dominant gradient directions (col 3) are well-reconstructed by **K** (col 4).

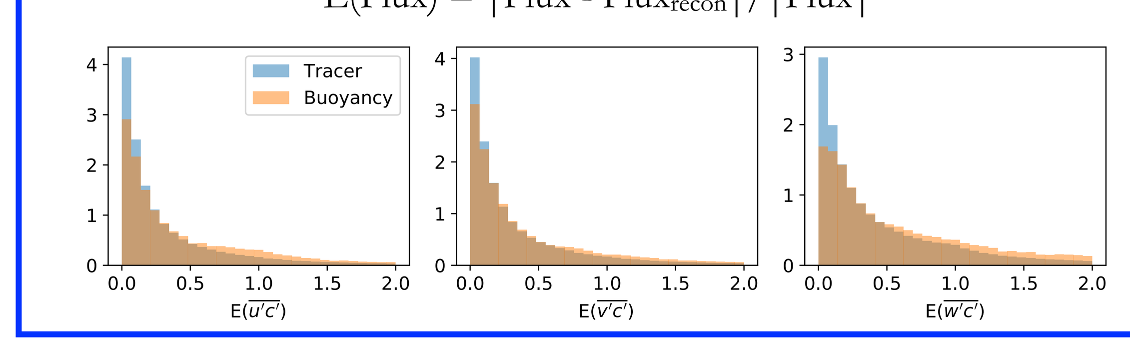


Stronger test: **K** constructed using tracers. Can it reconstruct T-fluxes?



Flux reconstruction error: For each tracer c_j, and buoyancy (temperature T), flux error is computed at each point in domain as

$$E(\text{Flux}) = |\text{Flux} - \text{Flux}_{\text{recon}}| / |\text{Flux}|$$



Parameterized eddy fluxes

Diffusivity K decomposed in symmetric S and antisymmetric A parts

$$\mathsf{K} = \mathsf{A} + \mathsf{S} \qquad \mathsf{S} = \frac{1}{2}(\mathsf{K} + \mathsf{K}^T) \qquad \mathsf{A} = \frac{1}{2}(\mathsf{K} - \mathsf{K}^T)$$

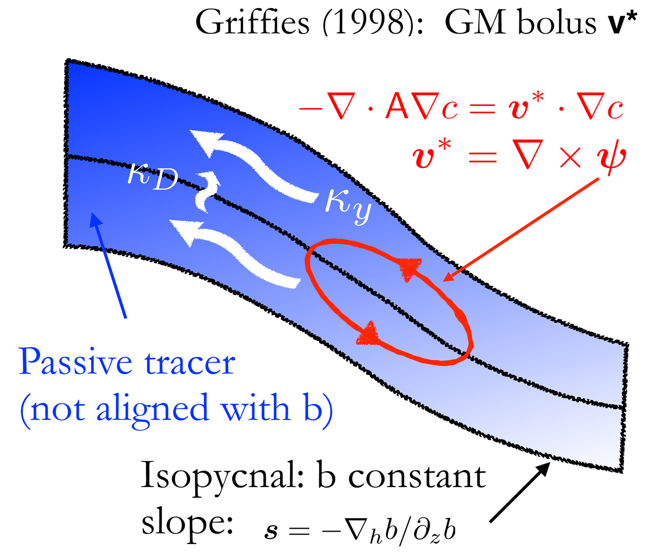
measured *parameterized*

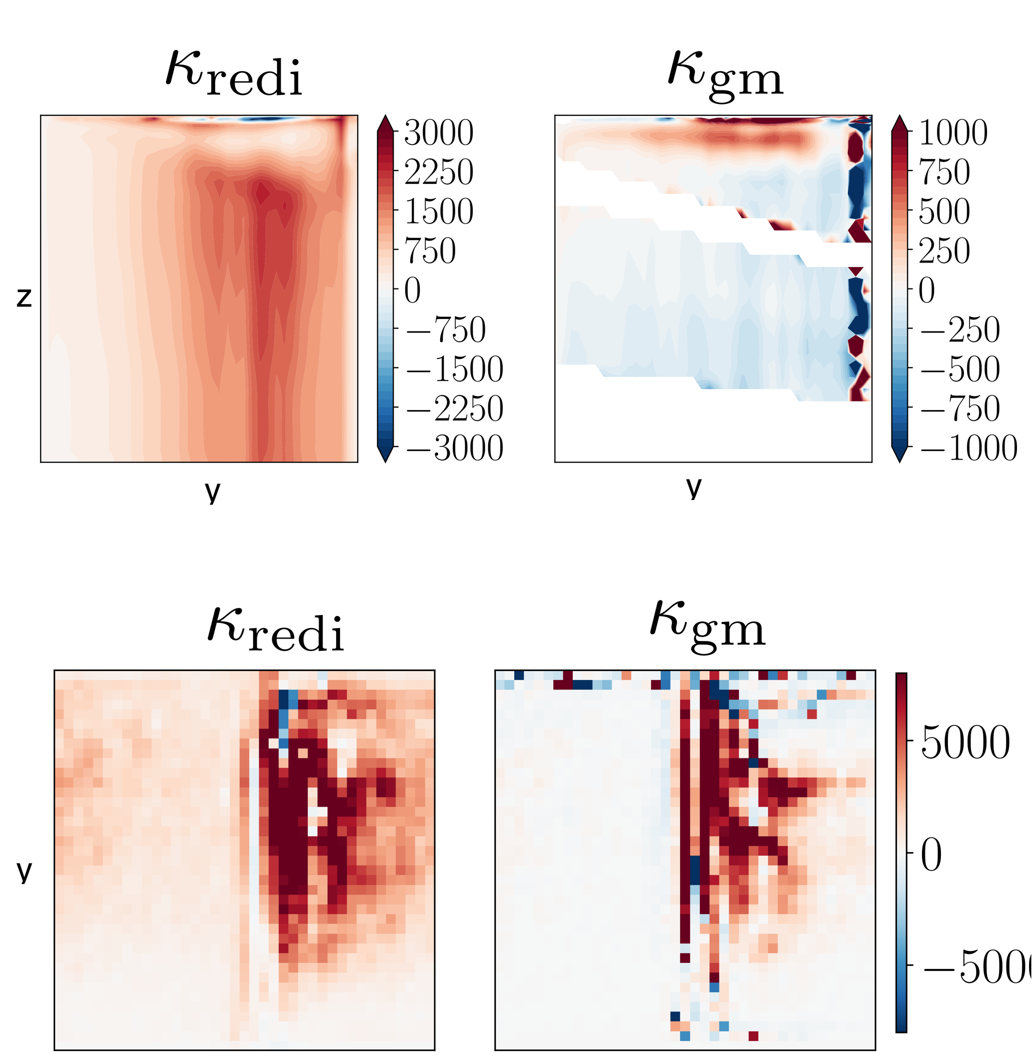
$$\mathsf{S} = \mathsf{V}\begin{bmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{bmatrix}\mathsf{V}^T \longleftarrow \mathsf{S}_{\text{redi}} = \mathsf{R}\begin{bmatrix} \kappa_{\text{redi}} & 0 & 0 \\ 0 & \kappa_{\text{redi}} & 0 \\ 0 & 0 & \kappa_D \end{bmatrix}\mathsf{R}^T$$

Redi (1982): set R as rotation to isopycnal coords; $\kappa_D \ll \kappa_{\text{redi}}$

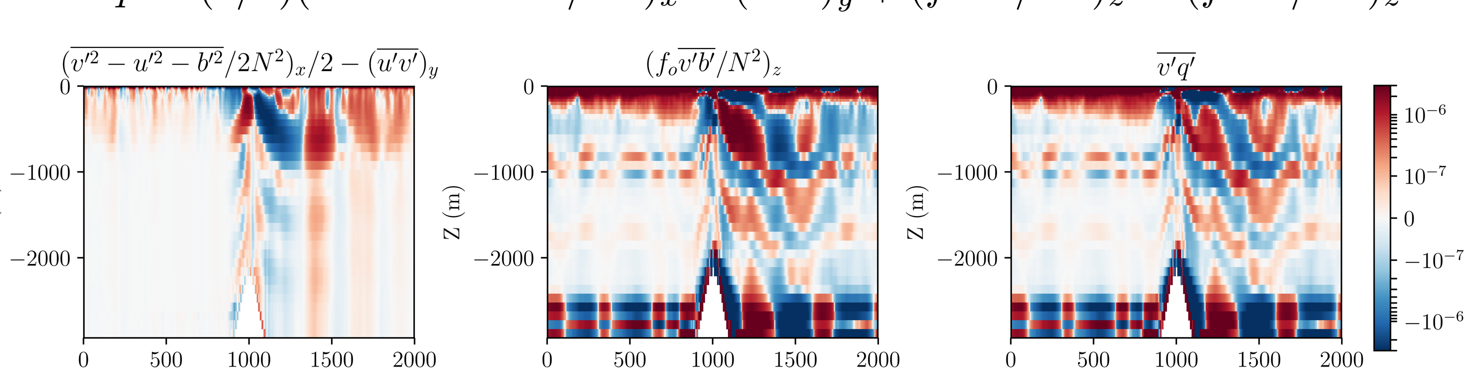
$$\mathsf{A} = \begin{bmatrix} 0 & \psi_3 & -\psi_2 \\ -\psi_3 & 0 & \psi_1 \\ \psi_2 & -\psi_1 & 0 \end{bmatrix} \longleftarrow \mathsf{A}_{\text{gm}} = \begin{bmatrix} 0 & 0 & -\kappa_{\text{gm}} s^x \\ 0 & 0 & -\kappa_{\text{gm}} s^y \\ \kappa_{\text{gm}} s^x & \kappa_{\text{gm}} s^y & 0 \end{bmatrix}$$

Griffies (1998): GM bolus **v*** can be written as A

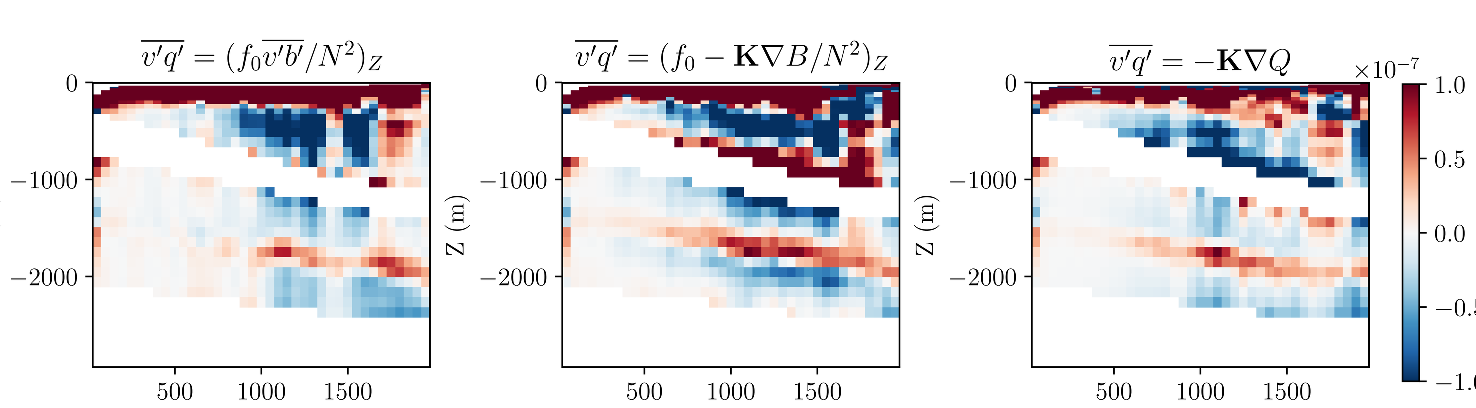




$$\overline{v'q'} = (1/2)(\overline{v'^2} - \overline{u'^2} - \overline{b'^2}/N^2)_x - (\overline{u'v'})_y + (f\overline{v'b'}/N^2)_z \approx (f\overline{v'b'}/N^2)_z\ ?$$



Components of QG PV flux reconstructed by K?



Summary

- With combined time- and spatial coarse-graining average, multi-tracer method can reveal 3D diffusivity matrix K.
- K includes first-order estimate of divergent and rotational components of flux — no need to remove rotational part
- K can reconstruct tracer and buoyancy fluxes with good accuracy, even in complex anisotropic, inhomogeneous, PE flow
- Ability to reconstruct QG PV flux from K, and momentum fluxes are relatively small, suggests possibility that PV and buoyancy diffusivity are related through QG relation
$$\overline{v'q'} \approx (f\overline{v'b'}/N^2)_z$$
- Fact that tracers and QG PV are advected similarly then suggests
$$\kappa_{\text{redi}}\partial_z \boldsymbol{s} \approx \partial_z\left(\kappa_{\text{gm}}\boldsymbol{s}\right)$$

Gent, P. R., and J. C. McWilliams, 1990: Isopycnal mixing in ocean cir- culation models. *J. Phys. Oceanogr.*, 20 (1), 150–155

Gnanadesikan, A., D. Bianchi, and M. A. Pradal, 2013: Critical role for mesoscale eddy diffusion in supplying oxygen to hypoxic ocean waters. *Geophys. Res. Lett.*, 40 (19), 5194–5198,

Gnanadesikan, A., M.-A. Pradal, and R. Abernathey, 2015: Isopycnal mixing by mesoscale eddies significantly impacts oceanic anthro- pogenic carbon uptake. *Geophys. Res. Lett.*, 42, 4249–4255,

Griffies, S. M., 1998: The Gent-McWilliams skew flux. *J. Phys. Oceanogr.*, 28, 831–841.

Bachman, S. D., B. Fox-Kemper, and F. O. Bryan, 2015: A tracer-based inversion method for diagnosing eddy-induced diffusivity and advection. *Ocean Modelling*, 86 (1), 1–1

Plumb, R. A., and J. D. Mahlman, 1987: The zonally averaged trans- port characteristics of the GFDL general circulation/transport model. *J. Atmos. Sci.*, 44 (2), 298–327

Redi, M. H., 1982: Oceanic isopycnal mixing by coordinate ro- tation. *J. Phys. Oceanogr.*, 12 (10), 1154–1158

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