Influence of The Scalar Physical Quantity Field on The Probability of an Outcome

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Key Points:

- Gradient of a scalar physical quantity field (i.e. entropy (∇S)) determines the direction and location of the most probable outcome.
- Non-uniform scalar field disrupts the equally likelihood for all possible outcomes.
- Gradient of a scalar field is independent of the definition of probability.

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Abstract

Probability allows predicting the most and least probable outcomes. However, the probability of an outcome is affected by the physical quantities that describe the universe. The certainty of a single outcome and uncertainties of many outcomes are determined by how uniformly a scalar field (i.e. Potential field, entropy, mass) is distributed over an entity. It is known that an increase in entropy increases the likelihood. In this paper, this knowledge is taken one step further to understand the likelihood of the possible outcomes within an entity which have either a uniform or non-uniform scalar field. Uniform scalar fields over an entity have net-zero scalar field value. An example of Uniform scalar fields over an entity is rolling an unloaded dice where every individual six outcomes have an equal likelihood. Uniform scalar fields over an entity are where most uncertainty occurs as all outcomes have an equal likelihood. The non-uniform scalar field over an entity is where there is most certainty towards a single outcome. For example, a loaded dice has the highest probability for a single outcome. The theoretical model created in this paper is based on two square six by 6 cm die, where one die is loaded non-uniformly with different chemical molecules of different entropy and mass value and represented with contour lines in a contour map. Another dice is loaded uniformly with the same chemical molecule of the same entropy and mass all over the die. As the distribution of the chemical molecule is uniform, this configuration represents an unloaded die. Neither entropy nor mass scalar fields alone are capable of determining the outcome of the dice alone. The outcome is also determined by the type of external force, energy acting on the entity (i.e. dice), and the definition of probability. All in all, the Important result is that regardless of the definition of probability, type of external force, energy, or internal scalar field within the entity, the most probable outcome, and the least probable outcome are determined and connected by the gradient of the scalar field (i.e. Gradient of Entropy, ∇S) within the entity.

1 Introduction

It is known that an outcome with the highest entropy is most probable as energy always tends to spread out (Županović & Kuić, 2018). In addition, this property is well described in the entropy equation with the use of a logarithmic function (Khinchin, 2013). In statistical entropy, the highest entropy also indicates the highest uncertainty between two available outcomes. The highest entropy indicating the highest uncertainty is shown by the binary entropy function of the Bernoulli trial with two possible outcomes (Kitto & Boschetti, 2013). The highest statistical entropy correlates with the minimum probability that an outcome can possess. This paper introduces a new idea of a gradient of the physical scalar quantity(i.e. entropy, mass) to reduce the uncertainty of a probability. In addition, this report describes physical quantity mass and entropy as a scalar field. Finally, Identifies two causes of the level of uncertainty where one is the non-uniform distribution of the entropy and mass. The non-uniform distribution of a scalar field lowers the uncertainty. Another is the uniform distribution of scalar mass and entropy field which is the same as saying the absence of entropy or mass as net entropy or the mass would be zero. Uniform distribution of the scalar fields increases the uncertainty. Finally, to determine the most probable outcome of the entity independent of the type of scalar field and the number of scalar fields that are present, the gradient of the scalar fields are taken to find the directions of the maximum change of the scalar quantity.

2 Theory of entropy influence on the probability of the possible outcomes

This probability analysis assumes entropy to be the scalar field out of other scalar quantities such as electric potential or mass etc.

Firstly, the highest entropy does not always mean the highest uncertainty. This is true when entropy is mostly localized on a single outcome rather than distributing evenly over many outcomes. If in the case that entropy distributes evenly on to the many outcomes, where the entropy values of each outcome are close to each other, the uncertainty increases. In this case, calculating the gradient of the entropy would help to find the direction of the highest entropy change which is the most probable outcome out of many possible outcomes.

In addition, if the scalar entropy field distributes evenly on to the possible outcomes, or if the scalar entropy field is zero and does not exist, it still leads to the highest uncertainty as all outcomes have an equal likelihood.

To explain the idea of a gradient of entropy, uniform (figure 1) and non-uniform (figure 2) scalar entropy field are created inside a six-centimeter square cube dice. Scalar entropy fields are plotted as contour plots. The contour plots of scalar entropy fields presented in figures 1 and 2 are the two-dimensional top view of the dice. In addition, the same field is projected in z-direction for the three-dimensional scalar entropy field inside the cubic dice.

Creating a scalar entropy field in dice to understand how it affects the probability of an outcome.

An equation describing entropy field in the unloaded dice is written as,

$$S_{Unloaded}(x,y) = x^2 + y^2 \tag{1}$$

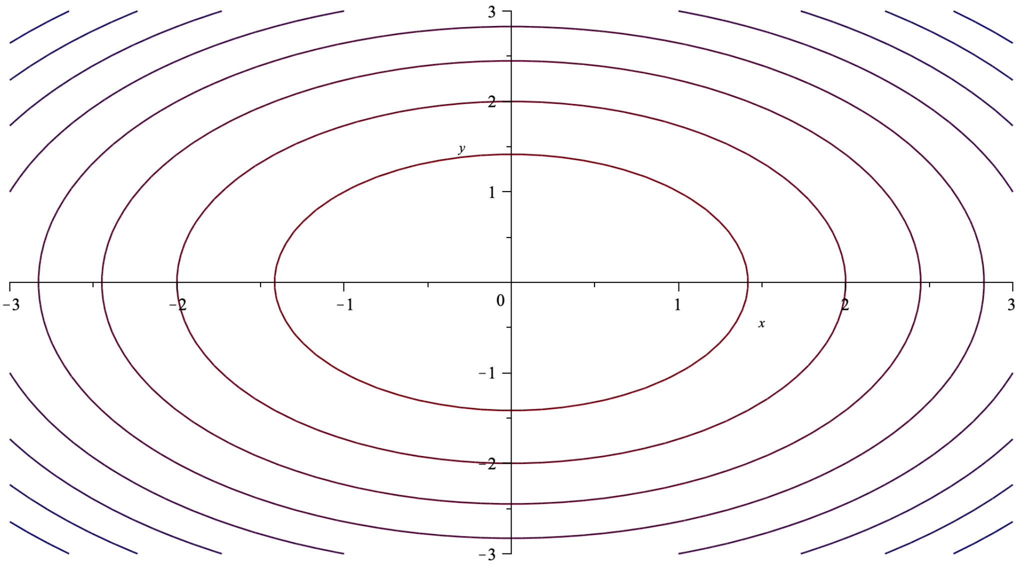

Figure 1. Top view of a square dice of 6cm in length. Uniform scalar entropy field, S is created in a square dice. Uniform scalar entropy field represents the unloaded dice, equally likelihood for all possible outcomes. In addition, x and y axes represents lengths in the units of cm.

Equation describing the entropy field in the loaded dice is written as,

$$S_{Loaded}(x,y) = x^2 - y^4 + e^{-x} + e^{-y} + 68.6 \tag{2}$$

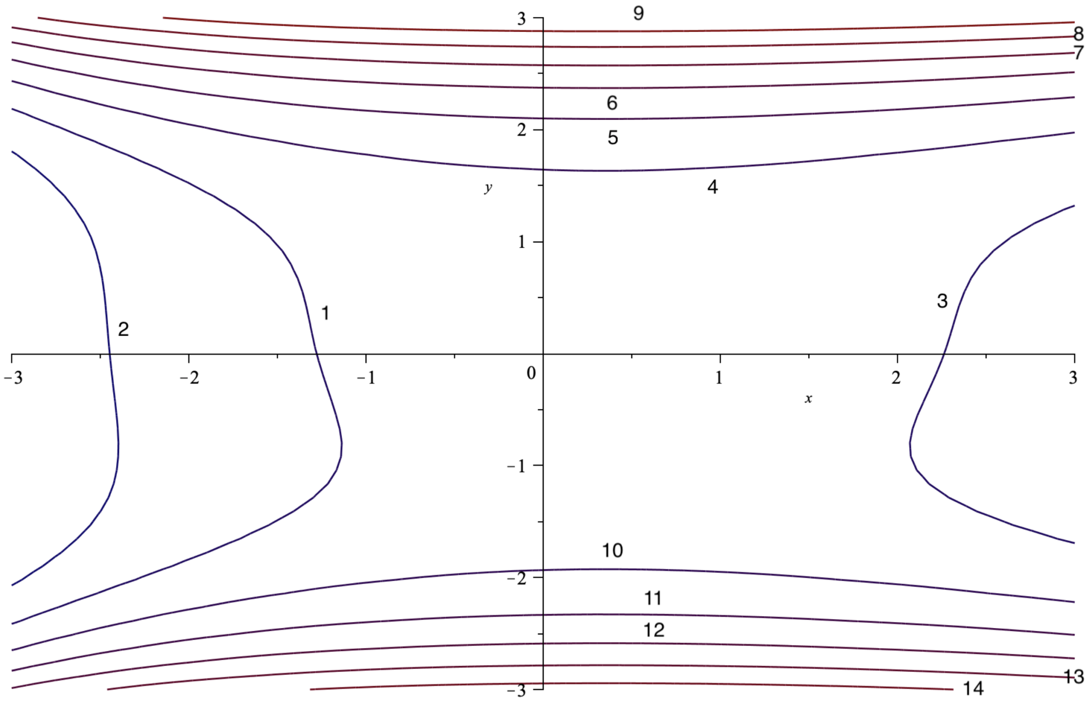

Figure 2. Top view of a square dice of 6cm in length. non-uniform scalar entropy field, S is created in a square dice. Non-uniform scalar entropy field represents the loaded dice, where likelihood of an one outcome is much higher. In addition, x and y axes represents lengths in the units of cm.

Figure 2 can be explained by naming each contour line with a chemical element that corresponds to the entropy value of contour lines presented in figure 2.

As can be seen in figure 2, the scalar values of the contour lines represent the entropy of the molecules. The assumed molecules also have mass as well as entropy. As the equations describing the contour lines describe only the entropy of the molecules, taking the gradient of the scalar entropy field would only provide probabilities for the actions that change the entropy of the molecules. Also, probability definition needs to be related to the entropy of the assumed molecules. In addition, the action is used to describe the external interference that starts the process of the probability towards a single outcome.

Therefore, under these assumptions, probability definition, and the action for this theory is set to ”which side of the cubic die would have the highest pressure when random, non-uniform external heat is applied”. Random (Direction and magnitude of heat changes all the time), non-uniform external heat is the action and it can resemble an action of rolling a die (Thrown randomly, i.e. not vertically dropped) which starts the probability process towards a single outcome.

Entropy, S values of the contour lines presented in figure 2 and corresponding chemical elements			
Contour line number	Entropy Value, S $Jmol^{-1}K^{-1}$	Corresponding Loading Element	Molar Mass, g/mol
1	74.8	Na_2O	61.9789
2	87.2	Fe_2O_3	159.69
3	74.8	Na_2O	61.9789
4	62.5	Pb	207.2000
5	50.2	Al_2O_5	133.9601
6	37.8	CaO	56.0774
7	25.5	Cr	51.99610
8	13.2	-	
9	0.815	-	
10	62.5	Pb	207.2000
11	50.2	Al_2O_5	133.9601
12	37.8	CaO	56.0774
13	25.5	Cr	51.99610
14	13.2	-	

Table 1. Each contour line have been assigned a chemical molecule that suits to the entropy value they represent. In addition, Na_2O: Sodium oxide, Fe_2O_3: Iron(III) oxide, Pb: Lead, Al_2O_5: aluminium(IIIII) oxide, CaO: Calcium oxide, Cr: Chromium. (Eboh et al., 2016)

Moreover, whatever the scalar field is, whether it is entropy or mass, or electric potential. The gradient of the scalar field would always link the highest and lowest probable outcomes together. Hence, if equations 1 and 2 are modified to represent the mass of the molecules (Just scaling the equation to the values of mass and keeping the shape of contour lines the same) for the same configuration presented in figure 2, the gradient would update to show the highest probable outcome. This would be for a different probability definition and action as the point of interest is a mass scalar field, hence action could be rolling (Rolled randomly) a die under gravity. Probability definition could be ordinary: which number on the die would face down to the ground.

Calculating the location of the most probable outcome for the loaded dice on the figure 2, by taking the gradient of the entropy of loaded dice, $\nabla S_{Loaded}(x, y)$

$$\nabla(x^2-y^4+e^{-x}+e^{-y}+68.6) = \frac{\partial(x^2-y^4+e^{-x}+e^{-y}+68.6)}{\partial x} + \frac{\partial(x^2-y^4+e^{-x}+e^{-y}+68.6)}{\partial y} \tag{3}$$

Hence,

$$\nabla(x^2-y^4+e^{-x}+e^{-y}+68.6) = 2x - e^{-x} - 4y^3 - e^{-y} \tag{4}$$

Rotating the entropy asymmetry about the y-axis for the loaded dice by changing the sign of the exponential function in x gives,

$$S_{Loaded,Rotated}(x,y) = x^2 - y^4 + e^x + e^{-y} + 68.6 \tag{5}$$

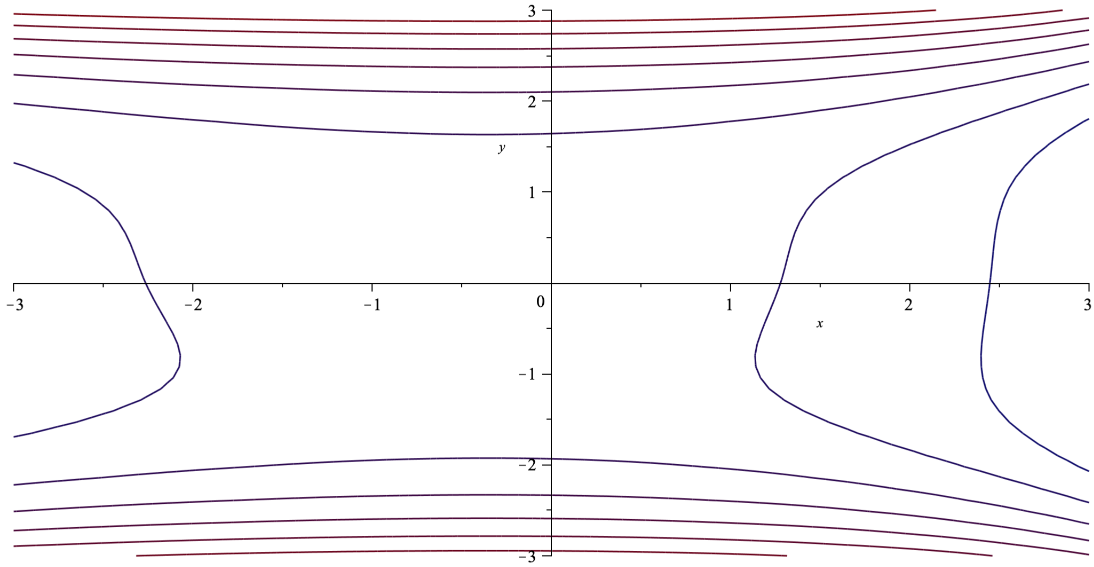

Figure 3. Top view of a square dice of 6cm in length. non-uniform scalar entropy field, S is rotated about the y-axis in a square dice. Non-uniform scalar entropy field represents the loaded dice, where likelihood of an one outcome is much higher.

Calculating the location of the most probable outcome for the loaded dice on figure 3 with a rotated scalar entropy field about the y-axis, by taking the gradient of the scalar entropy field, $\nabla S_{Loaded,Rotated}(x,y)$

$$\nabla(x^2 - y^4 + e^x + e^{-y} + 68.6) = 2x + e^x - 4y^3 - e^{-y} \tag{6}$$

3 Results

This section displays the results of probability prediction calculation is done using equations 5 and 6.

As can be seen in figure 4, the gradient of entropy has a negative value. This does not mean the entropy is decreasing. The negative sign is just the indication of the direction of the entropy lines.

Therefore, figure 4 tells that if theoretical dice are loaded with chemical elements and molecules presented in table 1 in the configuration presented in figure 2, the left-hand side of the die would have the highest probability of having the maximum pressure. Whereas, the right-hand side would have the least probability of having the highest pressure under non-uniform, random external heat apply.

Figure 4, is a contour plot and it links the points with constant entropy, same entropy values all over the given entity. Therefore, white, empty regions are the regions where the scalar entropy field changes the most. Hence, there is no line passing through that area as there is no constant, same entropy value to link together. All in all, white space is the region with the highest gradient of the scalar field where the maximum spatial change of entropy occurs.

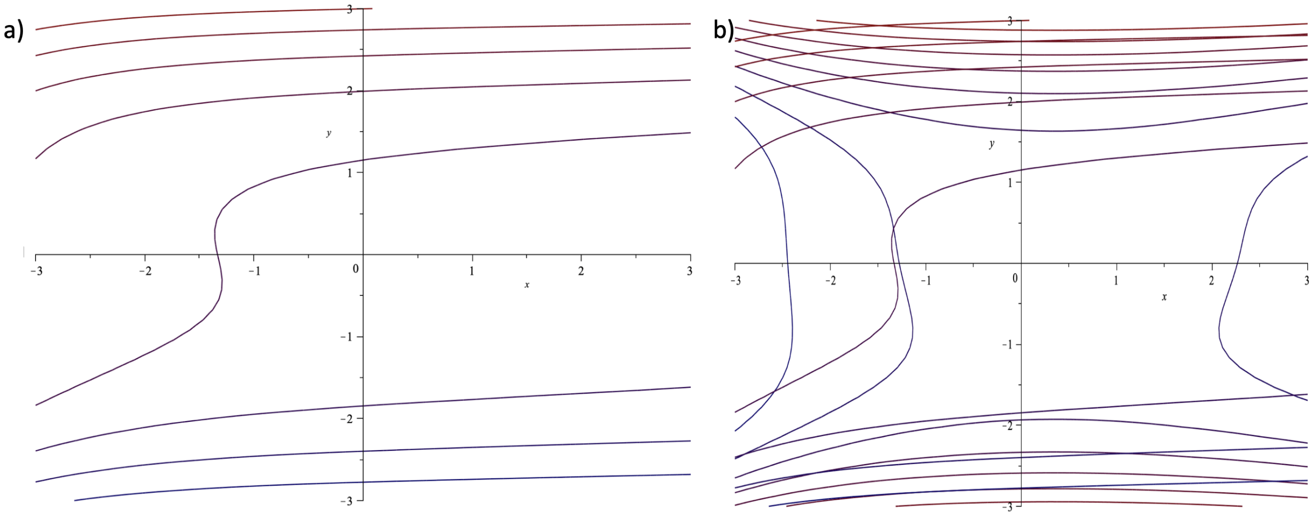

Figure 4. Top view of a square dice of 6cm in length. Result of non-uniform scalar entropy field, S in a square dice. The non-uniform scalar entropy field represents the loaded dice, where the likelihood of one outcome is much higher. In addition, the x and y axes represent lengths in the units of cm. a) Entropy gradient plot of a loaded die configuration in figure 2 displaying the highest probability region with the highest entropy gradient.

Looking at figure 2 loaded die configuration, assuming the action to be rolling die, probability definition to be the number facing down under gravity, die to be loaded with mass. At first instance, it can be thought that the upper side of the die in figure 2 has been loaded with chemical compounds that when summed, are much heavier compared to the other sides of the die. Hence, the upper side of the die is more likely to face down to the ground.

However, this is not the case as a mass on the upper side is balanced by the mass at the bottom side which has approximately the same mass value. Right, and left-hand faces have a larger mass ratio compared to each other. Probability can also be predicted by calculating the gradient of the scalar mass field. The gradient of the scalar mass field would link minimum and maximum likely outcomes together, which would be the regions of highest and lowest mass concentration.

Gradient, hence the maximum spatial change of properties mass, entropy, and moments, regardless of the choice of a physical quantity, determines and relates highest and lowest probability outcomes together.

Figure 5 shows the probability map for the loaded die in figure 2 in the form of a gradient of entropy. As can be seen in figure 5, region B and C has horizontal lines that connect the same entropy values. This means that the entropy gradient in these regions is constant. Looking at region D, this region only has a single entropy line connecting the same values of entropy and also the most and least probable outcome. Major white, space means there are no values of entropy that are the same to be connected on an entropy contour line. Therefore, this region has the highest gradient, hence the maximum scalar entropy field change. This loaded die is loaded from four sides. The sides are named, E, F, G, H, and displayed with solid grey lines, which are visible in figure 2. The white, space exists between four sides. However, as the sides G, and H have a uniform, constant entropy flow, white space then links outcomes of highest and lowest probability to be the sides E and F. Region A provides a section of the only entropy line that exist inside the region D and it has a value of " -7.47 J mol^{-1} K^{-1} cm^{-1}". The negative sign of the entropy value tells which side has the highest and which side has the lowest probability between E and F. Therefore, as the sign is negative, it means the gradient is pointing into the negative direction and the flow is from E to F. Hence, the outcome of side, E has the least and outcome of the side, F has the maximum likelihood to occur.

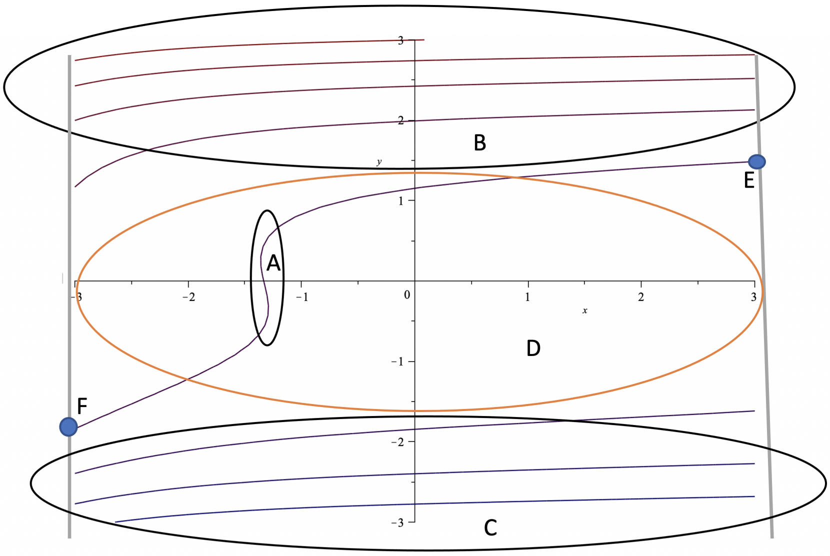

Figure 5. Top view of a square dice of 6cm in length. Result of non-uniform scalar entropy field, S in a square dice. In addition, the x and y axes represent lengths in the units of cm. The non-uniform scalar entropy field represents the loaded dice, where the likelihood of one outcome is much higher.

Another example of a loaded die is presented in figure 6. Equation describing contour lines in figure 5a is $e^x+x+y^4+e^y$ and figure 5c is $e^x+x^4+y^2+e^y$. Figure 6a and 6c are theoretical mass scalar fields presented in the form of contour lines on the same 6cm, loaded square dice. Contour lines represent the same value, assuming a normal board game played with a loaded die presented in figures 6a and 6c. It can easily be seen that, for the configuration presented in figure 6a, the mass of the top and bottom layer are almost balanced, leaving maximum mass change to occur between sides left and right. As the right-hand side is much heavier than the left-hand side, the right-hand side has the maximum likelihood of pointing towards the ground. This is shown in figure 5b, which represents the gradient map of the scalar mass field presented in figure 6a.

On the other hand, when the same configuration in figure 6a is rotated 90 degrees in an anti-clockwise direction, the new probability map shows that the top face is the most likely to face down to the ground. This is again shown in a scalar gradient map presented in figure 5d of mass configuration in figure 5c.

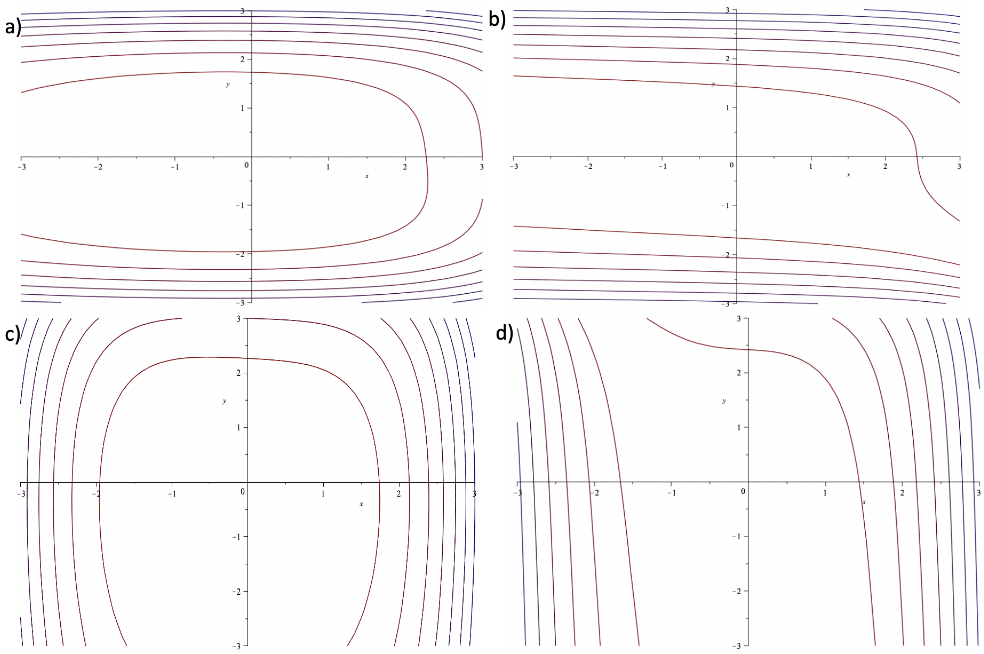

Figure 6. This example presents two differently loaded dice with a mass in figures 6a and 6c. The way the mass is distributed inside these two loaded dice is presented in contour plot format. Any point on a contour line represents the same mass value. Looking at figure 5a, without the need for numbers, it can be seen from the mass distribution that top and bottom layers balance each other out whereas, the right and left-hand sides cannot balance each other out. The right-hand side in figure 5a is heavier than the left-hand side. Hence, the moments that scale with mass. Intuitively, the expectation would that the most likely face to lie toward the ground would be the right-hand side face of the die in figure 6a. This expectation is satisfied by the prediction of the gradient of the scalar mass field, presented in figure 5b. Contour line of interest in figure 5b, representing the gradient of the scalar mass field is the one that goes into the white region as demonstrated in figure 5, region D. This is the contour line that exists in the white region of maximum change, and it gives the direction of where the gradient of the scalar mass field points. The direction that the gradient points to is the direction where the most likely outcome is. The same idea goes into dice configuration two in figure 6c, which is 90 degrees shifted in the anti-clockwise direction. Hence, the most likely outcome is located on the top face of the dice in figure 6c. This is again predicted in a gradient of the scalar mass field presented in figure 6d for the configuration in figure 6c.

As can be seen in figures 3 and 7, when the direction of the maximum entropy and minimum entropy is shifted, new gradients point in the positive direction, towards the right-hand side face indicating the maximum pressure would lie on the right-hand face (Where point E is located) face.

Finally, on the other hand, figure 8 shows an equal likelihood for all possible outcomes when the scalar entropy field is distributed uniformly over the die, representing unloaded die. This is where the overall gradient of the scalar entropy field balances out to give a net-zero gradient. Probability is distributed equally for all possible outcomes following $\frac{1}{N}$ where N, is the number of possible outcomes. The net-zero gradient of scalar

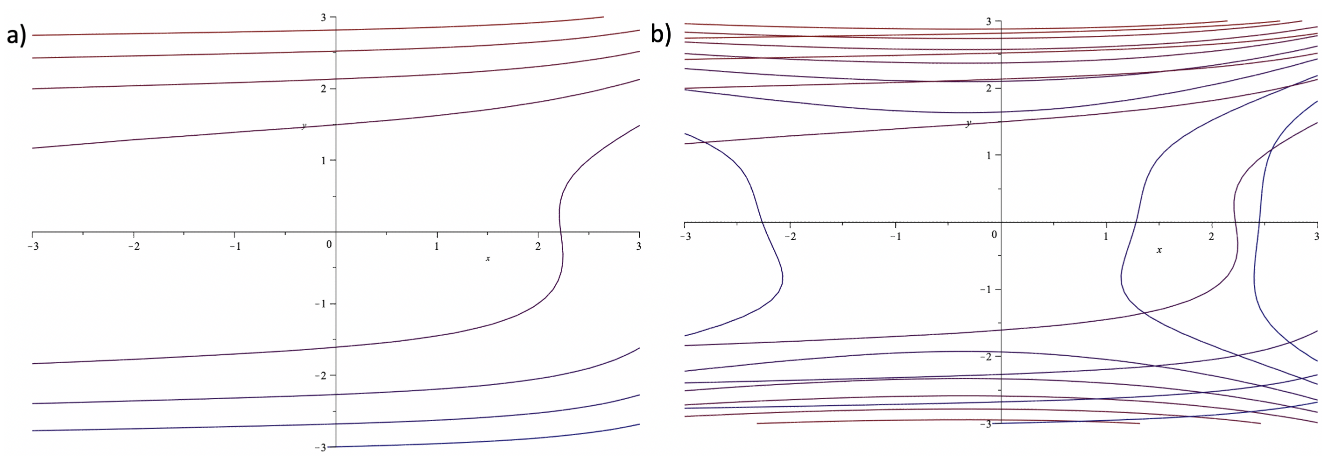

Figure 7. Top view of a square dice of 6cm in length. Result of when non-uniform scalar entropy field, S is rotated about the y-axis in a rectangular dice. In addition, the x and y axes represent lengths in the units of cm. The non-uniform scalar entropy field represents the loaded dice, where the likelihood of one outcome is much higher. a) Contour plot of the gradient of scalar entropy field. The right-hand side face has the highest likelihood to face down to the ground as it has the highest entropy gradient. b) Combined contour map of the gradient of the scalar entropy field with the scalar entropy field.

entropy field represents the highest uncertainty where no possible outcomes are in advantage compared to other possible outcomes.

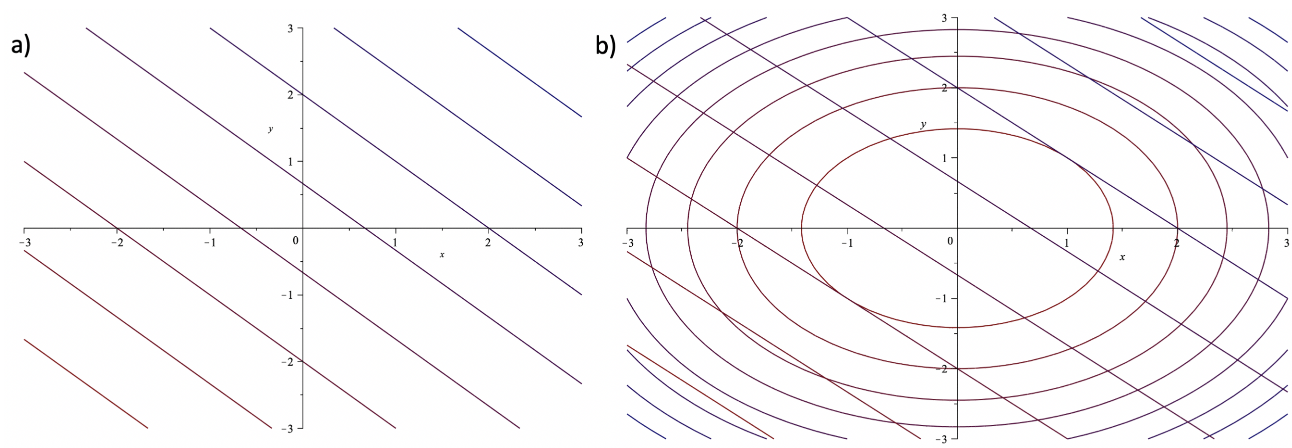

Figure 8. a) Represents the uniformly distributed entropy gradient all over the die. The entropy gradient is distributed equally for all possible outcomes such that the net entropy gradient is zero. This yields to the equal likelihood for all outcomes described statistically as, $\frac{1}{N}$. Where N is the number of outcomes. b) Gradient of the entropy is plotted over its scalar entropy field.

4 Discussion & Conclusion

This method of the gradient of a scalar field inside an entity is useful in the circumstances where the scalar field is so densely, closely, randomly, and non uniformly distributed such that the outcome of the process is not intuitively obvious or easy to predict.

Moreover, this technique can be used in any field including atmospheric electricity to determine the direction of leader growth or to determine the direction of lightning discharge between clouds of closely and densely packed scalar electric potential field.

The outcomes of all the events are determined by physical laws. However, it is hard to make a very detailed calculation and consider all the physical laws that act at the same time during the period until the outcome. Including the way the action is performed and all the physical forces induced at that moment in starting the event through a possible outcome. Therefore, to simplify the prediction of the most likelihood of possible outcomes, and to relate the statistic to the physical properties of the entity, a method of the gradient of a scalar quantity is introduced.

All in all, independent of the definition of a probability, the gradient of the scalar field tells that the most probable outcome lies on the line that exists in the region of maximum change of a physical scalar quantity (Figure 5, a line that exists in region D).

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