

Measurements of the Net Charge Density of Space Plasmas

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22 **Key Points:**

23 Charge densities in geomagnetopause have been calculated using MMS electric field
24 measurements.

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26 A method for extracting the charge density from 10-point electric potential
27 measurements is presented.

28

29 An additional scheme to measure the charge density using seven or eight electric
30 potential probes is explored.

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33 **Key Words:**

34 Charge Density, Electric Field, Electric Potential Measurements, Multi-Point
35 Measurements, Magnetopause

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44 **Abstract**

45 Space plasmas are composed of charged particles that play a key role in electromagnetic
46 dynamics. However, to date, there has been no direct measurement of the distribution
47 of such charges in space. In this study, three schemes for measuring charge densities in
48 space are presented. The first scheme is based on electric field measurements by
49 multiple spacecraft. This method is applied to deduce the charge density distribution
50 within Earth's magnetopause boundary layer using Magnetospheric MultiScale
51 constellation (MMS) 4-point measurements, and indicates the existence of a charge
52 separation there. The second and third schemes proposed are both based on electric
53 potential measurements from multiple electric probes. The second scheme, which
54 requires 10 or more electric potential probes, can yield the net charge density to first-
55 order accuracy, while the third scheme, which makes use of seven to eight specifically
56 distributed probes, can give the net charge density with second-order accuracy. The
57 feasibility, reliability, and accuracy of these three schemes are successfully verified for
58 a charged-ball model. These charge density measurement schemes could potentially be
59 applied in both space exploration and ground-based laboratory experiments.

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66 **1. Introduction**

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68 Electromagnetic fields are omnipresent in space. They control the motion of
69 plasmas, and the transportation, release, and transformation of energy in space, and
70 thereby are the key driver of space weather hazards. Charges and electric currents
71 (flows of charged particles) source the electromagnetic field, and therefore the
72 distribution and motions of charges determine its form. Charge separations occur in
73 electric double layers, which exist commonly in space plasmas (Block, 1975; Akasofu,
74 1981; Raadu, 1989). Net charges can appear in plasma boundary layers (Parks, 1991),
75 e.g., the magnetopause boundary layers and Alfvén layers (Hasegawa and Sato, 1989).
76 Charge separations can also occur during ambipolar diffusion processes (Alfvén, 1963;
77 Bittencourt, 2004), e.g., the Earth's polar wind (Axford, 1968; Lemaire and Pierrard,
78 2001; Yau et al., 2007). In macro-scale plasmas, flow shears or vorticities can
79 accumulate these net charges, driving the field-aligned currents (Michael, 2014).
80 Charge separations also play a key role in plasma instabilities, e.g., the Rayleigh-Taylor
81 instability (Treumann and Baumjohann, 1997; Michael, 2014) and the tearing
82 instability (Treumann and Baumjohann, 1997).

83 The charge separations in space plasmas can appear at various spatial scales. The
84 plasmas with no magnetic field are commonly electrically neutral when the spatial scale
85 is much larger than the Debye length and the temporal scale is rather longer than the
86 plasma oscillation time (Bittencourt, 2004). At the Debye length space scale or plasma

87 oscillation time scale, the electrical neutrality would be violated and charge separations
88 appear. On the other hand, the ambipolar diffusion takes place in inhomogeneous
89 plasmas due to the different thermal velocities of the electrons and ions, and
90 polarization electric fields will be created, which can span several Earth radii in the
91 Earth's polar wind regions (Axford, 1968; Lemaire and Pierrard, 2001). However, as
92 results of the difference between the parameters of electrons and ions, the charge
93 separations in magnetized plasmas at spatial scales much larger than the Debye length
94 can take place. As for the magnetopause boundary layers, the protons of solar wind can
95 penetrate more deeply into the magnetosphere than electrons because of their greater
96 gyroradius. Therefore, the magnetosphere and magnetosheath sides of the
97 magnetopause boundary layer are positively and negatively charged, respectively, and
98 the width of the magnetopause boundary layer is at the order of proton gyroradius
99 (several hundred Kilometers) (Parks, 1991; Kivelson and Russell, 1995). During the
100 magnetospheric substorms, the plasmas are injected from the magnetotail into the inner
101 magnetosphere, and the ions and electrons are energized and drift duskward and
102 dawnward, respectively. As a result, the duskside and dawnside of the inner
103 magnetosphere accumulate positive and negative charges, respectively, and a
104 dawnward shielding electric field with a spatial scale of several Earth radius is
105 established (Hasegawa and Sato, 1989).

106 The acquisition of a spatial distribution of electric charge density is of critical
107 importance for recognizing and understanding the dynamics of electromagnetic fields
108 and plasmas in space. However, there is still no equipment available for directly
109 measuring the net charge density in space, although measurements of the charge density
110 in the atmosphere near the ground have been achieved. The difficulty of such
111 measurements in space arises because the plasmas there are extremely thin, with only

112 a few charged particles per cm^3 , and the net charge density is even lower by several
113 orders. According to Harris (1962), the maximum charge density within the
114 magnetopause boundary layer is $|\rho|_{\max} \approx 2ne(1 - V^2/c^2)^{-2}V^2/c^2$, where n is the
115 number density of the plasmas, V is the drifting velocity of electrons and ions, c is the
116 free speed of light in vacuum. According to Lee and Kan (1979), the main carriers of
117 the current in the magnetopause are ions, whose temperature is about 300 eV and
118 thermal velocity is estimated to be $V \approx 200\text{km/s}$. Assume $n \approx 10\text{cm}^{-3}$ in the
119 magnetopause, then $|\rho|_{\max} \approx 10e/\text{m}^3$.

120 Cluster mission has first achieved the four-point measurements on the electric field
121 in space (Escoubet et al., 2001), with which the electric field structure of the
122 magnetopause boundary layer has been revealed (Paschmann et al., 2005; Haaland et
123 al., 2021 and references therein). The Magnetospheric MultiScale (MMS) constellation
124 (Burch et al., 2016) can measure the 3-dimensional electric field vector at four locations
125 in space so as to obtain the linear gradient of the electric field. By using this advantage,
126 Tong, et al. (2018) have deduced the spatial distribution of net charge within a magnetic
127 hole and found there are net positive charges in the center of the magnetic hole and an
128 electron sheath around the hole. With a similar approach Argall et al. (2019) have
129 investigated the distribution of charge density in the diffusion region of magnetic
130 reconnection. However, we still have no independent charge density measurement
131 equipment in space. In this article, we will explore how the charge density can be
132 deduced based on multiple-probe electric potential measurements on board a single
133 spacecraft.

134 In Section 2, we first discuss the method for deducing the charge density from 4-
135 point electric field measurements, which has been applied to analyze the charge density
136 distribution in the dayside magnetopause boundary layer during an MMS

137 magnetopause crossing event. In Section 3, a method for deducing the charge density
138 from ≥ 10 -point electric potential measurements is studied. Section 4 explores
139 measurements of the charge density based on seven or eight electric potential probes.
140 Section 5 gives a summary and some discussion.

141

142 **2. Deducing the charge density from multi-spacecraft electric field measurements**

143 The direct approach to obtain the net charge density is to sum up the charge
144 densities of positively and negatively charged particles with the formula

$$145 \quad \rho = -en_e + \sum_i q_i n_i, \quad (1)$$

146 where n_e and n_i are the densities of the electrons and the i -th ion, respectively, and
147 q_i is the charge of the i -th ion. However, the electric force is so strong that the plasmas
148 are always quasi-neutral, and the separation between the two types of charges is very
149 slight. Therefore, the charge densities in space plasmas are extremely small. It is almost
150 impossible to determine the net charge density by measuring the densities of charged
151 particles at the present stage of space exploration.

152 The most feasible and practicable method at present is to deduce the net charge
153 density by measuring the electric potentials or electric fields created by the net charges
154 at high accuracies with well-developed technology (Mozer et al., 1967; Mozer, 1973;
155 Paschmann et al., 1997; Pedersen et al., 1998; Michael, 2014). The Spin-plane Double
156 Probes (SDPs) and Axial Double Probes (ADPs) (Torbert et al., 2016; Lindqvist et al.,
157 2016; Ergun et al., 2016) onboard the four spacecraft of the MMS constellation (Burch
158 et al., 2016) yield four electric field vectors at four different locations separated by tens

159 of kilometers. With the Gaussian theorem, $\rho = \varepsilon_0 \nabla \cdot \mathbf{E}$, we can get the charge density at
 160 the center of the constellation, as illustrated in Fig. 1. Suppose that the four spacecraft
 161 of the MMS constellation are located at four different positions \mathbf{r}_α ($\alpha = 1, 2, \dots, 4$). The
 162 barycenter of the MMS constellation is $\mathbf{r}_c \equiv \frac{1}{4} \sum_{\alpha=1}^4 \mathbf{r}_\alpha$. It is convenient to assume that
 163 $\mathbf{r}_c = 0$, so that the barycenter of the constellation is the origin of the frame of reference.
 164 The four spacecraft yield four electric fields, $\mathbf{E}_\alpha = \mathbf{E}(\mathbf{r}_\alpha)$, $\alpha = 1, 2, \dots, 4$. Under the linear
 165 assumption, the i -th component of the gradient of the electric field at the barycenter can
 166 be calculated as (Harvey, 1998; Chanteur, 1998)

$$167 \quad (\nabla_i \mathbf{E})_c = \frac{1}{4} \sum_{\alpha=1}^4 \mathbf{E}_\alpha r_{\alpha j} \mathbf{R}_{ji}^{-1}, \quad (2)$$

168 where $\mathbf{R}_{ij} = \frac{1}{4} \sum_{\alpha=1}^4 r_{\alpha i} r_{\alpha j}$ is the volumetric tensor of the constellation (Harvey, 1998), and
 169 \mathbf{R}_{ji}^{-1} its inverse. By using the Gaussian theorem, we can get the charge density with
 170 the divergence of the electric field vector, i.e.,

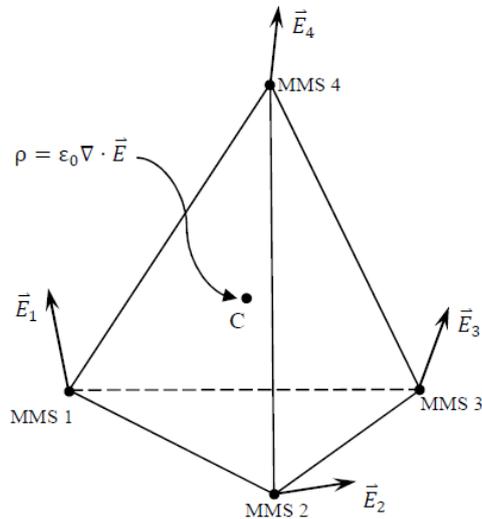
$$171 \quad \rho = \varepsilon_0 \nabla \cdot \mathbf{E} = \varepsilon_0 \sum_{i=1}^3 \nabla_i E_i, \quad (3)$$

172 The accuracy of the axial electric field measured by MMS is 1 mV/m (ADPs, Ergun
 173 et al., 2016), while the accuracy of the components of electric field in the spin plane is
 174 $< 0.5 mV/m$ (SDPs, Lindqvist et al., 2016). The two corresponding errors can be
 175 denoted as $\delta E_A \sim 1 mV/m$ and $\delta E_S \sim 0.5 mV/m$, respectively. It is known that the
 176 characteristic spatial scale of MMS is $L \approx 20 km$. Therefore, the error of the charge
 177 density calculated from the MMS 4 point electric measurements is estimated to be

178 $\delta \rho \approx \varepsilon_0 \left(\frac{\delta E_A}{L} + 2 \frac{\delta E_S}{L} \right) \approx 0.45 e/m^3$ which, as we will see in a case study, is much
 179 smaller than the observed charge density. The algorithm presented here is also evaluated
 180 and validated by a more sophisticated simulation shown in Figure S1 and S2 in the

181 supporting information file (jgra55009-sup-0001-2021JA029511-si).

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183

184 **Figure 1.** A schematic view of the measurements of the electric field by the MMS

185 constellation and the calculation of the charge density.

186

187 Here we will explore the net charge distribution within the magnetopause

188 boundary layer based on MMS electric measurements. It is well known that a charge

189 separation occurs in the magnetopause, brought about by the effects of inertia (because

190 there is a large difference between the masses of the electrons and ions). As a result of

191 that, the net positive charges accumulate at the magnetospheric side and the net negative

192 charges accumulate at the magnetosheath side of the magnetopause boundary. Because

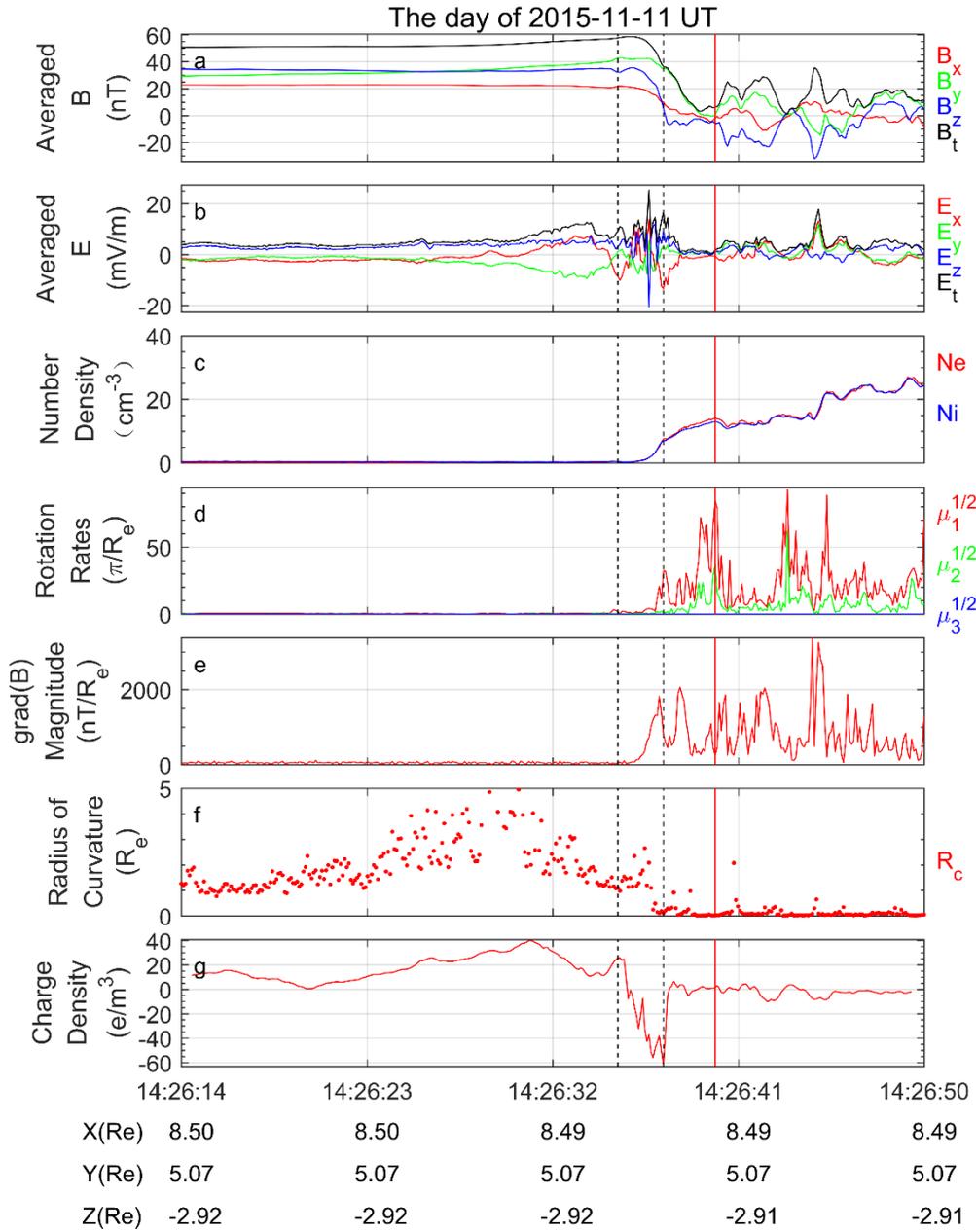
193 the MMS constellation has a rather small size (with the spacecraft separations being

194 several tens of kilometers) and can be well-embedded in the magnetopause boundary,

195 the charge density can be deduced from the MMS electric observations using the above

196 method. We investigate one MMS magnetopause crossing event at 14:26:14 on 11

197 November 2015 by examining the electric field and calculating the charge density,
198 whose values during the crossing event are shown in Fig. 2. It can be seen that the
199 rotational discontinuity (RD) appear at UT14:26:40 with the maximum magnetic
200 rotation rates (Panel (d)) (Shen et al., 2007), minimum value of the gradient of the
201 magnetic strength (Panel (e)), and smallest radius of curvature of the magnetic field
202 lines (Panel (f)). As shown in Panel (g), a charge separation is evident within the
203 magnetopause boundary, with the positive charges at the magnetospheric side and
204 negative charges at the magnetosheath side. The maximum value of the charge density
205 in the magnetopause is about $60 e/m^3$, which is much larger than the error ($\delta\rho \approx$
206 $0.45 e/m^3$) as given above. It is evident that the electric neutrality is kept in the
207 magnetosheath near to the magnetopause. These results are in agreement with the
208 conventional kinetic models of the magnetopause boundary layers (Harris, 1962; Lee
209 and Kan, 1979; Parks, 1991; Kivelson and Russell, 1995).



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211

Figure 2. The structure of the magnetopause during an MMS crossing event on 11

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November 2015. From top to bottom: (a) the magnetic flux density at the center of the

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constellation, (b) the electric-field at the center of the constellation, (c) the electron and

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ion number densities measured by MMS-1 (Pollock et al., 2016), (d) the rotation rates

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of the magnetic field (Shen et al., 2007), (e) $|\nabla|\mathbf{B}||$, (f) the radius of curvature of the

216

magnetic field lines (Shen et al., 2003), and (g) the charge distribution. The red vertical

217 line marks the largest rotation rates, and the black vertical dotted lines mark the largest
218 and the smallest charge densities.

219

220 **3. Charge density measurements from 10 probes on board a spacecraft – Stiff** 221 **Booms Method**

222

223 It is known that the linear gradient of a quantity can be estimated based on 4-point
224 measurements (Harvey, 1998; Chanteur, 1998; Shen et al., 2003), while the quadratic
225 gradient of a quantity can be calculated based on 10-point measurements (Chanteur,
226 1998). In the low Earth Orbit missions DEMETER (Berthelier, et al., 2005) and
227 Zhangheng-1 (Shen, et al., 2018), the electric field is measured with four probes
228 mounted at the ends of four stiff booms. We suggest to construct an electric equipment
229 composed of 10 or more electric probes so that both the electric field and charge density
230 can be measured. In a previous investigation (Shen et al., 2021), a new algorithm was
231 put forward to calculate the linear and quadratic gradients jointly based on 10 or more
232 measurements. It can be applied to obtain the quadratic gradients ($\nabla^2\varphi$) from 10-point
233 electric potential field (φ) measurements. Moreover, with the Poisson equation,

$$234 \quad \rho = -\varepsilon_0 \nabla^2 \varphi, \quad (4)$$

235 it yields the distribution of the electric charge density. For the processes with temporal
236 variations, the general governing equation is the d'Alembert equation,
237 $-c^{-2}\partial_t^2\varphi + \nabla^2\varphi = -\varepsilon_0^{-1}\rho$, instead. However, for slow varying structures or steady
238 structures and low-frequency plasma waves with their motion speeds much less than c ,

239 the first term at the right hand side of the d'Alembert equation can be neglected.

240 We can check the feasibility of this 10 probe scheme. The electric field generated
 241 by a uniformly charged ball will be used to test this approach. Supposing that the radius
 242 of the ball is r_0 and its charge density is ρ , we get the electric potential field
 243 analytically as,

$$244 \quad \varphi(\mathbf{r}) = \begin{cases} -\frac{1}{6}\epsilon^{-1}\rho r^2 + \frac{1}{2\epsilon}r_0^2\rho & \text{if } r \leq r_0, \\ -\frac{1}{4\pi\epsilon}\frac{Q}{r} & \text{if } r > r_0, \end{cases} \quad (5)$$

245 where $Q = \frac{4}{3}\pi r_0^3\rho$ is the total charge and r is the distance from the center of the ball
 246 to the measurement point. In the following modeling, constant values of 1 are assigned
 247 to ρ , r_0 , and ϵ , i.e., $\rho = r_0 = \epsilon = 1$. The positions of the 10 probes in the barycenter
 248 coordinates are generated randomly and presented in Tab. 1 and Fig. 3. The three
 249 characteristic lengths of the distribution of the 10 probes (Harvey, 1998; Robert, et al.,
 250 1998) are $a = 0.10$, $b = 0.06$, and $c = 0.03$. The reconstructed characteristic
 251 matrix \mathfrak{R}^{MN} is

$$252 \quad (\mathfrak{R}^{MN}) = \begin{pmatrix} 12.73 & -11.09 & -5.05 & 5.22 & 2.74 & 1.61 \\ -11.09 & 20.90 & 5.47 & -6.71 & -4.97 & -2.28 \\ -5.05 & 5.47 & 6.44 & -2.49 & -4.56 & -2.27 \\ 5.22 & -6.71 & -2.49 & 12.83 & -1.91 & 2.27 \\ 2.74 & -4.97 & -4.56 & -1.91 & 9.09 & 0.86 \\ 1.61 & -2.28 & -2.27 & 2.27 & 0.86 & 2.68 \end{pmatrix} 10^{-3}, \quad (6)$$

253 and its eigenvalues are given in Tab. 2.

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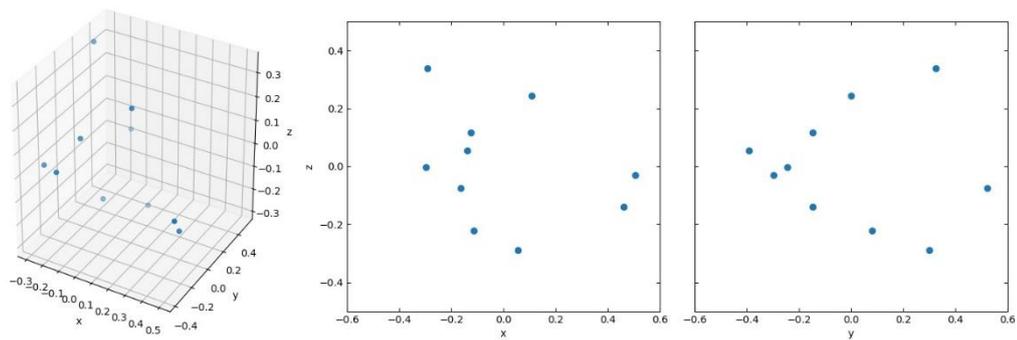
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Table 1. The locations of the 10 probes in the barycenter coordinates .

x	y	z
-0.16474	0.520923	-0.07516
-0.29774	-0.2433	-0.00151
0.107263	-0.00029	0.243785
-0.12458	-0.14707	0.116693
-0.11324	0.080113	-0.22108
0.505285	-0.29726	-0.0293
0.055479	0.300437	-0.28976
0.461577	-0.14647	-0.13865
-0.2916	0.323618	0.339179
-0.13771	-0.3907	0.055801



261

262

Figure 3. The distribution of the 10 probes.

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Table 2. The eigenvalues of the characteristic matrix \mathfrak{R}^{MN} .

0.03614	0.01326	0.00114	0.00235	0.00510	0.00668
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267

We first investigate the behavior of the resultants with the number of iterations.

268

D is the local characteristic scale of the electric field structure and is set equal to r in

269

this model. It is assumed that the barycenter of the constellation is at $[0.1,0,0]$, and the

270

probe separations L are reduced proportionally so that the relative measurement scale

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$L/D= 0.026$. The relative truncation error, $X_{algorithm}/X_{real} - 1$, is shown in Fig. 4.

272

With increasing numbers of iterations, the errors decrease and finally converge to

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certain fixed values. In this calculation, the solution converges after 100 iterations. By

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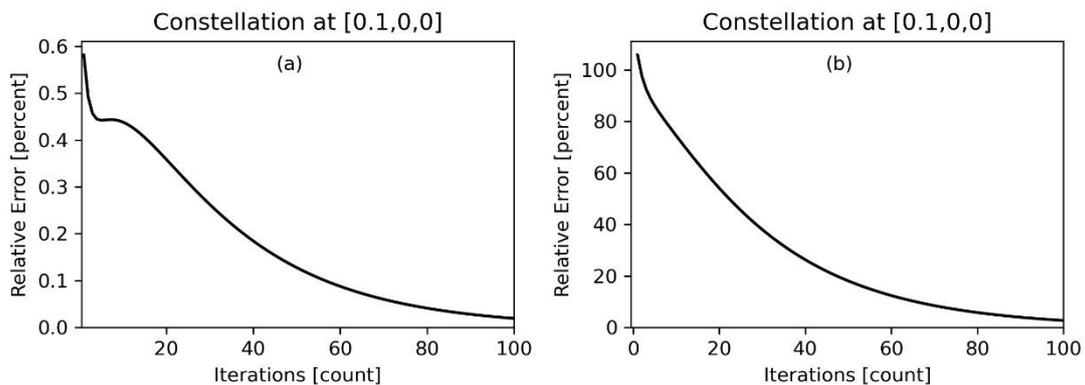
testing various fields, we found that the number of iterations required for convergence

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varies.

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Figure 4. The relative errors of the linear (a) and the quadratic (b) electric potential

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gradients, i.e., $\partial_x\phi$ and $\partial_x\partial_x\phi$, calculated for different numbers of iterations at

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$[0.1,0,0]$ within the uniformly charged ball.

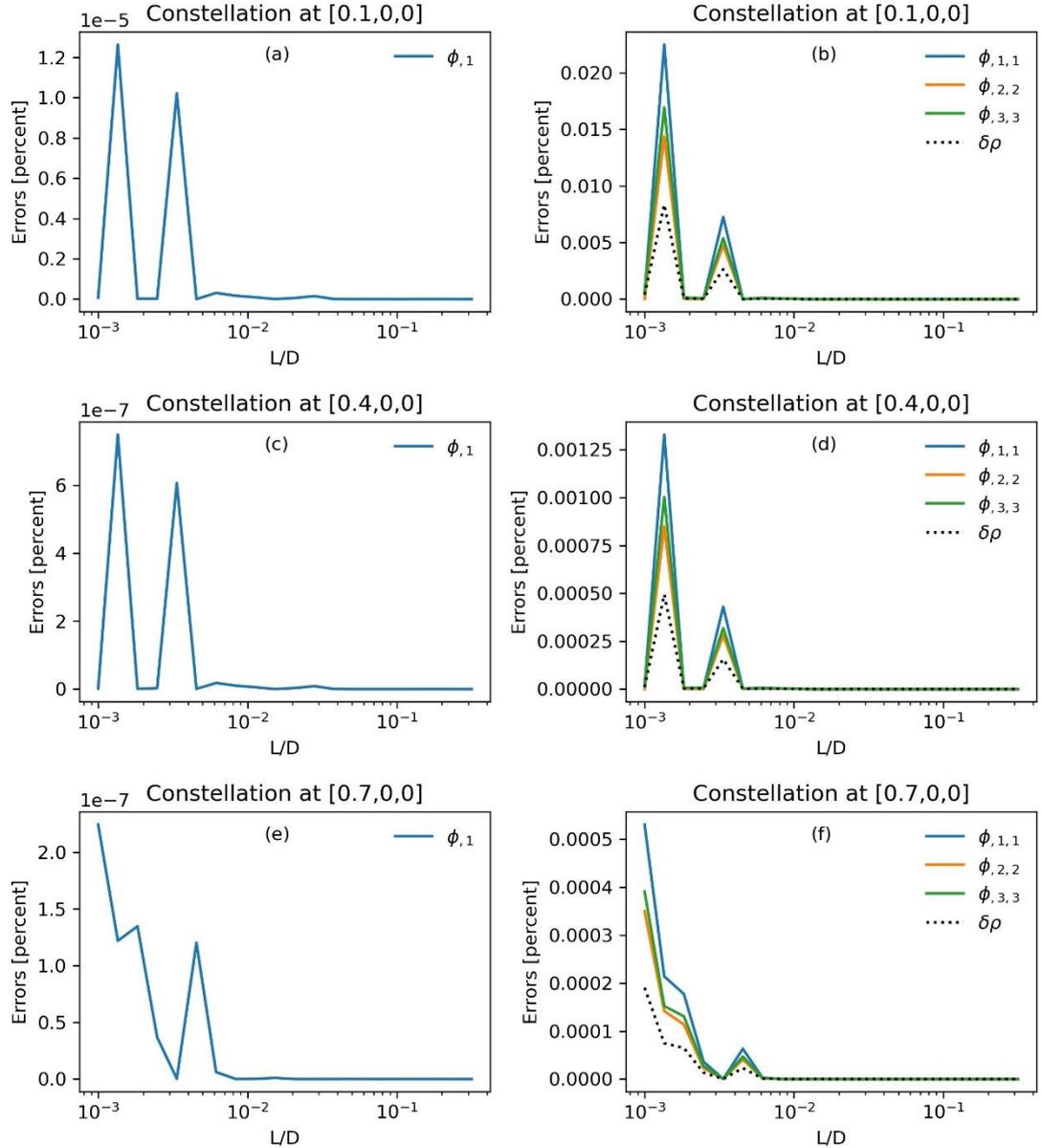
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283 Secondly, we investigate the dependence of the truncation errors on the relative
284 measurement scale L/D . We have tested six situations, with the barycenter of the 10
285 probes located at three representative points within the ball, $[0.1,0,0]$, $[0.4,0,0]$, and
286 $[0.7,0,0]$, and three points outside the ball, $[3,0,0]$, $[5,0,0]$, and $[8,0,0]$. We scale up
287 and down the size of the original 10 probes to adjust the characteristic size L and
288 therefore L/D .

289

290 Figure 5 shows the errors modeled in the ball. In general, the errors are less than
291 $10^{-5}\%$ for the linear gradients and less than 0.02% for the quadratic gradients. With
292 the same number of iterations, 1000, the errors at different positions vary by an order
293 of 2. The extremely accurate results arise from the fact that the charge density has been
294 assume homogeneous and electric field is linear varying within the charged ball. A
295 further check on the method for a charged ball model with a non-uniform charge density
296 ball has been performed in the Supporting Information file (jgra55009-sup-0002-
297 2021JA029511-si).

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299

300 **Figure 5.** The variation of the errors of the calculation by using the 10-probe scheme
 301 with the relative measurement scale L/D for the case of a uniformly charged ball. The
 302 measurements are performed inside of the charged ball. The left panels, (a), (c), and (e),
 303 show the truncation errors for the non-vanishing component of the linear gradient by
 304 L/D calculated for three different locations of the barycenter of the 10 probes inside
 305 the ball, $[0.1,0,0]$, $[0.4,0,0]$, and $[0.7,0,0]$. The right panels, (b), (d), and (f),
 306 illustrate the relative errors of the non-vanishing components of the quadratic gradient

307 and charge density (dashed line) calculated for the same three locations of the
308 barycenter. It is noted that $\phi_{,1} \equiv \partial_x \phi$ and $\phi_{,2,2} \equiv \partial_y \partial_y \phi$, where a comma denotes
309 partial differentiation.

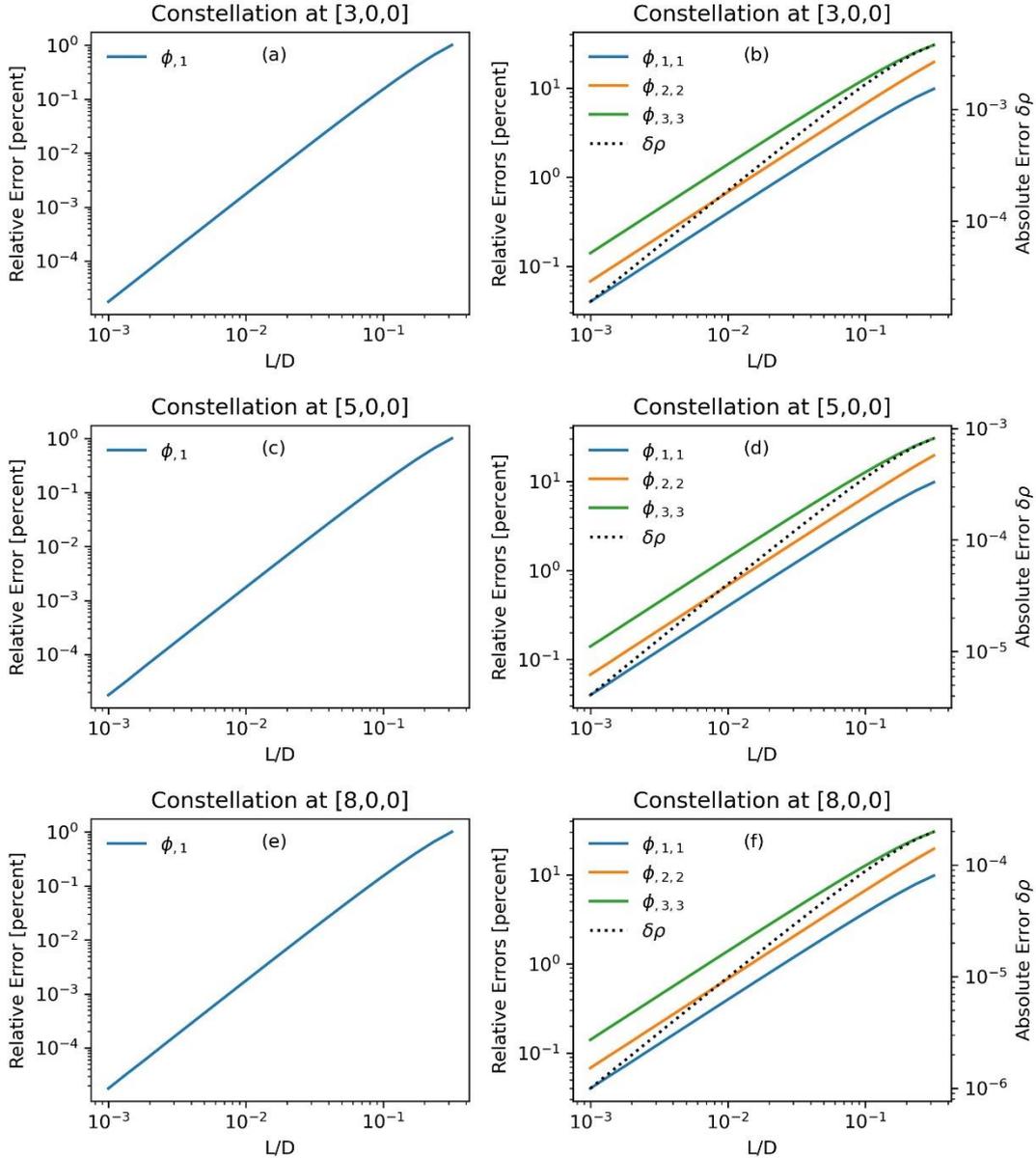
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311 Figure 6 shows the modeling results outside of the ball. As $L/D < 0.01$, the
312 relative errors of the non-vanishing quadratic gradient components are below 2%. The
313 attained linear and quadratic gradients are accurate to second order and first order,
314 respectively.

315 The same error analysis procedure for the 10-probe scheme has been applied to
316 another charged ball model in which the charge density is inversely proportional to the
317 square of the distance from the ball center, as shown in Figure S3 and S4 in the
318 Supporting Information file (jgra55009-sup-0002-2021JA029511-si), and a similar
319 conclusion has been reached.

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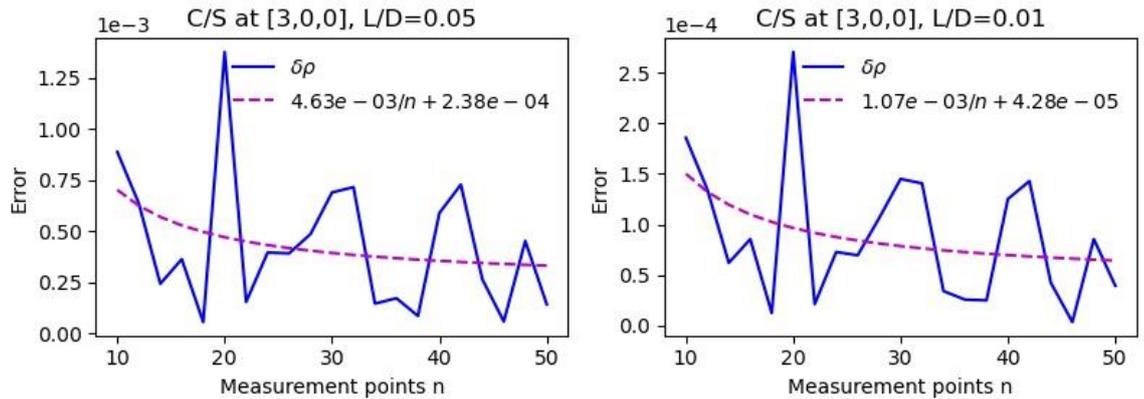
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323 **Figure 6.** The dependence of the truncation errors of the calculations by using the 10-
 324 probe scheme on the relative measurement scale L/D for the case of a uniformly charged
 325 ball. The measurements are performed outside of the charged ball. The left panels, (a),
 326 (c), and (e), show the truncation error for the non-vanishing component of the linear
 327 gradient as a function of L/D calculated for three different locations of the barycenter
 328 of the 10 probes outside of the ball, $[3,0,0]$, $[5,0,0]$, and $[8,0,0]$. The right panels, (b),
 329 (d), and (f), illustrate the relative errors of the non-vanishing components of the

330 quadratic gradient and the absolute value of the charge density (dashed line) calculated
 331 for the same three locations of the barycenter. It is noted that the real charge density
 332 outside of the ball is zero.
 333



334
 335 **Figure 7.** The relation between the absolute error of the charge density and the number
 336 of measurement points at $[3,0,0]$. The relative measurement scale is chosen as $L/D =$
 337 0.05 (left) and $L/D = 0.01$ (right). The dashed lines are fitted from the modeled
 338 errors.

339
 340 We further investigate the relationship between the accuracy of the density
 341 estimated and the number of the probes used. Figure 7 indicates that the accuracy of the
 342 charge density is not improved significantly as the number of probes is increased.
 343 Therefore, 10 probes with a proper spatial configuration will be sufficient for robust
 344 measurements of the charge density.

345 This scheme is possible to be used for the net charge measurements on the low
 346 Earth orbits at the altitudes of several hundred kms, for which the 10 probes are
 347 mounted at the ends of 10 booms with different lengths, and the spacecraft can be either

348 spinning or not.

349 The feasibility of the measurements at the low attitude Earth orbits can be shown
350 by including observational errors. The accuracy of the probes is assumed at
351 $\delta\phi \approx L\delta\nabla\phi \sim 10m \times 0.5mV/m \sim 5mV$. The electric potential at an arbitrary probe can
352 be expanded as the following.

$$\begin{aligned} \phi &= \phi_c + \Delta\mathbf{x} \cdot \nabla\phi + \frac{1}{2} \Delta\mathbf{x} \Delta\mathbf{x} \cdot \nabla\nabla\phi \\ &\sim \phi_c - E \cdot L + \frac{1}{2} \frac{1}{\epsilon_0} \rho L^2, \end{aligned}$$

354 where, $\Delta\mathbf{x}$ is the distance of the probe from the center, which is at the scale of L;
355 $\nabla\phi = -\mathbf{E}$, and $\nabla\nabla\phi$ is estimated by $\nabla^2\phi = -\rho/\epsilon_0$. The second term at the right
356 hand side (or the first order term) is the contribution of the electric field, which is about
357 $EL \sim 600mV/m \times 10m \sim 6.0V$. The third term (or the second order term) is the
358 contribution of the charge density, which is about $\frac{1}{2} \frac{1}{\epsilon_0} \rho L^2 \sim 50mV$ if the typical
359 value of the charge density at low Earth orbits is assumed to be $\rho \sim 5 \times 10^4 e/m^3$, which
360 is about three order higher than those at the high Earth orbits. They are both much larger
361 than the probe sensitivity ($5mV$), so that at low Earth orbits the charge density is
362 observable with the approach described above.

363

364 4. Measuring the charge density with seven or eight electric potential probes

365 Only three diagonal components of the quadratic gradient of the electric potential
366 are contained in the Poisson equation ($\rho \propto \nabla^2\phi = \partial^2/\partial x^2\phi + \partial^2/\partial y^2\phi + \partial^2/\partial z^2\phi$). The
367 three other cross-components of the quadratic gradient, $\partial_x\partial_y\phi$, $\partial_y\partial_z\phi$, and $\partial_z\partial_x\phi$,

368 are of no use for computing the charge density, so three independent parameters can be
369 neglected in this algorithm. Therefore, $10-3=7$ probes are sufficient to acquire the data
370 for the estimation of the Laplacian operator on the electric potential ($\nabla^2\phi$) as well as
371 the charge density.

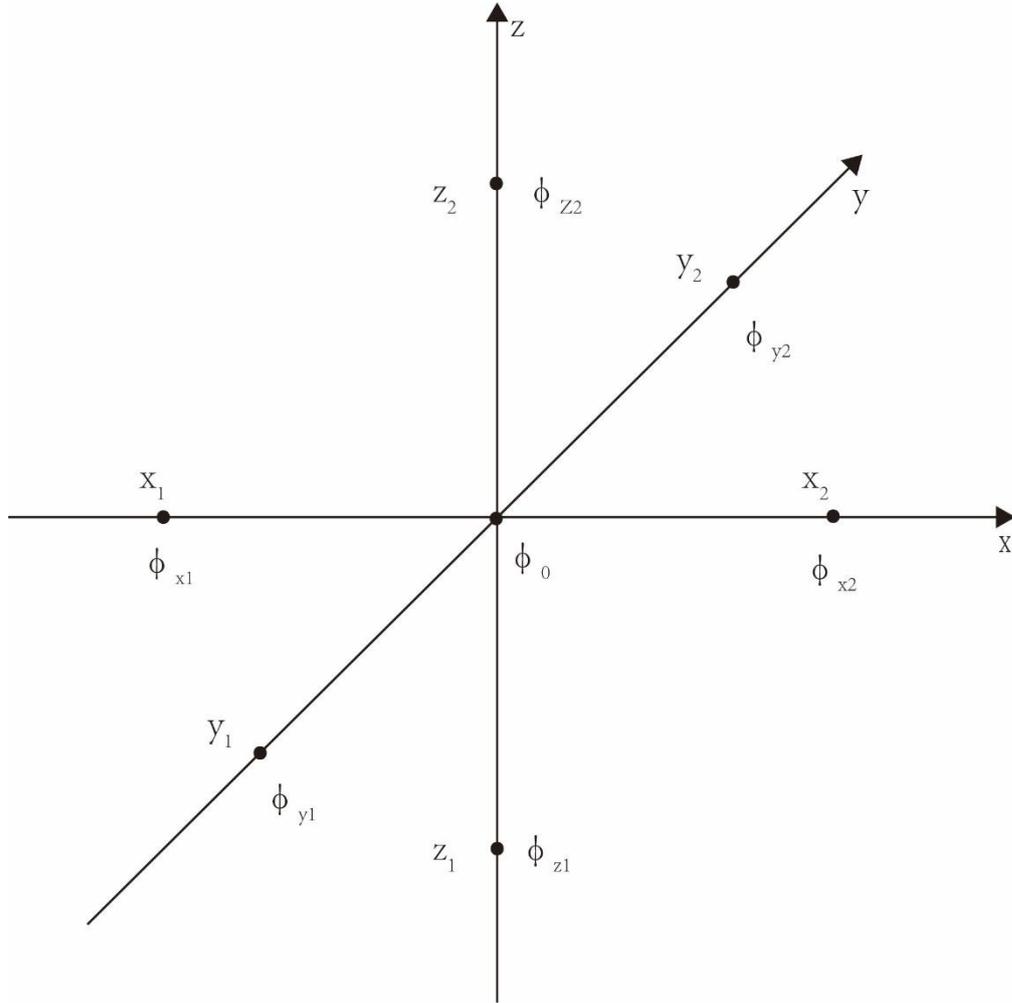
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373 **4.1 Seven-probe scheme**

374 A seven-probe scheme, which is similar to the electric potential measurement of
375 the MMS at high altitude orbits, is shown in Fig. 8. All probes are placed on three axes
376 of the Cartesian coordinate system. The spatial parameters are $x_2 = -x_1 = L_x$,
377 $y_2 = -y_1 = L_y$, and $z_2 = -z_1 = L_z$. By taking differences, the linear and quadratic
378 gradients at second-order accuracy can be obtained to estimate the charge density at the
379 center.

380

381



382

383 **Figure 8.** A schematic view of the seven-probe measurement of the charge density. The
 384 probes are indicated by black dots.

385

386 The linear and quadratic gradients along the x-axis are

$$\left\{ \begin{array}{l} \partial_x \phi = \frac{\phi_{x2} - \phi_{x1}}{2L_x} \quad (7) \\ \partial_x^2 \phi = \frac{\phi_{x2} - \phi_0}{L_x} - \frac{\phi_0 - \phi_{x1}}{L_x} = \frac{(\phi_{x2} + \phi_{x1}) - 2\phi_0}{L_x^2} \quad (8) \end{array} \right.$$

388 Similarly, the linear and quadratic gradients along the y-axis are

$$\left\{ \begin{array}{l} \partial_y \phi = \frac{\phi_{y2} - \phi_{y1}}{2L_y} \quad (9) \\ \partial_y^2 \phi = \frac{(\phi_{y2} + \phi_{y1}) - 2\phi_0}{L_y^2} \quad (10) \end{array} \right.$$

The linear and quadratic gradients along the z-axis are

$$\left\{ \begin{array}{l} \partial_z \phi = \frac{\phi_{z2} - \phi_{z1}}{2L_z} \quad (11) \\ \partial_z^2 \phi = \frac{(\phi_{z2} + \phi_{z1}) - 2\phi_0}{L_z^2} \quad (12) \end{array} \right.$$

The linear and quadratic gradients are both accurate to second order.

However, in actual measurements, the central probe is inside the spacecraft and cannot determine the electric potential accurately. To improve this measurement, the central probe is replaced by another two additional probes located on the z-axis. The algorithm for this is shown in the following section. It is noted the seven-probe scheme can be still applied to the electric field and charge density measurements in ground-based laboratory experiments.

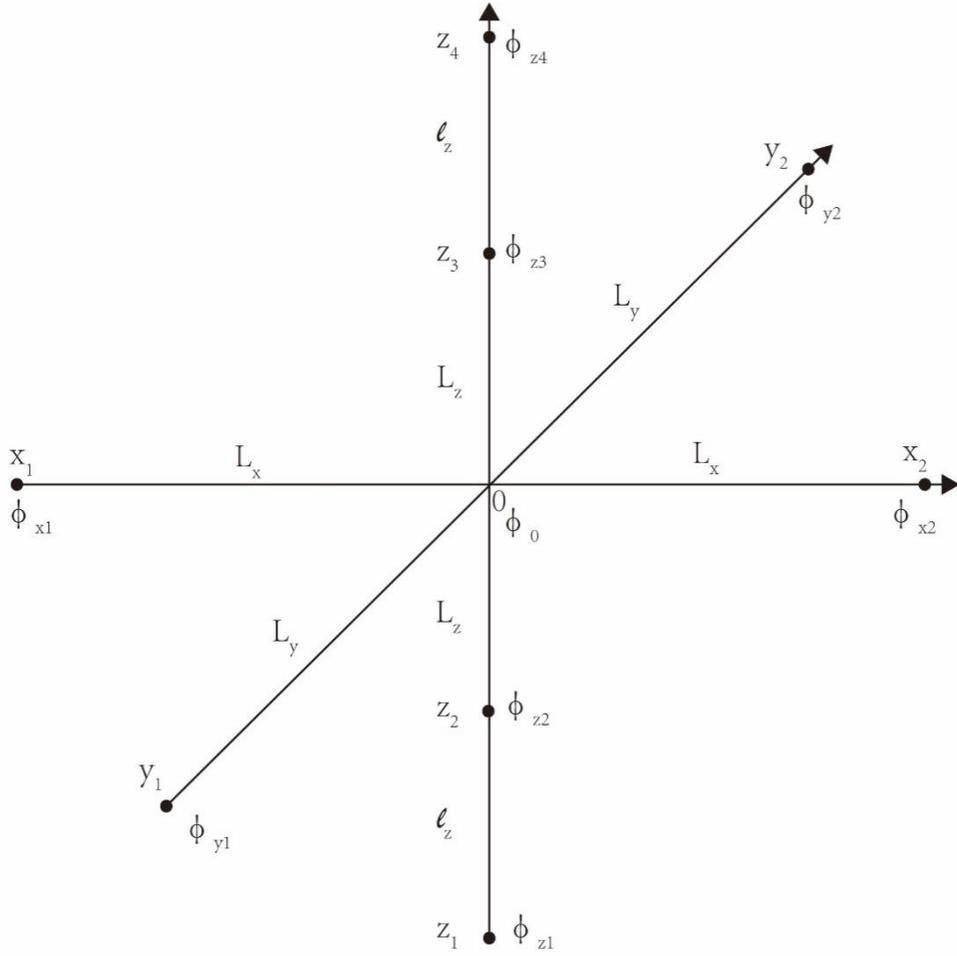
399

4.2 Eight-probe scheme

The eight-probe scheme is shown in Fig. 9 with $x_2 = -x_1 = L_x$, $y_2 = -y_1 = L_y$,

$z_3 = -z_2 = L_z$, and $z_4 = -z_1 = L_z + l_z$. The algorithm is constructed as follows.

403



404

405 **Figure 9.** A schematic view of the eight-probe measurement of charge density.

406

407

408 The four electric potentials observed by the probes on the z-axis can be expressed as a

409 Taylor series. By keeping the first five terms we get

410

$$\left\{ \begin{array}{l} \phi_{z1} = \phi_0 + z_1 \partial_z \phi + \frac{1}{2} z_1^2 \partial_z^2 \phi + \frac{1}{3!} z_1^3 \partial_z^3 \phi + \frac{1}{4!} z_1^4 \partial_z^4 \phi \end{array} \right. \quad (13)$$

$$\left\{ \begin{array}{l} \phi_{z2} = \phi_0 + z_2 \partial_z \phi + \frac{1}{2} z_2^2 \partial_z^2 \phi + \frac{1}{3!} z_2^3 \partial_z^3 \phi + \frac{1}{4!} z_2^4 \partial_z^4 \phi \end{array} \right. \quad (14)$$

411

$$\left\{ \begin{array}{l} \phi_{z3} = \phi_0 + z_3 \partial_z \phi + \frac{1}{2} z_3^2 \partial_z^2 \phi + \frac{1}{3!} z_3^3 \partial_z^3 \phi + \frac{1}{4!} z_3^4 \partial_z^4 \phi \end{array} \right. \quad (15)$$

$$\left\{ \begin{array}{l} \phi_{z4} = \phi_0 + z_4 \partial_z \phi + \frac{1}{2} z_4^2 \partial_z^2 \phi + \frac{1}{3!} z_4^3 \partial_z^3 \phi + \frac{1}{4!} z_4^4 \partial_z^4 \phi \end{array} \right. \quad (16)$$

412 Summing up the above four equations leads to

$$413 \quad (\phi_{z1} + \phi_{z2} + \phi_{z3} + \phi_{z4}) = 4\phi_0 + \frac{1}{2}(z_1^2 + z_2^2 + z_3^2 + z_4^2)\partial_z^2\phi + \frac{1}{4!}(z_1^4 + z_2^4 + z_3^4 + z_4^4)\partial_z^4\phi \quad .$$

414 The electric potential at the center is therefore

$$415 \quad \phi_0 = \frac{1}{4}(\phi_{z1} + \phi_{z2} + \phi_{z3} + \phi_{z4}) - \frac{1}{8}(z_1^2 + z_2^2 + z_3^2 + z_4^2)\partial_z^2\phi - \frac{1}{96}(z_1^4 + z_2^4 + z_3^4 + z_4^4)\partial_z^4\phi \quad (17)$$

416 Subtracting Eq. (13) from Eq. (16) and Eq. (14) from Eq. (15) gives

$$417 \quad \begin{cases} \phi_{z4} - \phi_{z1} = (z_4 - z_1) \partial_z\phi + \frac{1}{3!}(z_4^3 - z_1^3)\partial_z^3\phi \\ \phi_{z3} - \phi_{z2} = (z_3 - z_2) \partial_z\phi + \frac{1}{3!}(z_3^3 - z_2^3)\partial_z^3\phi \end{cases} \quad (18)$$

418 or

$$419 \quad \begin{cases} \phi_{z4} - \phi_{z1} = 2z_4\partial_z\phi + \frac{1}{3}z_4^3\partial_z^3\phi \\ \phi_{z3} - \phi_{z2} = 2z_3\partial_z\phi + \frac{1}{3}z_3^3\partial_z^3\phi \end{cases} \quad (18')$$

420 Then, we get the linear gradient along the z-axis at the center as

$$421 \quad \partial_z\phi = \frac{z_3^3(\phi_{z4} - \phi_{z1}) - z_4^3(\phi_{z3} - \phi_{z2})}{2z_4z_3^3 - 2z_3z_4^3} \quad (19)$$

422 The expression above is of fourth-order accuracy. On the other hand, from Equation

423 (18), the third-order derivative of electric potential along the z-axis is

$$424 \quad \partial_z^3\phi = \frac{3z_3(\phi_{z4} - \phi_{z1}) - 3z_4(\phi_{z3} - \phi_{z2})}{z_3z_4^3 - z_4z_3^3} \quad (20)$$

425 The expression above is of second-order accuracy.

426 Subtracting the sum of Eq. (14) and Eq. (15) from the sum of Eq. (13) and Eq. (16), we

427 get

$$428 \quad (\phi_{z4} + \phi_{z1}) - (\phi_{z3} + \phi_{z2}) = \frac{1}{2}(z_1^2 + z_4^2 - z_2^2 - z_3^2)\partial_z^2\phi + \frac{1}{4!}(z_1^4 + z_4^4 - z_2^4 - z_3^4)\partial_z^4\phi$$

429 The second-order derivative is, therefore,

$$430 \quad \partial_z^2 \phi = \frac{2(\phi_{z4} + \phi_{z1} - \phi_{z3} - \phi_{z2})}{(z_1^2 + z_4^2 - z_2^2 - z_3^2)} - \frac{1}{12} \frac{(z_1^4 + z_4^4 - z_2^4 - z_3^4)}{z_1^2 + z_4^2 - z_2^2 - z_3^2} \partial_z^4 \phi \quad (21)$$

431 The expression above is of second-order accuracy.

432 Substituting Eq. (21) into Eq. (17), we get the corrected potential ϕ_0 at the center
433 as

$$434 \quad \phi_0 = \frac{1}{4}(\phi_{z1} + \phi_{z2} + \phi_{z3} + \phi_{z4}) - \frac{1}{4} \frac{z_1^2 + z_2^2}{z_1^2 - z_2^2} (\phi_{z4} + \phi_{z1} - \phi_{z3} - \phi_{z2}) + \frac{1}{24} z_1^2 z_2^2 \partial_z^4 \phi \quad (17')$$

435 The above expression is of fourth-order accuracy because the expression is
436 truncated at the fourth-order term.

437 Furthermore, by neglecting high order terms, we get the estimators for the
438 potential and its linear and quadratic gradients at the center as

$$\left\{ \begin{array}{l} \partial_z^2 \phi = \frac{(\phi_{z4} + \phi_{z1}) - (\phi_{z3} + \phi_{z2})}{l_z(2L_z + l_z)} \quad (21') \\ \partial_z \phi = \frac{(L_z + l_z)^3 (\phi_{z3} - \phi_{z2}) - L_z^3 (\phi_{z4} - \phi_{z1})}{2L_z(L_z + l_z)(2l_z L_z + l_z^2)} \quad (19') \\ \phi_0 = \frac{1}{4}(\phi_{z1} + \phi_{z2} + \phi_{z3} + \phi_{z4}) - \frac{(L_z + l_z)^2 + L_z^2}{4l_z(2L_z + l_z)} (\phi_{z4} + \phi_{z1} - \phi_{z3} - \phi_{z2}) \quad (17'') \end{array} \right.$$

440 As stated above, the second-order derivative along the z-axis is of second-order
441 accuracy. The potential and its first-order derivative along the z-axis are of fourth-order
442 accuracy.

443 Similar to the seven-probe scheme, the first-order and second-order derivatives of
444 the potential along the x- and y-axis are subjected to Eqs. (7)-(10). The central potential
445 ϕ_0 is calculated with Eq. (17''). The first-order and second-order derivatives along the
446 x- and y-axis are of second order accuracy.

447 The electric field at the center is

$$448 \quad \mathbf{E} = -\hat{\mathbf{e}}_x \partial_x \phi - \hat{\mathbf{e}}_y \partial_y \phi - \hat{\mathbf{e}}_z \partial_z \phi \quad (22)$$

449 Using the Poisson equation (4), the charge density is obtained as

$$450 \quad \begin{aligned} \rho &= -\epsilon_0 (\partial_x^2 \phi + \partial_y^2 \phi + \partial_z^2 \phi) \\ &= -\epsilon_0 \left[\frac{(\phi_{x2} + \phi_{x1}) - 2\phi_0}{L_x^2} + \frac{(\phi_{y2} + \phi_{y1}) - 2\phi_0}{L_y^2} + \frac{(\phi_{z4} + \phi_{z1}) - (\phi_{z3} + \phi_{z2})}{l_z(2L_z + l_z)} \right] \end{aligned} \quad (23)$$

451 where ϕ_0 is given by Eq. (17’’).

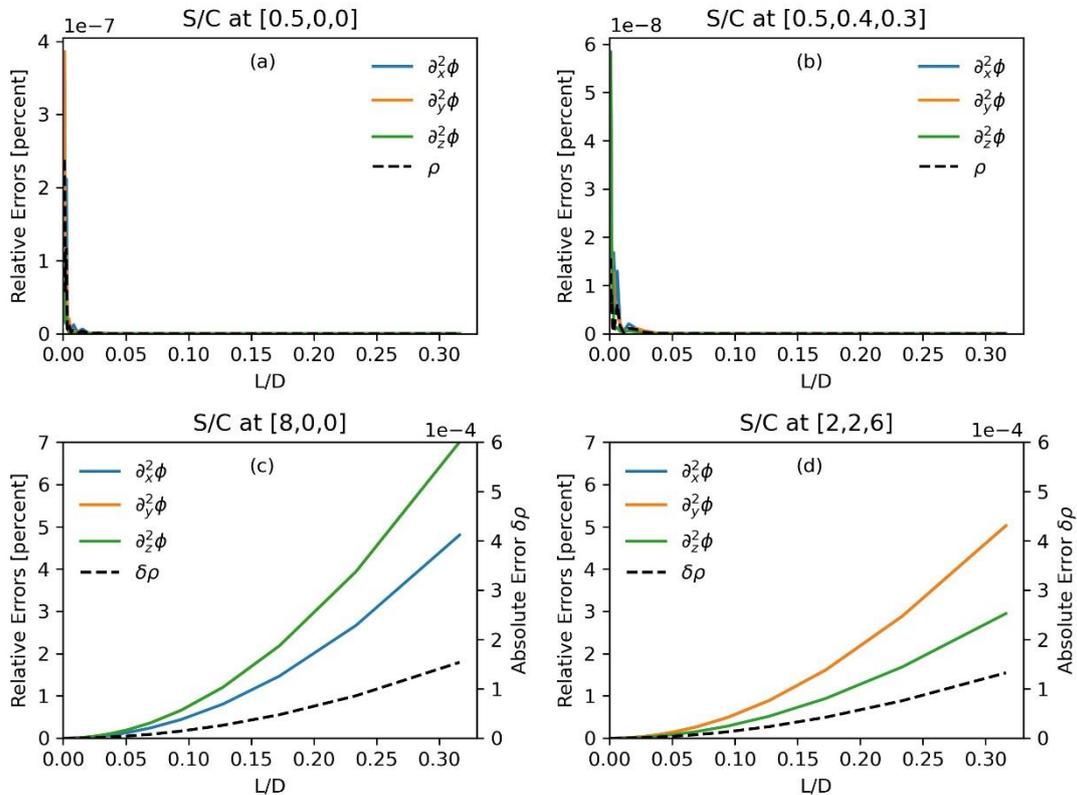
452

453 The eight-probe scheme will now be examined for the electric field produced by a
454 uniformly-charged ball.

455 The relationship between the relative truncation errors and the relative
456 measurement scale, L/D , is studied when we set $L_x = L_y = L_z = l_z$ and scale up and
457 down the distances between the spacecraft to adjust L/D . Due to the broken spherical
458 symmetry, two points inside the ball, $[0.5,0,0]$ and $[0.5,0.4,0.3]$, and two points
459 outside of the ball, $[8,0,0]$ and $[2,2,6]$, are chosen as the representative points. The
460 modeled results are shown in Fig. 10. The quadratic gradient in the ball is close to a
461 constant and the charge density here is a constant. The truncation errors given by the
462 algorithm, as shown in Fig. 10 (a,b), are negligible in this case. The charge density
463 outside the ball is zero, and the calculated density, amounting to 10^{-4} as shown by the
464 dashed lines in Fig 10 (c,d), is fairly close to zero. Note that the scale is one in the
465 modeled system. As $L/D < 0.1$, the truncation errors of the quadratic gradient are less
466 than 2%. It can be seen that the relative errors of the quadratic gradient and hence the
467 charge density are at second order in L/D .

468

469



470

471 **Figure 10.** The dependence of the truncation errors of the calculations by using the 8-

472 probe scheme on the relative measurement scale L/D for the case of a uniformly charged

473 ball. Panel (a) and (b) show the relative truncation errors of the quadratic gradient of

474 the electric potential (solid lines) and the charge density (dashed lines) at $[0.5,0,0]$ and

475 $[0.5,0.4,0.3]$ in the ball, respectively. Panel (c) and (d) show the relative truncation

476 errors of the quadratic gradient of the electric potential (solid lines and left vertical axis)

477 and the absolute errors of the charge density (dashed lines and right vertical axis) at

478 $[8,0,0]$ and $[2,2,6]$ out of the ball. In panel (c), the orange line orange line is overlaid

479 with the green line. In panel (d), the blue line is overlaid with the orange line.

480

481 For real measurements in space, the distances between the probes along the z-axis,
482 L_z and l_z , are much smaller than those along the other axes, L_x and L_y . The
483 truncation error in real case, therefore, should be less than evaluated when setting them
484 all equal.

485 An error analysis on the eight-probe scheme using the charged ball model of $\rho =$
486 b/r^2 is also conducted. The result as shown in Figure S5 in Supporting Information
487 (jgra55009-sup-0002-2021JA029511-si) has further confirmed the accuracy of this
488 algorithm. This 8 probe scheme is potentially applied for the net charge measurements
489 on the high altitude orbits, for which the spacecraft is spinning thus that the four probes
490 can stretch out at the ends of the four wire booms on the spin plane as shown in Fig. 9.
491 Performing similar error analysis as in Section 3, it is found the sensitivity of the probes
492 is required to reach 0.5mV, which still need technical efforts to achieve in the future.

493

494 **5. Summary and Discussions**

495 Preliminary explorations for measuring the net charge density in space have been
496 presented in this paper. Three schemes for the charge density measurements have been
497 developed.

498 The first scheme deduces the charge density based on four spacecraft electric field
499 measurements. Based on the electric fields (\mathbf{E}_α , $\alpha = 1,2,3,4$) observed at the four
500 spacecraft, we can obtain the gradient of the electric field at the barycenter of the
501 constellation, $(\nabla\mathbf{E})_c$, and furthermore, the divergence of the electric field, $(\nabla \cdot \mathbf{E})_c$.
502 The Gaussian theorem yields the charge density as $\rho = \epsilon \nabla \cdot \mathbf{E}$. This algorithm requires

503 the constellation not to be distributed in a plane or linearly. In other words, the three
504 eigenvalues of the volumetric tensor of the constellation should be non-vanishing.
505 Based on this algorithm, an analysis on the electric field data acquired during a dayside
506 magnetopause crossing event by the MMS constellation shows a charge separation in
507 the magnetopause boundary layer and that the positive charges are accumulated on the
508 magnetospheric side while the negative charges are accumulated on the magnetosheath
509 side. A normal electric field pointing at the magnetosheath is also discovered. This
510 confirms a previous theoretical prediction (Parks, 1991; Kivelson and Russell, 1995).

511

512 Another charge density measurement scheme is based on 10 or more electric
513 potential probes. By using a newly-developed algorithm [Shen et al., 2021], the linear
514 gradient, $(\nabla\phi)_c$, and the quadratic gradient, $(\nabla\nabla\phi)_c$, of the electric potential at the
515 center of the probes can be calculated from the $N \geq 10$ electric potentials,
516 $\phi_\alpha (\alpha = 1, 2, \dots, N)$, as measured at the N probes. Furthermore, the electric field and
517 the net charge density at the center of the probes can be calculated using $\mathbf{E} = -(\nabla\phi)_c$
518 and the Poisson equation, $\rho = -\epsilon\nabla^2\phi$, respectively.

519 This scheme requires the probes to be distributed uniformly. In other words, the
520 eigenvalues of the 6×6 matrix \mathfrak{R} should be non-vanishing (Shen et al., 2021). The
521 accuracy of the charge density estimated by the algorithm is of first order and that of
522 the electric field is of second order. Modeling also shows that more probes lead to
523 higher accuracy.

524

525 Finally, two other schemes are presented to measure the electric charge density,
526 which improve on the existing schemes for electric field observations onboard
527 spacecraft. If one more electric potential probe is added in addition to the six electric
528 potential probes of the electric field equipment on board the MMS spacecraft (that are
529 distributed symmetrically on the three axes of the Cartesian coordinate system), the
530 charge density can be derived along with the electric field vectors. The seventh probe
531 is placed at the origin of the coordinate system. Due to the shielding potential of the
532 spacecraft, this seven-probe scheme cannot be applied to measurements in space.
533 However, it can be utilized in charge density measurements in ground-based laboratory
534 experiments. Alternatively, by placing two more probes symmetrically on the two stiff
535 booms in the six-point scheme of the MMS constellation, the eight-probe scheme will
536 work for charge density measurements in space. The simulation test shows that the
537 estimated electric field is of fourth-order accuracy and the charge density is of second-
538 order accuracy. The truncation errors contained in this scheme are much less than those
539 in the 10 -probe scheme. The implementation of this scheme requires further
540 development in the future.

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549 **Acknowledgments**

550 This work was supported by National Natural Science Foundation (NSFC) of China

551 (Grant No. 41874190) and the Shenzhen Technology Project

552 (JCYJ20190806144013077). The MMS data were obtained from the MMS Science

553 Data Center (<http://lasp.colorado.edu/mms/sdc/>). We thank the MMS plasmas and

554 field terms for providing with these high quality data.

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681 **Figure Captions**

682

683 **Figure 1.** A schematic view of the measurements of the electric field by the MMS
684 constellation and the calculation of the charge density.

685

686 **Figure 2.** The structure of the magnetopause during an MMS crossing event on 11
687 November 2015. From top to bottom: (a) the magnetic flux density at the center of the
688 constellation, (b) the electric-field at the center of the constellation, (c) the electron and
689 ion number densities measured by MMS-1 (Pollock et al., 2016), (d) the rotation rates
690 of the magnetic field (Shen et al., 2007), (e) $|\nabla|\mathbf{B}||$, (f) the radius of curvature of the
691 magnetic field lines (Shen et al., 2003), and (g) the charge distribution. The red vertical
692 line marks the largest rotation rates, and the black vertical dotted lines mark the largest
693 and the smallest charge densities.

694

695 **Figure 3.** The distribution of the 10 probes.

696

697 **Figure 4.** The relative errors of the linear (a) and the quadratic (b) electric potential
698 gradients, i.e., $\partial_x\phi$ and $\partial_x\partial_x\phi$, calculated for different numbers of iterations at
699 $[0.1,0,0]$ within the uniformly charged ball.

700

701 **Figure 5.** The variation of the errors of the calculation by using the 10-probe scheme
702 with the relative measurement scale L/D for the case of a uniformly charged ball. The
703 measurements are performed inside of the charged ball. The left panels, (a), (c), and (e),
704 show the truncation errors for the non-vanishing component of the linear gradient by
705 L/D calculated for three different locations of the barycenter of the 10 probes inside
706 the ball, $[0.1,0,0]$, $[0.4,0,0]$, and $[0.7,0,0]$. The right panels, (b), (d), and (f),
707 illustrate the relative errors of the non-vanishing components of the quadratic gradient
708 and charge density (dashed line) calculated for the same three locations of the
709 barycenter. It is noted that $\phi_{,1} \equiv \partial_x \phi$ and $\phi_{,2,2} \equiv \partial_y \partial_y \phi$, where a comma denotes
710 partial differentiation.

711

712 **Figure 6.** The dependence of the truncation errors of the calculations by using the 10-
713 probe scheme on the relative measurement scale L/D for the case of a uniformly charged
714 ball. The measurements are performed outside of the charged ball. The left panels, (a),
715 (c), and (e), show the truncation error for the non-vanishing component of the linear
716 gradient as a function of L/D calculated for three different locations of the barycenter
717 of the 10 probes outside of the ball, $[3,0,0]$, $[5,0,0]$, and $[8,0,0]$. The right panels, (b),
718 (d), and (f), illustrate the relative errors of the non-vanishing components of the
719 quadratic gradient and the absolute value of the charge density (dashed line) calculated
720 for the same three locations of the barycenter. It is noted that the real charge density
721 outside of the ball is zero.

722

723 **Figure 7.** The relation between the absolute error of the charge density and the number
724 of measurement points at $[3,0,0]$. The relative measurement scale is chosen as $L/D =$
725 0.05 (left) and $L/D = 0.01$ (right). The dashed lines are fitted from the modeled
726 errors.

727

728 **Figure 8.** A schematic view of the seven-probe measurement of the charge density. The
729 probes are indicated by black dots.

730

731 **Figure 9.** A schematic view of the eight-probe measurement of charge density.

732

733 **Figure 10.** The dependence of the truncation errors of the calculations by using the 8-
734 probe scheme on the relative measurement scale L/D for the case of a uniformly charged
735 ball. Panel (a) and (b) show the relative truncation errors of the quadratic gradient of
736 the electric potential (solid lines) and the charge density (dashed lines) at $[0.5,0,0]$ and
737 $[0.5,0.4,0.3]$ in the ball, respectively. Panel (c) and (d) show the relative truncation
738 errors of the quadratic gradient of the electric potential (solid lines and left vertical axis)
739 and the absolute errors of the charge density (dashed lines and right vertical axis) at
740 $[8,0,0]$ and $[2,2,6]$ out of the ball. In panel (c), the orange line orange line is overlaid
741 with the green line. In panel (d), the blue line is overlaid with the orange line.

742