

1 A Long-Lived Lunar Magnetic Field Powered by Convection in the Core and a
2 Basal Magma Ocean

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4 Saira S. Hamid^{1,2}, Joseph G. O'Rourke², Krista M. Soderlund³

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6 ABSTRACT

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8 An internally generated magnetic field once existed on the Moon. This field
9 reached high intensities ($\sim 10\text{--}100\ \mu\text{T}$, perhaps intermittently) from $\sim 4.3\text{--}3.6$
10 Gyr ago and then weakened to $\lesssim 5\ \mu\text{T}$ before dissipating by $\sim 1.9\text{--}0.8$ Gyr ago.
11 While the Moon's metallic core could have generated a magnetic field via a
12 dynamo powered by vigorous convection, models of a core dynamo often fail to
13 explain the observed characteristics of the lunar magnetic field. In
14 particular, the core alone likely may not contain sufficient thermal,
15 chemical, or radiogenic energy to sustain the high-intensity fields for >100
16 Myr. A recent study by Scheinberg et al. suggested that a dynamo hosted in
17 electrically conductive, molten silicates in a basal magma ocean (BMO) may
18 have produced a strong early field. However, that study did not fully explore
19 the BMO's coupled evolution with the core. Here we show that a coupled BMO-
20 core dynamo driven primarily by inner core growth can explain the timing and
21 staged decline of the lunar magnetic field. We compute the thermochemical
22 evolution of the lunar core with a 1-D, parameterized model tied to extant
23 simulations of mantle evolution and BMO solidification. Our models are most
24 sensitive to four parameters: the abundances of sulfur and potassium in the
25 core, the core's thermal conductivity, and the present-day heat flow across
26 the core-mantle boundary. Our models best match the Moon's magnetic history
27 if the bulk core contains $\sim 6.5\text{--}8.5$ wt% sulfur, in agreement with seismic
28 structure models.

¹ Corresponding author sshamid1@asu.edu

² School of Earth and Space Exploration, Arizona State University, Tempe, AZ 85287, USA

³ Institute for Geophysics, Jackson School of Geosciences, University of Texas at Austin, Austin, TX 78758, USA

1. INTRODUCTION

Paleomagnetic analyses of lunar meteorites and Apollo samples suggest that a high-intensity magnetic field of $\sim 10\text{--}100\ \mu\text{T}$ existed $\sim 4.25\text{--}3.56$ billion years (Gyr) ago, followed by a weakened field of $\lesssim 5\ \mu\text{T}$ that persisted until $\sim 1.9\text{--}0.8$ Gyr ago (e.g., Tikoo et al. 2014, Tikoo et al. 2017, Mighani et al. 2020, Strauss et al. 2021, Wieczorek et al. 2022). Generation of an intrinsic magnetic field via dynamo action requires vigorous motion of an electrically conducting fluid such as the liquid portion of a metallic core (e.g., Bullard 1949, Elsasser 1950, Bullen 1954, Glatzmaier and Roberts 1995, Kageyama et al. 1995). Various observations indicate that the Moon has a metallic core, including seismic data from the Apollo missions (e.g., Garcia et al. 2011, Weber et al. 2011), electromagnetic sounding (e.g., Hood et al. 1999, Shimizu et al. 2013), and gravity data from the Gravity Recovery and Interior Laboratory (GRAIL) mission (e.g., Williams et al. 2014), which are all consistent with a core radius of $\sim 250\text{--}430$ km. Today, a solid inner core with a radius up to ~ 250 km may also exist (Williams et al. 2014, Weber et al. 2011).

Models of the thermal evolution of the lunar core have difficulty reproducing the history of the lunar magnetic field (e.g., Evans et al. 2018, Laneuville et al. 2014, Scheinberg et al. 2015). These models have two goals that often seem incompatible: 1) sustaining a long-lived field (e.g., multiple Gyr) and 2) sustaining an early strong field (i.e., $>10\ \mu\text{T}$, at least for the first ~ 1 Gyr). With available energy sources internal to the core (e.g., radiogenic, latent, and chemical energy, plus inner core precession), the Moon can sustain a low-intensity field for long durations (e.g., Laneuville et al. 2014; Scheinberg et al. 2015, Evans et al. 2018, Stys & Dumberry 2020). However, Evans et al. (2018) showed that those energy sources could only sustain a $>10\ \mu\text{T}$ field for <50 Myr, assuming that the radius of the core is ≤ 380 km as favored by Weber et al. (2011) and Williams et al. (2014). So, sustaining a $>10\ \mu\text{T}$ field for ~ 1 Gyr is highly improbable without an external mechanism, such as mechanical stirring between the solid mantle and the liquid core from precession of the lunar spin axis (e.g., Dwyer et al. 2011; Meyer & Wisdom 2010; Čuk et al. 2019) and/or impact-induced changes in the rotation rate of the solid mantle (e.g., Le Bars et al. 2011). Another solution to this seeming paradox is to invoke intermittency during the high-intensity epoch. For example, a recent study proposed that foundering of relatively cold material in the lunar mantle may have excited episodes of

67 rapid core cooling that lasted <1 Myr (Evans & Tikoo 2022). Finally, in this
68 study, we explore the idea that the core is not the only potential host for a
69 lunar dynamo as argued by Scheinberg et al. (2018).

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1.1. A Basal Magma Ocean

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Almost any scenario for the formation of the Moon involves enough energy to melt much of the newly formed Moon (e.g., Hartmann & Davis 1975, Warren 1985, Elkins-Tanton et al. 2011, Canup 2012, Čuk & Stewart 2012, Nakajima & Stevenson 2014). The resulting magma ocean is often modeled as solidifying in three primary stages (e.g., Elardo et al. 2011, Wieczorek et al. 2006, Hess & Parmentier 1995, Hamid & O'Rourke 2022). As the lunar magma ocean cooled, dense mafic cumulates (e.g., olivine and pyroxene) formed and sank towards the bottom. Once most of the lunar magma ocean solidified, anorthositic plagioclase with lower density began to crystallize, rising to form the lunar crust. The final, highly evolved liquids, "ur-KREEP" (enriched in uranium, thorium, potassium, rare earth elements, and phosphorus), alongside ilmenite-rich cumulates, would be gravitationally unstable because of their high densities. Some fraction of this ur-KREEP-ilmenite mixture eventually sank to the base of the mantle, ponding as a layer above the core-mantle boundary (CMB). Radiogenic heat from elements present in this fallen ur-KREEP layer, such as uranium, thorium, and potassium (with concentrations up to ~ 12 times higher than the bulk mantle), could fully melt this layer (e.g., Scheinberg et al. 2018). The result is a basal magma ocean (BMO) that persists until convective heat loss into the overlying mantle causes solidification. The nominal model of Scheinberg et al. (2018) had a 301-km peak thickness BMO; less conservative models had BMO thicknesses up to 450 km.

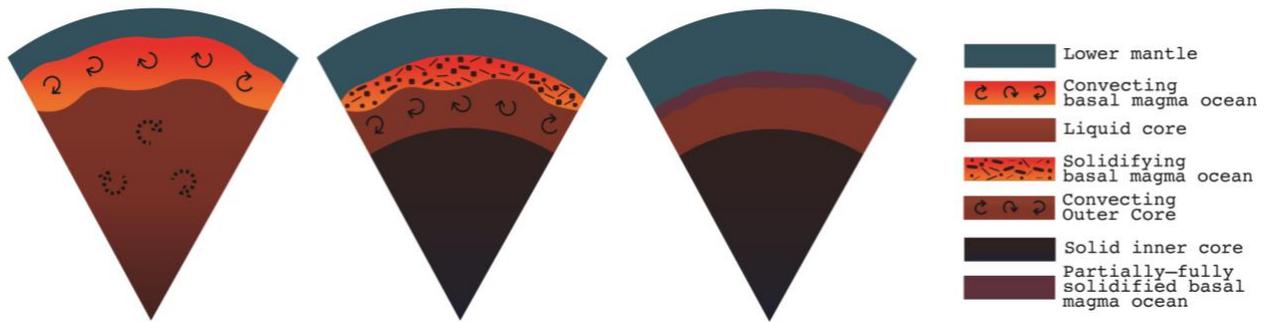
Models are equivocal about the lifetime of a BMO. For example, a small compositional density contrast between the BMO and the overlying mantle could make the BMO short-lived (Stegman et al. 2003). In this scenario, thermal expansion of the BMO can overcome the compositional density contrast between the BMO and the overlying mantle, causing the BMO to buoyantly rise and re-mix with the mantle. Conversely, the persistence of interstitial fluid trapped within the solidified cumulates could leave the BMO sequestered at the CMB (Elkins-Tanton et al. 2011, Scheinberg et al. 2018). Indeed, interpretations of geophysical data (Khan et al. 2014), seismic data (Weber et al. 2011), and gravity data (Williams et al. 2014) have indicated that a

105 deep-seated zone of partial melt at the CMB may exist today. This partial
106 melt could be the last remnant of a once-thicker BMO.

107 The lunar BMO could have sustained a dynamo if it was vigorously
108 convecting and had an electrical conductivity, σ , of several thousand S/m
109 (Scheinberg et al. 2018). Such a BMO dynamo would have an advantage over the
110 core in terms of generating strong crustal fields because it is closer to the
111 surface (e.g., Ziegler & Stegman 2013). Magnetic fields attenuate rapidly
112 with distance, so a magnetic field generated in the BMO would appear stronger
113 at the surface than a magnetic field generated with the same strength in the
114 core (e.g., Scheinberg et al. 2018, Stevenson 1983, Christensen 2010). While
115 sufficiently high conductivity is a challenge for this hypothesis, thermal
116 coupling between the BMO and core can fortunately be explored regardless of
117 this uncertainty.

118 Our study is built on the whole-Moon models presented in Scheinberg et al.
119 (2018). That study focused on the thermal evolution of the solid mantle and
120 BMO to explain the early, strong (i.e., $>10 \mu\text{T}$) lunar dynamo. Both the BMO and
121 the core were assumed to be well-mixed on the timescales of the overlying
122 solid mantle convection and to have an adiabatic temperature gradient, except
123 during the phase in which the magma ocean increases in temperature. That
124 study further tested the sensitivity of their model to the reference
125 viscosity in the solid mantle, the fraction of the KREEP layer that remained
126 near the surface, and the fraction of radioactive material concentrated in
127 the BMO. At the start of their simulations, the BMO exhibited a rapid
128 increase in heat flow from radiogenic heating, followed by a steady decline
129 to its solidus temperature. A detailed model of the core was not included
130 because the core is relatively small and does not strongly affect the thermal
131 evolution of the BMO and solid mantle. In this study, we do not directly
132 model the BMO-hosted dynamo, but rather focus on the core to test if models
133 of lunar evolution that feature a BMO as a boundary condition can explain
134 both the strong, early dynamo and the later dynamo that produced much weaker
135 fields (Figure 1).

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140 Figure 1: We study three stages in the coupled evolution of the lunar BMO and
 141 core. (Left) Convection in the BMO with the potential to produce an early, high
 142 intensity dynamo $\sim 4.25\text{--}3.56$ Gyr ago while the core was fully liquid. Dashed
 143 arrows indicate that in limited scenarios, thermal convection in the core may
 144 have occurred in tandem with the BMO-hosted dynamo. (Middle) Compositional
 145 convection in the core produced a late, low intensity dynamo until $\sim 1.9\text{--}0.8$ Gyr
 146 ago once the inner core started growing and the BMO began to solidify. (Right)
 147 The internal field ceased ~ 1 Gyr ago once the BMO solidified sufficiently, the
 148 inner core grew too large, and convection ceased in the liquid outer core.
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150 2. METHODS

151 2.1. Structure of the Metallic Core

154 We assume that the lunar core is an iron alloy that starts fully liquid
 155 with no chemical or thermal stratification. To build our models, we assume
 156 that sulfur is the major light element in the core, given its siderophile
 157 behavior and cosmochemical abundance (e.g., Pommier et al. 2018, Cameron
 158 1973). Our models also include trace amounts of potassium as a source of
 159 radiogenic heating. Other studies have speculated about the possible roles of
 160 other light elements in the lunar core, including carbon (e.g., Dasgupta et
 161 al. 2009), silicon (e.g., Berrada et al. 2020), and phosphorous (e.g., Yin et
 162 al. 2019). However, the complexities of a core with multiple light elements
 163 are beyond the scope of this study.

164 A 1-D, parameterized description of the structure of the core is the
 165 foundation of our models. As described in Appendix A, we used hydrostatic
 166 equilibrium and equations of state detailed in Khan et al. (2017) to
 167 calculate the radial profiles of density, pressure, temperature, and
 168 gravitational acceleration within the core. Our fiducial structural model
 169 assumes that the core contains 6 wt% sulfur and has a central pressure and
 170 temperature of 5.15 GPa and 1800 K, respectively, to match the core
 171 parameters described in Scheinberg et al. (2018). The radius of the core is
 172 then 350 km, which is also the same as in Scheinberg et al. (2018) and in
 173 agreement with available observational constraints. However, Scheinberg et
 174 al. (2018) used an average density for the core appropriate to a composition

175 of pure iron, which would increase the total mass of the core by ~20%.
 176 Fortunately, most of the structural parameters that are key to our
 177 thermodynamic calculations (e.g., K_0 , K_1 , L_p , and A_p in Table D1) are not
 178 sensitive to the bulk composition of the core. Sulfur is most important to
 179 the thermal evolution of the lunar core via its effect on the bulk liquidus.
 180 Using a fixed sulfur content to calculate other parameters (e.g., ρ_0 , P_0 , and
 181 M_C) should only introduce inaccuracies that are smaller than the observational
 182 uncertainties.

184 2.2. Energetics of the Metallic Core

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 186 The overlying BMO controls the evolution of the core. In the models of
 187 Scheinberg et al. (2018), the BMO is initially set to 1700 K at 4.2 Ga, heats
 188 up for ~200 Myr due to radiogenic heating, and subsequently cools until it
 189 reaches the initial temperature when the models are stopped. We start
 190 tracking the evolution of the core at the time when the BMO starts cooling
 191 again. At that time, we assume the core is fully molten and has an adiabatic
 192 temperature gradient throughout. We set that "initial" temperature at the top
 193 of the core equal to that at the bottom of the BMO. From the results of
 194 Scheinberg et al. (2018), we know the total heat flow across the core-mantle
 195 boundary (Q_{CMB}) over time:

$$196 \quad Q_{CMB} = Q_B - Q_{S_{BMO}} - H_{BMO} \quad (1)$$

197 Here, Q_B is the heat flow outward from the BMO into the solid mantle, $Q_{S_{BMO}}$ is
 198 heat associated with secular cooling, and H_{BMO} is the radiogenic heating in the
 199 BMO. In order to model the magnetic history of the Moon until present day
 200 (i.e. after the BMO model has stopped), we further assume that Q_{CMB} changes
 201 linearly to a specified present day value, which could be the same or (much)
 202 less than the value of Q_{CMB} when the BMO solidifies. With the boundary
 203 condition provided by the BMO model, we then use a well-established method,
 204 developed to study Earth's core (e.g., Labrosse 2015), to model the
 205 thermodynamic evolution of the lunar core once it starts cooling again.
 206 First, we can calculate the global heat budget of the core:

$$207 \quad Q_{CMB} = Q_L + Q_G + Q_R + Q_S \quad (2)$$

208 Here, Q_S represents the secular cooling of the core and is proportional to the
 209 core's specific heat. We assume that trace amounts of potassium produce
 210 radiogenic heating (Q_R). The remaining two terms are only relevant once the
 211 inner core nucleates: energy from latent heat (Q_L) and gravitational energy

212 from the exclusion of light elements into the outer core (Q_G) that are
213 released as the inner core grows.

214 Given the total heat flow, we solve for the rate of change in the CMB
215 temperature. As shown in Appendix B, most of the terms on the right side of
216 equation (2) are products of dT_{CMB}/dt and a term (\tilde{Q}) that depends only on the
217 thermodynamic properties of the core and its structural parameters. Each of
218 those terms is calculable using polynomial functions. We can thus rearrange
219 equation (2):

$$220 \quad \frac{dT_{CMB}}{dt} = \frac{Q_{CMB} - Q_R}{\tilde{Q}_S + \tilde{Q}_G + \tilde{Q}_L} \quad (3)$$

221 The growth rate of the inner core is directly proportional to dT_{CMB}/dt also
222 (see Appendix B). Because equation (2) does not include any secular cooling
223 of the inner core, we are implicitly assuming that the inner core is
224 perfectly insulating (i.e., with zero thermal conductivity). We could also
225 model a conductive inner core with infinite thermal conductivity, but the
226 associated heat flow is a minor contribution to the global heat budget if the
227 inner core extends to only <75% of the core radius, as expected at present
228 day. Technically, equations (2) and (3) are only valid if the liquid portion
229 of the core is convective and thus maintaining a nearly adiabatic thermal
230 profile. This assumption is not valid at present day since thermal
231 stratification probably exists since the core heat flux was likely lower than
232 the heat flux that can be conducted along the adiabat for most of the Moon's
233 evolution (e.g., Laneuville et al. 2014).

234 Our models use a liquidus for the core that depends on the bulk
235 composition. We adapted Equation 29 from Buono & Walker (2011), in which the
236 Fe-FeS liquidus is fit to a polynomial that is fourth-order in both pressure
237 and sulfur content. Our model uses an approximation of the liquidus that is
238 first-order in both pressure and sulfur content. Specifically, we estimated
239 the approximate pressure derivative (dT_L/dP) based on the difference in the
240 liquidus temperatures at 5.15 GPa at the center of the core versus 4.43 GPa
241 at the CMB for 6 wt% sulfur. We found the approximate compositional
242 derivative (dT_L/dc) based on the difference in liquidus temperatures for 0 vs.
243 25 wt% sulfur at 5 GPa (Table D1).

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245 2.3. Strength of a Core-Hosted Dynamo

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247 Vigorous convection in the core can produce a dynamo through the
248 conversion of kinetic to magnetic energy. In general, there are two types of

249 power sources for convection in the core. First, the buoyancy of light
 250 elements released from inner core solidification can drive compositional
 251 convection. Second, thermal buoyancy from secular cooling of the core,
 252 freezing of the inner core, and/or radiogenic heating can power thermal
 253 convection. For thermal convection to occur from secular cooling alone, Q_{CMB}
 254 must exceed the adiabatic heat flow (Q_{AD}), which equals the product of the
 255 thermal conductivity of the core and the adiabatic temperature gradient (see
 256 Appendix B). Once the inner core nucleates, the critical heat flow above
 257 which convection occurs is lowered.

258 We combined the energy and entropy budgets for the core to calculate the
 259 total dissipation available to power a dynamo (e.g., Labrosse 2015):

$$260 \quad \Phi_{CMB} = \Phi_L + \Phi_G + \Phi_R + \Phi_S - \Phi_K. (4)$$

261 Here, Φ_L , Φ_G , Φ_R , and Φ_S are the dissipation terms associated with Q_L , Q_G , Q_R ,
 262 and Q_S , respectively. The last term (Φ_K) corresponds to the entropy sink
 263 associated with thermal conduction in the core. Appendix B contains the
 264 polynomial expressions for each dissipation term, which, like the energy
 265 terms, depend on the thermophysical properties of the core and its overall
 266 cooling rate. Critically, we assume a dynamo exists if the dissipation is
 267 positive (i.e., if $\Phi_{CMB} > 0$ W). This criterion yields similar predictions as
 268 another often-used criterion, which is that the magnetic Reynolds number
 269 (defined below) exceeds a critical value of 50-100 (e.g., Roberts 2007).

270 Several scaling laws are available to convert the dissipation (in Watts)
 271 into the strength of the magnetic field at the equatorial surface of the Moon
 272 (in Teslas). First, we use a scaling law based on core energetics (see
 273 Appendix B) to calculate the total dipole moment (D_M) of the Moon (units of A
 274 m²). In this case, assuming the lunar magnetic field is dipolar, the surface
 275 field strength at the magnetic equator is

$$276 \quad B = \frac{\mu_0 D_M}{4\pi R_M^3}, (5)$$

277 where R_M is the radius of the Moon and μ_0 is the permeability of free space.
 278 Additionally, we estimate the magnetic field intensity using three scaling
 279 laws that relate the associated convective power to the anticipated
 280 convective velocities (e.g., Christensen 2010). These scaling laws use
 281 different force balances to calculate the strength of the magnetic field in
 282 the core (B_C). First, mixing length theory (ML) assumes a balance between
 283 inertial and buoyancy forces:

$$284 \quad B_{ML} = \left[2c\mu_0(\rho_0 R_C^2 \Phi_{CMB}^2)^{\frac{1}{3}} \right]^{\frac{1}{2}}, (6)$$

285 where $c \sim 0.63$ is a constant of proportionality, ρ_0 is the central density in
 286 the core, and R_c is the radius of the core. Second, assuming a balance of
 287 Coriolis, inertial, and gravitational (Archimedes) (CIA) forces yields:

$$288 \quad B_{CIA} = \left[2c\mu_0(\rho_0^2 R_c^4 \Omega \Phi_{CMB}^3)^{\frac{1}{5}} \right]^{\frac{1}{2}}, (7)$$

289 where Ω is the present-day angular velocity of the Moon, which may
 290 underestimate the field strength since the Moon likely rotated faster in the
 291 past. Third, the Magneto-Archimedes-Coriolis (MAC) scaling assumes a balance
 292 between Lorentz, gravitational, and Coriolis forces:

$$293 \quad B_{MAC} = \left[2c\mu_0(\rho_0 R_c^2 \Omega \Phi_{CMB})^{\frac{1}{2}} \right]^{\frac{1}{2}}. (8)$$

294 With these three scaling laws, we calculate the surface field strength of the
 295 dipole component as

$$296 \quad B_S = \frac{1}{7} B_C \left(\frac{R_C}{R_M} \right)^3. (9)$$

297 The ratio of the Moon's core radius to the Moon's radius (R_M) accounts for the
 298 fact that the dipole field at the surface is smaller than the dipole field at
 299 the core (Scheinberg et al. 2018). The pre-factor of 1/7 assumes an Earth-
 300 like power spectrum for the magnetic field and accounts for the fact that not
 301 all of the energy in the magnetic field is partitioned into the poloidal
 302 components that can reach the surface (e.g., Christensen et al. 2009,
 303 Scheinberg et al. 2018). Note that the core field is assumed to diffuse
 304 across an electrically insulating mantle in this approach, thus neglecting
 305 the contribution of the BMO. Because the BMO is argued to have a relatively
 306 large electrical conductivity, our surface field strength calculations may be
 307 considered as lower-bound estimates (discussed further in section 4.3).

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2.4. Local Rossby Number

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We further assess the dipolarity of the Moon's magnetic field,
 particularly whether a dipole-dominated or multipolar dynamo may be
 preferred. Although there are numerous hypotheses for what controls the
 breakdown of the dipole (e.g., Soderlund & Stanley 2020), we consider here
 the local Rossby number:

$$315 \quad Ro_l = \frac{U}{2\Omega l}, (10)$$

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where $\Omega = 2\pi/T$ is the angular velocity of the Moon, T is the rotation period
 in seconds, l is the characteristic length scale of the flow, and U is the
 characteristic fluid velocity. This dimensionless parameter measures the

319 relative importance of inertial to Coriolis forces at convective length
 320 scales. Numerical models of planetary dynamos indicate that dipole-dominated
 321 solutions tend to be found approximately when $Ro_l < 0.1$ (i.e., when inertial
 322 effects are relatively weak compared to rotation), with multipolar solutions
 323 occurring for larger Ro_l values (e.g., Christensen & Aubert 2006).

324 In order to estimate this parameter, we assume a characteristic fluid
 325 velocity and length scale following scaling law predictions as done for the
 326 magnetic field strengths (e.g., Christensen 2010). The mixing length (ML)
 327 scaling yields

$$328 \quad U_{ML} = \left(\frac{\Phi_{CMB} R_C}{\rho_0} \right)^{\frac{1}{3}}, \quad l_{ML} = R_C, \quad Ro_{l,ML} = \left(\frac{\Phi_{CMB} R_C}{\rho_0} \right)^{\frac{1}{3}} (2 \Omega R_C)^{-1}, \quad (11)$$

329 the Coriolis, inertial, and gravitational (Archimedes) (CIA) scaling yields

$$330 \quad U_{CIA} = \left(\frac{\Phi_{CMB}}{\rho_0} \right)^{\frac{2}{5}} \left(\frac{R_C}{\Omega} \right)^{\frac{1}{5}}, \quad l_{CIA} = \left(\frac{U_{CIA} R_C}{\Omega} \right)^{\frac{1}{2}}, \quad Ro_{l,CIA} = \left(\frac{\Phi_{CMB}}{\rho_0} \right)^{\frac{2}{5}} \left(\frac{R_C}{\Omega} \right)^{\frac{1}{5}} (4 \Omega U_{CIA} R_C)^{-\frac{1}{2}}, \quad (12)$$

331 and the Magneto-Archimedes-Coriolis (MAC) scaling yields

$$332 \quad U_{MAC} = \left(\frac{\Phi_{CMB}}{\rho_0 \Omega} \right)^{\frac{1}{2}}, \quad l_{MAC} = R_C, \quad Ro_{l,MAC} = \left(\frac{\Phi_{CMB}}{\rho_0 \Omega} \right)^{\frac{1}{2}} (2 \Omega R_C)^{-1}. \quad (13)$$

333 Here, $\phi = \Phi_{CMB} / V_{oc}$ is the volumetric thermodynamically available power over
 334 the fluid core. We could also use these velocity scalings to confirm that the
 335 magnetic Reynolds number, which relates the Ohmic diffusion timescale to the
 336 convective timescale, exceeds the critical value of ~ 50 for magnetic field
 337 generation to occur (e.g., Roberts 2007). With the definition

$$338 \quad Re_m = \mu_0 \sigma U l, \quad (14)$$

339 a flow velocity faster than ~ 0.1 -1 mm/s produces $Re_m > 50$ if we assume the
 340 length scale is equal to the core radius and the electrical conductivity is σ
 341 $\sim 10^5$ to 10^6 S m⁻¹ (e.g., Berrada et al. 2020, Pommier et al. 2020).

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343 2.5. Model Parameters

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345 Our model ingests the BMO model outputs from Scheinberg et al. (2018) and
 346 calculates the energy and dissipation budgets for the core to determine when
 347 the core may host a dynamo (see Table D2). Following the nomenclature of
 348 Scheinberg et al. (2018), naming of the BMO models corresponds to the
 349 parameters chosen to describe the mantle and the initial solidification of
 350 its magma ocean. For example, 'V19' indicates a reference mantle viscosity of
 351 10^{19} Pa s, 'K50' indicates that 50% of the KREEP layer remained trapped near
 352 the surface, and 'p54' indicates that 54% of the internal radiogenic heating

353 occurs in the sunken KREEP material. We focus on the BMO models that generate
354 magnetic fields with lifetimes of <2.9 Gyr for consistency with the
355 paleomagnetic record (e.g., Mighani et al. 2020). We adopt the nominal BMO
356 case, V19K50p54, as the basis for our nominal model of the core as it assumes
357 moderate yet reasonable values for the mantle parameters. To test the
358 sensitivity of our models to the properties of the core, we scan across four
359 different parameters: the abundance of sulfur and potassium in the core, the
360 thermal conductivity of the core, and the present-day heat flux at the CMB.

361 As with other planets, the Moon's core is expected to be an alloy of iron
362 and light elements, such as sulfur (e.g., Steenstra et al. 2016). Properties
363 of the FeS system are relatively well known (e.g., Fei et al. 1997, 2000,
364 Chudinovskikh & Boehler 2007, Morard et al. 2007, 2008, Stewart et al. 2007,
365 Chen et al. 2008, Bueno & Walker 2011, Pommier 2018) and concentrations of
366 sulfur in the lunar core are likely $<6-8$ wt% based on interpretations of
367 seismic data (e.g., Weber et al. 2011) and models of the lunar core (e.g.,
368 Scheinberg et al. 2015, Laneuville et al. 2014). We vary the sulfur
369 abundance, [S], in the bulk core from 1-9 wt% in increments of 0.5 wt%.

370 Potassium is a potential heat source in planetary cores and soluble in
371 iron alloys at planetary conditions (Murthy et al. 2003, Lee et al., 2004).
372 However, the potassium content of the lunar core remains uncertain. Based on
373 previous studies (e.g., Laneuville et al. 2014, Scheinberg et al. 2015), we
374 test a lower limit of 0 ppm, which assumes a complete lack of radiogenic
375 heating in the lunar core. Although the lower pressures and temperatures in
376 the lunar interior might lead to lower amounts of potassium in the lunar core
377 (e.g., Steenstra et al. 2018), we use plausible concentrations of potassium
378 in Earth's core as an upper limit (e.g., Hirose et al. 2013). In our models,
379 we assume that potassium is incompatible in the inner core, meaning that the
380 outer core becomes enriched in potassium as the inner core grows. We vary the
381 bulk potassium abundance, [K], from 0-50 ppm in increments of 25 ppm.

382 The thermal conductivity, k_c , of iron alloys defines the adiabatic heat
383 flux of the core. We assume that the maximum plausible value of k_c is ~ 50 W m^{-1}
384 K^{-1} , cited from thermal conductivity experiments on Fe-FeS alloys in the lunar
385 pressure and temperature range (e.g., Pommier 2018). Small amounts of
386 impurities, such as sulfur, can cause a large reduction in the thermal
387 conductivity. We investigate k_c and [S] independently in our models to isolate
388 the effects of each parameter, but they are coupled in reality. A minimum
389 value of 10 W m^{-1} K^{-1} is selected to represent relatively large impurities of
390 sulfur (e.g., Pommier 2018). Other proposed compositions for the lunar core,

391 such as Fe-Si alloys, have thermal conductivities that are intermediate
392 between these upper and lower bounds (Berrada et al. 2020). Overall, we vary
393 k_c from 10–50 $\text{W m}^{-1} \text{K}^{-1}$ in increments of 10 $\text{W m}^{-1} \text{K}^{-1}$.

394 The present-day heat flux at the CMB is highly uncertain and may have
395 been susceptible to higher heat fluxes out of the lower mantle from the
396 enrichment of water and other incompatible elements during solidification of
397 the lunar magma ocean (e.g., Elkins-Tanton & Grove 2011, Khan et al. 2014,
398 Evans et al. 2014, Weiss & Tikoo 2014, Dygert et al. 2017, Greenwood et al.
399 2018). To monitor how the core’s temperature evolves given a certain heat
400 flow, we test a range of values using thermal evolution models as a guide
401 (e.g., Laneuville et al. 2014). After the BMO solidifies, we assume that Q_{CMB}
402 decreases linearly from the final BMO simulation output ($\sim 0.90\text{--}3.70$ GW) to a
403 heat flux value specified at present. We therefore vary the present-day heat
404 flow, Q_c , from 0–2 GW in increments of 1 GW. While the lower limit of 0 GW may
405 represent an extreme scenario, we want to explore a full range of modeling
406 possibilities to account for multiple scenarios for the lunar solid mantle.
407 Furthermore, 1-D models for small planetary bodies typically indicate that
408 the heat flux varies slightly during most of the core’s evolution (e.g.,
409 Laneuville et al. 2014). We find that model outputs from simulations with a
410 Q_{CMB} equal to the final BMO simulation output are similar to those from models
411 where the Q_{CMB} slightly decreases.

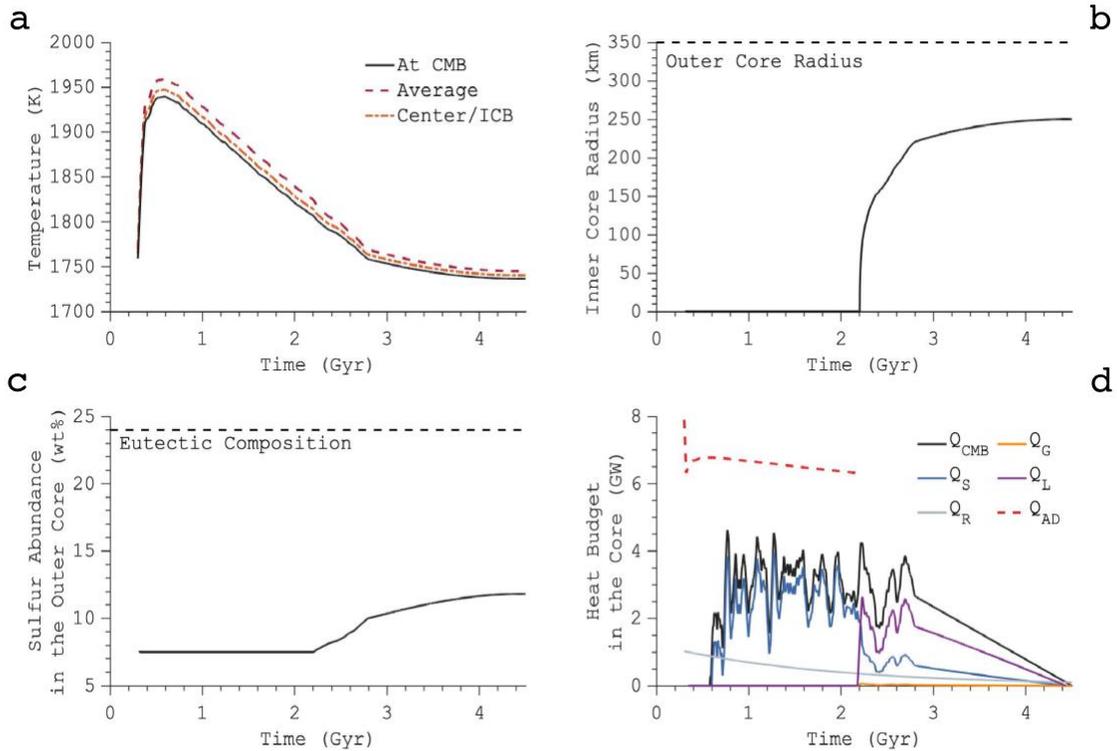
412 Astute readers will realize that our modeling approach makes the
413 cooling rate of the core seem artificially smooth over time after the BMO
414 solidifies. While the BMO exists, we use Q_{CMB} from the 3-D solid mantle models
415 of Scheinberg et al. (2018), which contain realistic time-variability and
416 fluctuations. Once the BMO has presumably solidified, our parameterized model
417 is effectively 1-D and uses a simplified approach for Q_{CMB} to capture the
418 average field strength and lifetime of the core dynamo. In reality, some
419 smaller-scale temporal variations in Q_{CMB} should be expected and the very last
420 time step is not necessarily representative of the end of the time series.

421 We ran a total of ~ 800 simulations to test the sensitivity of the core
422 model to $[S]$, $[K]$, k_c , and Q_c using BMO model outputs from Scheinberg et al.
423 (2018) as boundary conditions.

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3. RESULTS

3.1. Our Nominal Model for the Evolution of the Core

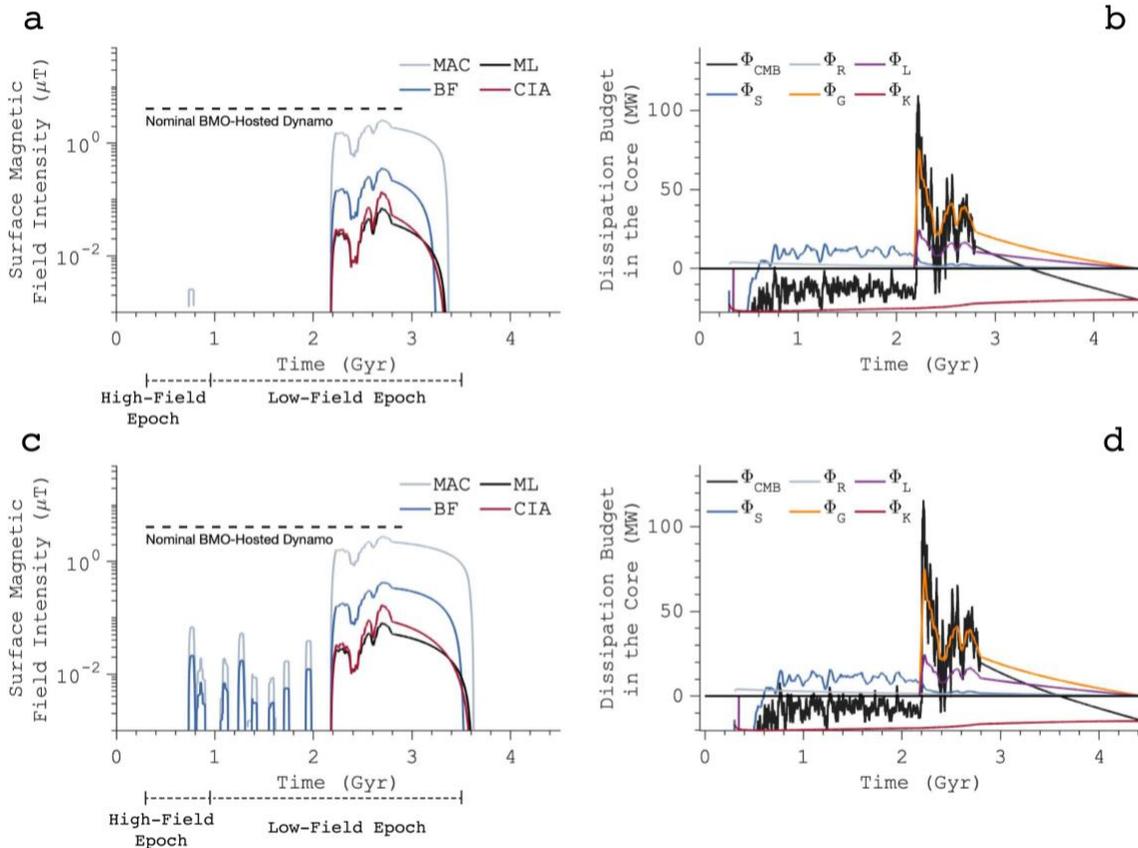


428 Figure 2: Results of the nominal core model with $k_c=40 \text{ W m}^{-1} \text{ K}^{-1}$, $Q_C = 0 \text{ GW}$,
 429 $[S]=7.5 \text{ wt}\%$, and $[K] = 25 \text{ ppm}$ coupled to the nominal BMO model
 430 (V19K50p54). All models began at 4.2 billion years before the present day.
 431 (a) Temperature at the core-mantle boundary (CMB), at the center of the
 432 core or near the inner core boundary (ICB), and the average temperature of the
 433 core. (b) Inner core radius with respect to time. (c) Sulfur abundance
 434 in the outer core with respect to time. (d) Heat budget given by latent
 435 heat, Q_L , radiogenic heating, Q_R , gravitational energy Q_G , adiabatic heat
 436 flow in the core, Q_{AD} , heat flow across the core-mantle boundary, Q_{CMB} , and
 437 secular cooling, Q_S .
 438

439 Our nominal values for the core parameters are $[S] = 7.5 \text{ wt}\%$, $[K] = 25$
 440 ppm, $k_c = 40 \text{ W m}^{-1} \text{ K}^{-1}$, and $Q_C = 0 \text{ GW}$ for the V19K50p54 BMO boundary condition
 441 (Table 1). Figure 2 details the outputs of our nominal model for the core
 442 coupled to the nominal BMO model (i.e., V19K50p54). The temperature at the
 443 CMB begins at $\sim 1760 \text{ K}$ and quickly spikes to $\sim 1940 \text{ K}$ due to radiogenic heating
 444 in the BMO (Fig. 2a). The BMO then begins solidifying as radiogenic heating
 445 declines over time, followed by the core cooling in tandem with the BMO. Once
 446 the BMO solidifies, an inner core forms at $\sim 2.2 \text{ Gyr}$ as relatively pure iron
 447 crystallizes from the inside out (Fig. 2b), expelling sulfur into the outer
 448 core (Fig. 2c). Our models assume that the lunar core always contains sub-
 449 eutectic amounts of sulfur. We verified that this assumption is consistent
 450 with our results, which track the sulfur content of the outer core over time

451 (e.g., Figure 2c). The liquidus temperature of the outer core is lowered as
 452 it is progressively enriched in sulfur. The result is a molten outer core and
 453 a growing inner core. The heat flow is always less than that transported by
 454 thermal conduction along the core adiabat, Q_{AD} . After the inner core
 455 nucleates, most extracted heat from the core arises from the release of Q_G and
 456 Q_L (Fig. 2d). The release of Q_G is nonzero, but small compared to Q_L . Following
 457 the release of Q_G and Q_L , there is a reduction in the core cooling rate due to
 458 these heat sources acting as a buffer to secular cooling. We note that the Q_{CMB}
 459 is much lower than the heat flow across the upper boundary of the BMO ($Q_B =$
 460 ~ 100 GW at 2.6 Ga) in Scheinberg et al. (2018) because Q_B includes radiogenic
 461 heating in the BMO and the heat associated with cooling the BMO itself.

462 Abundant sulfur influences the core's ability to drive a magnetic field
 463 by lowering its solidus temperature and controlling the onset of inner core
 464 crystallization (discussed further in section 3.2.1). The nominal model
 465 produces an inner core radius of 250 km at present day (Fig. 2b) and is
 466 consistent with core radii derived from calculated models of lunar gravity
 467 data (Williams et al. 2014) and reanalyzed Apollo seismic data (Weber et al.



468 2011).

469 Figure 3: a) Surface field intensities of the nominal core model where core
470 convection is driven by inner core growth relatively late in the Moon's
471 history. The buoyancy flux (BF), mixing length (ML), Coriolis, inertial,
472 gravitational (Archimedes) (CIA), and Magneto-Archimedes-Coriolis (MAC)
473 scaling laws are used to estimate surface field intensities of the dipole
474 component. Surface field intensities are compared to the nominal BMO magnetic
475 field intensity assuming the ML scaling law. (b) The dissipation budget of
476 the nominal core model includes the entropy sink associated with thermal
477 conductivity, Φ_K , the dissipation associated with secular cooling, Φ_S , latent
478 heat, Φ_L , gravitational energy, Φ_G , radiogenic heating, Φ_R , and the
479 dissipation available for a dynamo, Φ_{CMB} . (c) If k_C is lowered to $30 \text{ W m}^{-1} \text{ K}^{-1}$,
480 purely thermal convection occurs intermittently between ~ 0.7 and 2 Gyr . Those
481 resultant surface fields are several times weaker than the BMO-hosted field.
482 (d) Dissipation budget associated with a lower k_C of $30 \text{ W m}^{-1} \text{ K}^{-1}$.
483

484 The lunar BMO suppresses convection in the core by lowering its cooling
485 rate. The core produces a dynamo that begins near the cessation of the
486 nominal BMO-hosted dynamo and ends $\sim 1 \text{ Ga}$, consistent with the lower estimate
487 on the cessation of the lunar dynamo derived from radiometric dating of
488 Apollo 15 samples (e.g., Mighani et al. 2020) (Fig. 3a). The relatively weak
489 surface magnetic field strength of $\lesssim 2.55 \mu\text{T}$ is also consistent with
490 paleomagnetic data and intensities from previous models of the lunar core
491 dynamo (e.g., Laneuville et al. 2014, Tikoo et al. 2014, Tikoo et al. 2017,
492 Evans et al. 2018, Mighani et al. 2020).

493 We next consider different BMO conditions for our core model. Table 1
494 presents the nominal core input parameters for each BMO boundary condition
495 used in this study. BMOs with a smaller fraction of KREEP that remained near
496 the surface (i.e., V19K25p54 and V18K00p100 in Table 1) have greater initial
497 thicknesses and tend to require lower sulfur abundances (6.5–7 wt%) in the
498 bulk core to initiate dynamo action during the observed timing of the low-
499 intensity epoch. Because a BMO with a greater thickness will have a longer
500 lifetime (e.g., Scheinberg et al. 2018), the core will begin crystallizing at
501 a later time when the BMO eventually solidifies. Conversely, models with
502 shallower BMOs (i.e., 301 km) mostly require higher sulfur abundances in the
503 core (7–8.5 wt%) to achieve a core dynamo during the same period. BMO
504 boundary conditions with greater lifetimes additionally suppress inner core
505 growth for longer periods, resulting in smaller inner core radii at present
506 day. Furthermore, models that contain shallower BMOs match the estimated
507 timing of the lunar dynamo if balanced by less radiogenic heating in the core
508 (i.e., $\leq 25 \text{ ppm}$ of potassium). In general, BMO boundary conditions typically
509 require the core to have a higher thermal conductivity (i.e., $\geq 30 \text{ W m}^{-1} \text{ K}^{-1}$)
510 to match the estimated timing of the lunar dynamo.

Nominal Core Model Inputs

BMO Boundary Condition	V19K50p54	V19K50p36	V19K50p27	V19K25p54	V18.5K50p54	V18K00p100
BMO lifetime ¹ (Gyr)	2.6	2.0	1.6	2.9	1.2	2.1
BMO thickness ¹ (km)	301	301	301	383	301	450
[S] (wt%)	7.5	7.0	8.5	7.0	8.5	6.5
[K] (ppm)	25	0	0	50	0	50
\dot{Q}_c (GW)	0	0	0	0	0	0
k_c (W m ⁻¹ K ⁻¹)	40	10	30	40	30	30

Table 1: Nominal core parameters for each BMO boundary condition used in this study.

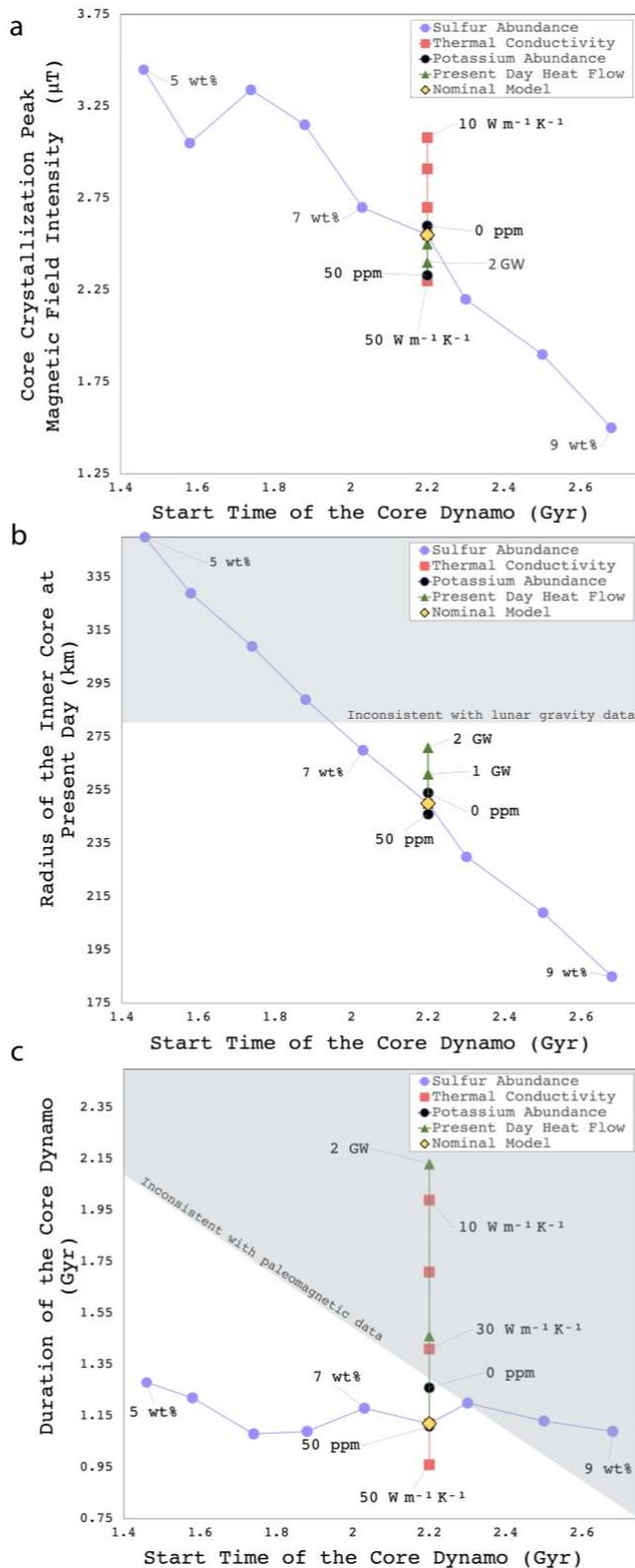
¹Values from Scheinberg et al. (2018), Table 1.

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Nominal Core Model Outputs

BMO Boundary Condition	V19K50p54	V19K50p36	V19K50p27	V19K25p54	V18.5K50p54	V18K00p100
Present-day inner core radius (km)	250	257	231	226	262	241
Compositional convection B_{\max} (μT)	0.07 (ML)	0.16	0.05	0.03	0.07	0.04
	0.13 (CIA)	0.45	0.09	0.04	0.13	0.08
	0.36 (BF)	0.77	0.27	0.20	0.40	0.28
	2.55 (MAC)	4.0	2.27	1.7	2.6	2.0
Thermal convection B_{\max} (μT)	3×10^{-6} (ML)	0.001	0.002	3×10^{-4}	0.001	0.002
	3×10^{-7} (CIA)	3×10^{-4}	0.001	9.3×10^{-5}	2×10^{-4}	0.001
	0.001 (BF)	0.08	0.07	0.02	0.06	0.06
	0.003 (MAC)	0.30	0.31	0.08	0.23	0.28
Combined B_{\max} (μT)	0.07 (ML)	0.16	0.052	0.03	0.071	0.042
	0.13 (CIA)	0.45	0.091	0.04	0.13	0.081
	0.36 (BF)	0.85	0.34	0.22	0.46	0.34
	2.55 (MAC)	4.3	2.58	1.78	2.83	2.28
Peak Local Rossby Number	0.02 (CIA)	0.03	0.02	0.02	0.02	0.03
	0.003 (ML)	0.004	0.003	0.003	0.003	0.003
	3×10^{-4} (MAC)	4×10^{-4}	2×10^{-4}	2×10^{-4}	2×10^{-4}	3×10^{-4}
Compositional convection duration (Gyr)	1.14 (ML)	1.83	1.90	0.66	2.23	1.42
	1.14 (CIA)	1.75	1.86	0.66	2.23	1.42
	1.06 (BF)	1.75	1.78	0.58	2.22	1.34
	1.12 (MAC)	1.89	1.95	0.71	2.33	1.49
Thermal convection duration (Gyr)	0.05 (ML)	0.87	0.08	0.12	0.85	1.65
	0.05 (CIA)	0.87	0.08	0.12	0.85	1.65
	0.05 (BF)	0.87	0.08	0.12	0.85	1.65
	0.05 (MAC)	0.87	0.08	0.12	0.85	1.65
Lifetime of core-hosted dynamo (Gyr)	1.19 (ML)	2.70	1.98	0.78	3.08	3.07
	1.19 (CIA)	2.62	1.94	0.78	3.08	3.07
	1.11 (BF)	2.62	1.86	0.7	3.07	2.99
	1.17 (MAC)	2.76	2.03	0.83	3.18	3.14

Table 2: Compositional and thermal convection in the core sustains low intensity magnetic fields following the cessation of a BMO-hosted dynamo. B_{\max} is the peak magnetic field intensity at the surface according to the ML, CIA, BF, and MAC magnetic field scaling laws, respectively, assuming that the mantle is electrically insulating. Thermal convection B_{\max} corresponds to the ML, CIA, BF and MAC scalings, respectively. The combined B_{\max} is the sum of surface fields generation from thermal and compositional convection. The peak local Rossby number corresponds to the CIA, ML, and MAC scaling laws, respectively. The thermal convection duration corresponds to the ML, CIA, BF, and MAC scalings, respectively.



3.2. Sensitivity Tests

Figure 4: The sensitivities of the nominal core model to core parameters k_c , $[K]$, $[S]$, and Q_c for the nominal V19K50p54 BMO model. (a) The surface magnetic field intensity is most sensitive to k_c and $[S]$ and less sensitive to $[K]$ and Q_c . (b) Our choice of $[S]$ controls the predicted timing of inner core growth and thus, a compositionally-driven core dynamo. The shaded region represents inner core radii that are probably inconsistent with lunar gravity data (e.g., Williams et al. 2014). (c) The duration of the dynamo is predicted to increase with increasing Q_c and decreasing k_c . High $[S]$ tends to delay the onset of inner core crystallization and result in a shorter field duration. The shaded region represents durations that are likely inconsistent with constraints on the end of the lunar dynamo (e.g., Mighani et al. 2020). The magnetic field intensity and the duration of the core dynamo are given by the MAC scaling law.

3.2.1. Influence of Sulfur in the Core

An inverse relationship exists between the sulfur content and the solidus temperature of the core. As the sulfur content increases, the solidus temperature of the Fe-S system decreases, delaying core

563 solidification until lower temperatures are reached. Therefore, the timing of
 564 inner core growth, and thus, the start time of compositional convection in

565 our models depends on the sulfur content of the bulk core (Fig. 4a). The
566 sulfur concentration is viable when the end of the core-hosted dynamo matches
567 the lower estimate on the cessation of the lunar dynamo at ~ 1 Gyr (e.g.,
568 Mighani et al. 2020). Initial sulfur abundances of 1–6.5 wt% result in inner
569 core nucleation at higher temperatures, causing the core to solidify rapidly
570 early in its history (Fig. 4b). Sulfur abundances from 7–8.5 wt% result in
571 the inner core nucleating near the cessation of the BMO-hosted dynamo.
572 Increasing the bulk sulfur content to > 8.5 wt% further delays inner core
573 growth and generally results in temporal gaps between the BMO-hosted and
574 core-hosted dynamo, a complete lack of core dynamo action, or contradictions
575 with timing estimates derived from paleomagnetic data (Fig. 4c). However, if
576 the BMO model assumes a lower solid mantle viscosity (i.e., $V_{18.5K50p54}$),
577 then convective heat transfer is more efficient and results in shorter BMO
578 lifetimes (Scheinberg et al. 2018). As a result, the inner core begins
579 crystallizing earlier and a bulk sulfur content of up to 12 wt% can produce
580 results consistent with the lower estimate on the cessation of the lunar
581 dynamo (e.g., Mighani et al. 2020). The trends outlined in Fig. 4 that arise
582 from variations in k_c , $[K]$, $[S]$, and Q_c continue under all other BMO boundary
583 conditions.

584 3.2.2. Influence of the Core's Heat Budget and Thermal Conductivity

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587 The duration and intensity of the core dynamo are also sensitive to k_c ,
588 Q_c , and $[K]$ (Figure 4). A potassium abundance of 50 ppm in the core
589 contributes thermal energy to the dynamo but suppresses growth of the inner
590 core, which can decrease the predicted intensity of the magnetic field
591 overall. Decreasing $[K]$ has a minimal effect on the field intensity because
592 radiogenic heating is nearly equivalent to secular cooling in the dissipation
593 budget. In contrast, increasing the total heat flow to 1–2 GW increases the
594 duration and strength of the core-hosted dynamo, unless a low sulfur
595 abundance leads to rapid core solidification. Furthermore, the duration and
596 intensity of the field generally increases with decreasing thermal
597 conductivity values. We find that purely thermal convection typically occurs
598 before the onset of inner core crystallization if the thermal conductivity is
599 low (i.e., $10\text{--}30 \text{ W m}^{-1} \text{ K}^{-1}$ as in Fig. 3c). As thermal conductivity decreases,
600 the super-adiabatic heat flow increases, leading to a stronger, more long-
601 lived dynamo. Thermal convection-driven dynamos typically occur
602 simultaneously with BMO-hosted dynamos as the core is still hot and fully

603 molten. Compared to the abundance of sulfur in the bulk core, our simulations
604 reveal that small variations in parameters such as k_c , Q_c , and $[K]$ play an
605 overall negligible role in the onset of a compositionally-driven dynamo,
606 whereas a thermal convection-driven dynamo is largely dictated by k_c .

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608 3.2.3. Strength and Timing of the Core Dynamo

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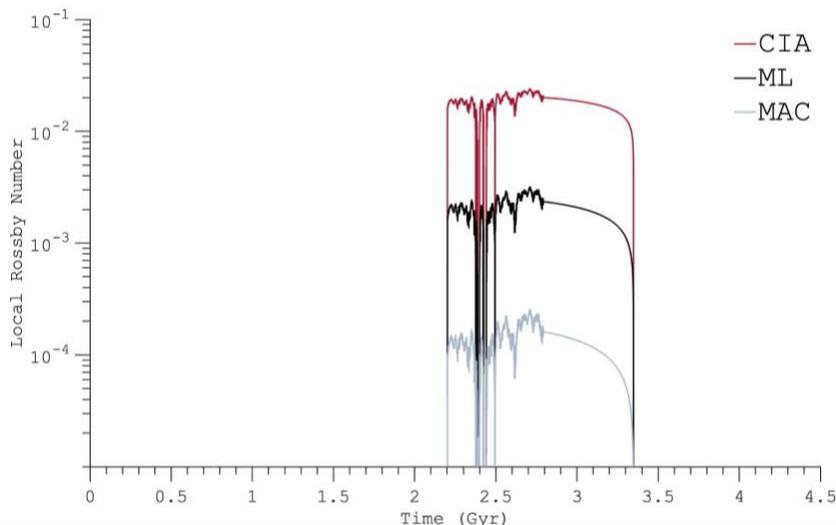
610 Depending on the BMO boundary condition, inner core crystallization can
611 produce fields ~ 0.7 – 2.3 Gyr in duration, with peak magnetic fields of 0.16,
612 0.45, 0.77, and 4 μT , for the ML, CIA, BF, and MAC scaling laws respectively
613 (Table 2). A general issue arises in the case of the CIA, ML, and BF scalings
614 laws in which the strengths are not sufficiently strong enough to reproduce
615 the first period of decline to ~ 4 – 7 μT by 3.19 Ga (e.g., Strauss et al. 2021)
616 or the second period of decline to $\sim 5 \pm 2$ μT by ~ 1 – 2 Ga (e.g., Tikoo et al.
617 2017, Mighani et al. 2020). However, intensities ranging from ~ 1.7 – 4 μT can be
618 achieved under all BMO boundary conditions if the MAC scaling law is assumed.
619 In particular, an intensity of 4 μT is achieved if the BMO boundary condition
620 contains a lower fraction of radioactive material concentrated in the BMO
621 (i.e., V19K50p36). However, this magnetic field weakens to a maximum of 4 μT
622 ~ 2 Gyr after accretion, which is ~ 0.7 Gyr later than what is observed in the
623 lunar paleomagnetic record (Strauss et al. 2021).

624 Surface magnetic fields are weaker if they are driven by thermal convection
625 rather than by inner core crystallization. The peak surface magnetic field driven
626 by thermal convection in the nominal core model is 3×10^{-6} , 3×10^{-7} , 0.001, and 0.003
627 μT for the ML, CIA, BF, and MAC scaling laws, respectively (Table 2). For all BMO
628 boundary conditions, thermal convection in the core is initiated ~ 3.7 Gyr ago
629 (albeit briefly in some models; e.g., Fig. 3a). Furthermore, depending on the BMO
630 boundary condition, thermal convection can persist intermittently for up to ~ 1.7
631 Gyr, resulting in an overlap with the BMO-hosted field (e.g., Fig. 3c). Thermal
632 convection produces intensities that are consistent with previous modeled
633 estimates of the core (e.g., Laneuville et al. 2014, Evans et al. 2018,
634 Scheinberg et al. 2015), but inconsistent with paleomagnetic analyses
635 constraining the initial and final decline of the lunar dynamo (e.g., Tikoo et
636 al. 2017, Mighani et al. 2020, Strauss et al. 2021). Furthermore, these results
637 are consistent with a low-intensity epoch that persisted from ~ 1.9 – 0.8 Ga (e.g.,
638 Mighani et al. 2020, Tikoo et al. 2017, Tikoo et al. 2014, Strauss et al. 2021).

639 An uneven heat flow across the CMB may make the magnetic field
 640 intermittent because dynamos can be sensitive to slight variations in heat
 641 flow (Scheinberg et al. 2015). As an artifact of our modeling approach, early
 642 magnetic fields produced via thermal convection are discontinuous due to
 643 fluctuations in the Q_{CMB} from mantle dynamics. In some cases, thermal
 644 convection generates fields that are predicted to drop to zero multiple times
 645 before rising again from inner core crystallization. The duration of these
 646 gaps in the magnetic field are much longer than the magnetic diffusion time
 647 (Appendix C). Using the nominal models but with core conductivity lowered to
 648 $k_c = 30 \text{ W m}^{-1} \text{ K}^{-1}$ as an example case (i.e., Fig 3c), gaps in thermal convection
 649 on average last $\sim 140 \text{ Myr}$, whereas the magnetic diffusion time is only a few
 650 hundred years. Alternatively, dynamos induced by thermal convection can
 651 transition directly into those induced by inner core crystallization,
 652 compounding the resultant fields.

653 3.3. Local Rossby Number

654 In order to make initial predictions for the magnetic field
 655 morphologies in our models, we estimate the local Rossby number as a proxy
 656 for whether the core dynamos would be dipole-dominated or multipolar, as for
 657 example has been done previously for Ganymede's dynamo (Rückriemen et al.
 658 2015). The CIA scaling law predicts higher values of the local Rossby number
 659 ($Ro_l \sim 10^{-2}$) relative to the ML ($Ro_l \sim 10^{-3}$) and MAC ($Ro_l \sim 10^{-4}$) scaling laws



660 since inertia plays a
 661 larger role in the
 force balance
 (Christensen & Aubert
 2006). However, for the
 nominal core model, all
 scaling laws predict
 that the local Rossby
 number is below the
 threshold value of ~ 0.1
 throughout the lifetime
 of the core dynamo,
 suggesting a prevailing

674 dipole-dominated magnetic field (Table 2 and Figure 5).

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685 Figure 5: Predictions of the local Rossby number for the nominal core model
686 estimated from CIA, ML, and MAC scaling laws.

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4. DISCUSSION

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In this study, we demonstrated that a BMO dynamo could naturally dovetail with a core dynamo. Future studies should further explore this hypothesis by addressing the following important issues.

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4.1. Other Modes of Crystallization in the Core

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Future studies could model more complex modes of crystallization in the lunar core. To recap, we made two relevant assumptions. First, we assumed that the core always contains sub-eutectic amounts of sulfur, which most of our models indeed predict (section 3.3). Second, we assumed that the core solidifies from the center outwards. We set the liquidus temperature to increase faster than the adiabatic temperature with pressure (e.g., with gradients of 30 K/GPa versus ~23–25 K/GPa, respectively).

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Future studies could relax these two assumptions, which would produce more complicated behavior in models (e.g., Hauck et al. 2006). First, FeS rather than Fe could crystallize from the outer core as it cooled if the sulfur content were super-eutectic. Being sulfur-rich compared to the residual liquid, solid FeS would float to the top of the liquid rather than sink to form an inner core like solid Fe. Second, solidification could occur at the top or middle of the outer core, rather than at its bottom. For example, “iron snow” could occur in metallic cores if the liquidus crosses the adiabat above the base of the outer core. This process could help drive a dynamo as the solidified iron sinks and remelts in the warmer fluid below,

713 leading to compositional convection (e.g., Williams 2009, Breuer et al.
714 2015). Whether the Moon's core entered an FeS crystallization or Fe snow
715 regime at any time remains an ongoing question.

716 Scientists might make more realistic models of the thermal evolution of
717 sulfur-rich cores if they include these processes. Such models require
718 detailed phase diagrams for the Fe-FeS system. The neglect of Fe snow and FeS
719 crystallization in our models does not change our takeaway message, however,
720 that the presence of a basal magma ocean overlying the core may influence the
721 timing and intensity of the core dynamo. Our models may interface with
722 future, more detailed descriptions of Fe snow and FeS in the core.

723

724 4.2. Morphology of the Lunar Dynamo

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726 The geometry and paleo-orientation of the Moon's magnetic field remains
727 largely uncertain. Estimates of paleoinclinations from five Apollo samples
728 suggest the existence of a dipolar field and a paleopole located at $\sim 75^\circ\text{N}$
729 between 3.8 and 3.3 billion years ago (e.g., Cournède et al. 2012). These
730 findings are possibly best explained with a paleofield geometry close to the
731 present-day rotation axis of the Moon. Assumptions of the paleopole were made
732 based on the location of Apollo samples: samples collected from the northern
733 hemisphere were given a positive declination while samples collected from the
734 southern hemisphere were given a negative declination. However, the sign of
735 the inclination remains largely unknown and more data is required to confirm
736 interpretations made from lunar samples. Studies of Apollo 17 mare basalts
737 estimated an inclination of $\sim 34^\circ$ based on the layering of its parent boulder
738 (Nichols et al. 2021). This inclination is consistent with, but does not
739 require, a dipole in the center of the Moon aligned along its rotation axis.

740 Conversely, Olson & Christensen (2006) hypothesized that the Moon's
741 magnetic field may have been multipolar rather than dipole-dominated. The
742 critical difference between our studies is the amplitude of buoyancy flux in
743 the core. Their study assumed that the average buoyancy flux associated with
744 convection in the lunar core was 0.3 times the terrestrial value. That is,
745 $F_{\text{Moon}} = 0.3 F_{\text{Earth}}$, where $F = \alpha g Q / (\rho C_p)$ with thermal expansivity α , gravitational
746 acceleration g , convective heat flux Q , density ρ , and specific heat capacity
747 C_p . This assumption was based on the idea that tidal dissipation could add
748 several TW of power to the ancient lunar core (e.g., Williams et al. 2001).
749 This larger heat flow leads to larger estimates of the local Rossby number
750 (e.g., $Ro_1 \sim 2$), which would shift the lunar dynamo into a multipolar regime.

751 In contrast, our models do not include tidal heating in the lunar core. So,
752 the total power available for convection is only several GW in our models, as
753 shown in Figure 2d.

754 It is also possible that the directional magnetization of lunar rocks
755 does not record a long-term orientation of the lunar magnetic field since
756 differential rotations between the mantle and core would cause a core dipole
757 field to drift across the lunar surface (e.g., Čuk et al. 2019). Relative
758 motions of the core and mantle or misalignment between the lunar dynamo and
759 spin axis may further explain the great variability in the inferred
760 orientation of the lunar dynamo from proposed paleopole locations (e.g.,
761 Oliveira & Wieczorek 2017, Nayak et al., 2017). Future missions sampling the
762 lunar bedrock along varying latitudes will allow for more precise geometric
763 determinations of the Moon's magnetic field.

764

765 4.3. Electromagnetic Core - Mantle Coupling

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767 The effects of an electrically conducting lower mantle on the core
768 dynamo is not considered in our study. This limitation is significant for
769 several reasons. First, as noted in Section 2.3, the relatively large
770 conductivity of the BMO, especially when it is fully liquid, will likely
771 cause our estimates of surface magnetic field strengths to be artificially
772 small compared to if this conductivity were taken into account. Our estimates
773 for the core field strength assume that the entire mantle, including the BMO,
774 is electrically insulating such that the core-generated magnetic field
775 becomes a potential field that diffuses upward through the mantle. Given the
776 anticipated higher conductivity of metalliferous silicate melts compared to
777 solid mantle rocks (e.g., Scheinberg et al. 2018), the top of the dynamo
778 region may effectively be the top of the BMO, rather than the top of the
779 core, even if the BMO is subcritical for dynamo action.

780 Second, fluid flows within the BMO may also modulate the core field
781 itself (e.g., Gómez-Pérez et al. 2010). Conversely, if the BMO fluid is
782 stably-stratified, its presence may still filter out small-scale components
783 of the core field that rapidly vary via the magnetic skin effect (e.g.,
784 Christensen 2006). Third, the BMO may have resulted in larger magnetic
785 coupling between the core-mantle in the past, relevant to studies of the
786 Moon's rotational dynamics over time (e.g., Dumberry & Wieczorek 2016).
787 Further work, such as numerical dynamo modeling, is needed to better
788 understand the full degree of coupling between the BMO and core of the Moon.

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4.4. Thermal Stratification in the Core

791 The effects of thermal stratification in the lunar core are not
792 considered in this study. The inclusion of thermal stratification can have
793 several effects on the heat flux at the CMB. Studies of Mercury's core (e.g.,
794 Knibbe and Westrenen 2018, Knibbe and van Hoolst 2021) found that thermal
795 stratification can lead to an increased inner core size, higher temperatures,
796 and a larger heat flux at the CMB, which together results in an early start to
797 the magnetic field. Subsequent heat released upon core solidification would
798 enable slow core growth and an active magnetic field until present day.
799 Future work could apply these models of Mercury to the Moon.

800

4.5. The Early Evolution of the Moon

801 Thermal stratification is probably inevitable at present day, but could
802 also exist early in the Moon's history. In this study, we assumed that the
803 core was initially fully molten and had an adiabatic temperature gradient. If
804 radiogenic heating in the BMO ever made the bottom of the BMO hotter than the
805 top of the core, then heat would move from the BMO into the core, which would
806 cause thermal stratification at the top of the core that may delay the start
807 of a core-hosted dynamo. However, the Moon could have formed with "superheat"
808 (such that the core was initially hotter than the BMO) (e.g., Evans et al.
809 2018), in which case the core could deliver heat to the BMO even while the
810 BMO heats up radiologically. Neither our study nor Scheinberg et al. (2018)
811 modeled these two, countervailing possibilities in detail. Further work is
812 thus needed to better understand the formation and early evolution of the
813 Moon.

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815

5. CONCLUSIONS

816 Our model for the coupled evolution of a basal magma ocean and the core
817 places estimates on the abundance of sulfur in the core (i.e., 6.5-8.5 wt% in
818 Table 1) and can explain the timing and relative intensity of the lunar
819 magnetic field consistent with other models of the lunar core (e.g.,
820 Laneuville et al. 2014, Evans et al. 2018, Scheinberg et al. 2015). The basal
821 magma ocean does not need to be electrically conductive to explain the
822 results presented here, even if it was required to explain the results of

823 Scheinberg et al. (2018). While that may mean the early, intense lunar dynamo
824 remains unexplained, we find that the predicted timing of the lunar dynamo in
825 our models is most consistent with observational constraints of the long-
826 lived low-intensity period when moderate abundances of sulfur and potassium
827 are assumed in the core, the core's thermal conductivity is high, and if the
828 present-day CMB heat flow is assumed to be low (or even zero). Excessively
829 high values of Q_{CMB} at present day (i.e., 1-2 GW) tends to increase the
830 duration of the magnetic fields longer than is consistent with timing
831 constraints on the end of the lunar dynamo (e.g., Mighani et al. 2020).
832 Modeled intensities are most consistent with paleomagnetic analyses
833 constraining the initial and final decline of the lunar dynamo (e.g., Tikoo
834 et al. 2017, Mighani et al. 2020, Strauss et al. 2021) when the BMO boundary
835 condition is assumed to have less radiogenic heating concentrated in the BMO
836 or when the MAC scaling is assumed. Other scaling laws (i.e., CIA, ML, and
837 BF) predict that magnetic field intensities would be $\sim 1-2$ orders of magnitude
838 weaker at the surface than inferred from paleomagnetic data (although recall
839 that our intensities may be higher if electrical conductivity of the BMO is
840 taken into account).

841 Thermal convection can briefly exist with the BMO, but is generally
842 short-lived (Fig 3a) or intermittent (Fig 3c), generating magnetic field
843 intensities of up to $\sim 0.3 \mu\text{T}$ that persist for $\lesssim 1.7$ Gyr (Table 2). Near
844 cessation of the lunar BMO dynamo, heat flows are too low for purely thermal
845 convection and later dynamo action requires inner core crystallization.
846 Magnetic fields generated from the onset of inner core crystallization can
847 reach intensities of up to $\sim 4 \mu\text{T}$ and can persist for $\lesssim 2.3$ Gyr (Table 2).
848 Temporal gaps may arise between dynamos powered by different types of energy
849 in the core (i.e., thermal vs. compositional), which are neither confirmed
850 nor excluded by extant data. Temporal gaps in the magnetic field can lead to
851 complications in interpretations of the paleomagnetic record and may indicate
852 that a portion of Apollo samples with null paleointensities (e.g., Tarduno et
853 al. 2021) may not result from poor magnetic recording properties.

854 Estimates of the core sulfur abundance from our model can further
855 translate into predictions of the radius of the inner core. These predictions
856 can be verified with future missions, such as the Farside Seismic Suite
857 (e.g., Panning et al. 2021), which will provide new constraints on the
858 internal structure of the Moon, and the Lunar Geophysical Network (e.g.,
859 Weber et al. 2021), which aims to understand the size, state, and composition
860 of the lunar core and the chemical and physical stratification of the mantle.

861 Together, these findings will help discriminate between hypotheses that seek
862 to explain the high-low intensity epoch. Research on the Moon's magnetic
863 history should remain fruitful for decades.

864

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869

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APPENDIX A. RADIAL STRUCTURE OF THE LUNAR CORE

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We approximated the lunar core as a mixture of liquid Fe and liquid Fe-10

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wt% S to make structure models. We followed the procedure detailed in Khan et

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al. (2017), especially in their Appendix A, to calculate radial profiles of

1199 density, pressure, and temperature. We use the mass-weighted averages of the
 1200 depth-dependent values of the Grüneisen parameter and the coefficient of
 1201 thermal expansion. We then performed a least-squares fit to parameterize the
 1202 radial density using a fourth-degree polynomial:

$$1203 \quad \rho(r) = \rho_0 \left[1 - \left(\frac{r}{L_\rho} \right)^2 - A_\rho \left(\frac{r}{L_\rho} \right)^4 \right], \quad (A1)$$

1204 where ρ_0 is density at the center of the core, L_ρ is a length scale, and A_ρ is
 1205 a constant. The effective bulk modulus is then $K_0 = 2\pi G(L_\rho \rho_0)^2/3$, where G is
 1206 the gravitational constant. The derivative of the effective bulk modulus is K_1
 1207 $= (10 A_\rho + 13)/5$. Finally, the adiabatic thermal gradient in the core is then
 1208 $T_a(r) = T(0) [\rho(r)/\rho_0]^\gamma$.

1210 APPENDIX B. ENERGETICS OF A DYNAMO IN THE LUNAR CORE

1211
 1212 Section 2.2 describes the heat budget of the lunar core. For completeness,
 1213 we list here the polynomial equations used to calculate the different terms.
 1214 Analogous equations that were developed to model Earth's core can be found in
 1215 Labrosse (2015), albeit with slightly different notation and additional
 1216 complexities added to the analytic formulation, and in the Supporting
 1217 Information for Blaske & O'Rourke (2021).

1218 In our models, the total heat flow across the core/mantle boundary can be
 1219 partitioned into four different terms, each of which is proportional to the
 1220 overall cooling rate of the core (dT_{CMB}/dt). First, we have the heat flow
 1221 associated with secular cooling of the fluid portion of the core. Before the
 1222 inner core nucleates, we have

$$1223 \quad \tilde{Q}_S = -\frac{4}{3}\pi\rho_0 C_C L_\rho^3 f_c \left(\frac{R_C}{L_\rho}, \gamma \right) \left[1 - \left(\frac{R_C}{L_\rho} \right)^2 - A_\rho \left(\frac{R_C}{L_\rho} \right)^4 \right]^{-\gamma}, \quad (B1)$$

1224 where

$$1225 \quad f_c(x, \delta) = x^3 \left[1 - \frac{3}{5}(\delta + 1)x^2 - \frac{3}{14}(\delta + 1)(2A_\rho - \delta)x^4 \right]. \quad (B2)$$

1226 After the inner core nucleates,

$$1227 \quad \tilde{Q}_S = -\frac{4}{3}\pi\rho_0 C_C L_\rho^3 \left[1 - \left(\frac{R_I}{L_\rho} \right)^2 - A_\rho \left(\frac{R_I}{L_\rho} \right)^4 \right]^{-\gamma} \left[\frac{dT_L}{dR_I} + \frac{2\gamma T_L(R_I) \left(\frac{R_I}{L_\rho} \right) \left(1 + 2A_\rho \left(\frac{R_I}{L_\rho} \right)^2 \right)}{1 - \left(\frac{R_I}{L_\rho} \right)^2 - A_\rho \left(\frac{R_I}{L_\rho} \right)^4} \right] \left[f_c \left(\frac{R_C}{L_\rho}, \gamma \right) \right. \\ 1228 \quad \left. - f_c \left(\frac{R_I}{L_\rho}, \gamma \right) \right] \left(\frac{dR_I}{dT_C} \right), \quad (B3)$$

1229 where $T_L(R_I)$ is the liquidus temperature evaluated at the inner core boundary
 1230 given by

$$1231 \quad T_L(R_I) = T_L(0) - K_0 \left(\frac{dT_L}{dP} \right) \left(\frac{R_I}{L_\rho} \right)^2 + \frac{c_0}{f_c \left(\frac{R_C}{L_\rho}, 0 \right)} \left(\frac{dT_L}{dc} \right) \left(\frac{R_I}{L_\rho} \right)^3. \quad (B4)$$

1232 Here c_0 is the mass fraction of sulfur in the outer core, which increases as
 1233 the inner core grows. Differentiating this equation yields the slope of the
 1234 liquidus at the inner core boundary:

$$1235 \quad \frac{dT_L}{dR_I} = -2K_0 \left(\frac{dT_L}{dP} \right) \left(\frac{R_I}{L_\rho^2} \right) + \frac{3c_0}{f_c \left(\frac{R_C}{L_\rho}, 0 \right)} \left(\frac{dT_L}{dc} \right) \left(\frac{R_I^2}{L_\rho^3} \right). \quad (B5)$$

1236 Following Nimmo (2015), we use this slope and the adiabatic thermal gradient
 1237 to calculate the growth rate of the inner core

$$1238 \quad \frac{dR_I}{dT_C} = - \frac{1}{\left(\frac{dT_L}{dP} - \frac{dT_a}{dP} \right)_{R_I}} \left(\frac{T_L(R_I)}{T_{CMB} \rho(R_I) g(R_I)} \right). \quad (B6)$$

1239 The growth of the inner core also releases latent heat

$$1240 \quad \tilde{Q}_L = 4\pi R_I^2 \rho(R_I) T_L(R_I) \Delta S_c \left(\frac{dR_I}{dT_C} \right), \quad (B7)$$

1241 where $\Delta S_c = 200$ J/K/kg is the entropy of melting for the inner core (Nimmo
 1242 2015). Next, we compute the gravitational energy related to the exclusion of
 1243 sulfur from the inner core as it freezes:

$$1244 \quad \tilde{Q}_G = \frac{8\pi^2 G \rho_0 c_0 \alpha_I R_I^2 L_p^2}{f_c \left(\frac{R_C}{L_\rho}, 0 \right)} \left[f_\chi \left(\frac{R_C}{L_\rho} \right) - f_\chi \left(\frac{R_I}{L_\rho} \right) \right] \left(\frac{dR_I}{dT_C} \right), \quad (B8)$$

1245 where $\alpha_I = 2.3$ is the coefficient of compositional expansion for enriching the
 1246 outer core in sulfur (Nimmo 2015). Here we leverage another useful function:

$$1247 \quad f_\chi(x) = x^3 \left\{ -\frac{1}{3} \left(\frac{R_I}{L_\rho} \right)^2 + \frac{1}{2} \left[1 + \left(\frac{R_I}{L_\rho} \right)^2 \right] x^2 - \frac{13}{70} x^4 \right\}. \quad (B9)$$

1248 Last and easiest, the radiogenic heat in the core is

$$1249 \quad Q_R = M_c H_K [K] \exp(-\lambda_K t), \quad (B10)$$

1250 where $\lambda_K = 1.76 \times 10^{-17}$ s⁻¹ and $H_K = 4.2 \times 10^{-14}$ W/kg/ppm are the decay constant
 1251 and the heat production rate at $t = 0$ for potassium-40, respectively.

1252 The energy budget by itself does not reveal whether a dynamo may exist in
 1253 the lunar core. We must compute the dissipation budgets, again following
 1254 Labrosse (2015) and studies such as Blaske & O'Rourke (2021). First, we
 1255 expand equation 3 in the main text as

$$1256 \quad \Phi_{CMB} = \left(\frac{T_D [T_L(R_I) - T_{CMB}]}{T_L(R_I) T_{CMB}} \right) Q_L + \left(\frac{T_D}{T_{CMB}} \right) Q_G + \left(\frac{T_D - T_{CMB}}{T_{CMB}} \right) Q_R + \left(\frac{T_D (T_S - T_{CMB})}{T_S T_{CMB}} \right) Q_S - \Phi_K. \quad (B11)$$

1257 Here we use the average temperature in the outer core:

$$1258 \quad T_D = \frac{T(R_I)}{\left[1 - \left(\frac{R_I}{L_\rho}\right)^2 - A_\rho \left(\frac{R_I}{L_\rho}\right)^4\right]^\gamma} \left[\frac{f_c\left(\frac{R_C}{L_\rho}, 0\right) - f_c\left(\frac{R_I}{L_\rho}, 0\right)}{f_c\left(\frac{R_C}{L_\rho}, -\gamma\right) - f_c\left(\frac{R_I}{L_\rho}, -\gamma\right)} \right], \quad (B12)$$

1259 The effective temperature associated with dissipation from secular cooling is
1260 almost identical to T_D but slightly hotter:

$$1261 \quad T_S = \frac{T(R_I)}{\left[1 - \left(\frac{R_I}{L_\rho}\right)^2 - A_\rho \left(\frac{R_I}{L_\rho}\right)^4\right]^\gamma} \left[\frac{f_c\left(\frac{R_C}{L_\rho}, \gamma\right) - f_c\left(\frac{R_I}{L_\rho}, \gamma\right)}{f_c\left(\frac{R_C}{L_\rho}, 0\right) - f_c\left(\frac{R_I}{L_\rho}, 0\right)} \right]. \quad (B13)$$

1262 Finally, we can calculate the dissipation sink associated with the thermal
1263 conductivity of the core fluid:

$$1264 \quad \Phi_K = 16\pi\gamma^2 k_C L_\rho \left[f_k\left(\frac{R_C}{L_\rho}\right) - f_k\left(\frac{R_I}{L_\rho}\right) \right] T_D, \quad (B14)$$

1265 where our last useful function is

$$1266 \quad f_k(x) = 0.2x^5 \left[1 + \frac{10}{7}(1 + 2A_\rho)x^2 + \frac{5}{9}(3 + 10A_\rho + 4A_\rho^2)x^4 \right]. \quad (B15)$$

1267 Note that we can then write the total adiabatic heat flow in terms of Φ_K :

$$1268 \quad Q_{AD} = \left(\frac{T_S T_{CMB}}{T_D (T_S - T_{CMB})} \right) \Phi_K \quad (B16)$$

1269 which is an energy-based definition that is basically equivalent to the usual
1270 formula, $Q_{AD} \sim 4\pi R_C^2 k_C (dT_a/dr)$, derived from Fourier's law.

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1272 APPENDIX C. MAGNETIC DIFFUSION TIME

1273 We determine the time it takes for the field to decay after convection
1274 ceases following the procedure detailed in Stevenson (2003) to approximate
1275 the magnetic diffusion time:

$$1276 \quad \tau = \frac{R_C^2}{\pi^2 \lambda}. \quad (C1)$$

1277 Here R_C is radius of the electrically conducting region (i.e., the core) and λ
1278 is magnetic diffusivity given by:

$$1279 \quad \lambda = \frac{1}{\mu_0 \sigma}, \quad (C2)$$

1280 where μ_0 is the permeability of free space and σ is the electrical
1281 conductivity. We assume $\lambda \sim 1 \text{ m}^2/\text{s}$, appropriate for terrestrial planets with a
1282 liquid iron alloy core (e.g., Schubert and Soderlund 2011), such that the
1283 magnetic field will diffuse across the core in $\tau \sim 400$ years.

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1285 APPENDIX D: TABLES

Table D1

Description of Model Constants

Term	Description	Value
μ_0	Permeability of free space	$1.257 \times 10^{-6} \text{ H}\cdot\text{m}^{-1}$
G	Gravitational constant	$6.67 \times 10^{-11} \text{ m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$
R	Universal gas constant	$8.3145 \text{ J}\cdot\text{K}^{-1}\cdot\text{mol}^{-1}$
R_M	Radius of the Moon	1737 km
R_C	Radius of the core	350 km
Ω	Angular velocity of the Moon	$2.66 \times 10^{-6} \text{ rad}\cdot\text{s}^{-1}$
K_0	Effective modulus	$121.4 \times 10^9 \text{ Pa}$
K_1	Effective derivative of effective modulus	5.7871
A_p	Constant in density profile	1.59
ρ_0	Central density	$6477 \text{ kg}\cdot\text{m}^{-3}$
P_0	Central pressure	$5.15 \times 10^9 \text{ Pa}$
M_c	Mass of the core	$1.16 \times 10^{21} \text{ kg}$
V_c	Volume of the core	$3.95 \times 10^{16} \text{ m}^3$
g	Gravitational acceleration near the core-mantle boundary	$0.6311 \text{ m}\cdot\text{s}^{-2}$
γ	Grüneisen parameter for the core	1.65
C_c	Specific heat of the core	$835 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$
ΔS_c	Entropy of melting for the inner core	$200 \text{ J}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$
α_I	Coefficient of compositional expansion for enriching the outer core in sulfur	2.3
λ_K	Average decay constant for potassium-40	$1.76 \times 10^{-17} \text{ s}^{-1}$
H_K	Heat production rate for potassium-40	$4.2 \times 10^{-14} \text{ W}\cdot\text{kg}^{-1}\cdot\text{ppm}^{-1}$
c	Constant of proportionality in equations 5-7	0.63
dT_L/dc	Compositional dependence of liquidus temperature	-2500 K
dT_L/dP	Pressure dependence of liquidus temperature	$3 \times 10^{-8} \text{ K}\cdot\text{Pa}^{-1}$

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Table D2		
Definition of Model Inputs and Outputs		
Variable	Definition	Values
Input parameters		
[S]	Abundance of sulfur in the core ^a	1–6 wt%
[K]	Abundance of potassium in the core ^b	0–50 ppm
k_c	Thermal conductivity of the core ^c	10–50 W m ⁻¹ K ⁻¹
Q_c	Present-day heat flow across the core-mantle boundary ^d	0–2 GW
Energy budget outputs of the core		
Q_{CMB}	Heat flow across the core-mantle boundary	GW
Q_L	Latent heat from inner core nucleation	GW
Q_G	Gravitational energy released from inner core nucleation	GW
Q_R	Radiogenic heating in the core	GW
Q_S	Secular cooling of the core	GW
Entropy budget outputs of the core		
Φ_{CMB}	Dissipation available to power a dynamo	MW
Φ_L	Dissipation associated with latent heat	MW
Φ_G	Dissipation associated with gravitational energy	MW
Φ_R	Dissipation associated with radiogenic heating	MW
Φ_S	Dissipation associated with secular cooling	MW
Φ_K	Dissipation sink associated with thermal conductivity	MW

1292 ^aWeber et al. 2011.

1293 ^bLaneuville et al. 2014, Scheinberg et al. 2015, Hirose et al. 2013.

1294 ^cPommier 2018.

1295 ^dLaneuville et al. 2014.