# AE11B-2703: Physical principles that determine electric streamer parameters

Nikolai G. Lehtinen
Nikolai.Lehtinen@uib.no

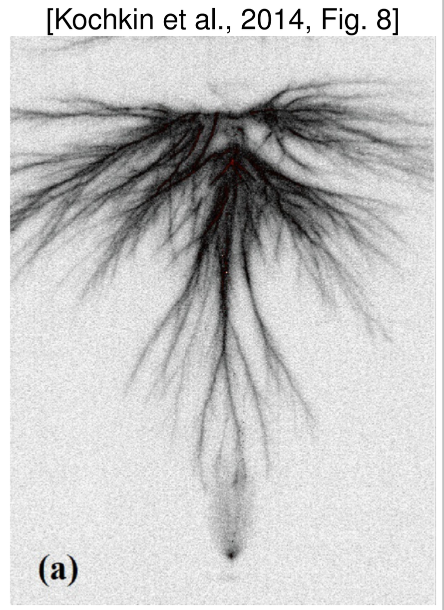
BIRKELAND CENTRE FOR SPACE SCIENCE

European Research Council
Established by the European Commission

## What is a streamer?

**Electric streamer discharges** are ionized columns in gas (or liquid) which advance by ionizing the material in front of them with the enhanced field at the streamer tip.

[Kochkin et al., 2014, Fig. 8]




Shown here is a laboratory ~MV, 1 m gap discharge, with a complicated branched streamer tree.

Applications:
- Lightning, sprites
- Industry (suprathermal electrons)
- In the future: x-ray production (still unexplained)

Streamer system is
- highly non-uniform
- thin streamers, $10^{-4}$–$10^{-2}$ m [Bazelyan and Raizer, 1998, p. 47]
- branching
- extinguishing and re-igniting
- reverse streamers (negative)
- stepping (negative)

## Streamer mechanism [Loeb and Meek, 1941]

- Electron drift (forward for negative streamer)
- Electron diffusion (if no photoionization)
- Photoionization (the main mechanism in air)
  Photons with wavelength between 980 Å and 1025 Å are produced by electron collisions with $N_2$ which then travel ahead of the streamer and ionize $O_2$.

$$e^- + N_2 \to e^- + N_2^*,\quad N_2^* \to N_2 + h\nu(980\text{–}1025\ \text{Å}),\quad h\nu + O_2 \to O_2^+ + e^-$$

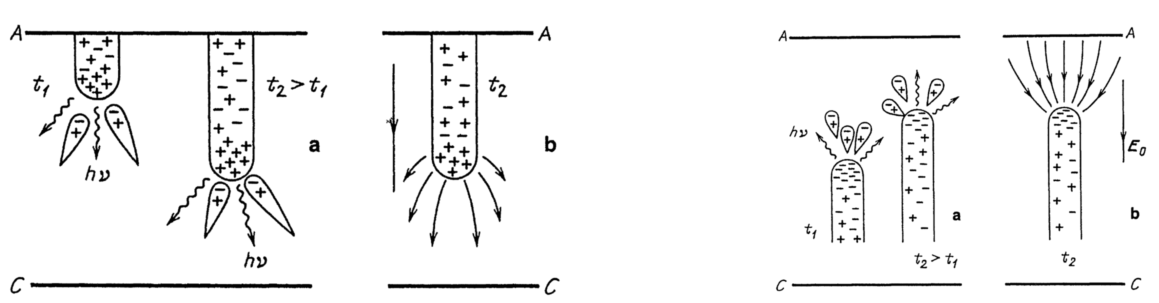


Figure 1: Positive streamer [Raizer, 1991, p. 335]

Figure 2: Negative streamer [Raizer, 1991, p. 338]

**Note:** the avalanches started by photoelectrons are directed outward, but the streamer moves so fast that it catches up with them.

**The goal**

To identify physical principles that determine and to calculate streamer **radius, speed** and other parameters (e.g., field in front of the streamer and inside the channel)

This task had not been solved before now. Another important unsolved task in streamer physics is understanding the physical principles of streamer **branching**.

## Hydrodynamic simulations

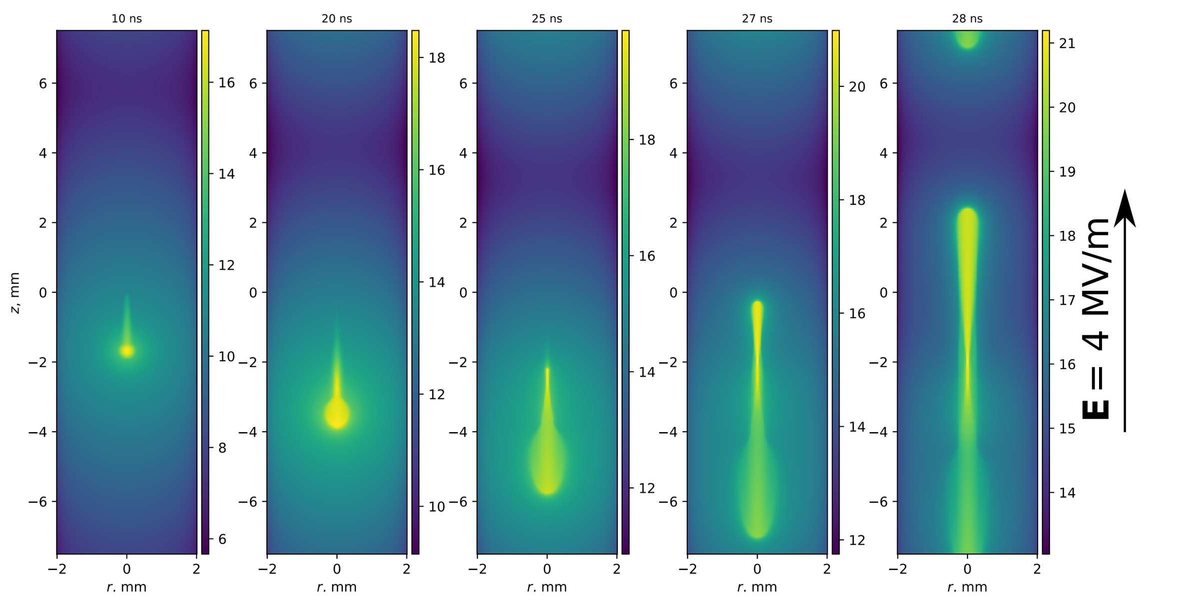



Figure 3: Evolution of $\log_{10} n_e$ in the avalanche-to-streamer transition. [Lehtinen and Østgaard, 2018]

They give a quantitative answer, but ...
- offer no insight into the physical reasons for the parameters;
- possibly have numerical errors such as instabilities and numerical diffusion:
  *... the authors [have] disagreement on whether the branching of model streamers ... is a consequence of a numerical instability.* — Pasko [2006, p. 291]

We prefer to understand the physical principles that quantitatively determine streamer propagation (and branching) than to rely blindly on possibly incorrect computer calculations.

## Flat front perturbations

- field at the protrusions increases, perturbations grow
- fastest-growing perturbation survives, determines transverse size
- **electron drift + diffusion:** this size is $a \propto D^{1/4}$ [Derks et al., 2008]
- no theory yet with **photoionization**

## Other proposed explanations

The following hypothesis were proposed:
- **Electron diffusion** causes transverse spreading of the streamer and therefore determines radius. Produces a radius that is too small [Dawson and Winn, 1965; Gallimberti, 1972; Qin and Pasko, 2014].
- Streamer **"chooses" the field** at the tip at which it "wants" to propagate [Bazelyan and Raizer, 1998, p. 277] (which also determines all other parameters).
  The physical reason why this particular field should be chosen is not apparent.

## Streamer model and relations between parameters

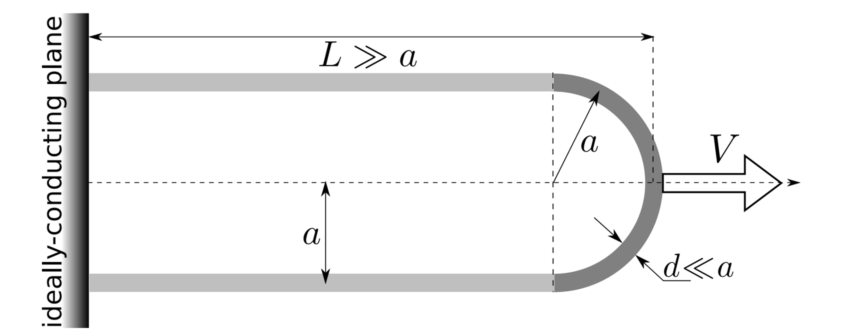



Figure 4: The streamer model (sizes)

**Sizes:**
- Streamer is a cylinder (**channel**) with a hemispherical cap (**head**)
- Length $L$ (given!) $\qquad L = \int V\,dt$
- Radius $a \ll L$
- Thickness $d \ll a$

## Equation 1: Fields

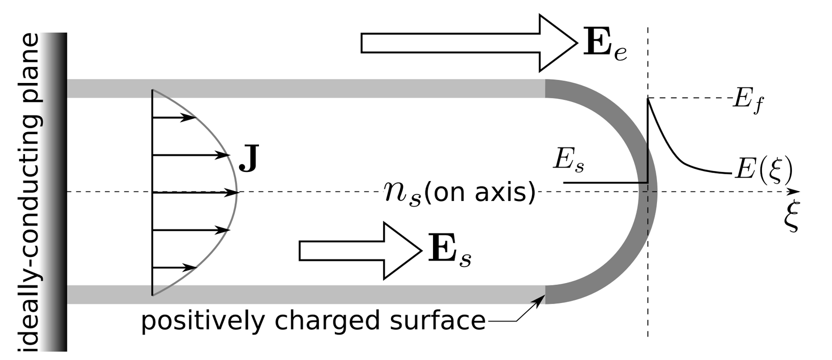



Figure 7: The streamer model (fields)

- External $E_e$ (given!)
- Inside $E_s \ll E_e$ due to high conductivity, all charges are at surface
- Still $E_s > 0$ because there is a current in the channel $\propto n_s$.
- Just outside $E_f \gg E_e$

An electrostatic model (solved using the **method of moments** or **MoM**) determines relation between $E_e$, $E_s$, $E_f$ (but $E_s$ and $E_f$ separately are still unknown!).

## Equation 2: Currents

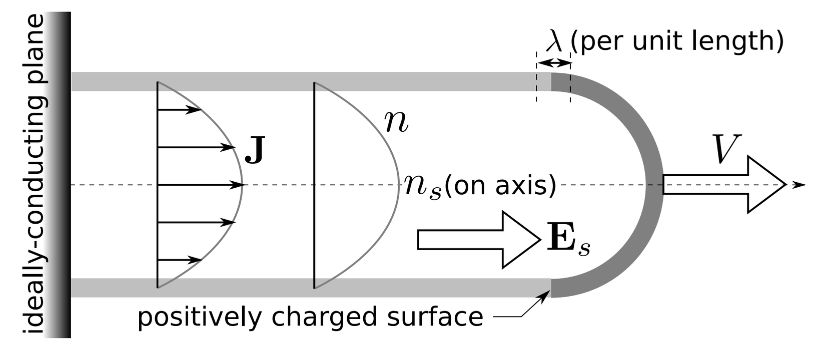



Figure 8: The streamer model (currents)

- Charge on the surface per unit length $\lambda$ is from MoM and $E_s$
- The total current is $I = \lambda V$
- It is also calculated from $n_s$ and $E_s$ as $I = \int \mathbf{J}\,dA_\perp$

Thus, we have a relation between $E_s$, $n_s$, $V$.

## Equation 3: Ionization front

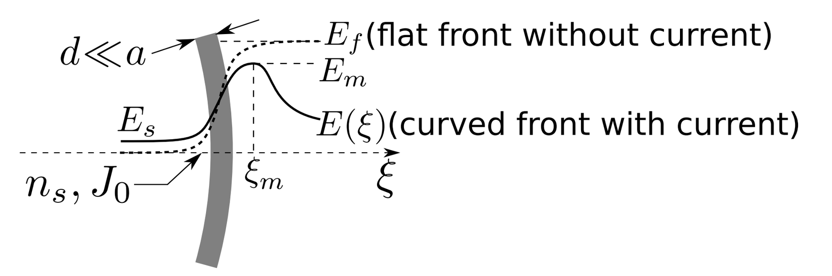



Figure 11: The streamer model (the front)

The flat front theory is used to relate $n_s$ to $E_f$. We also have corrections to this theory:
- to include the current $J_0$ (on the axis)
- to include curvature
- maximum field is not $E_f$ but corrected value $E_m$ (depending on $d$)

## Equation 4: Photoionization

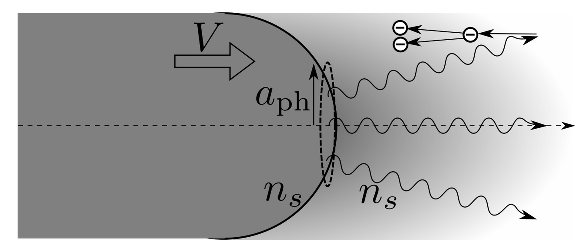



Figure 12: The streamer model (photoionization)

- Ionizing photons are produced in the front $\propto$ ionization rate
- Photon production volume (and the number) $\propto \pi a_{\rm ph}^2$, $a_{\rm ph} \sim a$
- Ionization occurs remotely, described by Zheleznyak et al. [1982]
- Electron avalanche has length $d$ in streamer frame, which depends on $V$ and $E_f$
- The electron density in the end of avalanche must match $n_s$

Thus, we relate $V$, $E_f$ and $a_{\rm ph} \sim a$.

## System of equations for $E_s$, $E_f$, $a$, $n_s$, $V$

- An electrostatic model determines relation between $E_e$ and $E_s$, $E_f$
- The ionization $n_s$ is related to velocity $V$: the current flowing in the streamer channel ($\propto n_s$) is equal to "flying charged rod" current $\propto V$
- The ionization $n_s$ is related to $E_f$ by the results of the flat front theory [Lagarkov and Rutkevich, 1994].
- Velocity $V$ is related to the radius $a$ (and field $E_f$) through the photoionization mechanism [Naidis, 2009; Pancheshnyi et al., 2001].

We notice:
- # unknowns = # constraints + 1
- it is convenient to choose the streamer radius $a$ as a free parameter
- velocity $V$ and other parameters are functions of $a$
- everything is also a function of **laboratory-condition fixed $E_e$ and $L$**

## Maximum velocity hypothesis

The number of these constraints is, however, smaller by one than the number of unknowns. This means that we still cannot fix all the streamer parameters based only on $E_e$ and $L$, and the streamer radius $a$ is still a free parameter. However, we can express $V$ and other parameters as functions of $a$. All parameters except $V$ are monotonous functions of $a$. But $V(a)$ has a maximum!

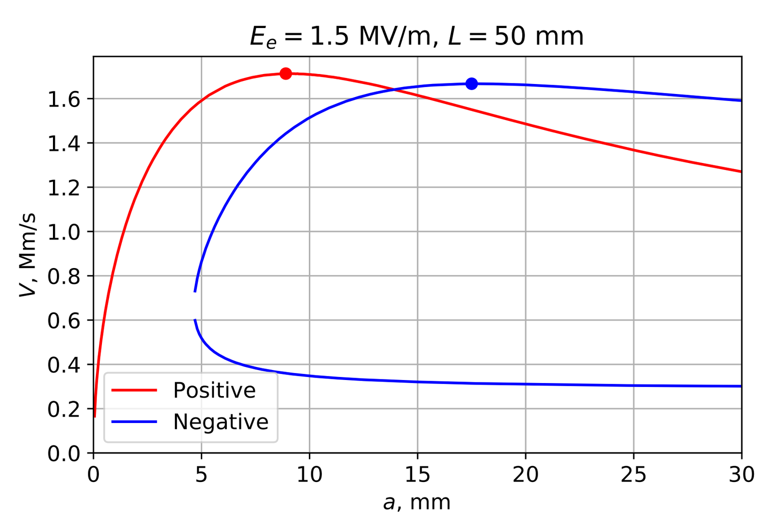



Figure 13: An example of solutions for $V(a)$ showing one solution for positive and two or zero solutions for negative streamers in air. The large dot shows the value at which $V$ is maximized.

**Intuitively:** The velocity drops for narrow streamers because there is not enough photon production, and for wide streamers because the field enhancement at its tip becomes insufficient.
- in flat-front perturbation theory, the surviving transverse size was the fastest-growing instability
- by analogy, choose the maximum $V$ (and the corresponding radius value $a_s$) as the correct parameter values
- thus, the streamer is treated as a **nonlinear instability**

**Equation 5**

We have identified one more constraint: $V \to \max$.
Now, # unknowns = # constraints and we can unambiguously determine all streamer parameters.

## The calculated parameters for lab conditions

We compare to experimental result of Allen and Mikropoulos [1999].

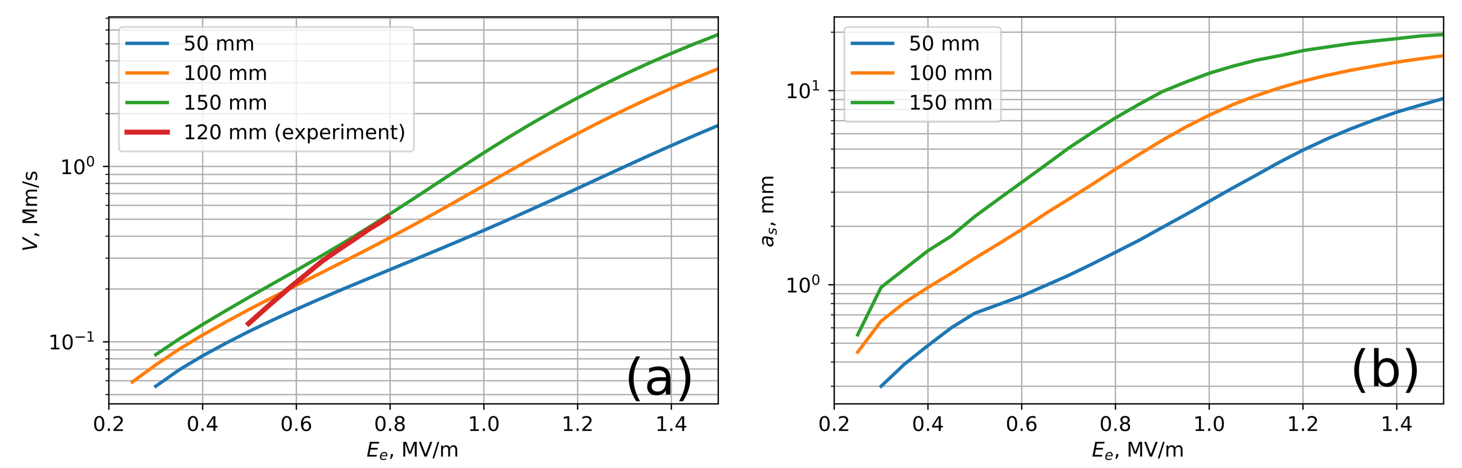



Figure 14: Results for positive streamers as functions of external field $E_e$, for three different values of $L$ = 50, 100 and 150 mm: (a) velocity $V$, (b) radius $a_s$. The measurements of Allen and Mikropoulos [1999, Fig. 10] at $L$ = 120 mm are presented together with calculated $V$ results in panel (a).

## Negative streamers

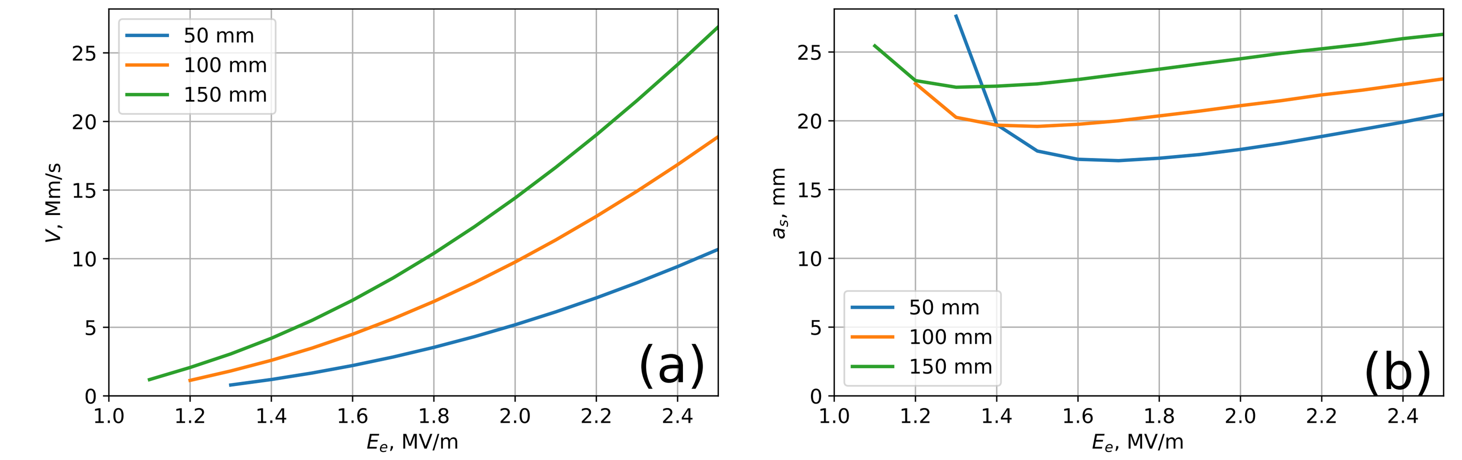



Figure 15: Results for negative streamers (same lab conditions).

## Streamer threshold fields [Raizer, 1991, p. 362]

Interestingly, the physical reason for the minimum external field $E_e = E_{\pm t}$ at which the streamer still can propagate is different for different streamer polarities.

**Positive streamers:** As the external field $E_e$ decreases, the internal field $E_s$ also decreases, and the three-body attachment becomes increasingly important. The number of attachment lengths determines the threshold $E_{+t} \approx 0.45$ MV/m. The threshold is **lower** for shorter streamers.

**Negative streamers:** $a \to \infty$ at threshold $E_{-t} \approx 0.75$–$1.25$ MV/m. The threshold is **higher** for shorter streamers.

The predicted dependence on streamer length may be checked experimentally.

## Summary

**We identified the physical principles and found a way to calculate streamer parameters, such as its velocity and radius, on the basis of the given external electric field and the streamer length.**

**Possible future directions:**
- More comparisons, with both experimental data and hydrodynamic simulations.
- Hydrodynamic simulation study of the parameter stability.
  I.e., try to vary the parameters and see if they come back to the fixed value.
- Nonuniform fields and branching.
  The presented theory produces no branching in uniform fields.
- What happens when the streamer becomes too wide (nothing/branching/something else)?

## Acknowledgements

This study was supported by the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013)/ERC grant agreement n. 320839 and the Research Council of Norway under contracts 208028/F50, 216872/F50 and 223252/F50 (CoE).

## References

N. L. Allen and P. N. Mikropoulos. Dynamics of streamer propagation in air. *J. Phys. D: Appl. Phys.*, 32(8):913, 1999. doi:10.1088/0022-3727/32/8/012.
E. M. Bazelyan and Y. P. Raizer. *Spark Discharge*. CRC Press, New York, 1998.
G. A. Dawson and W. P. Winn. A model for streamer propagation. *Z. Phys.*, 183(2):159–171, Apr 1965. doi:10.1007/BF01380792.
G. Derks, U. Ebert, and B. Meulenbroek. Laplacian instability of planar streamer ionization fronts—an example of pulled front analysis. *Journal of Nonlinear Science*, 18(5):551, Jun 2008. doi:10.1007/s00332-008-9023-0.
I. Gallimberti. A computer model for streamer propagation. *J. Phys. D: Appl. Phys.*, 5:2179, 1972. doi:10.1088/0022-3727/5/12/307.
P. O. Kochkin, A. P. J. van Deursen, and U. Ebert. Experimental study of the spatio-temporal development of metre-scale negative discharge in air. *J. Phys. D: Appl. Phys.*, 47(14):145203, 2014. doi:10.1088/0022-3727/47/14/145203.
A. N. Lagarkov and I. M. Rutkevich. *Ionization Waves in Electrical Breakdown of Gases*. Springer-Verlag, New York, 1994.
N. G. Lehtinen and N. Østgaard. X-ray emissions in a multi-scale fluid model of a streamer discharge. *J. Geophys. Res.*, 2018. doi:10.1029/2018JD028646.
L. B. Loeb and J. M. Meek. *The Mechanism of the Electric Spark*. Stanford University Press, Stanford University, California, 1941.
G. V. Naidis. Positive and negative streamers in air: Velocity-diameter relation. *Phys. Rev. E*, 79:057401, May 2009. doi:10.1103/PhysRevE.79.057401.
S. V. Pancheshnyi, S. M. Starikovskaia, and A. Y. Starikovskii. Role of photoionization processes in propagation of cathode-directed streamer. *J. Phys. D: Appl. Phys.*, 34(1):105, 2001. doi:10.1088/0022-3727/34/1/317.
V. P. Pasko. Theoretical modeling of sprites and jets. In M. Füllekrug, E. A. Mareev, and M. J. Rycroft, editors, *Sprites, Elves and Intense Lightning Discharges*, volume 225 of *NATO Science Series II: Mathematics, Physics and Chemistry*, chapter 12. Springer, 2006.
J. Qin and V. P. Pasko. On the propagation of streamers in electrical discharges. *J. Phys. D: Appl. Phys.*, 47:435202, 2014. doi:10.1088/0022-3727/47/43/435202.
Y. P. Raizer. *Gas discharge physics*. Berlin: Springer, 1991.
M. B. Zheleznyak, A. K. Mnatsakanyan, and S. V. Sizykh. Photo-ionization of nitrogen and oxygen mixtures by radiation from a gas-discharge. *High Temp. (USSR)*, 20(3):357–362, 1982.