

1 **Role of fluid injection on earthquake size in dynamic rupture**  
2 **simulations on rough faults**

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6 **Key Points:**

- 7 • Rupture size is not necessarily limited by the volume of injected fluid in earthquake  
8 simulations with imposed pore pressure perturbations
- 9 • Stress heterogeneity arising from geometric roughness may be the primary cause  
10 for rupture termination on rough faults
- 11 • High initial shear stress or pore pressure can trigger a rupture larger than the volume-  
12 based magnitude limit on faults with low roughness

**Abstract**

An outstanding question for induced seismicity is whether the volume of injected fluid and/or the spatial extent of the resulting pore pressure and stress perturbations limit rupture size. We simulate ruptures with and without injection-induced pore pressure perturbations, using 2-D dynamic rupture simulations on rough faults. Ruptures are not necessarily limited by pressure perturbations when 1) background shear stress is above a critical value, or 2) pore pressure is high. Both conditions depend on fault roughness. Stress heterogeneity from fault roughness primarily determines where ruptures stop; pore pressure has a secondary effect. Ruptures may be limited by fluid volume or pressure extent when background stress and fault roughness are low, and the maximum pore pressure perturbation is less than 10% of the background effective normal stress. Future work should combine our methodology with simulation of the loading, injection, and nucleation phases to improve understanding of injection-induced ruptures.

**Plain Language Summary**

Earthquakes can be induced or triggered by fluid injected deep underground, if the fluid encounters faults. Previous studies of induced seismicity at different injection sites around the world have empirically found that in many cases the maximum magnitude earthquake may be predicted from the total volume of injected fluid. However, this is not always the case, and the level and heterogeneity of pre-existing stress on faults likely plays an important role in determining the final earthquake size. In this paper, we use numerical simulations of earthquakes to quantify one source of stress heterogeneity - that arising from geometric roughness - and study how changes in pore pressure and stress from fluid injection interact with pre-existing stress to influence earthquake size. We find that earthquakes are not limited by the injected volume, except under specific conditions. Instead, earthquakes stop where pre-existing conditions are unfavorable for continued rupture; in our case because of bends in the fault geometry. Earthquakes can well exceed the predicted maximum magnitude, depending on the pre-existing stress on the fault, how rough it is, and the magnitude and extent of the perturbation from injection.

**1 Introduction**

An important question in the study of induced seismicity is whether earthquake magnitudes are limited by the volume of injected fluid or some other injection-related

parameter [e.g., *Baisch et al.*, 2010; *Shapiro et al.*, 2011, 2013; *McGarr*, 2014; *McGarr and Barbour*, 2017; *Maurer and Segall*, 2018], or follow naturally-occurring (Gutenberg-Richter) size variability [*van der Elst et al.*, 2016]. For example, *McGarr and Barbour* [2017] propose an upper bound on seismic moment released by induced earthquakes,  $M_0^{\max}$ , defined by

$$M_0^{\max} = 2G\Delta V \quad (1)$$

where  $G$  is shear modulus, and  $\Delta V$  is injected volume. The premise of such an approach is that pore pressure diffuses through the medium, perturbing the effective stress in a finite volume of crust sufficient to induce and maintain rupture, while stress conditions outside the perturbed region do not allow rupture. To evaluate this hypothesis, we consider the behavior of individual simulated ruptures perturbed by spatially-variable pore pressure increases.

Linear elastic fracture mechanics predicts that under uniform background stress conditions and constant fracture energy, a crack introduced to an elastic solid will grow unstably if its length exceeds a critical value  $a_c$ . Assuming linear slip-weakening friction on a pre-existing fault,  $a_c$  is proportional to the ratio of peak minus residual strength ( $\tau^P - \tau^r$ ) and the square of the static stress drop  $\Delta\tau$  [*Andrews*, 1976]:

$$a_c = \frac{(\tau^P - \tau^r)G}{(\Delta\tau)^2} \frac{f}{(1 - \nu)} d_c \quad (2)$$

where  $G$  is shear modulus,  $\nu$  is Poisson's ratio,  $d_c$  is the slip-weakening distance, and  $f$  is a factor related to the geometry of the problem. In this scenario, there are two possibilities: a crack that does not reach half-length  $a_c$  will naturally self-arrest, while a crack that does will slip indefinitely. *Galis et al.* [2017] applied this reasoning to fluid-induced earthquakes to estimate the size of the largest self-arresting ruptures for spatially-variable peak strength. They considered a stress perturbation due to pore pressure in an otherwise-uniform background stress, approximated as a point load. Since background stress is uniform, when the stress is low the localized strength drop provided by pore pressure drives slip into the (unfavorable) stress environment beyond the pressurized zone. If the background shear stress is high enough, the rupture will continue to grow without limit.

*Norbeck and Horne* [2018] considered quasi-dynamic simulations of induced earthquakes on flat faults with linear slip weakening friction. Based on their simulations, they proposed that induced earthquakes are governed by the ratio  $\tau^b / f^D \bar{\sigma}_0 = f^b / f^D$ . ( $f^b$  is the initial background shear to effective normal stress ratio,  $f^D$  is dynamic friction, and

74  $\bar{\sigma}_0 = \sigma_0 - \Delta p$ , where  $\sigma_0$  is the total normal stress and  $\Delta p$  is the pore pressure.) Events  
 75 on faults for which  $f^b/f^D < 1$  were limited to the pressurized zone, while  $f^b/f^D > 1$   
 76 resulted in runaway ruptures, irrespective of volume injected.

77 In these studies, the only source of stress heterogeneity is that of the perturbations  
 78 in pore pressure. However, pre-existing stress heterogeneity on faults occurs due to geo-  
 79 metric roughness and past fault slip, among other sources. *Dempsey and Suckale* [2016]  
 80 and *Dempsey et al.* [2016] investigated the role of heterogeneity on the size distribution of  
 81 induced earthquakes on 1-D flat faults using a fracture mechanics approach. They solved  
 82 the crack equation of motion numerically [*Freund*, 1998] for a suite of stochastic (frac-  
 83 tal) shear stress profiles. Ruptures arrest naturally due to variations in shear stress, and  
 84 *Dempsey and Suckale* [2016] showed that the distribution of rupture size was controlled by  
 85 the interaction between the spatial distribution of pore pressure and the statistical charac-  
 86 teristics of the fractal stress profiles. In their model, stress heterogeneity was imposed as  
 87 an initial condition, and the rupture size calculation did not account for the potential ef-  
 88 fects of fault roughness (which influences both shear and normal tractions) and off-fault  
 89 plasticity. These effects result in fracture energy that cannot be predicted *a priori*, and  
 90 higher background stress required for rupture [*Dieterich and Smith*, 2009; *Fang and Dun-*  
 91 *ham*, 2013].

92 In this study, we address these issues and explore the hypothesis that induced earth-  
 93 quakes are limited in size by the magnitude and/or spatial extent of the pore pressure  
 94 perturbation, in the context of 1-D rough (fractal) faults embedded in a 2-D elasto-visco-  
 95 plastic medium and obeying a rate-state friction law with strong dynamic weakening [*Dun-*  
 96 *ham et al.*, 2011a,b]. In contrast to the slip-weakening models discussed above, rate-state  
 97 friction does not have a well-defined residual strength. However, for strong rate weaken-  
 98 ing friction there exists a critical stress level  $\tau^{\text{pulse}}$ , at which self-sustaining rupture on  
 99 flat faults is just possible [*Zheng and Rice*, 1998; *Dunham et al.*, 2011a]. When the back-  
 100 ground shear stress is close to  $\tau^{\text{pulse}}$  (referred to here as “low-stress”), ruptures are pulse-  
 101 like: slip occurs in a narrow pulse just behind the rupture front, and shear strength recov-  
 102 ers behind the rupture tip [e.g., *Cochard and Madariaga*, 1994; *Beeler and Tullis*, 1996;  
 103 *Zheng and Rice*, 1998].

104 We simulate earthquakes with and without pore pressure and stress perturbations  
 105 to determine whether rupture size is limited by the volume of injected fluid and/or the

106 spatial extent of the stress changes. Since faults are geometrically rough, we generate sev-  
 107 eral thousand stochastic realizations in order to characterize results statistically. At low  
 108 background shear stress, one might expect the extent of the stress and pore pressure per-  
 109 turbations to exert some control on rupture lengths. However, we find that events may be  
 110 larger than the pressurized region even at low stress if the magnitude of the perturbation  
 111 is sufficiently large. Ruptures are not confined when stress is high, consistent with *Nor-*  
 112 *beck and Horne* [2018] and *Galis et al.* [2017]. Our results suggest that dynamic effects  
 113 and *in situ* stress conditions interact with pore pressure and poroelastic stress perturbations  
 114 to influence rupture size, and that low stress conditions may not be sufficient to guarantee  
 115 ruptures smaller than an injection-related threshold.

## 116 2 Modeling

### 117 2.1 2-D dynamic earthquake simulations

118 We use the 2-D plane strain rupture dynamics code FDMAP [*Kozdon et al.*, 2012,  
 119 2013; *Dunham et al.*, 2011a,b] (see Data and Resources). The model employs a rate-and-  
 120 state friction formulation in the slip law form with strong rate weakening on the fault and  
 121 Drucker-Prager visco-plasticity in the off-fault material [*Rice*, 1983; *Noda et al.*, 2009;  
 122 *Dunham et al.*, 2011a]. There is no quasi-static nucleation phase; events are artificially ini-  
 123 tiated by adding a Gaussian shear stress perturbation at the first time step. Once initiated,  
 124 the rupture process is entirely self-governed. Faults are 1-D self-similar fractal profiles,  
 125 and are oriented such that they lie along the  $y = 0$  line of the model domain; flat faults  
 126 are on the line exactly while rough faults follow it on average. Roughness, parameter-  
 127 ized by amplitude to wavelength ratio  $\alpha$  (Supp Fig. 1), is band-limited, with minimum and  
 128 maximum wavelengths of 300 m and 60 km. Values of  $\alpha$  on natural faults are thought to  
 129 vary over an order of magnitude or more, ranging from 0.001 or less on mature faults like  
 130 the San Andreas, up to perhaps 0.01 [e.g., *Candela et al.*, 2009, 2012; *Sagy and Brodsky*,  
 131 2009; *Brodsky et al.*, 2016; *Fang and Dunham*, 2013]. The initial stress is spatially uni-  
 132 form in the medium; pore pressure can be spatially variable as described in Section 2.3.  
 133 Resolved tractions on rough faults varies along the fault (See Section 2.2), so prior to sim-  
 134 ulation, the fault profile is shifted such that the least stable part of the fault is located at  
 135 the origin, where the initiating stress perturbation is applied.

## 2.2 Stress and slip on geometrically-rough faults

Fault roughness provides additional resistance to slip above that of friction, hence rougher faults require higher stress levels for events to propagate [Dieterich and Smith, 2009; Fang and Dunham, 2013]. This effect is termed “roughness drag” by Fang and Dunham [2013], and is proportional to slip ( $s$ ), roughness level ( $\alpha$ ), and inversely proportional to the minimum roughness wavelength,  $\lambda_{\min}$ . In most of our simulations,  $\lambda_{\min} = 300$  m and  $\tau^{\text{drag}}$  is approximately 10 MPa ( $s/\lambda_{\min}$ ) ( $\alpha/10^{-3}$ )<sup>2</sup>; however,  $\tau^{\text{drag}}$  increases as  $\lambda_{\min}$  decreases (see Supplemental Material). In comparison with the flat-fault simulations (Figure 1), ruptures on rough faults arrest over a wider range of initial background stress ratios, and may even arrest and then re-nucleate due to interacting stresses around fault bends [Bruhat *et al.*, 2016].

## 2.3 Pore Pressure Models

FDMAP does not model the nucleation phase of rupture; therefore, we run experiments imposing several different pore pressure distributions as part of the initial conditions. We simulate pore pressure and poroelastic stress changes based on an injector location centered with respect to the fault but offset by 2 km. Events are initiated at the origin, where both the resolved stress ratio (see Section 2.1) and the pore pressure are highest. Figure 1a-c and Supp. Fig. 2 shows pressure and poroelastic stress changes along the  $y = 0$  line of the model domain for each pore pressure model.

1. Pressure Model 0 (PM0) is the reference case with no pore pressure perturbation.
2. Pressure Models 1 and 2 (PM1 and PM2; Fig. 1a and b respectively) are two realizations of injection into an infinite 2-D (plane strain) poroelastic medium with uniform poroelastic and hydraulic properties, using line source solutions from Rudnicki [1986]. We account for the change in total stress from both poroelasticity and pore-pressure in the medium and on the fault. Pressure decays with distance from the origin  $r$  as  $\exp(-r^2/4ct)$ , with diffusivity  $c$  and time  $t$ . (Parameters for the simulations are given in Supp. tables 1-2.) The pore pressure profiles used in our simulations are for 1000 days of injection with different rates and diffusivities. Peak pore pressure on the  $y = 0$  plane (max  $\Delta p$ ) is 2 MPa for PM1 and 19 MPa for PM2, and drops to 10 kPa at 19 km from the origin for PM1 and 12.5 km for PM2 (Figure 1(a-b)).

167 3. In Pressure Model 3 (PM3; Fig. 1c), we introduce a high-permeability ( $k$ ) zone 20  
 168 km wide, oriented perpendicular to the fault in the out-of-plane direction and cen-  
 169 tered at the origin (initiation region), between two symmetric outer regions with  
 170 low permeability (Supp. Figs. 3-4). We simulate the same volume of injection as  
 171 in PM1, the only difference being the presence of the high permeability zone. The  
 172 resulting pressure distribution drops sharply at the boundaries by  $\sim 4$  MPa on the  
 173  $y=0$  line, introducing an additional length scale into the problem. We solve nu-  
 174 merically for the pressure distribution [Elsworth and Suckale, 2016] (details in the  
 175 Supplemental material) and use the pressure to calculate the effective stress in the  
 176 medium, and ignore poroelastic stress perturbations.

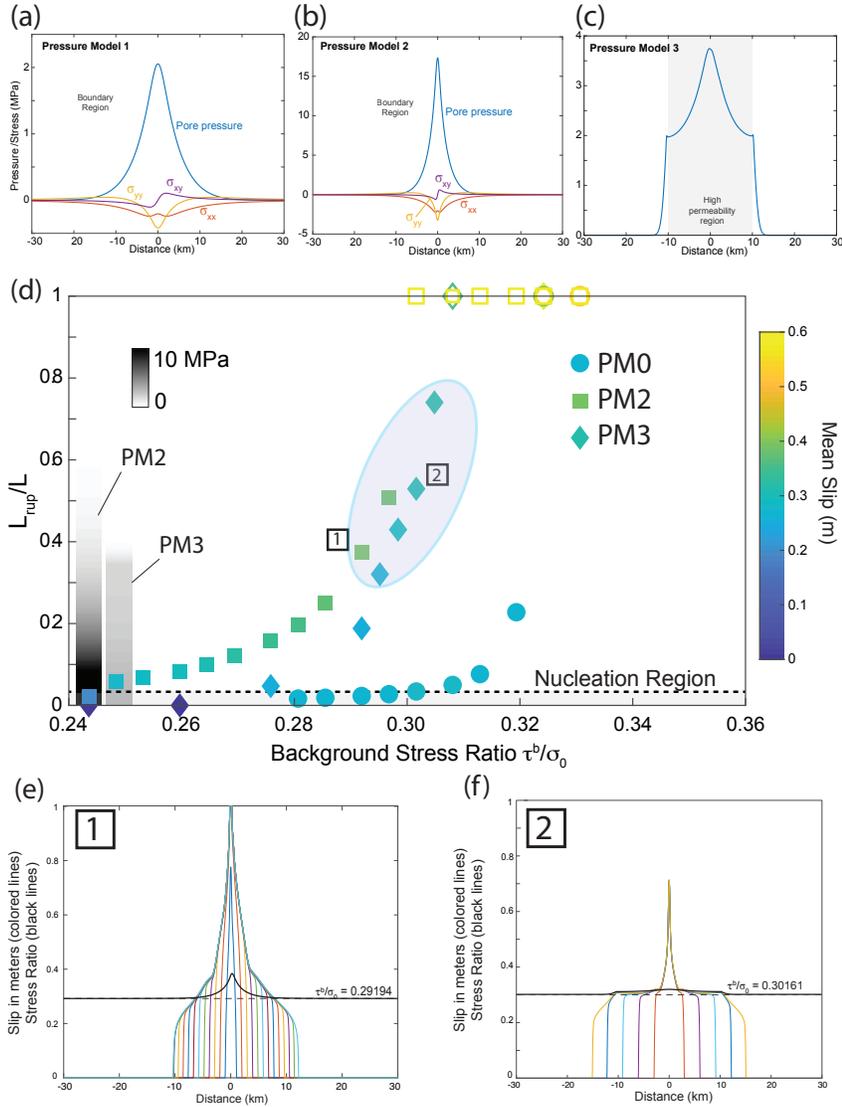
### 177 3 Results

#### 178 3.1 Flat faults with Strong Rate-weakening friction

179 As a reference, we ran a suite of simulations on flat faults. We show results for  
 180 PM0, PM2, and PM3 in Figure 1a; note that PM1 ruptures behave qualitatively similar  
 181 to PM3 but with a smaller effect, so are omitted for clarity. For these simulations,  $\bar{\sigma}_0 = 62$   
 182 MPa. The stress perturbation required to initiate events results in an slip peak at the origin  
 183 (see Fig. 1e,f). Ruptures may arrest immediately or transition to a pulse-like or crack-like  
 184 rupture mode, depending on the stress ratio  $f^b$  (Fig. 1d).

185 For PM0 events (solid circles in Fig. 1a), there is a narrow transition near  $\tau^{\text{pulse}}$   
 186 from self-arresting ruptures to full-fault ruptures, over a range less than 3% of  $\tau^b/\bar{\sigma}_0$ . At  
 187 low background shear stress ( $\lesssim 0.32\bar{\sigma}_0$ ) and no pore pressure perturbation, ruptures arrest,  
 188 while at higher stress ruptures are self-sustaining, consistent with previous work [Zheng  
 189 and Rice, 1998; Dunham et al., 2011a; Gabriel et al., 2012].

199 PM2 ruptures initiate, grow, and become full fault at lower levels and over a broader  
 200 range of background stress ratios than PM0 simulations, due to the decreased strength  
 201 from pore pressure in the nucleation region. Ruptures become self-sustaining at  $\tau^b/\bar{\sigma}_0 \approx$   
 202 0.30, lower than the reference case, even though the stress beyond  $\pm 10$  km from the origin  
 203 ( $L_{\text{rup}}/L = 0.33$ ) is very similar to the unperturbed model. That is, the decrease in pore  
 204 pressure towards the boundaries results in an increase in fault strength, such that away  
 205 from the origin the fault is nearly as strong as the unperturbed case. Rupture are able to  
 206 propagate through the strong region (once initiated inside the weaker perturbed zone), at



190 **Figure 1.** Dynamic ruptures on flat faults. (a-c) Pore pressure and poroelastic perturbation along the  $y = 0$   
 191 line for (a) Pressure Model 1 (PM1), (b) Pressure Model 2 (PM2), and (c) Pore pressure perturbation only for  
 192 Pressure Model 3 (PM3). The shaded region is the high- $k$  zone. (d) Rupture length normalized by fault length  
 193 (60 km) versus background stress ratio  $f^b = \tau^b/\bar{\sigma}_0$  for PM0, PM2, and PM3. Open symbols are for full fault  
 194 ruptures. The gray-scale colorbars on the left show how pore pressure decays with distance (saturated at 10  
 195 MPa for clarity). The colored oval highlights events that are possibly limited by the spatial extent of the pres-  
 196 sure perturbation. (e-f) Example ruptures corresponding to numbered boxes in (d). Dashed line is background  
 197 stress level and perturbed initial effective stress ratio is the solid line. Colored lines show cumulative slip at  
 198 0.7 second-intervals.

207 stress levels where they could not initiate. This is due partially to the larger shear stress  
 208 drop in the nucleation region for the perturbed case, and partially to the strong dynamic  
 209 weakening. Ruptures at lower stress may arrest due to the increase in fault strength en-  
 210 countered outside the perturbed region (Fig. 1e), consistent with Eq. 1.

211 PM3 ruptures (solid diamonds in Fig. 1a) show evidence of arresting due to spatially-  
 212 variable pore pressure. Fig. 1f shows an example, where the rupture begins to propagate at  
 213 a constant rate, then dies out upon reaching the edge of the perturbed zone. This is the  
 214 clearest example of pressure controlling where the rupture stops. At higher background  
 215 stresses, ruptures grow beyond the pressurized zone to the edge of the computational do-  
 216 main. Thus, for PM3 the increase in frictional strength at the edge of the pressurized re-  
 217 gion may influence rupture arrest for a small range of stress ratios  $\sim 0.27 - 0.30$ .

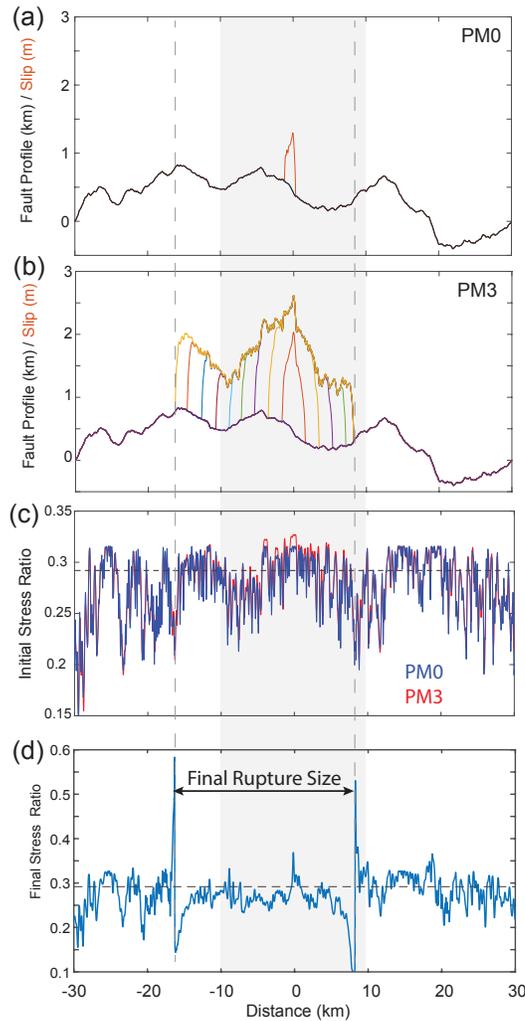
218 To summarize, flat fault simulations show that 1) pore pressure perturbations leads  
 219 to rupture at lower shear stress (or larger ruptures) relative to the reference case, and 2)  
 220 the spatial extent of pore pressure perturbations may limit ruptures in a narrow range of  
 221 stress conditions, but 3) at high shear stress ( $\tau^b / \bar{\sigma}_0 > \tau^{\text{pulse}}$ ) ruptures are unbounded,  
 222 consistent with the results of *Galis et al.* [2017]. The question we consider next is how  
 223 geometric roughness impacts rupture size.

### 224 3.2 Results on rough faults

225 Results for rough faults at a background effective normal stress of 62 MPa are shown  
 226 here; results for 126 MPa are shown in the Supplemental Material. For these simulations,  
 227  $\alpha = 0.004 - 0.012$  and  $f^b \sim 0.015 - 0.45$ . Note that the values of  $f^b$  are lower than  
 228 inferred in previous studies of induced seismicity (0.6-0.8; e.g. *Walsh and Zoback* [2016]),  
 229 which is because the minimum roughness wavelength in the simulations is much larger  
 230 than that expected on natural faults (see Supplemental Material). Fault strength at high  
 231 slip speed depends on fault roughness (due to  $\tau^{\text{drag}}$ ), thus faults with smaller minimum  
 232 roughness wavelength require higher stress to rupture (see the Supplemental material for  
 233 more details).

234 Figure 2 shows two example simulations on the same fault with identical parameters,  
 235 one with no pressure perturbation (PM0) and one with perturbed pressure model PM3.  
 236 Slip in Fig. 2a, without a perturbation, does not extend outside the nucleation region, and  
 237 therefore is considered an ‘arrested’ rupture, while the simulation with PM3 in Fig. 2b

238 ruptures  $\sim 40\%$  of the fault. In this simulation, stress perturbations due to fault geome-  
 239 try dominate the initial stress heterogeneity on the fault (10x larger than the pore pressure  
 240 perturbation). However, the perturbed rupture propagated outside the nucleation region,  
 241 suggesting that the length scale over which the pressure perturbation acts is an important  
 242 factor in determining final rupture size. Comparing the initial and final stresses in Fig-  
 243 ure 2c and d shows that the PM3 rupture arrests due to encountering low-stress barriers at  
 244 restraining bends. Supp. Figs. S5 and S6 show additional simulation examples.



245 **Figure 2.** (a) Unperturbed (PM0) and (b) perturbed (PM3) ruptures for the same fault and background  
 246 stress ratio ( $\bar{\sigma}_0 = 126$  MPa,  $f^b = 0.282$ ,  $\alpha = 0.006$ ). The shaded area is the high permeability zone  
 247 and the vertical dashed lines give the termination of the PM3 rupture. (c) Initial stress for both simulations;  
 248 blue=PM0, red=PM3. Horizontal line is the background stress ratio ( $f^b = \tau^b / \sigma_0$ ). (d) Final stress ratio for  
 249 PM3 rupture. Horizontal dashed line is  $\tau^{\text{pulse}} / \sigma_0$ .

250 In Figure 3 we show summary results for several hundred simulations, illustrating  
 251 two background stress ratios and roughness levels. The left column in Fig. 3 shows empiri-  
 252 cal frequency-length distributions, while the right column shows frequency-moment distri-  
 253 butions. The gray-scale bars at the top left show the spatial extents of pore-pressure per-  
 254 turbation for the different models. Additional event size distributions are shown in Supp.  
 255 Figs. S7-S10. Figure 3 demonstrates the importance of the length scale of the pressure  
 256 perturbation. Pressure models PM1 and PM3 have the same total injected volume, but  
 257 PM3 ruptures propagate farther than PM1. Pore pressure has less of an impact on rupture  
 258 size at high roughness.

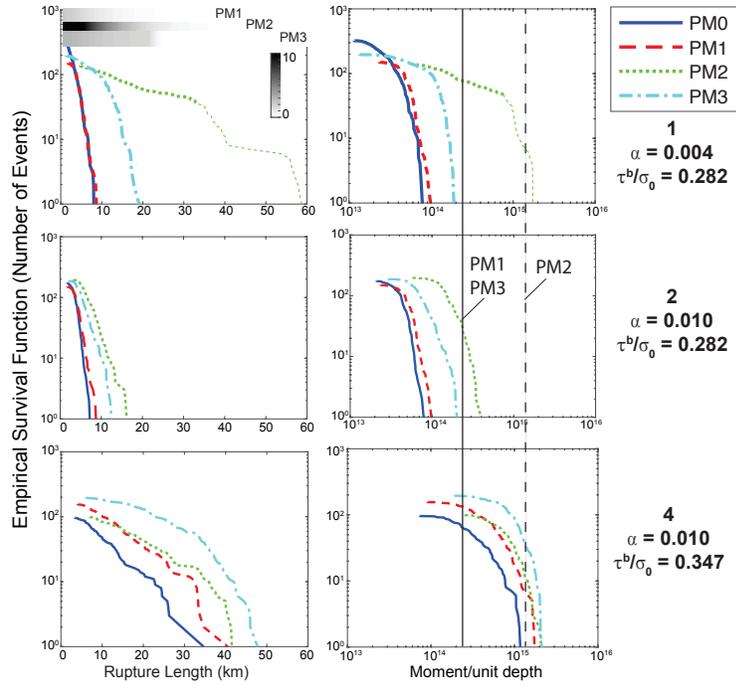
259 The right column of Fig. 3 shows frequency-moment distributions. Moment per unit  
 260 length in the out-of-plane direction ( $D$ ), is defined as the product of the shear modulus  $G$   
 261 with the length-averaged slip  $s(\xi)$ , where  $\xi$  is arclength along the fault trace of length  $L$ :

$$\frac{M}{D} = G \int_L s(\xi) d\xi \quad (3)$$

262 There is a minimum moment imposed by the initiation process of approximately  $2 \times 10^{13}$   
 263 N m /m, while the upper bound on moment corresponds to a full fault rupture (60 km)  
 264 times a few meters of slip, giving a “full-fault” moment between  $\sim 10^{15} - 10^{16}$  N m /m,  
 265 depending on the amount of slip. The injected volume (see Supp. Tables 3-5 for relevant  
 266 parameters) is  $\Delta V = 4 \times 10^3$  m<sup>3</sup>/m for PM1 and PM3, and  $2 \times 10^4$  m<sup>3</sup>/m for PM2.  $M_0^{\max}$   
 267 from Eq. 1 is then  $2.8 \times 10^{14}$  N m/m for PM1 and PM3, and  $1.55 \times 10^{15}$  N m /m for PM2.

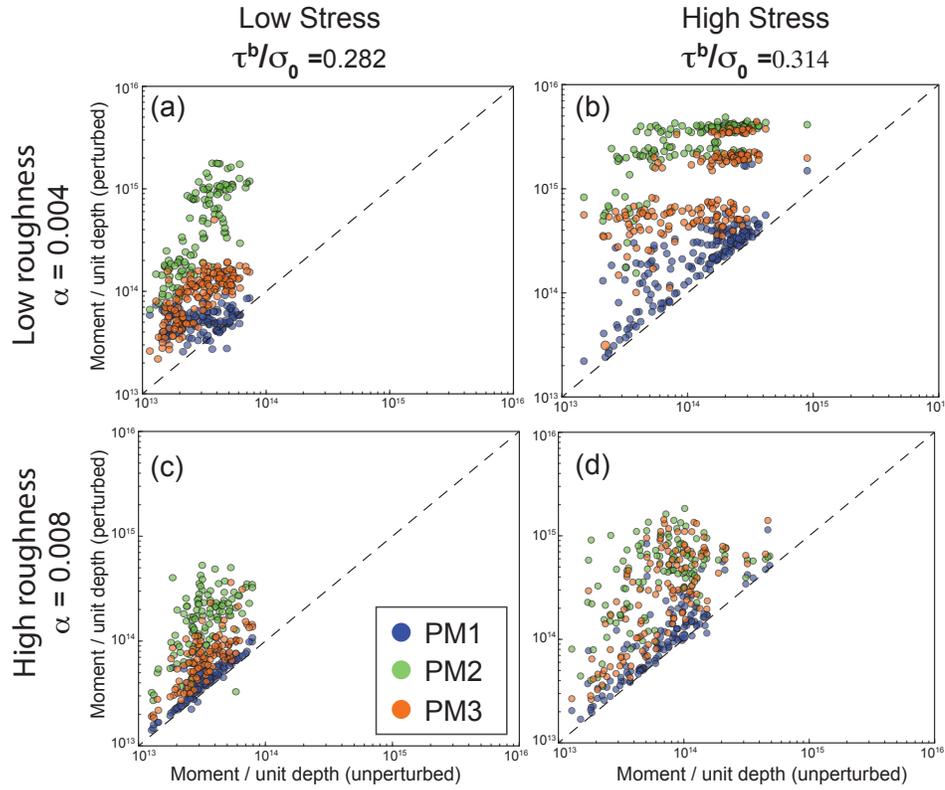
268 At high background stress ( $f^b = 0.347$ ) all of the moment distributions exceed the  
 269 hypothesized bounds. At low background stress ratios ( $f^b = 0.282$ ) the distributions tend  
 270 to tail off well before reaching the hypothesized bounds. At best, PM3 ruptures at low  
 271 stress ( $f^b = 0.282$ ) arrest close to the magnitude limit theorized by *McGarr and Barbour*  
 272 [2017], which may indicate that pore pressure may have a secondary role in stopping rup-  
 273 tures when roughness and stress are low (and compare to Figure 1f for PM3 rupture on  
 274 a flat fault). Even at low background stress, the strong pore pressure perturbations (max  
 275  $\Delta p \sim 30\%$  of the background normal stress) of PM2 are sufficient to induce large rup-  
 276 tures greater than the *McGarr and Barbour* [2017] limit in our simulations (Figure 3, top  
 277 panel).

285 Figure 4 and Supp. Figure 11 show perturbed vs. non-perturbed moment for several  
 286 roughness/stress combinations. As with Figure 3, at higher roughness (Fig. 4c,d), the max-  
 287 imum size of perturbed events is controlled primarily by roughness and background stress,



278 **Figure 3.** Frequency-length distributions (first column) and frequency-moment distributions (second  
 279 column) for two roughness and and background stress levels. The y-axis is one minus the cumulative distri-  
 280 bution function, times the number of non-failed ruptures. Low roughness/high stress results are not shown;  
 281 all events are full-fault ruptures. The gray-scale bars at the top of the left column show the spatial extent  
 282 of the pore pressure perturbation, saturated at 10 MPa. Vertical solid line in the right column represents  
 283  $M_0^{\max} = 2.8 \times 10^{14}$  N m/m for PM1/PM3, and dashed is  $1.55 \times 10^{15}$  N m/m (PM2). The thinner dashed line  
 284 segment in the top two panels are simulations that reach one or both ends of the fault.

288 and secondarily by the injection-induced stress perturbation. In particular, for high stress  
 289 and high roughness, the largest perturbed event (i.e., out of the whole population of events  
 290 with the same stress conditions and fault roughness) is less than four times larger than the  
 291 largest non-perturbed event out of the whole population. The perturbation has a stronger  
 292 impact on rupture size at low roughness. At low stress and roughness (Fig. 4a), strongly-  
 293 perturbed events (PM2) tend to be much larger (by more than an order of magnitude in  
 294 moment) than non-perturbed events, while moderate pressure changes (PM1) result in little  
 295 difference between perturbed and non-perturbed ruptures.



296 **Figure 4.** Perturbed versus non-perturbed moment per unit out-of-plane distance for identical fault geome-  
 297 tries and background stress. Columns are the same background stress, rows are the same roughness. The two  
 298 lines of clustered events in the upper-right plot are ruptures that reach one or both ends of the fault and thus do  
 299 not naturally arrest.

#### 4 Discussion

On flat faults, we find empirically (Fig. 1) that the criteria for when ruptures exceed the pressurized zone is related to the ratio of the background shear stress and  $\tau^{\text{pulse}}$ :

$$f^b = \tau^b / \bar{\sigma}_0 > \tau^{\text{pulse}} / \bar{\sigma}_0 \approx 0.3$$

$$\rightarrow \frac{f^b}{(\tau^{\text{pulse}} / \bar{\sigma}_0)} \leq 1 \quad (4)$$

where the bar in  $\bar{\sigma}_0$  emphasizes that this is the effective normal stress. *Zheng and Rice* [1998] showed that faults for which  $f^b \approx \tau^{\text{pulse}} / \bar{\sigma}_0$  could sustain pulse-like ruptures, while *Norbeck and Horne* [2018] showed that if this criteria is met only locally inside of a pressurized zone, ruptures would be limited by the spatial extent of the zone. Replacing  $f^D$  in their slip-weakening simulations with  $\tau^{\text{pulse}} / \bar{\sigma}_0$  as a modified criteria, our results qualitatively agree with this conclusion.

In contrast to flat faults, on rough faults (with the parameter ranges we have considered:  $10^{-3} < \alpha < 10^{-2}$ ,  $\bar{\sigma}_0 \sim 100$  MPa,  $\Delta p \sim 1 - 10$  MPa), pore pressure plays a less important role compared to stress perturbations from geometry. Comparison of rupture magnitudes with those predicted by the *McGarr and Barbour* [2017] relationship indicates that ruptures are not limited by the volume injected; either ruptures arrest due to local high-strength patches, or ruptures exceed the hypothesized boundary. The exception is at low roughness and low background stress, where pore pressure decay may result in ruptures arresting in some cases (Figure 3, low stress PM3 ruptures; cf. Figure 1f). These results suggest that the role of pore pressure in limiting rupture size is secondary to that of the *in situ* stress level and heterogeneity.

The results shown in Figure 3 demonstrate that stress heterogeneity arising from fault roughness exerts primary control on stopping ruptures. However, the spatial distribution of pore pressure clearly plays an important role. Comparing PM1 with PM3 ruptures, which have identical injected volume, PM3 ruptures can reach larger size than PM1 ruptures regardless of stress and roughness, and can be larger than PM2 ruptures at high roughness. This may be because the higher available stress drop from the perturbation distributed over a smaller region is not able to overcome the resistance to slip of very poorly oriented fault segments. Thus, the pore pressure perturbations does impact rupture size, but not in the simple manner suggested by Equation 1. Instead, the pre-existing stress state, including both the mean value and the heterogeneity in stress and interactions with

329 the spatial distribution and magnitude of the pore pressure perturbation to impact rupture  
330 size.

331 The results presented in this study demonstrate that the addition of pore pressure to  
332 a given background stress state encourages larger ruptures. However, the results do not  
333 address whether the pore pressure distributions considered in this study are realistic in  
334 natural settings. For example, perhaps events in Figure 3 exceeding the moment limits of  
335 Eq. 1 would have nucleated a smaller event at a lower pore pressure. While it is possi-  
336 ble to reach high pore pressure consistent with PM2 in localized areas around an injector  
337 [Häring *et al.*, 2008], this level of pore pressure would not be expected at large depths  
338 and/or distances from the injector. Thus, care must be taken in interpreting the results.  
339 However, no events at low stress exceed the hypothesized limits without additional pore  
340 pressure, so the artificial initiation alone is not sufficient to produce large events.

341 Future research should address the limitations of this study and focus on sequence  
342 simulations of induced earthquakes that account for nucleation and aseismic slip processes  
343 explicitly, and allow rupture to occur naturally, rather than artificially imposing a particular  
344 pressure perturbation and comparing rupture size. Simulations that account for both grad-  
345 ual pressure build-up as well as the dynamic effects that occur during rupture are required  
346 to fully resolve how stress and frictional strength change throughout the earthquake cycle,  
347 and determine whether the results presented here are relevant in more realistic scenarios.

## 348 **5 Conclusions**

349 We have conducted an extensive set of simulations to explore how injection-induced  
350 pore pressure and poroelastic stress changes impact the size of dynamic ruptures on rough  
351 faults. We find that rupture size is not limited by injected volume except when roughness,  
352 background stress, and the pressure perturbation are all low. Events can grow beyond the  
353 pressurized zone and exceed published magnitude limits if  $\tau^b > \tau^{\text{pulse}}$  or the pore pressure  
354 perturbation is large. Higher pore pressure tends to result in larger ruptures; however, at  
355 low background stress and high roughness events never grow as large as published limits.  
356 Only in the limited case of low to no roughness and low background stress ( $\tau^b \leq \tau^{\text{pulse}}$ )  
357 do events appear to ever be limited in size by the size of the perturbed region. Instead, the  
358 results indicate that rupture size is primarily controlled by the *in situ* stress level and het-  
359 erogeneity, and only secondarily by pressure. This is likely partly due to the stress ratio

360 on geometrically-rough faults varying up to 30-70% from the background level for the  
361 parameter ranges considered here, compared to 15% or less for the modeled pressure-  
362 induced perturbations. Future research is required to determine whether our results hold  
363 for naturally-nucleated earthquakes, but at present we suggest that, once nucleated by fluid  
364 injection, induced earthquakes are not required to stop at the boundaries of the pressurized  
365 region.

## 366 **6 Data and Resources**

367 The code for FDMAP is available from <https://bitbucket.org/ericmdunham/fdmap>.  
368 Data from the simulations is available from *Maurer* [2020], last accessed April 13, 2020.

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## 374 **References**

- 375 Andrews, D. J. (1976), Rupture velocity of plane strain shear cracks, *Journal of Geophys-*  
376 *ical Research*, *81*(32), 5679–5687, doi:10.1029/JB081i032p05679.
- 377 Baisch, S., R. Vörös, E. Rothert, H. Stang, R. Jung, and R. Schellschmidt (2010), A  
378 numerical model for fluid injection induced seismicity at Soultz-sous-Forêts, *In-*  
379 *ternational Journal of Rock Mechanics and Mining Sciences*, *47*(3), 405–413, doi:  
380 <http://dx.doi.org/10.1016/j.ijrmms.2009.10.001>.
- 381 Beeler, N. M., and T. E. Tullis (1996), Self-healing slip pulses in dynamic rupture mod-  
382 els due to velocity-dependent strength, *Bulletin of the Seismological Society of America*,  
383 *86*(4), 1130–1148.
- 384 Brodsky, E. E., J. D. Kirkpatrick, and T. Candela (2016), Constraints from fault roughness  
385 on the scale-dependent strength of rocks, *Geology*, doi:10.1130/G37206.1.
- 386 Bruhat, L., Z. Fang, and E. M. Dunham (2016), Rupture complexity and the supershear  
387 transition on rough faults, *Journal of Geophysical Research B: Solid Earth*, *121*(1), 210–  
388 224, doi:10.1002/2015JB012512.

- 389 Candela, T., F. Renard, M. Bouchon, A. Brouste, D. Marsan, J. Schmittbuhl, and C. Voisin  
390 (2009), Characterization of fault roughness at various scales: Implications of three-  
391 dimensional high resolution topography measurements, *Pure and Applied Geophysics*,  
392 *166*(10-11), 1817–1851, doi:10.1007/s00024-009-0521-2.
- 393 Candela, T., F. Renard, Y. Klinger, K. Mair, J. Schmittbuhl, and E. E. Brodsky (2012),  
394 Roughness of fault surfaces over nine decades of length scales, *Journal of Geophysical*  
395 *Research: Solid Earth*, *117*(B8), doi:10.1029/2011JB009041.
- 396 Cochard, A., and R. Madariaga (1994), Dynamic faulting under rate-dependent friction,  
397 *pure and applied geophysics*, *142*(3), 419–445, doi:10.1007/BF00876049.
- 398 Dempsey, D., and J. Suckale (2016), Collective properties of injection-induced earthquake  
399 sequences: 1. Model description and directivity bias, *Journal of Geophysical Research:*  
400 *Solid Earth*, *121*(5), 3609–3637, doi:10.1002/2015JB012550.
- 401 Dempsey, D., J. Suckale, and Y. Huang (2016), Collective properties of injection-induced  
402 earthquake sequences: 2. Spatiotemporal evolution and magnitude frequency dis-  
403 tributions, *Journal of Geophysical Research: Solid Earth*, *121*(5), 3638–3665, doi:  
404 10.1002/2015JB012551.
- 405 Dieterich, J. H., and D. E. Smith (2009), Nonplanar Faults: Mechanics of Slip and  
406 Off-fault Damage, *Pure and Applied Geophysics*, *166*(10-11), 1799–1815, doi:  
407 10.1007/s00024-009-0517-y.
- 408 Dunham, E. M., D. Belanger, L. Cong, and J. E. Kozdon (2011a), Earthquake Rup-  
409 tures with Strongly Rate-Weakening Friction and Off-Fault Plasticity, Part 1: Pla-  
410 nar Faults, *Bulletin of the Seismological Society of America*, *101*(5), 2296–2307, doi:  
411 10.1785/0120100076.
- 412 Dunham, E. M., D. Belanger, L. Cong, and J. E. Kozdon (2011b), Earthquake rup-  
413 tures with strongly rate-weakening friction and off-fault plasticity, part 2: Nonpla-  
414 nar faults, *Bulletin of the Seismological Society of America*, *101*(5), 2308–2322, doi:  
415 10.1785/0120100076.
- 416 Elsworth, W. C., and J. Suckale (2016), Rapid ice flow rearrangement induced by sub-  
417 glacial drainage in West Antarctica, *Geophysical Research Letters*, *43*(22), 697–11, doi:  
418 doi:10.1002/2016GL070430.
- 419 Fang, Z., and E. M. Dunham (2013), Additional shear resistance from fault roughness and  
420 stress levels on geometrically complex faults, *Journal of Geophysical Research: Solid*  
421 *Earth*, *118*(7), 3642–3654, doi:10.1002/jgrb.50262.

- 422 Freund, L. B. (1998), *Dynamic Fracture Mechanics*, Cambridge UP, Cambridge.
- 423 Gabriel, A.-A., J.-P. Ampuero, L. A. Dalguer, and P. M. Mai (2012), The transition of  
424 dynamic rupture styles in elastic media under velocity-weakening friction, *Journal of*  
425 *Geophysical Research: Solid Earth*, 117(B9), doi:10.1029/2012JB009468.
- 426 Galis, M., J. P. Ampuero, P. M. Mai, and F. Cappa (2017), Induced seismicity provides  
427 insight into why earthquake ruptures stop, *Science Advances*, 3(12).
- 428 Häring, M. O., U. Schanz, F. Ladner, and B. C. Dyer (2008), Characterisation of  
429 the Basel 1 enhanced geothermal system, *Geothermics*, 37(5), 469–495, doi:  
430 <http://dx.doi.org/10.1016/j.geothermics.2008.06.002>.
- 431 Kozdon, J. E., E. M. Dunham, and J. Nordström (2012), Interaction of waves with fric-  
432 tional interfaces using summation-by-parts difference operators: Weak enforcement of  
433 nonlinear boundary conditions, *Journal of Scientific Computing*, 50(2), 341–367, doi:  
434 10.1007/s10915-011-9485-3.
- 435 Kozdon, J. E., E. M. Dunham, and J. Nordström (2013), Simulation of dynamic earth-  
436 quake ruptures in complex geometries using high-order finite difference methods, *Jour-*  
437 *nal of Scientific Computing*, 55(1), 92–124, doi:10.1007/s10915-012-9624-5.
- 438 Maurer, J. (2020), 2D Rough Fault Simulations of Induced Earthquakes, doi:  
439 <https://doi.org/10.17605/OSF.IO/TMWCF>.
- 440 Maurer, J., and P. Segall (2018), Magnitudes of Induced Earthquakes in  
441 Low-Stress Environments, *Bulletin of the Seismological Society of America*, doi:  
442 10.1785/0120170295.
- 443 McGarr, A. (2014), Maximum magnitude earthquakes induced by fluid injection, *Journal*  
444 *of Geophysical Research: Solid Earth*, 119(2), 1008–1019, doi:10.1002/2013JB010597.
- 445 McGarr, A., and A. J. Barbour (2017), Wastewater Disposal and the Earthquake Se-  
446 quences During 2016 Near Fairview, Pawnee, and Cushing, Oklahoma, *Geophysical*  
447 *Research Letters*, 44(18), 9330–9336, doi:10.1002/2017GL075258.
- 448 Noda, H., E. M. Dunham, and J. R. Rice (2009), Earthquake ruptures with thermal weak-  
449 ening and the operation of major faults at low overall stress levels, *Journal of Geophysi-*  
450 *cal Research*, 114(B7), B07302, doi:10.1029/2008JB006143.
- 451 Norbeck, J. H., and R. N. Horne (2018), Maximum magnitude of injection-induced earth-  
452 quakes: A criterion to assess the influence of pressure migration along faults, *Tectono-*  
453 *physics*, 733, 108–118, doi:<https://doi.org/10.1016/j.tecto.2018.01.028>.

- 454 Rice, J. R. (1983), Constitutive relations for fault slip and earthquake instabilities, *Pure*  
455 *and Applied Geophysics PAGEOPH*, 121(3), 443–475, doi:10.1007/BF02590151.
- 456 Rudnicki, J. W. (1986), Fluid mass sources and point forces in linear elastic diffusive  
457 solids, *Mechanics of Materials*, 5(4), 383–393, doi:http://dx.doi.org/10.1016/0167-  
458 6636(86)90042-6.
- 459 Sagy, A., and E. E. Brodsky (2009), Geometric and rheological asperities in an ex-  
460 posed fault zone, *Journal of Geophysical Research: Solid Earth*, 114(B2), doi:  
461 10.1029/2008JB005701.
- 462 Shapiro, S. a., O. S. Krüger, C. Dinske, and C. Langenbruch (2011), Magnitudes of  
463 induced earthquakes and geometric scales of fluid-stimulated rock volumes, *GEO-*  
464 *PHYSICS*, 76(6), WC55–WC63, doi:10.1190/geo2010-0349.1.
- 465 Shapiro, S. A., O. S. Krüger, and C. Dinske (2013), Probability of inducing given-  
466 magnitude earthquakes by perturbing finite volumes of rocks, *Journal of Geophysical*  
467 *Research: Solid Earth*, 118(7), 3557–3575, doi:10.1002/jgrb.50264.
- 468 van der Elst, N. J., M. T. Page, D. A. Weiser, T. H. Goebel, and S. M. Hosseini (2016),  
469 Induced earthquake magnitudes are as large as (statistically) expected, *Journal of Geo-*  
470 *physical Research: Solid Earth*, 121(6), 4575–4590, doi:10.1002/2016JB012818.
- 471 Walsh, F. R. I., and M. D. Zoback (2016), Probabilistic assessment of potential fault slip  
472 related to injection-induced earthquakes: Application to north-central Oklahoma, USA,  
473 *Geology*, 44(12), 991–994, doi:10.1130/G38275.1.
- 474 Zheng, G., and J. R. Rice (1998), Conditions under which velocity-weakening friction al-  
475 lows a self-healing versus a cracklike mode of rupture, *Bulletin of the Seismological So-*  
476 *ciety of America*, 88(6), 1466–1483.

Figure 1.

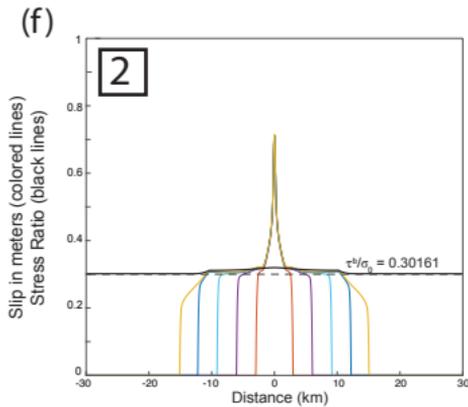
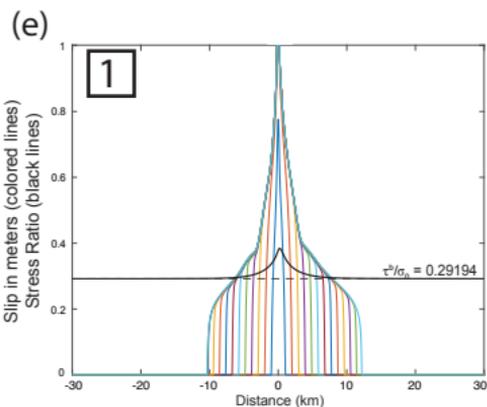
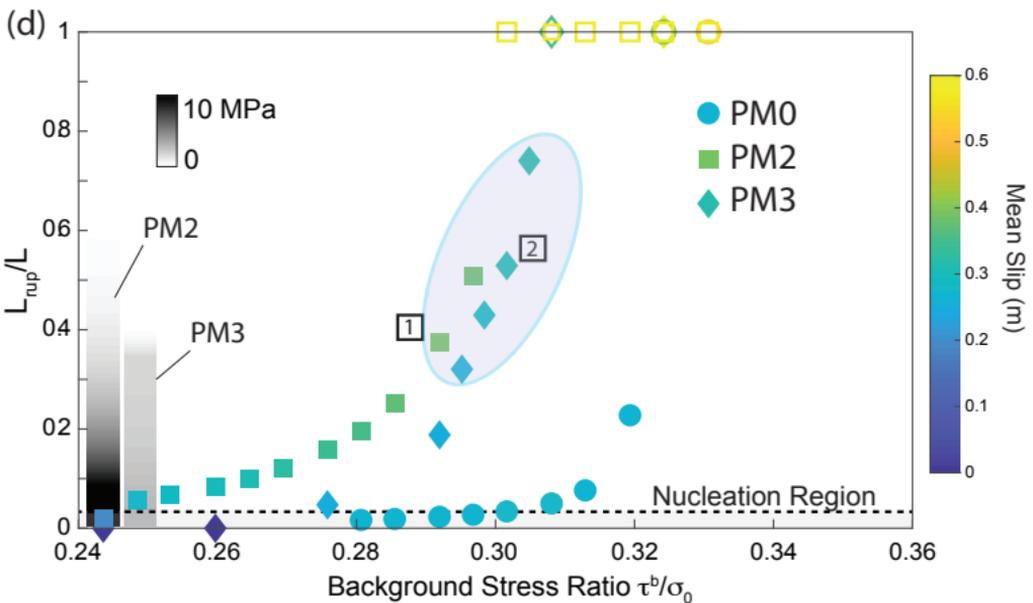
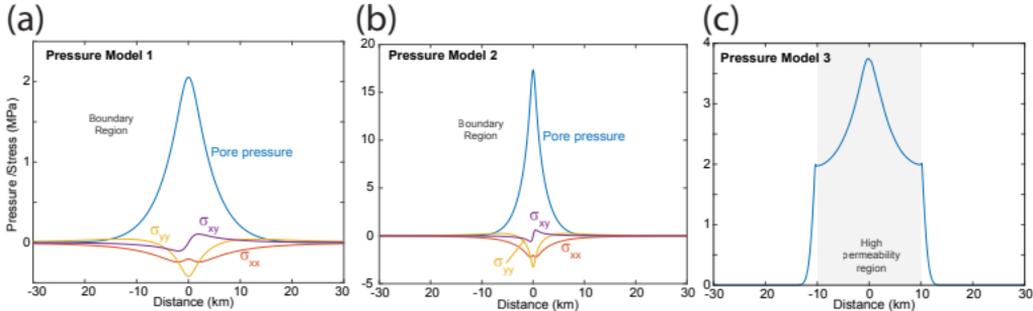


Figure 2.

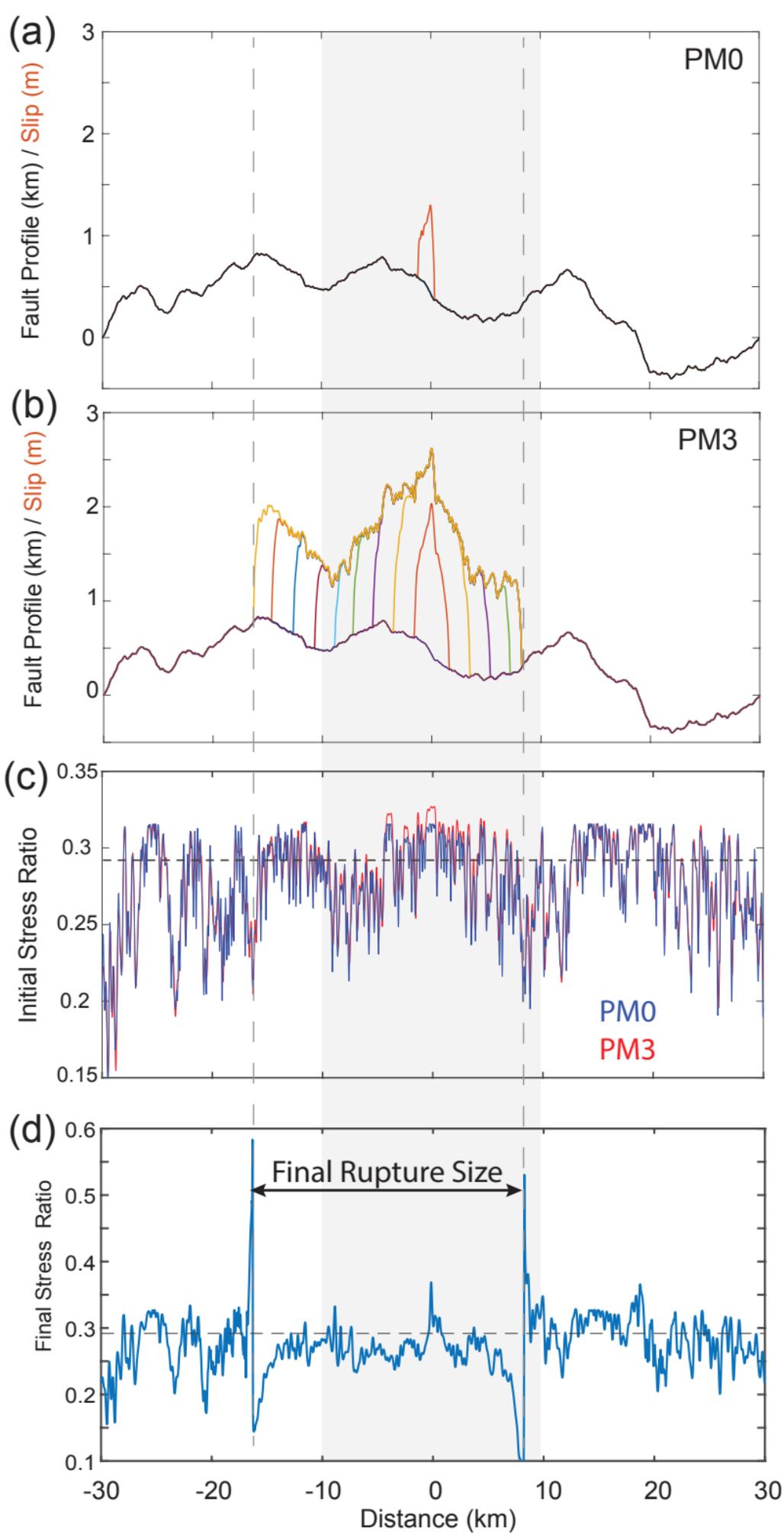
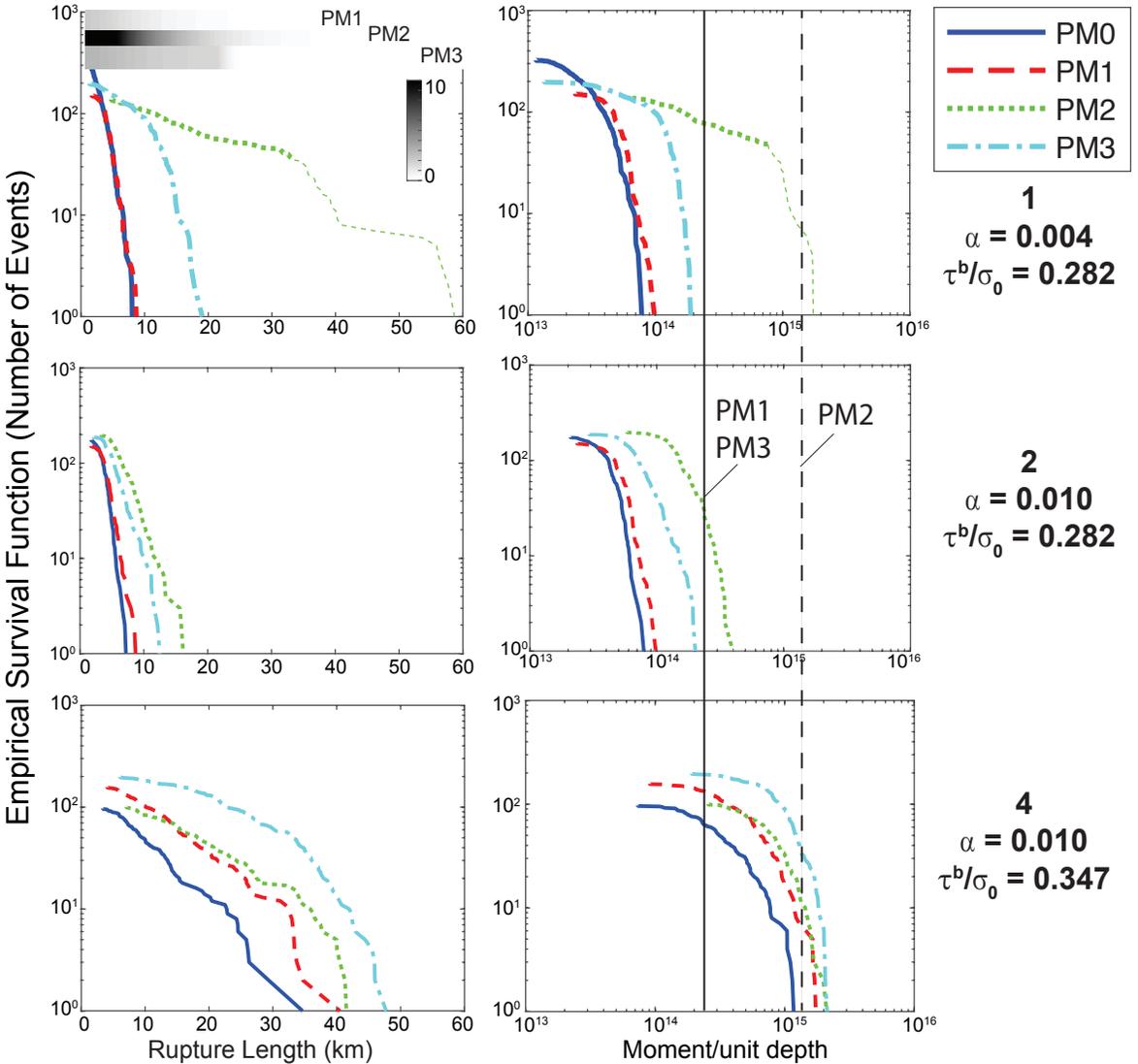


Figure 3.



**Figure 4.**

