An Analysis of Magnetosphere-Ionosphere Coupling That Is Independent of Inertial Reference Frame

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Key Points:

- Relativistic transformations applied to electrodynamics can differ from what are typically used in the context of MI-coupling.
- We present an alternative "Ohm's law" for horizontal ionospheric currents that do not require a large scale electric field.
- Electrodynamic theories of MI coupling that do not account for the relative motion of ions and neutrals are not quantitatively accurate.

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Abstract

This paper analyses magnetosphere-ionosphere (MI) coupling from a perspective that is independent of inertial reference frame, explicitly acknowledging the role of special relativity in MI coupling. We review the theory of special relativity in the context of MI coupling, and discuss how the MI coupling literature has used a particular low-velocity limit of special relativity known as the "magnetic limit". We discuss how purely electrodynamic approaches to MI-coupling, where the high latitude electric field plays a central role, depend on inertial reference frame, so descriptions of MI-coupling involving the electric field depend on what reference frame is used to build the physical description. Choosing different reference frames leads to different descriptions of the physics, and essential physics common to all reference frames may be missed by tying the physical description to a specific reference frame. Reference frame-independent descriptions require that ion-neutral relative velocities and ion-neutral collisions are central to MI-coupling. Yet, the literature contains several examples of MI coupling theories that ignore the neutrals and focus instead on the electric field. Whereas neutral wind effects have been reported to modify electrodynamic effects such as Joule heating by ~25%, we show that the consequences of relative motion between ions and neutrals result in much larger impacts for significant geomagnetic storms when ion-neutral velocity differences are largest near the initiation of large-scale ion convection.

Plain Language Summary

Interactions between the magnetized and ionized solar wind, the magnetospheric cavity surrounding Earth, and the Earth's ionized upper atmosphere (ionosphere) can create fast (~1 km/sec) large-scale motions of the ionosphere during periods known as geomagnetic storms. To gain insight into the complex physics of these interactions, we use the principle of special relativity, which states that the physics must be independent of the velocity of an observer making measurements of the system. We write key equations governing interactions of the system in terms of quantities that do not depend on the observer's motion. In doing so, we find that previous theories had over-emphasized the importance of a large-scale electric field that grows during storms. Instead, we show that the important physics is related to collisions between the ionized portion of the atmosphere and the un-ionized "neutral" component that contains much more mass. These collisional interactions create upper atmospheric heating and expansion, and cause large-scale currents to flow betwen the ionosphere and magnetosphere, resulting in a

multitude of impacts to our technological society. Using the principle of relativity, and isolating the physics that is independent of observer motion, led us to a deeper understanding of key interactions during storms.

1 Introduction

Electrodynamics as it pertains to magnetosphere-ionosphere (MI) coupling is a critical aspect of the ionospheric response during geomagnetic storms. Large-scale convection of the ionospheric plasma during disturbed geomagnetic conditions at auroral latitudes and higher, and heating of the atmosphere due to collisions between charged and neutral species, originates with electric and magnetic forces acting on the plasma that change dramatically when solar wind conditions and the state of the MI system(?) lead to geomagnetic storms. Scientific consensus on fundamental aspects of the physical processes that occur at high latitude is not yet achieved, including the definition of Joule heating (Vasyliunas and Song, 2005; Verkhoglyadova et al. 2017). In this paper, we provide insight into the physical processes occurring during geomagnetic storms by emphasizing an approach that is independent of the inertial reference frame used to describe the phenomena.

The principle of special relativity (PSR) states that the laws of physics are independent of inertial frame. The PSR has important implications for high latitude electrodynamics due to the electric field in the high-latitude ionosphere being dependent on inertial reference frame. The large-scale electric field at high latitudes is directed predominantly in the horizontal direction and the magnetic field tends to be predominantly in the vertical direction. The PSR requires that the electric field component perpendicular to the magnetic field vary according to inertial reference frame. There are several examples in the literature where electric fields are used to describe MI coupling physics. For example, the concept that horizontal currents originating in the magnetosphere and closing in the ionosphere results in electric fields that cause high velocity **ExB** plasma flow has been invoked for phenomena such as sub-auroral ion drifts (Anderson et al., 1993). Another example is the relationship between the ionospheric electric field and field-aligned Region 1 currents closing in the ionosphere (Fedder and Lyon, 1987; Siscoe et al., 2002; Wiltberger et al., 2003; Rothwell and Jasperse, 2006). Since the electric field depends on reference frame, such examples would seem to obscure essential physics because the same phenomena can be viewed in a reference frame for which the electric field vanishes. The notion of a “preferred inertial reference frame” for high latitude electrodynamics has been discussed (Vasyliūnas and Song, 2005; Leake et al., 2014; Strangeway, 2012) and is useful when considering that high latitude phenomena occur within the physical media of plasma and neutral gases. A reference frame where the electric field vanishes is also useful to consider in understanding the physical basis of MI coupling.

In the remainder of this paper, we apply the PSR in the context of high latitude electrodynamics. We first review the PSR and how it is typically expressed in the low velocity limit. We next discuss an equation for the high latitude horizontal current density as an alternative to Ohm's law that contains only terms that do not depend on reference frame. In doing so, we clarify that the high-latitude current is primarily the result of the relative bulk motion between ions and neutrals resulting from large-scale convection of the ions initiated during a geomagnetic storm by the solar wind. Horizontal ionospheric currents are the by-products of ion-neutral velocity differences, rather than drivers of an electric field. We discuss the implications of ignoring the role of neutral dynamics in magnetosphere-ionosphere coupling.

2 The Principle of Special Relativity and its Low Velocity Limit

In this section, we review the literature of how electric and magnetic fields transform in the low velocity limit of the PSR. The invariance of physical laws in this limit is known as the "Principle of Galilean Relativity" (PGR). While the PGR might appear to be a well-settled topic, there are subtleties to the low-velocity limit of the PSR, which applies to most MI coupling studies where relative velocities tend not to exceed a few km/s. To our knowledge, references to the literature describing Galilean relativity applied to electrodynamics has not appeared in the literature of MI coupling, so we devote some discussion to this topic.

Throughout the text, we use the symbol $\mathbf{v}_r$ to indicate the relative velocity between two inertial reference frames, for example frames $\mathcal{A}$ and $\mathcal{B}$. We use primed variables to refer to physical quantities measured in the inertial reference frame $\mathcal{B}$ moving with velocity $\mathbf{v}_r$ relative to reference frame $\mathcal{A}$.

As first deduced by Einstein in 1905, electric and magnetic fields transform between inertial reference frames according to the following relationships (Rousseaux, 2014; Heras, 2010):

$$\mathbf{E}' = \gamma\left(\mathbf{E} - \frac{\gamma - 1}{\gamma}\frac{\mathbf{v}_r(\mathbf{v}_r \cdot \mathbf{E})}{v_r^2} + \mathbf{v}_r \times \mathbf{B}\right) \tag{1}$$

$$\mathbf{B}' = \gamma\left(\mathbf{B} - \frac{\gamma - 1}{\gamma}\frac{\mathbf{v}_r(\mathbf{v}_r \cdot \mathbf{B})}{v_r^2} - \frac{1}{c^2}\mathbf{v}_r \times \mathbf{E}\right) \tag{2}$$

where $(\mathbf{E}', \mathbf{B}')$ are the electric and magnetic fields in the new inertial reference frame, $(\mathbf{E}, \mathbf{B})$ are the fields in the original frame, and $\gamma = 1/\sqrt{1 - v_r^2/c^2}$ with *c* being the speed of light.

It is clear from these equations that there is not a unique low-velocity limit because the field transformations do not depend on velocity exclusively, but also on the electric and magnetic fields themselves. Le Bellac and Lévy-Leblond (1973) identified two low-velocity limiting cases, the "electric" and "magnetic" limits of PGR, and this spawned a literature on the topic of "Galilean electromagnetism". The electric and magnetic limits correspond to which field magnitude is dominant. The electric limit applies when $E \gg cB$, and the magnetic limit applies when $cB \gg E$. Fortunately, high latitude electrodynamics and MI coupling usually encompasses only one of these limits: the magnetic. The magnetic limit applies because the Earth's magnetic field is relatively strong and plasmas are quasi-neutral. Charge neutrality, corresponding to equal numbers of positive and negative charges occupying the same volume, reduces the strength of electric fields, while the ability of charges to move freely in tenuous plasmas creates currents that are the source of magnetic fields, adding to the Earth's geomagnetic field.

For these reasons, we focus here on the magnetic limit of the PGR ($cB \gg E$) and assume that terms of order v_r/c are small. The Lorentz transformation laws (Equations (1) and (2)) become (Le Bellac and Lévy-Leblond, 1973):

$$\mathbf{E}' = \mathbf{E} + \mathbf{v}_r \times \mathbf{B} \tag{3}$$

$$\mathbf{B}' = \mathbf{B} \tag{4}$$

which are familiar transformation rules in the context of space physics (e.g. Parks, 2007; Vasyliūnas and Song, 2005). We refer to these low-velocity limit equations as comprising a Galilean transformation, by analogy to the more general Lorentz transformation.

An alternative derivation of the Galilean transformation of electric and magnetic fields is possible by considering the Lorentz force law (Preti et al., 2009; Heras, 2010). The Lorentz force $\mathbf{F}$ in inertial reference frame $\mathcal{A}$ is given by:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{5}$$

where $\mathbf{F}$ is the force on a charge q moving with velocity $\mathbf{v}$ in frame $\mathcal{A}$, and where the electric field is $\mathbf{E}$ and the magnetic flux density is $\mathbf{B}$. If applied to a charged particle of mass m, the Lorentz force will result in acceleration $\mathbf{F}/m$. This acceleration is independent of inertial reference frame. Therefore, if $\mathbf{F}'$ represents the force measured in an inertial reference frame $\mathcal{B}$ moving with velocity $\mathbf{v}_r$ with respect to the original frame $\mathcal{A}$, we know that $\mathbf{F}' = \mathbf{F}$. In reference frame $\mathcal{B}$ the particle velocity is $\mathbf{v}' = \mathbf{v} - \mathbf{v}_r$, and the particle's mass and charge are invariant with respect to inertial reference frame (Galilean low-velocity limit, to first order in v_r/c). The equality of forces between the two inertial reference frames requires that:

$$(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = (\mathbf{E}' + (\mathbf{v} - \mathbf{v}_r) \times \mathbf{B}') \tag{6}$$

which is achieved if the transformation equations (3) and (4) are used.

Equation 3 shows that the component of the electric field parallel to $\mathbf{B}$ is unchanged under a Galilean transformation, whereas the electric field component perpendicular to both $\mathbf{v}_r$ and $\mathbf{B}$ changes depending on the relative velocity of frame $\mathcal{B}$. Equation (4) states that $\mathbf{B}$ is a Galilean invariant (GI) and the same in both inertial frames, but of course the invariance of $\mathbf{B}$ only holds if there is a very small net charge density (see below), a condition generally consistent with MI-coupling theories. The frame-variant nature of the electric field has significant implications within the context of high-latitude electrodynamics, as we discuss below. Table 1 summarizes how different physical quantities relevant to high latitude electrodynamics vary under a Galilean transformation in the magnetic limit.

The source terms of the fields, net charge density ρ and current density $\mathbf{J}$, also transform according to the PSR. As shown by Rousseaux (2014), they transform as a four vector according to:

$$\mathbf{J}' = \mathbf{J} - \gamma \mathbf{v}_r \rho + (\gamma - 1)\frac{\mathbf{v}_r(\mathbf{v}_r \cdot \mathbf{J})}{v_r^2} \tag{7}$$

$$\rho' = \gamma\left(\rho - \frac{(\mathbf{v}_r \cdot \mathbf{J})}{c^2}\right) \tag{8}$$

In the magnetic limit, the transformation equations become (Le Bellac and Lévy-Leblond, 1973):

$$\rho' = \rho - \frac{(\mathbf{v}_r \cdot \mathbf{J})}{c^2} \tag{9}$$

$$\mathbf{J}' = \mathbf{J} \tag{10}$$

For the magnetic limit to apply, the condition $c\rho \ll J$ must hold also. The Galilean invariance of current expressed by Equation (10) is familiar in the context of high-latitude (?) ionospheric electrodynamics (e.g. Thayer and Semeter, 2004) and is intuitive when net charge density is zero or very small ($c\rho \ll J$). It is intuitively obvious that invariance of the magnetic field (Equation (4)) *requires* very small charge densities, because to first order in $\mathbf{v}_r$, a charge density becomes a current in the moving reference frame, and such a current will modify the magnetic field to first order in v_r/c. Therefore, MI coupling theories that implicitly rely on magnetic field invariance cannot admit the existence of significant charge densities. We note also (Table 1) that zero charge density in the original reference frame leads to a small charge density in the moving frame according to Equation (9), but this is a second-order effect in v_r/c.

The literature of ionospheric electrodynamics discusses non-zero charge densities or "charge accumulation" (e.g. Figure 3 of Vasyliunas, 2012), leading to "polarization electric fields" (Richmond and Thayer, 2000). To our knowledge, the inconsistency between non-zero charge densities and frame-invariant currents has not been discussed in the MI coupling literature. This inconsistency requires further analysis to ensure that the condition $c\rho \ll J$ is satisfied for these treatments.

It is important to understand the magnitude of electric field changes that can occur when considering relative velocities that are typical at high latitudes where MI coupling occurs. In Figure 1 we depict an idealized situation where electric and magnetic fields are at perpendicular angles to each other, to represent approximately the high latitude ionosphere where the Earth's magnetic field is close to vertical and large-scale convection electric fields are predominantly horizontal. We will refer to this geometry elsewhere in the text. For typical high latitude electric fields of magnitude ~25 mV/m as viewed in an Earth-fixed frame, and typical magnetic field magnitudes of ~50,000 nT (near 110 km altitude), the electric field will be nearly zero in a reference frame moving with a speed of ~0.5 km/s relative to the Earth. The direction of such a moving frame is shown in Figure 1. A speed of ~0.5 km/s relative to the Earth is consistent with the low velocity limit of Galilean electrodynamics and consistent with velocities of the plasma or neutral species relative to the Earth. An electric field that is non-negligible observed from an Earth-fixed frame can be zero viewed from an inertial reference frame moving at speeds consistent with the low velocity limit of the PSR, and with the Galilean transformation rules of Table 1. Although an Earth fixed frame is not inertial, the small acceleration in this frame is typically ignored for the purposes of these estimates.

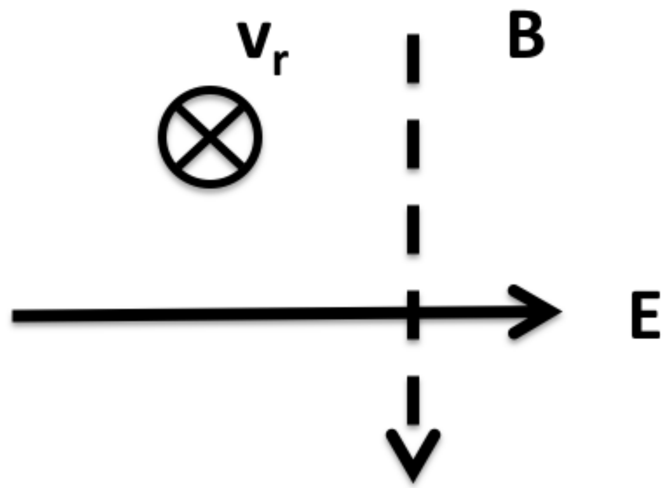

Figure 1. Schematic representation of electric and magnetic fields at northern high latitudes. Magnetic fields are directed vertically (B). Electric fields are directed horizontally (e.g. in an Earth fixed frame, resulting from magnetospheric convection). In a reference frame moving with velocity $\mathbf{v}_r$ in the direction shown (into the page), the electric field is zero if the reference frame moves with speed $\|\mathbf{E}\|/\|\mathbf{B}\|$. For "typical" disturbed conditions, non-relativistic speeds of ~ 1.6 km/s are sufficient.

Table 1. Transformation of physical quantities under change of inertial reference frame in the Galilean magnetic limit, which applies when $cB \gg E$ and $c\rho \ll J$. The primed quantities are in the new reference frame moving at velocity $\mathbf{v}_r$ relative to the original frame. All quantities are assumed to be quasi-static or slowly varying. Galilean invariance refers to quantities that are the same in all inertial reference frames.

Physical Quantity	Transformation	Comment
Electric field **E**	$\mathbf{E}' = \mathbf{E} + \mathbf{v}_r \times \mathbf{B}$	The electric field in a direction parallel to **B** is invariant, as is the electric field in the absence of a magnetic field
Magnetic field **B**	$\mathbf{B}' = \mathbf{B}$	
Current density **J**	$\mathbf{J}' = \mathbf{J}$	Assumes no net charge. Not invariant if charges are present.
Charge density ρ	$\rho' = \rho - \frac{1}{c^2}\mathbf{v}_r \cdot \mathbf{J}$	Charge per unit volume.
Heat energy **Q**	$\mathbf{Q}' = \mathbf{Q}$	Heat energy and temperature are Galilean invariant (GI)
Velocity **v**	$\mathbf{v}' = \mathbf{v} - \mathbf{v}_r$	
Relative velocity between two species	Invariant	
Collisional forces	Invariant	Depends on relative velocities, which are invariants

There are several implications of the electric and magnetic field transformation rules in Table 1. First, Maxwell's equations are not invariant under these transformations, unless the displacement current term is neglected in Ampere's law (Le Bellac and Lévy-Leblond, 1973; Preti et al., 2009; Heras, 2010). Second, currents in a moving reference frame that arise from accumulated charges in the original frame do not generate magnetic fields (Le Bellac and Lévy-Leblond, 1973). The theoretical implications of these inconsistencies are not discussed further here. The PGR is mentioned in textbooks on electrodynamics such as Jackson (1975) and Pollock and Stump (2001), primarily to show how the PGR fails in the context of electromagnetism. A point is made, however, that the physical principle of relativity demonstrates that "**E** and **B** have no independent existence" (Jackson, 1975), which is true for Galilean as well as special relativity.

3 The PSR in Geospace

Discussions of the PSR in the context of geospace are diverse. On the one hand, widely-used textbooks that describe ionospheric electrodynamics (Kivelson and Russell, 1995; Kelley, 2009; Brekke, 2013) do not refer to the PSR or its Galilean limit. In the journal-based literature, several authors discuss inertial reference frames in the context of high latitude electrodynamics (examples are provided in the text below). The reference frame-dependent property of the electric field is mentioned on occasion, but not emphasized or exploited in many cases.

We note that a "preferred reference frame" is a useful construct in plasma physics because of the importance of material media that obey the laws of classical mechanics, such as the plasma and the neutral atmosphere (Vasyliunas and Song, 2005; Leake et al., 2014). Song et al. (2001) derive different versions of Ohm's law appropriate to reference frames that move with either the plasma bulk flow or the neutrals, including different expressions for the conductivities in these two reference frames.

An important construct to examine from the perspective of the PGR, and widely seen in the literature, is based on the following quantity: $\mathbf{E} + \mathbf{V} \times \mathbf{B}$, where $\mathbf{V}$ is the velocity of a constituent of the material medium (ions, electrons, neutrals, etc.). In several publications the quantity $\mathbf{E} + \mathbf{V} \times \mathbf{B}$ is referred to as "the electric field in the reference frame of species X" where $\mathbf{V}$ is the bulk velocity of that species (Vasyliūnas, 2012; Vasyliūnas and Song, 2005; Thayer and Semeter, 2004; Richmond, 1995; Leake et al., 2014; Hidekatsu, 2009; Strangeway, 2012). The symbol $\mathbf{E}'$ is often used to denote this quantity, but in this work, we will use the convention $\mathbf{E}^* = \mathbf{E} + \mathbf{V} \times \mathbf{B}$ (following Vasyliūnas and Song, 2005) to maintain use of the prime symbol to refer to transformations between inertial reference frames. The second term in the expression for $\mathbf{E}^*$ has been referred to as a dynamo electric field when $\mathbf{V} = \mathbf{V}_n$, the velocity of the neutral species (Richmond, 1995). However, $\mathbf{V}_n \times \mathbf{B}$ has the units of electric field, but not the transformation properties of an electric field, so this second term in the expression for $\mathbf{E}^*$ does not refer to a physical electric field.

The GI property of $\mathbf{E}^*$ holds no matter what the velocity $\mathbf{V}$ refers to, whether that of the neutral species, or the ions, etc. The use of $\mathbf{E}^*$ has encouraged the exploration of reference frames tied to material media (e.g. Leake et al., 2014), whereas for electrodynamic quantities there is no need for a preferred inertial reference frame. In many situations applicable to the high latitude ionosphere, *there exists a reference frame for which the electric field is close to zero* (Figure 1). This frame is not tied to any particular medium, but is instructive to consider. As we show in the discussion of Ohm's law, currents can arise in the absence of electric fields for precisely the reason that currents depend on $\mathbf{E}^*$ rather than $\mathbf{E}$. Considering this special inertial reference frame – where the electric field is close to zero – provides insight into the physical basis for momentum and energy changes at high latitudes.

3.1 The PSR and Magnetosphere-Ionosphere Coupling

We focus in this paper on the quasi-static electrodynamic perspective where inductive electric fields and inertial effects are not important and where force balance applies between the different species: ions, electrons and neutrals. Even within this restricted domain, where the MI coupling theories describe large-scale plasma convection and slowly-varying currents, the MI coupling literature contains several examples of theoretical formulations that do not adhere to the PSR or PGR. Formulations that do not follow the PSR tend to de-emphasize the important role of the neutral wind – specifically the relative velocity of the neutrals and ions – in MI coupling.

A widely cited foundational MI coupling publication is Fedder and Lyon (1987) that couples an ideal magnetohydrodynamic (MHD) model for the magnetosphere with the ionosphere, via the equation (7) in that paper:

$$\nabla \cdot \bar{\bar{\Sigma}} \cdot \mathbf{E} = \mathrm{J}_{\parallel} \tag{11}$$

where $\mathbf{E}$ is the electric field in the ionosphere, $\bar{\bar{\Sigma}}$ is the ionospheric conductivity tensor, and $\mathrm{J}_{\parallel}$ is the field aligned current entering the ionosphere from the magnetosphere. This equation expresses the fact that the currents flowing between ionosphere and magnetosphere are divergence-free and thus there is no net charge accumulation. The divergence of the horizontally flowing current in the ionosphere given by $\bar{\bar{\Sigma}} \cdot \mathbf{E}$ must be balanced by the field aligned current $\mathrm{J}_{\parallel}$. While Fedder and Lyon (1987) addresses fundamental aspects of MI coupling, Equation (1) does not adhere to the PSR. This contrasts with an earlier paper by Vasyliunas (1970) where the MI coupling is expressed as follows (equations 7 and 8 in that paper):

$$\nabla \cdot \bar{\bar{\Sigma}} \cdot (\mathbf{E} + \mathbf{V}_{\boldsymbol{n}} \times \mathbf{B}) = \mathrm{J}_{\parallel} \tag{12}$$

where we have assumed that the magnetic field inclination is vertical to match the approximation in Fedder and Lyon (1987). Equation (12), unlike Equation (11), is consistent with the PSR.

3.2 The PSR and Ohm's Law

Ohm's law is derived from the plasma force balance equations taking collisions into account (Song et al., 2001; Richmond, 1995). These equations can be written, for ions and electrons, as:

$$qN_e(\mathbf{E} + \mathbf{u}_{\boldsymbol{i}} \times \mathbf{B}) = N_e m_i \upsilon_{in}(\mathbf{u}_{\boldsymbol{i}} - \mathbf{u}_{\boldsymbol{n}}) + N_e m_i \upsilon_{ie}(\mathbf{u}_{\boldsymbol{i}} - \mathbf{u}_{\boldsymbol{e}}) \tag{13}$$

$$-qN_e(\mathbf{E} + \mathbf{u}_{\boldsymbol{e}} \times \mathbf{B}) = N_e m_e \upsilon_{en}(\mathbf{u}_{\boldsymbol{e}} - \mathbf{u}_{\boldsymbol{n}}) - N_e m_e \upsilon_{ei}(\mathbf{u}_{\boldsymbol{i}} - \mathbf{u}_{\boldsymbol{e}}) \tag{14}$$

Charge neutrality is assumed such that N_e is the charge density of either electrons or ions. q is the elementary charge, $\mathbf{u}_e$, $\mathbf{u}_n$ and $\mathbf{u}_i$ are the electron, neutral and ion velocities, respectively, and m_i and m_e are the ion and electron masses, respectively. We assume a single ion species for simplicity. υ_{in}, υ_{ie}, υ_{en}, and υ_{ei} are the ion-neutral, ion-electron, electron-neutral and electron-ion momentum transfer rates, respectively. These rates are sometimes referred to as "collision rates". The following reciprocity relation applies to these rates: $m_k \upsilon_{kl} = m_l \upsilon_{lk}$, where k, l represent one of the species: electron, ion, or neutral (see Gombosi, 2004), and $k \neq l$. We are ignoring pressure forces and other forces such as gravity, centrifugal, etc. Since these are force balance equations, the acceleration of each species is zero. It is clear these equations are consistent with PGR and are valid in any inertial reference frame, as can be readily seen by applying the transformations from Table 1. A simplification is that the momentum transfer rates do not depend on the relative velocities; see Richmond (1995) for a statement regarding this limitation. Equations (13) and (14) apply under quasi steady-state conditions and do not apply during rapid changes or in the presence of dynamical processes associated with reconnection-driven substorms.

A form of Ohm's law that is derived from these force balance equations is as follows (see derivation provided by Song et al., 2001):

$$\mathbf{J} = \sigma_{\parallel}\mathbf{E}_{\parallel} + \sigma_P(\mathbf{E}_{\perp} + \mathbf{V} \times \mathbf{B}) + \sigma_H \mathbf{B} \times (\mathbf{E} + \mathbf{V} \times \mathbf{B})/B \qquad (15)$$

where the current density $\mathbf{J}$ is given by $qN_e(\mathbf{u}_i - \mathbf{u}_e)$ (charge neutrality is assumed). $\sigma_{\parallel}, \sigma_P$ and σ_H are the parallel, Pedersen and Hall conductivities, respectively. $\mathbf{E}_{\perp}$ is the component of the electric field perpendicular to $\mathbf{B}$. $\mathbf{V}$ might be the bulk plasma velocity or the neutral wind velocity (see Song et al., 2001, Equation (17). In our Equation (15), we have corrected typographical errors of Song et al.'s (2001) equation (17)). The conductivities typically involve terms that are GI, such as collision frequencies and gyrofrequencies that depend on the invariant magnetic field. Since conductivities are GI, this form of Ohm's law is explicitly GI, since both the left-hand side and right-hand side do not change under the Galilean transformation.

Ohm's law Equation (15) could be interpreted to imply that currents can be generated by 1) electric fields alone, 2) neutral winds alone, or 3) a combination of the two. The choice of inertial reference frame influences these three different possibilities. Inferring a causal connection between electric fields and currents depends on reference frame. If an inertial reference frame is chosen such that the perpendicular electric field is zero, then in that reference frame the currents are not associated with electric fields. Strangeway (2012) also questions whether electric fields cause the currents, instead suggesting that both the electric field and the currents are a consequence of the plasma flow. From the perspective of the PSR, it must be concluded that it is inconsistent to assert that electric fields cause plasma flow, at least in certain reference frames. As we discuss below, it is consistent with the PSR to suggest that plasma motion relative to neutrals (magnetospheric dynamo) lead to flow differences between the plasma and neutrals that are responsible for currents.

Further clarification is found in Vasyliunas' (2012) work on the "physical basis for ionospheric electrodynamics". Vasyliunas (2012) suggests that the ionospheric current is primarily a stress-balance current ultimately due to the relative motion between plasma and neutrals. That paper thus makes the direct link between two GI quantities, currents and relative velocities, without the problematic intermediary of the reference frame-dependent electric field. In the discussion section, we remark on the implications of a direct relationship between currents and electric fields, as has been proposed in the context of Subauroral Polarization Stream (SAPS) electric fields.

We are motivated to seek an alternative to the traditional Ohm's law, starting with the same force balance equations (13) and (14), but with all quantities being explicitly invariant to reference frame. Equations (13) and (14) can be written as:

$$qN_e\mathbf{F}_i = N_e m_i v_{in}\boldsymbol{\delta}\mathbf{v}_{in} + \left(\frac{m_i v_{ie}}{q}\right)\mathbf{J} \tag{16}$$

$$-qN_e\mathbf{F}_e = N_e m_e v_{en}\boldsymbol{\delta}\mathbf{v}_{en} - \left(\frac{m_i v_{ie}}{q}\right)\mathbf{J} \tag{17}$$

where now all terms in these expressions are explicitly GI. $\mathbf{F}_i$ and $\mathbf{F}_e$ are the Lorentz force per unit volume and per unit charge on the ions and electrons, respectively, and $\boldsymbol{\delta}\mathbf{v}_{in}$ and $\boldsymbol{\delta}\mathbf{v}_{en}$ are the relative velocities between ions and neutrals and electrons and neutrals, respectively. Recognizing that the electron and ion Lorentz forces are related as follows:

$$\mathbf{F}_e = \mathbf{F}_i - \left(\frac{1}{qN_e}\right)\mathbf{J}\times\mathbf{B} \tag{18}$$

and substituting $\mathbf{F}_e$ into Equation (17) we find that:

$$\mathbf{J}\times\mathbf{B} = N_e m_i v_{in}\boldsymbol{\delta}\mathbf{v}_{in} + N_e m_e v_{en}\boldsymbol{\delta}\mathbf{v}_{en} \tag{19}$$

which shows directly the dependence of the current on the relative velocities of ions and electrons to neutrals, independent of reference frame. Multiplying Equation (19) on the left by **B** and using the appropriate vector identity, we find that:

$$\mathbf{J} = \mathbf{B} \times (N_e m_i \upsilon_{in} \boldsymbol{\delta}\mathbf{v}_{in} + N_e m_e \upsilon_{en} \boldsymbol{\delta}\mathbf{v}_{en})/B^2 + \mathbf{B}(\mathbf{B} \cdot \mathbf{J})/B^2 \qquad (20)$$

It is clear from Equation (19) that **J** is defined up to an arbitrary vector parallel to **B**, so we can write:

$$\mathbf{J} = \mathbf{B} \times (N_e m_i \upsilon_{in} \boldsymbol{\delta}\mathbf{v}_{in} + N_e m_e \upsilon_{en} \boldsymbol{\delta}\mathbf{v}_{en})/B^2 + k\mathbf{B} \qquad (21)$$

where k is an arbitrary constant. The component of **J** perpendicular to **B** is of primary interest in the high latitude E-region ionosphere where collisions predominate. Equation (21) shows explicitly that electric fields are not required for horizontal currents to flow at high latitude. What is required is that ionosphere-magnetospheric interaction results in relative motion between ions, electrons and neutrals. We note that Equation (21) is consistent with the physical interpretation of Vasyliūnas (2012) that the "ionospheric current is … not an Ohmic current in the usual sense."

For comparable relative velocities with the neutrals, the ion term in Equation (21) dominates by three orders of magnitude over the electron term, for typical conditions at local noon near ~110 km altitude where horizontal currents typically flow (see Figure 1 of Song et al., 2001 for quantitative estimates. At 110 km altitude, the constant preceding the ion-neutral relative velocity is ~2.1×10^{-12} versus ~1.0×10^{-15} for the electron-neutral term (SI units)). Although FACs originating in the magnetosphere that close horizontally in the ionosphere are often carried by electrons due to their high mobility (Carlson et al., 1998; Sugino et al., 2002), these currents likely do not play an important role in the force balance leading to Ohm's law. The primary factor is ion-neutral relative velocity.

3.3 The PSR and High Latitude Heating

Energy input from the magnetosphere to the ionosphere increases significantly during geomagnetic disturbances. Poynting's theorem (PT) and Poynting flux are often used in the context of high latitude electrodynamics to understand energy deposition (Kelley, 1989; Thayer and Semeter, 2004; Richmond and Matsuo, 2008).

Poynting's theorem is:

$$\frac{\partial W}{\partial t} = -\nabla \cdot \mathbf{S} - \mathbf{J} \cdot \mathbf{E} \tag{22}$$

where W is the energy density of the electromagnetic (EM) field:

$$W = \frac{1}{2}\left(\epsilon_0 \mathbf{E} \cdot \mathbf{E} + \frac{1}{\mu_0} \mathbf{B} \cdot \mathbf{B}\right) \tag{23}$$

and $\mathbf{S}$ is the Poynting vector:

$$\mathbf{S} = \mathbf{E} \times \frac{1}{\mu_0} \mathbf{B} \tag{24}$$

Physically, Equation (22) represents energy conservation. It states that the rate of change of EM energy density W within a volume equals the energy leaving that volume, via divergence of Poynting flux ($\nabla \cdot \mathbf{S}$), plus the rate of work done by the EM field within the volume ($\mathbf{J} \cdot \mathbf{E}$).

Matsuo and Richmond (2008) divide the electromagnetic work done into two terms as follows:

$$\mathbf{J} \cdot \mathbf{E} = \mathbf{J} \cdot (\mathbf{E} + \mathbf{V}_{\boldsymbol{n}} \times \mathbf{B}) + \mathbf{V}_{\boldsymbol{n}} \cdot (\mathbf{J} \times \mathbf{B}) \tag{25}$$

where the first term on the RHS represents the Joule heating or thermal energy contribution (e.g. Cole, 1962; Thayer and Vickrey, 1992) and the second term on the RHS represents the rate at which mechanical energy is transferred to the neutrals. The following standard expression for Joule heating (JH)

$$\mathrm{JH} = \mathbf{J} \cdot (\mathbf{E} + \mathbf{V}_{\boldsymbol{n}} \times \mathbf{B}) \tag{26}$$

is composed of two terms, each of which depends on reference frame. Following in the same spirit as Equation (19), we wish to write JH in terms of quantities that are independent of reference frame. Another expression for JH is given by $\eta J_\perp^2$ (Cole, 1962) where $J_\perp$ is the current perpendicular to $\mathbf{B}$ and η is the inverse of the Cowling conductivity (see eqn. 11 in Vasyliūnas and Song, 2005 for an expression for η). Therefore, using the component of the current perpendicular to $\mathbf{B}$ from Equation (21), JH can be written as:

$$\mathrm{JH} = \eta \, |\mathbf{B} \times (N_e m_i \nu_{in} \boldsymbol{\delta} \mathbf{v}_{in} + N_e m_e \nu_{en} \boldsymbol{\delta} \mathbf{v}_{en})|^2 / B^4 \qquad (27)$$

where all terms are now invariant to Galilean reference frame.

Equation (27) explicitly relates heating to the relative motions of the different plasma species. Such collisional heating has also been termed "frictional heating" in the literature. The discussion in the appendix of Thayer and Semeter (2004) starts with the more conventional Equation (26) and concludes that heating at high latitudes is due to friction between plasma and neutrals. Vasyliunas and Song (2005) reach the same conclusion and state that the heating is not "Joule heating in the physical sense" (see also Brekke and Rino, 1978). Strangeway (2012) considers additional forces than are implicit in the momentum balance Equations (13) and (14), but concludes that ionospheric heating predominantly increases plasma temperature due to plasma motion relative to the neutrals.

4 Discussion

A standard interpretation of magnetosphere-ionosphere coupling is that magnetospheric currents lead to electric fields that in turn lead to plasma motion, ion-neutral velocity differences and finally heating (Milan et al., 2017; Cowley, 2000; Kan, 1997). The electric field is not an invariant, so explanations linking currents to electric fields are problematic from the perspective of the PSR. In addition, Vasyliunas (2001) has shown that electric fields do not cause bulk plasma motion (see also the introduction of Strangeway, 2012). However, the paper by Cole (1962) states that electric fields induce plasma motion, in contradiction to Vasyliunas (2001) and Strangeway (2012).

Consistency with the PSR is achieved if ion-neutral velocity differences are viewed as the primary causative factor of heating (as per the appendix in Thayer and Semeter (2004) or Vasyliūnas and Song (2005)) and currents (Vasyliūnas (2012)). In the introduction section of Vasyliūnas and Song (2005) is it stated that "by virtue of the plasma momentum equation" the ion-neutral velocity difference is proportional to the current density $\mathbf{J}$. From a cause-and-effect perspective, ion-neutral velocity difference *causes* the existence of $\mathbf{J}$.

The PSR is relevant to explanations for the high velocity ion flows known as Subauroral ion drifts (SAID), which are often considered as a consequence of Sub-auroral Polarization Stream (SAPS) electric fields. SAPS are postulated to be the result of magnetospheric currents closing in a low-conductivity region of the ionosphere, thus leading to large electric fields (Anderson et al., 1993; Clausen et al., 2012). Currents as the sole driver for high velocity plasma flows in the lower ionosphere is problematic from the perspective of the PSR, because the flow velocity depends on inertial reference frame, but the currents do not. From the perspective of the PSR, it is more satisfactory to suggest that large-scale ionospheric convection is driven by the imposed velocity differences between the solar wind and magnetospheric plasmas without requiring an electric field as intermediary in the causal chain.

One might well ask how the velocity of the neutral species (Equation (26)) is relevant when calculating the heating that results from work done by the electromagnetic field, since the neutral species do not experience the electromagnetic force. The answer is that this equation is based on the single-fluid magnetohydrodynamic (MHD) energy equation (Richmond, 1983), for which the velocity of the fluid is the mass-weighted velocity of the individual species. In the ionosphere, the neutral species dominate in the collisional E-region, so the mass weighted fluid velocity is nearly equal to the velocity of the neutral species. Considering only the electrodynamic contributions to energy, we can equate the energy lost by the electromagnetic field, $\mathbf{J} \cdot \mathbf{E}$, to the work done by the $\mathbf{J} \times \mathbf{B}$ force on the fluid, plus the heating of the fluid:

$$\mathbf{J} \cdot \mathbf{E} = \mathbf{V}_{\boldsymbol{n}} \cdot (\mathbf{J} \times \mathbf{B}) + \mathrm{JH} \tag{28}$$

which immediately implies Equation (26). Thus, the standard expression for Joule heating is closely tied to single fluid MHD theory, whereas the expression for JH embodied in Equation (27) admits of multiple species explicitly, and is simpler to reconcile with the concept of “frictional heating” between species. The appendix of Thayer and Semeter (2004) goes to considerable lengths to reconcile the single fluid and multi-species treatments as they pertain to high latitude heating.

The chain of interactions that lead to large-scale MI coupling would then appear to be:

- Coupling between the solar wind and magnetosphere imparts momentum to the magnetospheric plasma at altitudes of a few Earth radii. This coupling is primarily electrodynamic in character, since collisions between solar wind and magnetospheric ions are infrequent at such altitudes. This is referred to as "mechanical coupling" in Strangeway and Raeder (2001).
- This collisionless interaction, which also leads to magnetic flux being frozen into the magnetospheric plasma, causes magnetospheric plasma motion at progressively lower altitudes, eventually driving ion motion at low altitudes in the ionosphere.
- The momentum imparted to the ionospheric plasma by the magnetospheric plasma motion creates a velocity difference between the ionospheric plasma and the neutral species at ionospheric altitudes, leading to transfer of momentum to the neutral species via collisional processes that become important in the lower ionosphere. The ion-neutral velocity differences lead to the presence of horizontal currents and heating in the ionosphere (Vasyliūnas, 2012; Mannucci et al., 2018).

In this picture, partly alluded to in Vasyliunas and Song (2005), the currents are by-products of momentum transfer (as the plasma equations suggest) and not the primary drivers in the causal chain that couples the ionosphere to the magnetosphere (see also Vasyliunas, 2012).

Momentum transfer between ions and neutral species appears central to understanding cause and effect relationships in magnetosphere-ionosphere coupling (e.g. using the multispecies plasma equations in Vasyliunas and Song, 2005). Momentum transfer is independent of inertial reference frame, and so can be considered a primary focus for understanding high latitude processes without difficulty. Conversely, in the high latitude ionosphere, the electric field depends on the inertial reference frame, so electric fields as the primary cause of high latitude electrodynamics can be problematic.

Physical insight from the PSR is particularly relevant to models that relate region 1 currents directly to the convection electric field. For example, Siscoe et al. (2002) writes in the introduction that "Φ_m (magnetospheric convection potential) is then impressed via equipotential magnetic field lines onto the ionosphere, where it becomes the Φ_{pc} (transpolar ionospheric potential) that generates region 1 currents." This statement suggests a causative relation between the high latitude convection electric field and region 1 currents. We have discussed above how Ohm's law (Equation (15)) is not in all cases a relationship between currents and electric fields, and that ion-neutral velocity differences can play an important role in generating currents. Consistency with the PSR does not permit a direct relationship between currents (Galilean invariant) and electric fields alone (not invariant).

The role of the neutrals seems to have become de-emphasized in several studies of magnetosphere-ionosphere coupling, where the following equation (or close versions) is often used to relate field aligned currents $J_{||}$ to electric fields:

$$\nabla \cdot (\bar{\bar{\Sigma}} \cdot \mathbf{E}) = J_{||} \qquad (29)$$

where $\bar{\bar{\Sigma}}$ is the conductance tensor (Fedder and Lyon, 1987). Siscoe et al. (2002) use a simplified version of this equation. Raeder (2003), and Rothwell and Jasperse (2006) use similar equations to (29), citing Vasyliunas (1970) in that context. However, the relevant equation (8) in Vasyliunas (1970) has the term $\mathbf{E} + \mathbf{V_n} \times \mathbf{B}$ (SI units) in the above equation, instead of $\mathbf{E}$ alone. In the derivation in Wiltberger et al. (2003), the starting point is $\mathbf{J} = \bar{\bar{\Sigma}} \cdot \mathbf{E}$, (their Equation 1), which differs from Ohm's law (our Equation (15)). In these works, the neutral wind term, needed to maintain consistency with the PSR, has been dropped despite its being in the original reference from Vasyliunas (1970).

In a review of large-scale high-latitude ionospheric electrodynamic fields and currents focused on the Assimilative Mapping of Ionospheric Electrodynamics (AMIE) procedure, Lu (2017) discusses the neglect of neutral wind effects that can have a significant impact on JH, citing examples where the neutral winds can enhance or decrease JH by 400% or 40% respectively. In an Earth fixed frame, high latitude convection velocities can reach 1 km/s, which implies that velocity differences between the ions and neutrals can reach similar magnitudes, particularly in the early phases of storms. The $\boldsymbol{\delta}\mathbf{v} \times \mathbf{B}$ magnitudes can reach values of 50 mV/m, comparable to the largest electric fields during intense storms. Therefore, MI coupling theories based on formulations that use Equation (29) will be quantitatively challenged by neglecting the relativistic implications of (29) where a velocity term is missing (see Equation (12)).

5 Conclusions

We have reviewed the principle of special relativity in the context of high latitude electrodynamics, where large-scale ionospheric convection is occurring, to show that commonly-used relationships that transform electric and magnetic fields between inertial frames correspond to a low-velocity limiting case of the PSR known as the "magnetic limit" of Galilean electromagnetism. This limit is used in the literature related to magnetosphere-ionosphere coupling (Equations (3) and (4)). The magnetic limit of special relativity applies when electric and magnetic field magnitudes are related by $cB \gg E$ which is valid in the collisional region of the high latitude ionosphere (altitudes near 110 km). We have noted that previous authors have shown that Maxwell's equations are invariant under the Galilean transformation only if the displacement current term is removed.

The PSR provides insight into the physical basis for magnetosphere-ionosphere coupling. We have considered Ohm's law from the perspective of the PSR and noted that Ohm's law need not be considered as a relationship between currents and electric fields. We suggest that physical insight is gained by considering an inertial reference frame that is not tied to a particular species of the material medium: the reference frame in which the large-scale electric field is zero. This provided motivation to develop a version of "Ohm's law" that does not depend on the electric field and for which all terms in the equation are Galilean invariants (Equation (21)). Currents arise due to velocity differences between ions, neutrals and electrons. We suggest that more accurate modelling of the coupled MI-system will occur when divergence of horizontal ionospheric currents is based on Equation (21), versus equations such as (29) that are not compliant with the PSR. However, this requires information on neutral winds. Undersampling of neutral winds, as noted by Lu (2017), is a significant scientific issue that needs to be addressed to improve understanding of MI coupling and to predict space weather. A planned NASA mission, the Geospace Dynamics Constellation (GDC STDT, 2019) may provide valuable information. A scientific objective of GDC is: "Determine how high-latitude plasma convection and auroral precipitation drive thermospheric neutral winds." The scientific emphasis of GDC on the interactions between charged and neutral particle populations is a needed scientific basis from the perspective of special relativity.

Joule heating at high latitudes can be understood without reference to an electric field, as being due to friction between plasma and neutrals, as shown by detailed multi-species plasma calculations (Brekke and Rino, 1978, Vasyliunas and Song, 2005 and Thayer and Semeter, 2004). JH can be expressed in a form for which all terms are Galilean invariants, by combining the MHD-based formulation that JH is proportional to $J_\perp^2$ and using Equation (21) to compute $J_\perp$ (set $k = 0$). Despite the term "Joule heating" that is associated with experiments where heating is caused by currents generated by electric fields, high latitude heating cannot depend exclusively on electric fields, the latter being reference frame-dependent.

The Galilean transformation rules for currents and electric fields are very different from each other. This suggests it is problematic to assert that currents of magnetospheric origin *cause* horizontal electric fields in the ionosphere. Such a causal relation would seem to be reference frame dependent. At E-layer altitudes in the ionosphere, currents are independent of inertial reference frame, whereas perpendicular electric fields are not. Thus it is problematic to assert that horizontal currents closing in the ionosphere are the primary driver of high-velocity plasma flows known as SAID. Similarly, equations that relate field aligned currents to the divergence of high latitude horizontal currents must invoke the full Ohm's law, that shows the currents are not dependent solely on high latitude electric fields, but also on the neutral wind.

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