


## LIP flows may not have been as thick as they appear

Jonas Katona (1, 3), Xiaojing Fu (2, 4), Tushar Mittal (2, 5), Michael Manga (2), Stephen Self (2)

(1) Department of Mathematics and (2) Earth and Planetary Science at University of California Berkeley, Berkeley, CA; (3) Department of Mathematics at Yale University; (4) Department of Civil and Environmental Engineering at California Institute of Technology, Pasadena, CA; (5) Department of Earth, Atmospheric, and Planetary Sciences at Massachusetts Institute of Technology, Cambridge, MA



### Preliminary questions

What happens when another lava lobe is emplaced upon a partially or fully-solidified preexisting lava lobe?

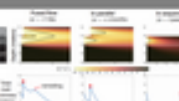
How do secondary (or tertiary, quaternary, etc.) emplacement affect the cooling of the lobes in the stack?

How well do our predictions compare with real-life observations in LIP?

How well do our predictions confirm other well-accepted relationships between  $t_{\text{emp}}$  and  $h$ ?


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### Three sample regions at $h=10\text{m}$



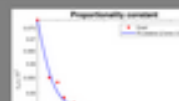
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### Solidification dynamics



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### Nonlinear corrections to $t_{\text{emp}} \sim h^{2/\alpha}$



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### Mathematical formulation

Phase-field equations

$$\partial_t \phi = -\frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)$$

$$\partial_t \theta = -\frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial x} \right)$$

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### Conclusions

There are roughly three qualitative regions that correspond to different emplacement times. Let  $t_{\text{emp}}$  be the time it takes for a single lobe of size  $h$  to completely solidify, and  $t_{\text{emp}}$  the time of emplacement.

- Merged ( $t_{\text{emp}} < t_{\text{emp}}$ ): After emplacement, the solidified portion of the lower lava lobe eventually remelts completely, and then

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Departments of (1) Mathematics and (2) Earth and Planetary Science at University of California Berkeley, Berkeley, CA; (3) Department of Mathematics at Yale University; (4) Department of Mechanical and Civil Engineering at California Institute of Technology, Pasadena, CA; (5) Department of Earth, Atmospheric, and Planetary Sciences at Massachusetts Institute of Technology, Cambridge, MA



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## PRELIMINARY QUESTIONS

What happens when another lava lobe is emplaced upon a partially or fully-solidified preexisting lava lobe?

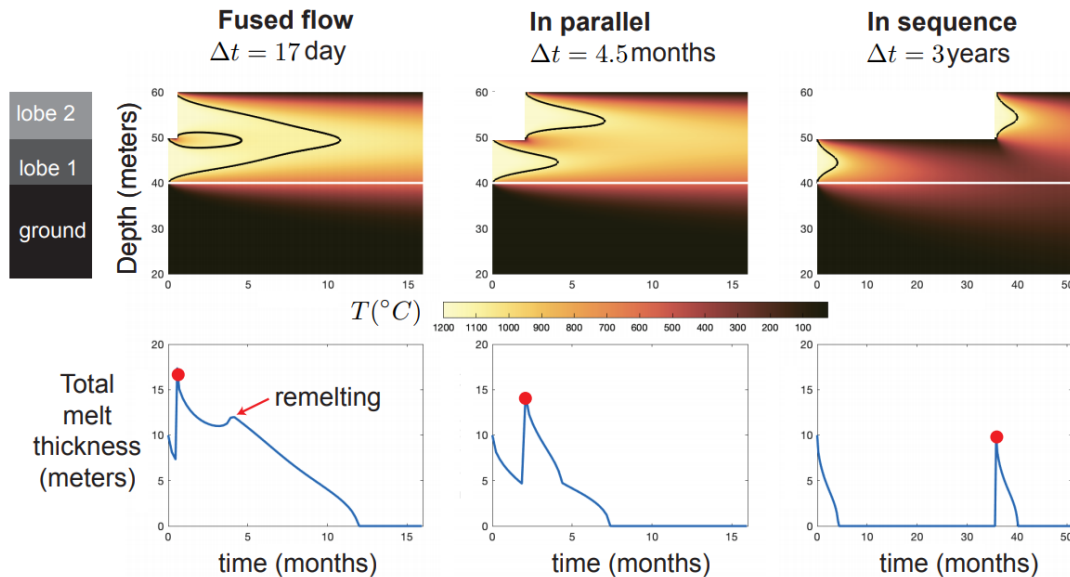
How do secondary (or tertiary, quaternary, etc.) emplacements affect the cooling of the lobes in the stack?

How well do our predictions compare with real-life observations in LIPs?

How well do our predictions refine and confirm other, well-accepted assumptions, such as  $t_h \sim h^2$  and  $S(t) \sim \sqrt{t}$ ?

**Note:** We are not trying to challenge the more intricate models which include seasonal rainfall, wind speeds, temperature-dependent specific heat, etc.

## THE THREE SAMPLE REGIONS AT H=10 METERS



The dotted contours in the temperature contour plot represent when  $T=T_m$ . We have confirmed that these coincide precisely with where  $\phi=0.5$ , and hence, these contours represent where the solid-liquid interface is. Also, in our simulations, we used a domain that goes down to a depth of 0 meters, but we exclude this part of the plot because it adds little to the qualitative importance of the plot.

Let  $t_h$  be the solidification time for a single lobe of size  $h$ , and let  $t_{emp}$  be the emplacement time interval, i.e., the time between when the first and second lobes are emplaced.

Regardless of the lobe size, we identify **three** qualitative regions which indicate different regimes for the interlobe dynamics (inequalities are approximate):

- **Fused:** ( $0 < t_{emp} < 0.15t_h$ ) After emplacement, the solidified portion of the lower lava lobe eventually remelts completely, and then both lobes combine to form one large lobe which solidifies as one.
- **Fused or in parallel:** ( $0.15t_h < t_{emp} < 0.27t_h$ ) The results depend on the size of the lobe. Smaller lobes will be more likely to exhibit fused flow, while larger lobes will be more likely to flow in parallel in this region.
- **In parallel:** ( $0.27t_h < t_{emp} < t_h$ ) The two lava lobes solidify simultaneously for some period of time yet do not combine into a single, larger lobe.
- **In sequence:** ( $t_{emp} > t_h$ ) The first lava lobe completely solidifies before the second is even emplaced.

For all cases, as  $t_{emp}$  increases, the dynamics go in the order outlined above, but the boundaries between the regions occur at different ratios of  $t_{emp}/t_h$  for each lobe height.

By looking at the temperature contour plot for a specific emplacement event, we can readily determine whether or not emplaced flows have fused, flowed in parallel, or flowed in sequence.

- For the fused case, the dotted contours marking the solid-liquid interface are nested, i.e., one lies within the other. This indicates that during some period of time, there are four solid-liquid interfaces, but the two middle interfaces disappear once the solid layer between the two flows remelts. Afterwards, there are only two interfaces since the two lobes have combined (which eventually combine as well), and this causes the nested contours to appear.
- For flows in parallel, there are two distinct interfacial contours which outline region that do not overlap, i.e., there is no nesting of contours. What differentiates this case from

flows in sequence is that there are four solid-liquid interfaces for some period of time, indicating when the two lava lobes cool simultaneously.

- Finally, for flows in sequence, the interfacial contour representing the lower (first) lobe closes up at a time  $t < t_{\text{emp}}$  before the contour representing the upper (second) lobe even appears. This represents how the lower (first) lobe cooled before the upper (second) one was even emplaced.

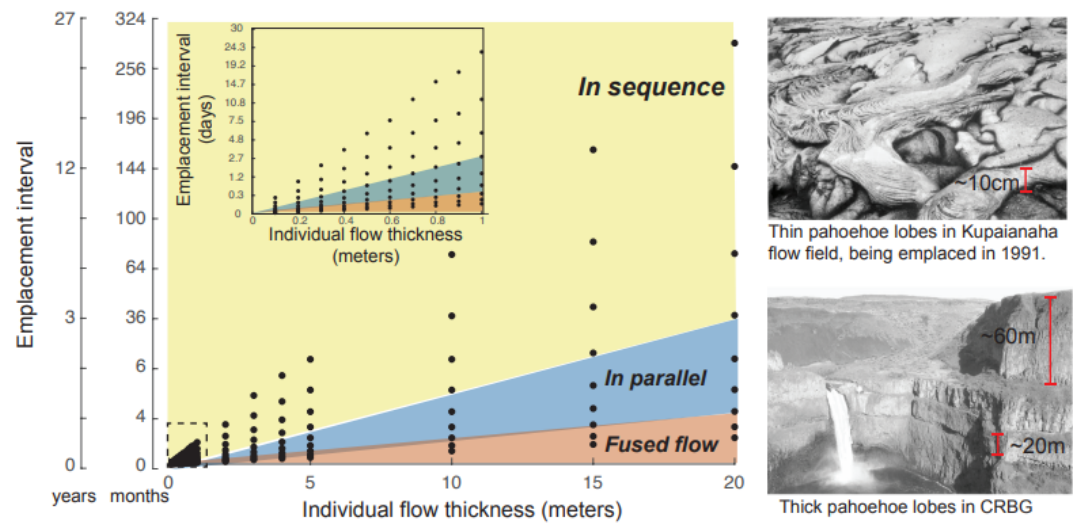
Alternatively, we can also look at the temperature contour plots to determine the region for the dynamics of a given emplacement event. There is an initial peak where the second lobe is emplaced at  $t_{\text{emp}}$ . Then,

- For the fused case, after the initial peak, the total melt thickness increases slightly once the solidified portion in between the two lobes remelts completely and the lobes combine. In all other regions below, the melt thickness never increases after the initial peak.
- For flows in parallel, the thickness decreases relatively quickly after secondary emplacement. Then, after the lower lobe solidifies, the thickness decreases as a slower rate to zero as the upper lobe eventually solidifies as well and the thickness decreases down to zero.
- For flows in sequence, before the initial peak even occurs, the thickness decreases down to and remains at zero for some period of time as the lower lobe has completely solidified. Then, after this, secondary emplacement finally occurs as the upper lobe solidifies in sequence.

Of course, these above observations are generalizable to tertiary, quaternary, etc. emplacement events, as well as emplacement events involving lobes of arbitrary sizes.

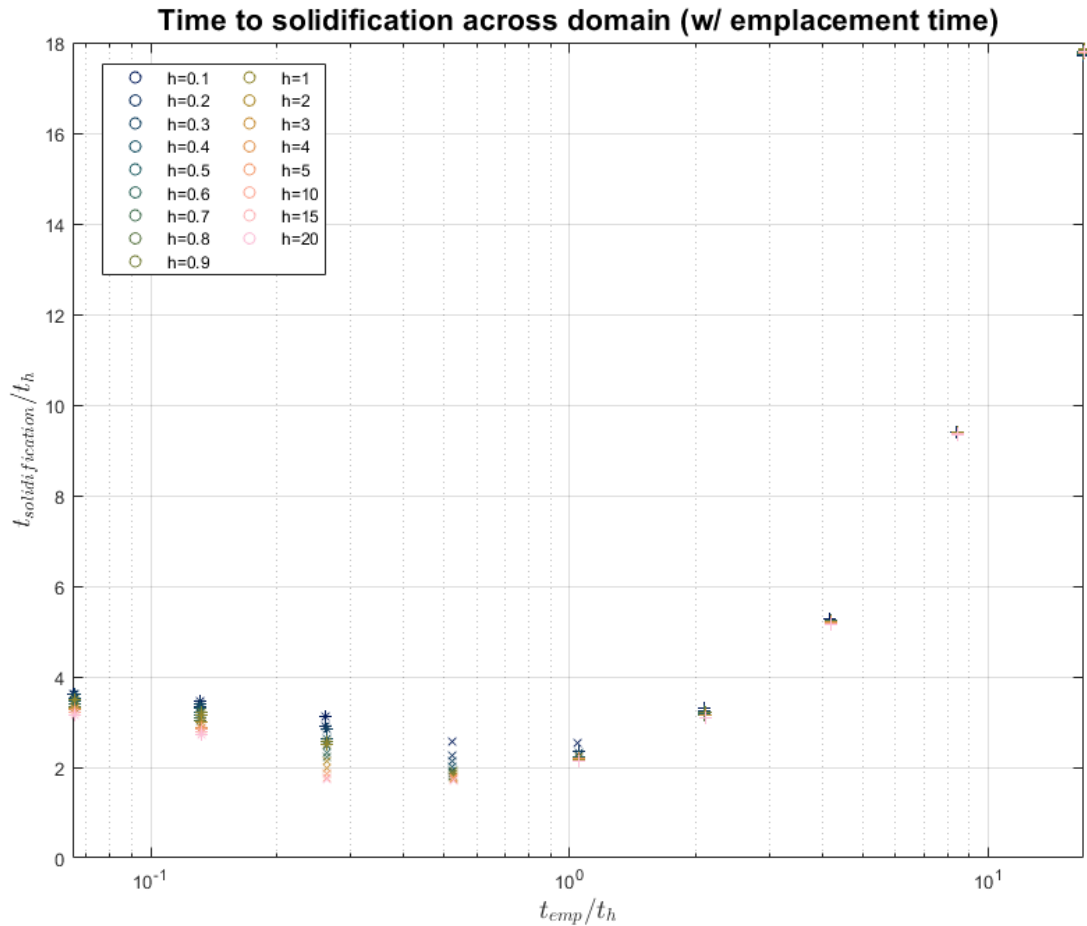


# SOLIDIFICATION DYNAMICS

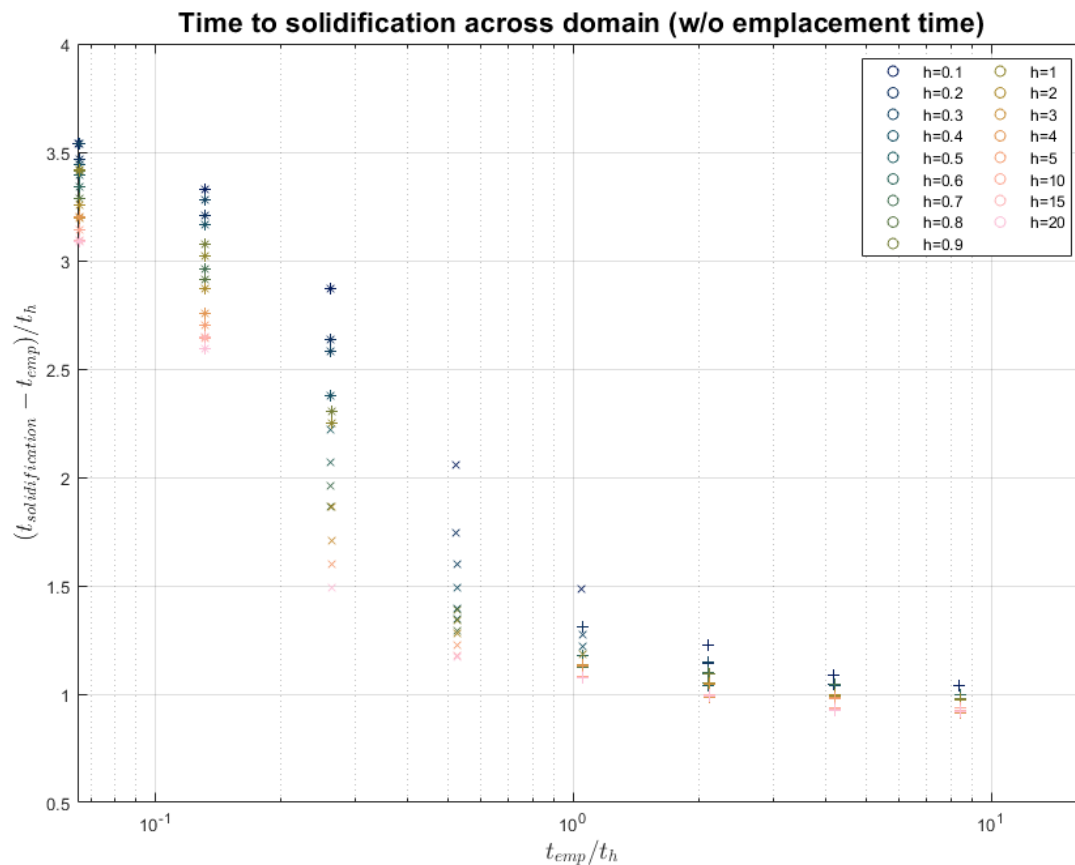


The dimensional plot above shows the relationship between the three regions for the interlobe dynamics and the flow thickness. In particular, it demonstrates the qualitative relationship without scaling the emplacement time interval by  $t_h$ .

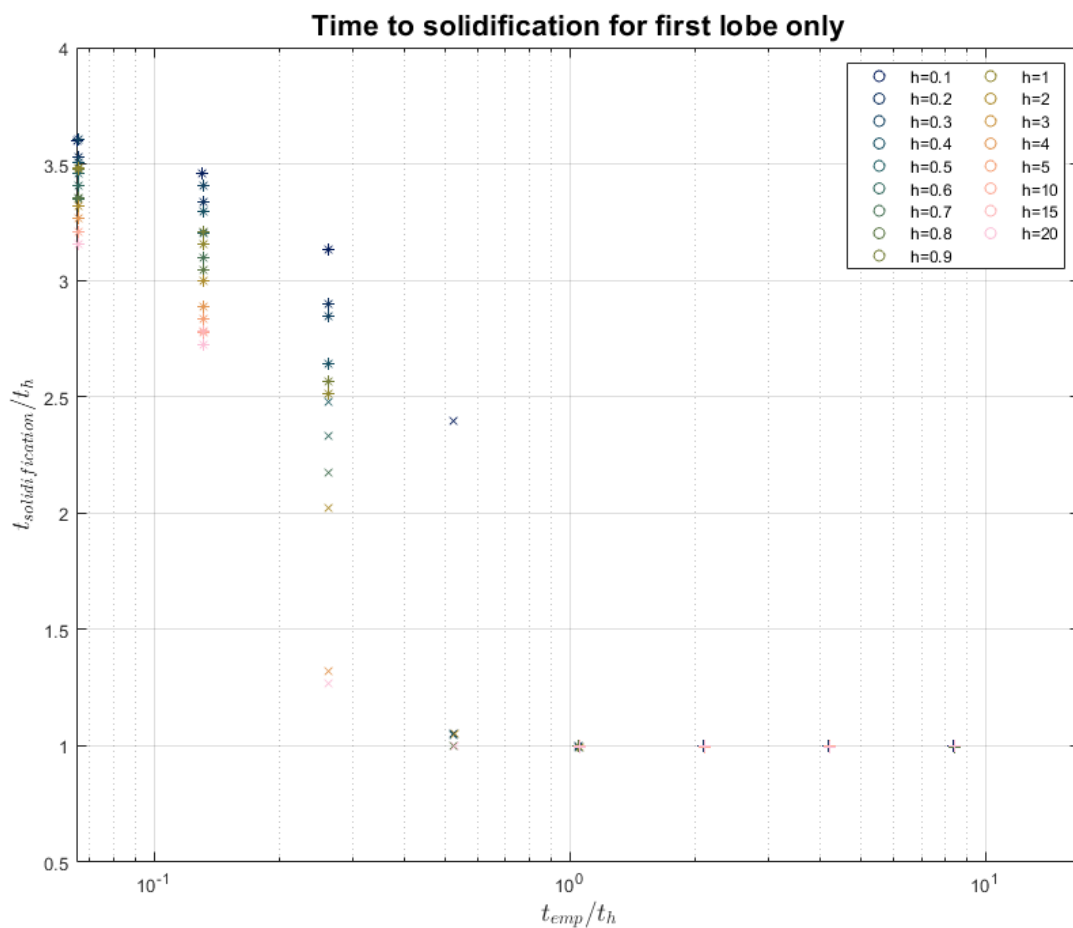
For the dimensionless plots below, we will consider the trends between different lobe thicknesses once we weight the emplacement time by  $t_h$ . For every dimensionless plot, the stars (\*) represent merged cases, the crosses (x) represent in parallel cases, and the pluses (+) represent in sequence cases.



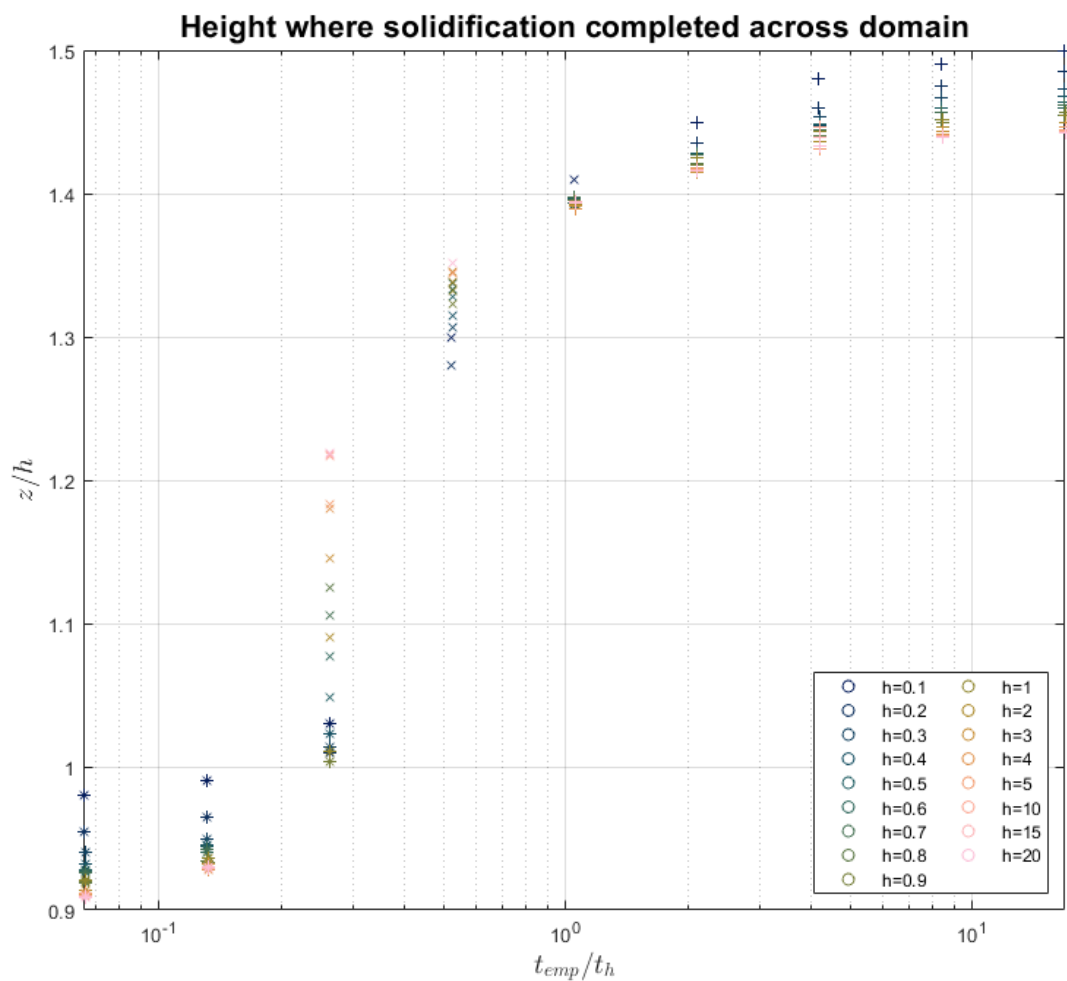
The plot above shows the solidification time including the time between emplacement,  $t_{\text{solidification}}$ , as a function of the emplacement time interval,  $t_{\text{emp}}$ , with both axes scaled by  $t_h$ . Note in particular that the graph at any lobe size has a minimum near or slightly below  $t_{\text{emp}}=t_h$ . This minimum reflects some optimal balance between the emplacement time and the thermal/phase interaction between the two lobes which minimizes the solidification time across the domain. This optimal balance lies within the "in parallel" region.



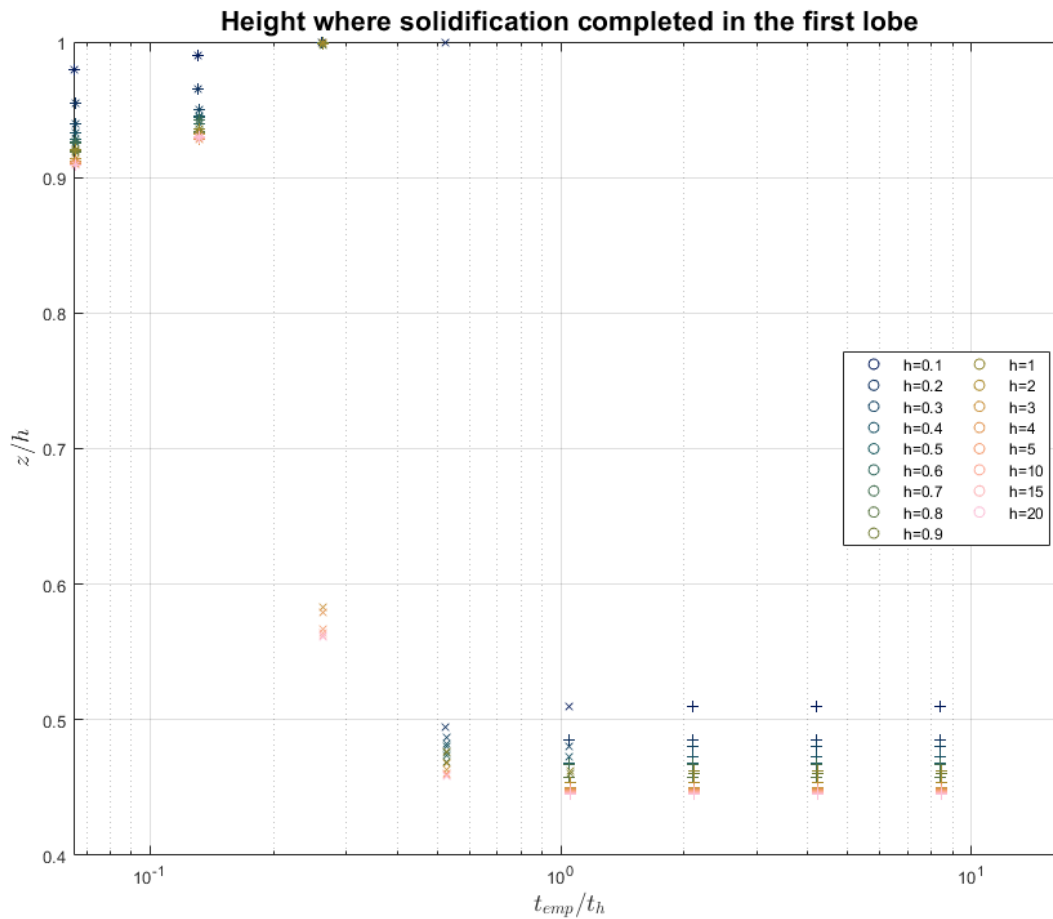
The above plot highlights an alternate interpretation of the solidification time in which we neglect the time between emplacements. On either plot, we note that as  $t_{\text{emp}} \rightarrow 0$ ,  $t_{\text{solidification}} > 4t_h$ . This reflects how, since  $t_h \sim h^2$ ,  $t_{2h} \sim (2h)^2 = 4h^2$ . Meanwhile, in comparison to the first solidification plot, this plot better demonstrates how as  $t_{\text{emp}} \rightarrow \infty$ ,  $t_{\text{solidification}} > t_h + t_{\text{emp}}$ .



The above plot only considers the time for the first lobe to solidify vs.  $t_{emp}/t_h$ . This plot highlights the thermal influence of the upper lobe upon how the lower lobe solidifies, relative to  $t_h$ . As expected,  $t_{solidification} \rightarrow t_h$  when  $t_{emp} \rightarrow \infty$ , which indicates how if the lower lobe has fully solidified before the upper lobe is replaced, the upper lobe will have no influence on the solidification of the lower lobe.



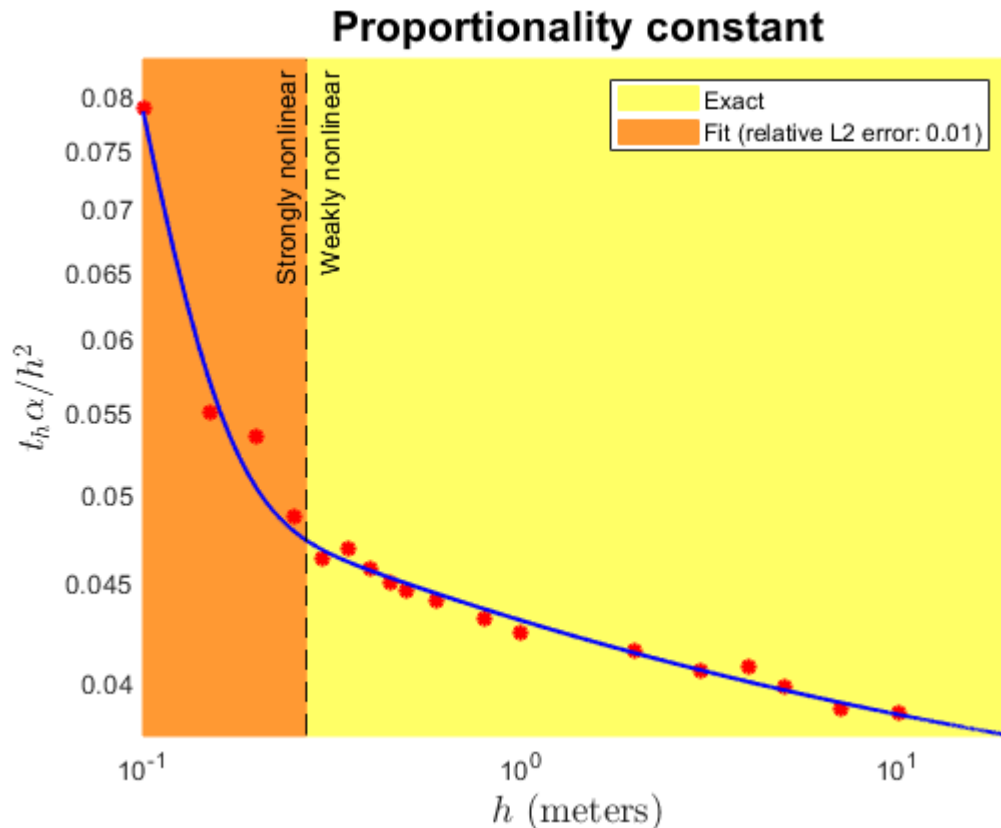
This plot indicates the height, scaled to the lobe size, at which solidification completed across the entire two-lobe system vs.  $t_{emp}/t_h$ . This variable is significant because certain horizontal fractures form where solidification completes in a lava lobe. Note in particular that the smaller lobe sizes appear to have higher solidification heights in the merged and in sequence regions, while they appear to have lower solidification heights in the in parallel region.



This final plot is the same as the one above, except that this plot measures solidification in the first lobe only, i.e., where the first lobe solidified. Note that for a given height, the graph appears to increase during the fused region, decrease sharply during the in parallel region, and then finally level out during the in sequence region. The trend in the in parallel region appears to be sharper the smaller the lobe size is, which indicates how the thermal influence of the upper lobe on the lower lobe increases as the lobe size decreases, assuming that the lobes do not just merge entirely.

For both height plots, the quantitative differences in behavior across different lobe sizes appears to be greatest for the in parallel region.

# NONLINEAR CORRECTIONS TO $t_h \sim h^2$



Using nonlinear least squares, we fit the solidification data for a *single* lobe cooling alone to the function

$$t_h = \frac{h^2}{\alpha} \left( \frac{A}{h^B} + C e^{-Dh} + E \right).$$

The best-fit parameters we find are  $A \approx 0.0110$ ,  $B \approx 0.2294$ ,  $C \approx 0.3346$ ,  $D \approx 24.8922$ , and  $E \approx 0.0320$ . With these parameters, the relative  $L^2$  error between  $t_h \alpha / h^2$  as fitted above and the actual data is  $\approx 9.8237 \times 10^{-3} \approx 1\%$ .

By using the term "strong nonlinearity" in the log-log plot above, we are referring to how there is a qualitative difference in the curve for small enough lobe sizes. This difference is best explained by the quick decay of the exponential term in our curve fit: For  $h$  not too large, the exponential term quickly disappears and the trend becomes primarily dominated by the power law term. Hence, motivated by how the relative  $L^2$  error between our best-fit curve and numerical solution is just under 1%, we heuristically draw the line between the "strongly nonlinear" and "weakly nonlinear" regions by indicating where the relative error between the fit with and without the exponential term falls below 1%. That point is at roughly  $h=0.26344$ , after which the exponential term contributes an error which is below 1% and decreases further as  $h$  increases.

We label these two regions above to give a rough estimate of where the usual  $t_h \sim h^2$  scaling relationships are mostly valid, and show how for small enough lobe sizes, deviations from this trend begin to dominate significantly. The physical interpretation of these regions is as follows: As we work with smaller and smaller lobes, the nonlinear effects of convection cooling and radiative heat loss at the lava's surface begin to dominate the time it takes for a lobe of that size to cool. The usual Stefan problem formulation often ignores these nonlinear effects in the boundary condition at the lava-air interface, but based off of our results here, we suggest that these will contribute a non-negligible effect to the solution when the lobe size is too small.



## QUANTITATIVE FORMULATION OF THE MODEL

Phase-field equations:

$$\begin{aligned}\tau \partial_t \phi &= \omega_\phi^2 \nabla^2 \phi - g'(\phi) - \frac{L}{H} \frac{(T - T_m)}{T_m} P'(\phi), \\ \partial_t T &= \alpha \nabla^2 T + \frac{L}{c_p} h'(\phi) \partial_t \phi,\end{aligned}$$

Let  $t_1 > 0$  be the time at which a secondary emplacement occurs, and let  $h_1 > 0$  be the height of the first lobe. Then,

$$\begin{aligned}\phi_1(x, 0) &= \begin{cases} 1 & x \in [0, D) \\ 0 & x \in [D, D + h_1] \end{cases}, \\ \phi_1(0, t) = \phi_1(D + h_1, t) &= 1, \quad t \in [0, t_1), \\ T_1(x, 0) &= \begin{cases} T_g & x \in [0, D) \\ T_0 & x \in [D, D + h_1] \end{cases}, \\ T_1(0, t) &= T_g, \quad t \in [0, t_1), \\ k \frac{\partial T_1(D + h_1, t)}{\partial x} &= -h_c (T_1(D + h_1, t) - T_s) \\ &\quad - \sigma_s \varepsilon (T_1^4(D + h_1, t) - T_s^4), \quad t \in [0, t_1),\end{aligned}$$

describes the initial-value problem, which we solve from  $t = 0$  up to  $t = t_1$ .

$T_1$  and  $\phi_1$  solve the PDE system from earlier, and  $T_g$ ,  $T_s$ , and  $T_0$  are ground temperature, surface temperature, and initial temperature of the lava, respectively. We can let  $T_g = 20 + 273.15$  K,  $T_s = 30 + 273.15$  K, and  $T_0 = 1200 + 273.15$  K.



Let  $t_2 > t_1$  be the time of the end of the simulation (or tertiary emplacement), and let  $h_2 > 0$  be the height of the second lobe.

$$\begin{aligned}\phi_2(x, t_1) &= \begin{cases} \phi_1(x, t_1) & x \in [0, D + h_1) \\ 0 & x \in [D + h_1, D + h_1 + h_2] \end{cases}, \\ \phi_2(0, t) &= \phi_2(D + h_1 + h_2, t) = 1, \quad t \in [t_1, t_2), \\ T_2(x, t_1) &= \begin{cases} T_1(x, t_1) & x \in [0, D + h_1) \\ T_0 & x \in [D + h_1, D + h_1 + h_2] \end{cases}, \\ T_2(0, t) &= T_g, \quad t \in [t_1, t_2), \\ k \frac{\partial T_2(D + h_1 + h_2, t)}{\partial x} &= -h_c (T_2(D + h_1 + h_2, t) - T_s) \\ &\quad - \sigma_s \varepsilon (T_2^4(D + h_1 + h_2, t) - T_s^4), \quad t \in [t_1, t_2),\end{aligned}$$

describes the initial-value problem for  $t \geq t_1$ , which we solve from  $t = t_1$  up to  $t = t_2$ .

$T_2$  and  $\phi_2$  also solve the PDE system from earlier. We can repeat the procedure outlined in the last few slides ad nauseum for an arbitrary number of emplacements with arbitrary sizes, provided that  $D$  is large enough such that our finite domain is valid, i.e.,  $|T(x, t) - T_g| \ll 1$  whenever  $x \approx 0$  for all  $t \in [0, t_2]$ .

For the numerical scheme, we solved this initial-value problem using the usual MOL (method of lines).

- 4th-order explicit central differences in space
- 4th-order AB4/AM4 predictor-corrector method in time with adaptive time step control
- Ralston's 4th-order Runge-Kutta method for restarting the scheme

For our domain size, we choose a ground which is **four** times as large as the size of each lobe, and in all cases,  $h_1 = h_2$ , i.e., we suppose that both lobes are of the same size.









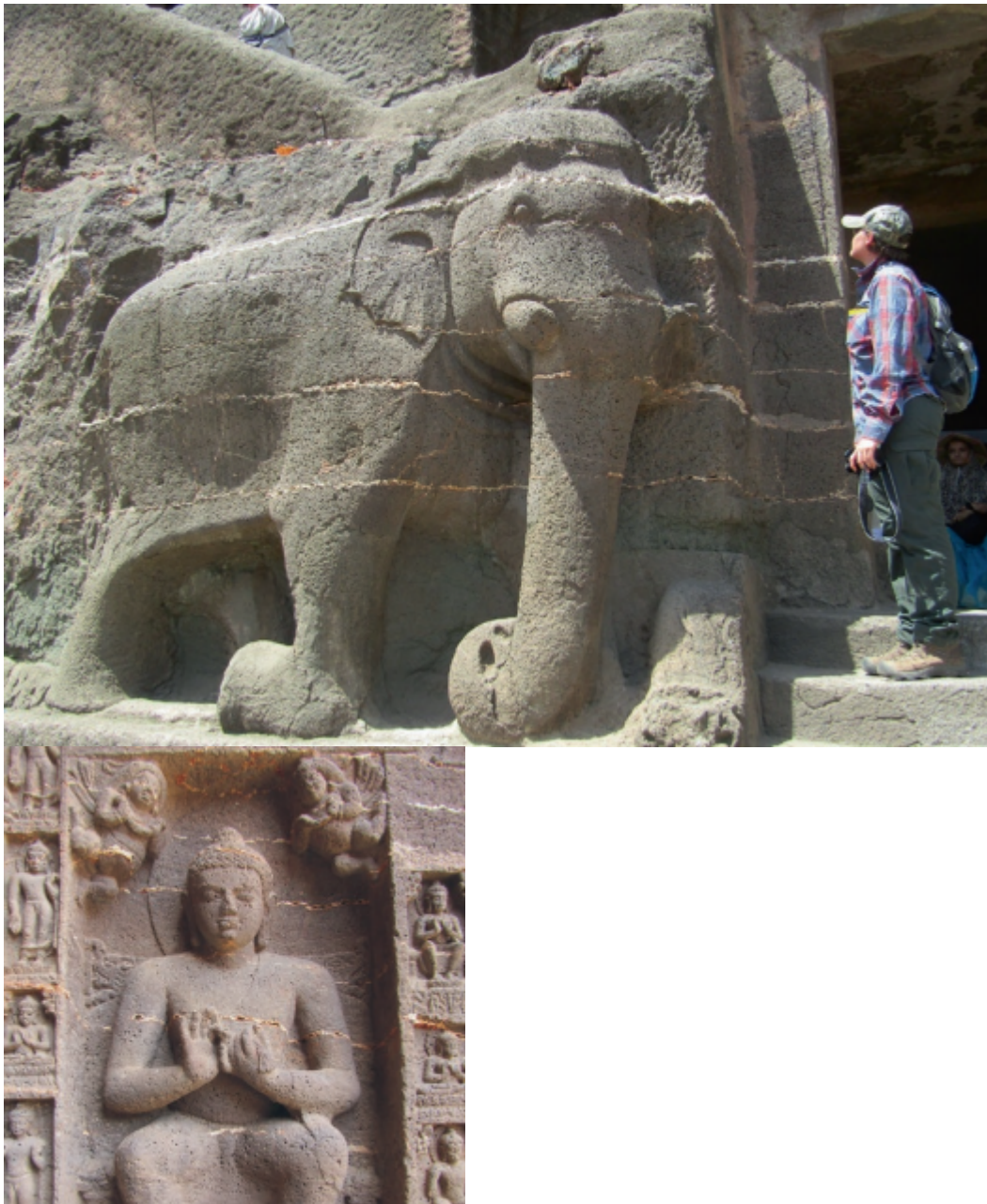




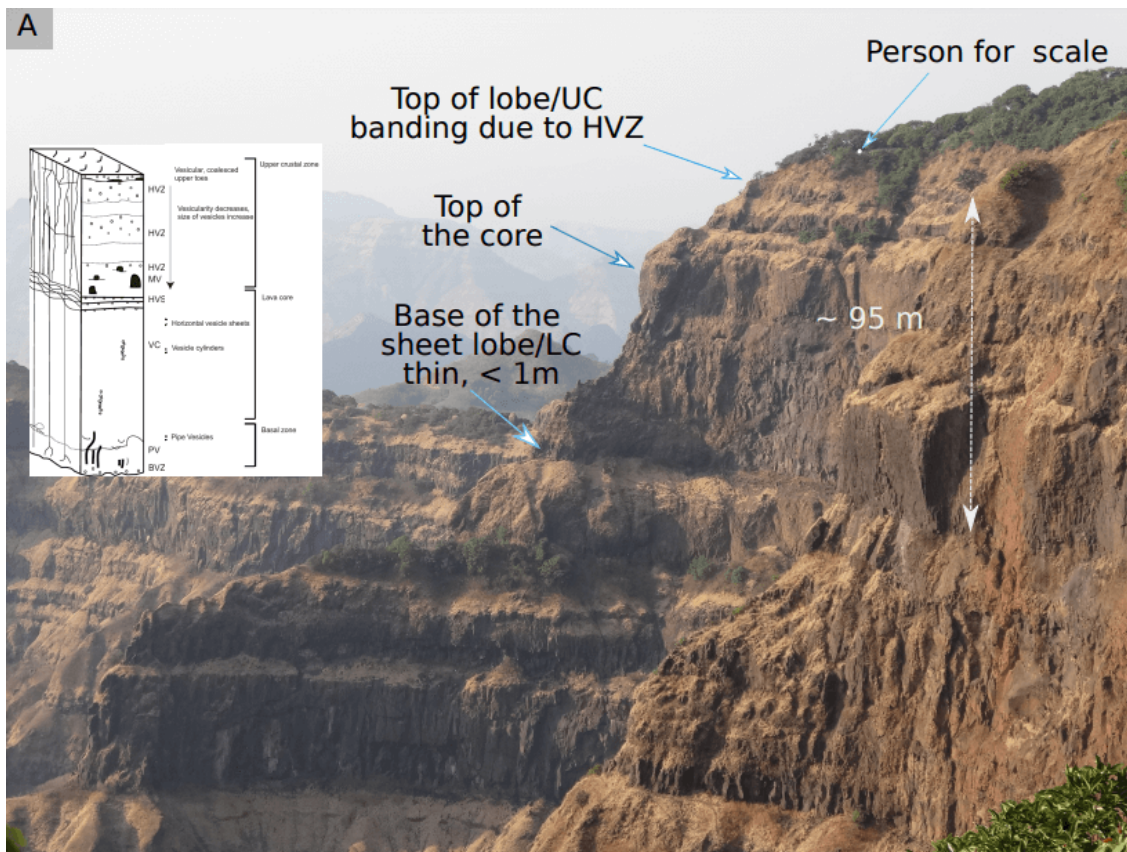
















## CONCLUSIONS

There are roughly **three** qualitative regions that correspond to different emplacement time. In increasing order with the emplacement time interval, these are fused, in parallel, and in sequence, irregardless of the lobe size.

The nonlinear effects and deviation from  $t_h \sim h^2$  are more apparent as the lobes decrease in size, primarily due to the effects of radiation and convection at the surface.

$S(t) \sim \sqrt{t}$  is only valid when a lava lobe is not near complete solidification. Near solidification, the geometry of the solidification profile begins to dominate. (Solving an adequately elaborate Stefan problem also reproduces these same results.)

There is an optimal balance between the emplacement time interval and temporal solidification dynamics which leads to the shortest solidification time overall for the two lava lobes placed on top of each other.

- In particular, this balance is **not** achieved when the lobes merge quickly, nor is it achieved when the lobes are placed one after another. Rather, this balance is best achieved when the lobes in parallel, but near to where they become in sequence.
- If we ignore the time between emplacements, then as expected from  $t_h \sim h^2$ , the solidification time across the domain is smallest when the lobes cool in sequence.

The empirical cooling rates of LIPs suggest that these solidified flows, while uniform in appearance, could actually consist of multiple, emplaced smaller lava flows which combined before they solidified. In prior literature, most volcanologists and geologists assume that LIPs were formed due to inflation, but our results also indicate that these could have been formed from fused flows which would erase the trace of their prior separation.

Finally, using the qualitative and quantitative results we propose, we provide a possible way to reconstruct the solidification history of an LIP by looking at its fractures, its geological and mineral composition, etc., since these are related to the variables we measured.

## ABSTRACT

Large Igneous Provinces (LIPs) are among the greatest magmatic events in Earth history with volumes in excess of  $\sim 500,000 \text{ km}^3$  of predominantly basaltic lavas covering huge continental and ocean regions ( $>100,000 \text{ km}^2$ ). Field observations suggest that lava flow fields in LIPs are made largely of sheet pāhoehoe lava lobes and the 10-100 m thick flows are formed by inflation. Understanding the emplacement history of these lava lobes can help us infer the magnitude and temporal dynamics of past events.

We use a phase-field model to describe solidification and re-melting of sequentially emplaced lava flows. We calibrate model parameters using field measurements at Makaopuhi lava lake and perform extensive numerical simulations by varying the thickness of individual flow and the time intervals between eruptions. These results help quantify the complex interplay between thermal evolution, flow thickness and emplacement frequency. If flows are thick enough and the interval between emplacement short enough, reheating and re-melting may remove the textural record of flow contacts – making flows appear thicker than they actually were. Guided by field observations in Columbia River Basalt and Deccan Traps, we illustrate how the final morphology of sequentially emplaced lava is controlled by both the time scale of emplacement intervals and the time scale of cooling. We summarize our results to provide theoretical constraints on the thickness and emplacement intervals of individual LIP lava flows.