Theoretically Proving the Quantized Wave-Particle Duality

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Abstract A century ago, the energy quantization hypothesis and wave-particle duality have been proposed as the foundations of the quantum theory, but still lacking complete theoretical proof and interpretation. For the first time, the "quantized wave particle duality equation" is expressed by a way of physical and mathematical formulas, which simply compose of three factors: the quantized resonant eigenvalue, interaction or transition time, and sum of translational kinetic energy; and accompanied by a full interpretation of the related physical laws. From now on, they are no longer the premise hypothesis of quantum theory, but the inevitable conclusion. The long-term conflict between quantum theory and classical physics has been resolved, which is actually a self-consistent and coherent theoretical system.

I. Introduction

Reviewing the development history of quantum theory, in 1901 Planck [1] deduced the blackbody radiation law and formula, and for this purpose put forward the assumption of energy quantization, afterwards as the fundamental theoretical basis of the quantum theory system. On this basis, in 1905, Einstein continued to improve Planck's quantization hypothesis, put forward the light quantum hypothesis, and perfectly explained the photoelectric effects. In 1916, Robert Milligan made experiments to prove Einstein's theory of photoelectric effects.

Then in 1924, De Broglie boldly put forward the "material wave" hypothesis, believing that all matter has wave particle duality. It was verified by two independent electron diffraction experiments in 1927. George Thomson irradiated the electron beam through the thin metal sheet, and observed the predicted interference patterns. And Clinton and Lester made experiments to incident low-speed electrons into nickel crystals, and obtained the experimental conclusion of electron diffraction patterns. Similarly, in 1969, Willis and Scully applied the energy level transition mechanism of bound electrons in atoms to prove the same argument.

Similarly, through the upgraded double slit interference experiment, such as experiments with electrons, neutrons, atoms, and even molecules, such strange quantum behaviors can be demonstrated. In 1961, Claus Jö nsson pioneered the double slit interference experiment to test the physical behavior of electrons, and found that electrons can also interfere. In 1974, Pier Merli successfully emitted electrons one by one, and the interference phenomenon was also clearly observed on the detection screen.

De Broglie's wave particle duality hypothesis was experimentally verified to be able to describe the quantum behavior of electrons. Then Bohr put forward the "complementary principle" to describe the quantum behavior in a more detailed and perfect way: the wave and particle properties of micro particles will not appear in the same measurement. The wave and particle properties are mutually exclusive at the same time and unified at a higher level.

It is known that in today’s quantum theory, the quantization hypothesis and wave particle duality have become the two core basic hypotheses in quantum theory. Although it was proved to be correct by experiments at the beginning, or that quantum theory itself originated from experimental science, it has never been directly proved in a theoretical way consistent with the physical essence since its birth for more than a century, that is, it lacks the understanding and complete interpretation of its mechanism. There is a serious conflict with classical physics, that is why there are many quantum-supernatural events and some unexplained physical experimental phenomena in history.

It has also become the focus of debate between Bohr school and Einstein school. The Copenhagen School (including Bonn, Heisenberg, Pauli, Dirac, etc.) led by Bohr explained the structure and motion law of the quantum world with probability theory, mathematical statistics and contingency philosophy. The classical school of thought

led by Einstein (including Planck, Bose, De Broglie, Schrodinger, etc.) explained the incredible and strange behaviors of the quantum world with geometry, differential equations and the philosophy of necessity.

Therefore, it is necessary to reveal the physical mechanism of quantum theory and the essential truth behind the phenomenon in order to answer these questions and settle the disputes. The quantization hypothesis is put forward based on the empirical formula that the experimental conclusions are consistent, and other scholars also use some indirect theories to derive seemingly reasonable conclusions [2]-[5], but they do not reveal the physical truth. Based on this hypothesis, the Heisenberg uncertainty principle, De Broglie matter wave, Schrodinger probability wave equation and other basic quantum theoretical systems have been successively established. When applying to atoms, De Broglie qualitatively believes that the bound electrons form standing waves, and their rotation frequencies can only take on some discrete values. These quantized orbits correspond to discrete energy levels, thus confirming the energy level quantum phenomenon of Bohr model.

These are also hypotheses that lack theoretical proof. It can be said that the whole quantum theory originated from experimental science. So far, the experimental verification has shown that its results are correct and have been widely applied in sciences and technologies [6]-[7]. Unfortunately, it has never been systematically proved by theory, especially the lack of satisfactory explanation of physical mechanism; any truth needs more exhaustive, scientific logical and complete theoretical proof to be perfect. Can't answer Einstein's question: can God roll the dice? Who (the hand of God, the palm of Buddha) controls the micro world? That is, the problem of local implicit variables.

II. Case of a Single Electron or Particle

We know that in nature, the generation and propagation of any waves originate from oscillations, which in turn originate from an oscillator or a stimulator, a resonator or a periodic motion (circling). This is the basic mechanism requirement of classical physics and the common senses, for example, resonators such as mechanical oscillator, electromagnetic carrier, resonant circuit, etc.; and periodic motions such as circular orbit motion, elliptical orbit motion, spiral orbit motion, etc.

Especially the helical orbital motion of electrons in atom is the most typical universal case representation. This typical representation is taken as the object of this paper study. It can be covered or simplified into other motion in mathematical complex spatial geometry, such as linear oscillator, circular motion, elliptical motion, helical motion.

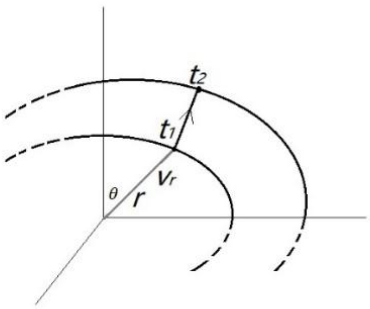

Fig.1 Periodic Motion of Electrons in Helical Orbits and its Coordinate System

A large number of experimental observations and validations in today's physics have led to the atomic standard model [8]. Especially in the advanced atomic model represented by scientists such as Bohr, it is pointed out that electrons move in helical orbits around the center of the nucleus, and the spherical coordinate system of

their orbits is shown in Fig.1, which represents the orbital mathematical equation of the atomic standard model.

$$r = (R_0 + V_r t)\, e^{j\omega t}$$

Where R_0 is the initial radial coordinate, V_r is the radial velocity, ω is the angular velocity, t is time.

The velocity of electron motion is derived from the derivatives:

$$\mathrm{V} = \frac{dr}{dt} = e^{j\omega t}\ \frac{d}{dt}(R_0 + V_r t)\ +\ (R_0 + V_r t)\ \frac{d}{dt}(e^{j\omega t})$$
$$= [V_r + j\omega(R_0 + V_r t)\,]e^{j\omega t}$$

The electrons are instantly timed at the two coordinate points（t_1, t_2）, from $t_1 = t_0$ to $t_2 = t_0 + t_\Delta$ motion, which kinetic energy increment is:

$$\mathrm{E} = \frac{1}{2}m{V_2}^2 - \frac{1}{2}m{V_1}^2$$

When $t_1 = t_0$,

$${V_1}^2 = [V_r + j\omega(R_0 + V_r t_0)]^2\, e^{j2\omega t_0}$$

When $t_2 = t_0 + t_\Delta$,

$${V_2}^2 = \{V_r + j\omega[R_0 + V_r(t_0 + t_\Delta)]\}^2 e^{j2\omega(t_0 + t_\Delta)}$$

Subtract the above two equations:

$${V_2}^2 - {V_1}^2 = e^{j2\omega t_0}\{\{V_r + j\omega[R_0 + V_r(t_0 + t_\Delta)]\}^2 e^{j2\omega t_\Delta} - [V_r + j\omega(R_0 + V_r t_0)]^2\}$$

From the classical physics theory, it is known that periodic motion must satisfy the resonance condition before the eigen-solutions can have physical significance (or observability) and scientific application value. It is known from a large number of experimental validations of electron motion models in atoms and the wide applications of atomic spectroscopy. From the derivations of the corresponding mathematical physical equations, the preconditions for the eigen-solutions are as follows: $e^{j2\omega t_\Delta} = 1$, also the resonance condition, where ω is the resonance angle frequency.

Substitute $e^{j2\omega t_\Delta} = 1$ for further simplification:

$${V_2}^2 - {V_1}^2 = e^{j2\omega t_0}\{\{V_r + j\omega[R_0 + V_r(t_0 + t_\Delta)]\}^2 - [V_r + j\omega(R_0 + V_r t_0)]^2\}$$
$$= e^{j2\omega t_0}\,(j\omega t_\Delta V_r)\,\{2V_r + j\omega[2R_0 + V_r(2t_0 + t_\Delta)]\}$$

Let $R_r = R_0 + V_r(t_0 + t_\Delta/2)$, then

$${V_2}^2 - {V_1}^2 = j2\omega t_\Delta V_r\, e^{j2\omega t_0}(V_r + j\omega R_r)$$
$$= j2\omega t_\Delta V_r\,[cos(2\omega t_0) + jsin(2\omega t_0)](V_r + j\omega R_r)$$

The magnitude of the energy (amplitude for a complex) is:

$$|\mathrm{E}| = \frac{1}{2}m\left|{V_2}^2 - {V_1}^2\right|$$

$$= \omega t_\Delta\, m{V_r}^2\,\{\,[(\omega R_r/V_r)\,cos\,(2\omega t_0) + sin(2\omega t_0)]^2 + [\cos\,(2\omega t_0) -\ (\omega R_r/V_r)\ sin(2\omega t_0)]^2\,\}^{\frac{1}{2}}$$

$$= \omega t_\Delta\, m{V_r}^2\,\{\,1 + (\omega R_r/V_r)^2\,\}^{\frac{1}{2}}$$

Periodic formula, $\omega = 2\pi\nu = 2\pi/T_c$, where ν is the resonant frequency, T_c is the motion cycle, then the above formula is derived:

$$|\mathrm{E}| = 2\pi\nu t_\Delta\ m{V_r}^2\ \left\{1 + \left(\frac{2\pi}{T_c}\frac{R_r}{V_r}\right)^2\right\}^{\frac{1}{2}}$$

The above formula shows that the contribution of energy amplitude is decomposed into circular motion component, which is the main motion component, and its radial motion component. For simplicity, the initial time can be set $t_1 = t_0 = 0$. The radial movement or drift distance is simplified as: $R_r = R_0 + V_r(t_0 + t_\Delta/2) = R_0 +$

$t_\Delta\,(V_r/2)$. Pay particular attention to the term of $V_r/2$ can be regarded as the drift rate corresponding to the average kinetic energy of random thermal motion in the case of many micro-electrons, and $t_\Delta\,(V_r/2)$ is the drift distance between two orbits. Considering that there are no any conditions for V_r in terms of whether a positive or negative number in the initial orbital parameter equations, it means that the electrons can drift up and down between the two orbits, so the rate mean square value of the average kinetic energy is equivalent to $V_r/2$.

The radial motion component can be regarded as the perturbation correction term of the energy amplitude, which is the mechanism of the orbital transition of electrons or particles. Because the electron or particle is mainly in approximate circular motion, and the transition orbit is usually close enough, so the disturbance term is usually a small quantity, which can be approximately treated as random thermal motion in statistical mechanics. However, this radial disturbance component is extremely important, which is the inducement for electrons or particles to transition from one orbit to another, and it is a necessary element for discretization and quantization.

It is known from the measurement results obtained from some known atomic energy spectra, that are separated, not continuous, and radiated by electromagnetic waves. This means in most cases, electrons or particles move periodically self-optimizing selective orbits, that is, the eigen-solution conditions corresponding to the electronic motion eigen-states. Between two time points: $t_2 - t_1 = t_0 + t_\Delta - t_0 = t_\Delta$, the corresponding phase difference is: $\Delta\varphi = \omega t_\Delta$; if $\Delta\varphi = 2\pi n$, where n=1,2,3..., is a positive integer; it means if fitting in-phase conditions, it will reach the state of synchronous resonance orbit.

In this way, when the resonance of phase synchronization and resonance in-phase condition $2\pi n$ is reached, it is called the radiation in-phase mode (resonator principle). That is $\Delta\varphi = \omega t_\Delta = 2\pi\nu t_\Delta = 2\pi t_\Delta/T_c = 2\pi n$, where $\nu = 1/T_c$ as the resonant frequency, T_c as period, then $t_\Delta/\,T_c = n$, or $t_\Delta = n\,T_c$, based on this condition, the common resonance orbit is selected (as if the hand of God had made a choice).

The resonant orbit is clearly defined by the mathematical equation as follow:

$$r = (R_0 + V_r t)\,e^{j(2\pi/\,T_c)t} = (R_0 + V_r t)\,e^{j(2\pi n)t/t_\Delta}$$

Therefore, a series of phase synchronous resonant orbits are generated, that is, when $t = n\,T_c$, and then $\Delta\varphi = 2\pi n$ in phase, which means that the in-phase points/orbits are quantized because of integer n at this time.

It is completely consistent with the previously: $e^{j2\omega t_\Delta} = e^{j2(2\pi t_\Delta/T_c)} = e^{j2(2\pi n)} = 1$, set by eigen-solution conditions, so that the energy amplitude value can be written as:

$$|\mathrm{E}| = 2\pi\nu\, nT_c\, mV_r{}^2 \left\{1 + \left(\frac{2\pi}{T_c}\frac{R_r}{V_r}\right)^2\right\}^{\frac{1}{2}}$$

$$= n\nu\bullet 2\pi m V_r{}^2\, T_c \left\{1 + \left(\frac{2\pi}{T_c}\frac{R_r}{V_r}\right)^2\right\}^{\frac{1}{2}}$$

So far, we have discussed the solution $e^{j\omega t}$ of the positive direction spiral motion of electrons or particles, in fact, there should be another solution of the reverse direction spiral motion $e^{-j\omega t}$, so the contribution to the energy amplitude value is less than half, which needs to be multiplied by 2 for correcting back:

$$|\mathrm{E}| = n\nu\bullet 4\pi m V_r{}^2\, T_c \left\{1 + \left(\frac{2\pi}{T_c}\frac{R_r}{V_r}\right)^2\right\}^{\frac{1}{2}}$$

By the way, it is estimated that the difference between the results derived by Planck himself (the experimental results of Planck's formula) and the theoretical results derived by Bose and Einstein and also Poincaré is the relationship of factor 2. However, it does not affect the substantive conclusion and application, as long as the real physical mechanism being revealed, the specific numerical relationship can easily be verified and calibrated by experiments.

If the initial time and time error are considered, $t_1 = t_0 = or \approx 0$, the initial radial coordinate $R_r = R_0 + V_r(t_0 + t_\Delta/2)$ is also treated in initial drift case: set $R_0 = V_r t_0$, then $R_r = V_r\,(2t_0 + 0.5t_\Delta)$. For the drift or transition time t_Δ,the energy amplitude difference between the two resonance orbits in is further simplified as:

$$|\mathrm{E}| = n\nu \bullet 4\pi m {V_r}^2\, T_c \left\{ 1 + \left(\frac{2\pi}{T_c}\frac{R_r}{V_r}\right)^2 \right\}^{\frac{1}{2}}$$

$$= n\nu \bullet 4\pi m {V_r}^2\, T_c \left\{ 1 + \left(2\pi\frac{2t_0 + 0.5t_\Delta}{T_c}\right)^2 \right\}^{\frac{1}{2}}$$

It is obvious that even in the case of single electron motion, the energy difference released between two resonant orbits with the same phase is an integer portion of differences ($n = 1, 2, 3$..... as a positive integer). For further simplification, the initial time is set to $t_0 = or \approx 0$, when the resonance condition is met, $t_\Delta / T_c = n$, or $t_\Delta = n\,T_c$, and then make approximate treatments:

$\left(2\pi\frac{2t_0+0.5t_\Delta}{T_c}\right)^2 \gg 1$, then,

$$\left\{ 1 + \left(2\pi\frac{2t_0 + 0.5t_\Delta}{T_c}\right)^2 \right\}^{\frac{1}{2}} \approx \pi\frac{t_\Delta}{T_c}$$

Thus, the energy amplitude released between the resonant orbits is simplified again as:

$$|\mathrm{E}| = n\nu \bullet (8\pi^2 t_\Delta)\left(\frac{1}{2} m {V_r}^2\right)$$

It is shown again that the energy released between two resonant eigen-orbits with the synchronous phase is integer multiple one by one even in the case of a single electron motion.

III. Case of Numerous Electrons or Particles

In the case of multiple electrons or particles, the contributions of all electrons to the above energy amplitude values are summed, and common physical parameters such as drift time t_Δ and resonance frequency ν (for satisfying the resonance conditions of the eigen-solutions), with being extracted as common factors in front of the right side of the equation:

$$|E| = n\nu \bullet (8\pi^2 t_\Delta)\left\{ \sum_{i=1}^{I} \frac{1}{2} m_i {V_{ri}}^2 \right\}$$

The physical meaning of the above formula is obvious: in multiple electrons cases, energy is still quantized in integer multiple one by one. And the above formula is derived by a classical theory and mathematical method, and has obvious physical meaning as: for a given potential well system space and quantum transition time, the quantized energy is a kind of integer multiple conversion energy by fusing interactions through summing particle thermos-kinetic energies under the quantum wave eigenstates. The energy amplitude value is determined main by only two factors, the former being $n\nu$, mathematically, the common factor of all items is listed separately ahead. Physically, it is the intrinsic resonance eigen-value common to all electrons, and it is also the essence of quantum theory, that is, energy is the most important feature of integer multiple one by one, quantized emissions and propagations. And the latter part is the collective effect of many micro-electronic movements, i.e. given time t_Δ, the sum of thermal kinetic energy of many electrons or particles within $\sum_{i=1}^{I} \frac{1}{2} m_i {V_{ri}}^2$, which parameters mainly

consist of mass, time of motion drift, average thermal drift rate, radius or distance of motion, number of electrons and so on.

In addition, the mathematical and physical conditions (including the various force fields in the nucleus and the resulting boundary conditions) that define the given or constituted potential well boundaries (including natural boundaries and initial conditions, boundary dimensions, initial moments, etc.), so that relatively isolated from the outside world. The parameters and potential trap boundaries of these electrons are determined when a specific object of study or observation (for example, a specific atom or material chemical element) is given, and the interactions between these electrons are weakly coupled and extremely weak (because common factors for much stronger interactions have been already extracted). The collective or cumulative effects of this weak coupling can be linear and precisely measured at the minimum scale (very similar to the minimum difference unit in mathematical calculus) and are constants.

For example, there is no any specific volume space limit to define for the above theoretical derivations. So here we can re-construct Boltzmann volume or Planck blackbody volume for the whole derivations. The Boltzmann constant in the ideal gas (or electrons) in the minimum cell volume also meets and perfectly constitutes the ideal Boltzmann condition, revealing the collective or accumulated thermal motion effects of many random electrons. Similarly, the Planck constant in the ideal blackbody radiation unit at the smallest Planck scale reveals the collective or cumulative effect of electron thermal radiations.

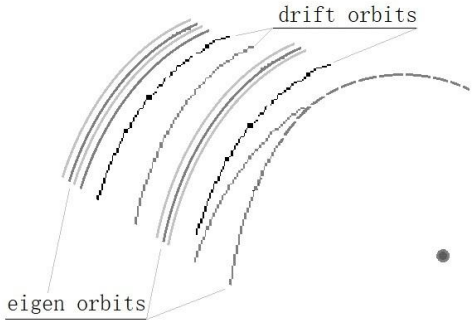

Fig.2 The resonant mechanism of electron motions and the explanatory diagram of quantization process

In the unit volume with potential well constraints, it has been described with electrons gathering on eigen orbitals (as shown in Fig. 2), and a small number of electrons or particles may drift or transition back and forth from one quantum orbit to another due to external forces or disturbances in the initial state or later. This is no problem but subject to the phase quantization condition of resonances, that is, the phase differences before and after the transitions to satisfy: $\Delta\varphi = 2\pi n$. That is, the departure and arrival must be at the same quantization phase point/orbit, otherwise, the electron or particle will get lost its quantum orbit and continue to drift or transition until the next quantized orbit to find out.

The collective or cumulative thermal motion effect of many random micro-electrons, that is, the condition that the above drift motion effect can be equivalent to the ideal gas in the minimum unit volume, is derived, measured and verified by introducing the Boltzmann constant. If the above potential well cavity is constructed into a Boltzmann thermal motion unit volume, the corresponding unit energy is:

$$|E_{0k}| = \sum_{i=1}^{Ik} E_{ki} = \frac{3}{2}kT$$

$$= n\nu\,(8\pi^2)\,t_\Delta \left\{ \sum_{i=1}^{Ik} \frac{1}{2} m_i {V_{ri}}^2 \right\} = n\nu(8\pi^2) \left| \sum_{i=1}^{k-vol} E_{0ki} \right|$$

Where $k = 1.380649 \times 10^{-23}$ J/K is Boltzmann constant, E_{ki} is the average translational kinetic energy of the ith electron or particle, and T is the thermodynamic temperature of the system. In other words, the absolute thermodynamic temperature T of the unit and the amplitude value $|E_{0k}|$ of the released energy of the corresponding unit can be measured by re-constructing a Boltzmann ideal thermal motion unit volume （$k - vol$ micro system space cavity) to obtain the constant of unit energy.

Similarly, if the above potential well cavity is also re-constructed into Planck's ideal blackbody radiation unit volume ($h - vol$ micro system space cavity), the corresponding unit energy is:

$$|E_{0h}| = \sum_{i=1}^{Ih} E_{hi} = \frac{3}{2} kT$$

$$= n\nu\,(8\pi^2)\,t_\Delta \left\{ \sum_{i=1}^{Ih} \frac{1}{2} m_i {V_{ri}}^2 \right\} = n\nu(8\pi^2) \left| \sum_{i=1}^{h-vol} E_{0hi} \right|$$

If using a constant h is introduced into the blackbody radiation element of the minimum Planck scale, the equation is actually the equivalent form of the Planck blackbody radiation equation:

$$|E_{0h}| = n\nu h = nh\,c/\lambda\,,\ n = 1, 2, 3.....$$

Here ν is the radiation propagation frequency of quantum wave, λ is the wavelength, c is the speed of light, h is the Planck constant, $h = 6.6260755 \times 10^{-34}$ Js, which can be measured and calibrated repeatedly, and with $n = 1, 2, 3.....$ taking only positive integers.

Of course, it can be concluded that by re-constructing Planck's ideal blackbody radiation unit volume, the absolute thermodynamic temperature T of the unit and the energy amplitude $|E_{0h}|$ released by the corresponding unit can be measured to obtain the constant of unit energy, which is presenting another choice of the measurement methods for the constant Planck constant h. That is, Planck's Law: energy is radiated and propagated with integer multiple one by one, and each energy quantum is νh. So here is the point to prove the Planck's quantization hypothesis of quantum theory.

IV. Expanding Methods to the General

1. Reviewing the whole derivation and demonstration process of above chapters, we can summarize as the following standardized processes or steps：

(1) It is not difficult in classical physics and traditional mathematics to write down the orbital equations of a single electron or particle based on the original atomic model.

(2) From the unique characteristic of the periodic motion of a single electron or particle, and under the condition that the eigen motion orbit is satisfied, we obtain the eigen-solutions (only worthy attention in phase synchronization) by rigorous mathematical methods.

(3) Then, in the cases of many electrons or particles, it is assumed that the collective or cumulative thermal motion effects of random micro-electrons or particles can be equivalent to the Boltzmann constant condition of the ideal gas in the minimum unit volume. When electrons move periodically around the nucleus in an open atom, they are free and random, which obviously meets these conditions.

(4) When electrons move periodically around the atomic nucleus in an open atom, due to the internal

interaction forces, they cannot escape to the area outside the internal forces. Therefore, the physical constraints of the natural potential well boundary (natural boundary and initial conditions, boundary size and initial time, that are relatively isolated from the outside world) are clearly given or formed, that is, the physical conditions similar to those of a closed resonant cavity.

(5) Such a cavity condition can resonantly amplify the eigenstates of electrons or particles, and make a common choice of collective optimizations, that is, the law of quantization. This may be a response to Einstein's question: the problem of local implicit variables in quantum theory.

(6) This kind of quantum waves naturally radiate and spread out, which is also convenient for observers outside the system to discover and measure. Only then can it be possible to obtain Planck's law by a pure experimental way. In turn, it also brings convenience to this research work since Planck constant is directly quoted here with no need to repeat the same measurement again. Meanwhile, it is consistent with the accurate and reasonable experimental results, which also proves the correctness of the theory in this paper. It means the experimental and theoretical results are mutually verified smoothly.

(7) The microscopic physical quantities (such as the position and momentum of a single electron or particle) that cannot be measured or uncertain in the system have become quantities with no need attention, but have been converted into physical quantities (such as the frequency and energy of quantum waves) that can be accurately measured to study the propagations of radiations to quantum states outside the system.

(8) It is concluded that the whole derivation and proof process or steps above are rigorous, and all the preconditions for derivations are obviously tenable, and the constants involved can be measured and verified repeatedly. Therefore, the classical theoretical derivations and accompanying interpretations of physical mechanism presented in this paper are self-consistent universal theories and methods, which can be extended to any other similar physical systems or physical phenomenon.

2. The quantized wave particle duality equation

After deriving from the previous chapter, the physical and mathematical expression of the energy quantization formula (new expression of Planck's law) with physical meaning is (or later referred to as "quantized wave particle duality equation"):

$$|E_0| = n\nu\,(8\pi^2)\,t_\Delta \left\{ \sum_{i=1}^{I} \frac{1}{2} m_i {V_{ri}}^2 \right\} = n\nu h,$$

at $t_\Delta / T_c = n$, or $t_\Delta = n\,T_c$, the quantum orbitals; where $n = 1, 2, 3.....$ is positive integer.

That is, the energy amplitude (or quantized energy level) $|E_0|$ of the above formula is simply composed of three factors: the quantized resonant eigenvalue $n\nu$, interaction or transition time t_Δ, sum of translational kinetic energy $\sum_{i=1}^{I} \frac{1}{2} m_i {V_{ri}}^2$. If it is confined in the element cavity of Planck's law, the total result is simplified to $n\nu h$. This is the new expression and significance of Planck's law with physical interpretation in quantum theory.

In physics, the resonant eigenvalue ($n\nu$) of all particles or quanta reflects the essence of quantum theory, that is, energy is the most important quantum wave characteristic of integer multiple one by one, and quantized emissions and propagations. At the same time, the latter part of the equation reflects the collective effect of the movement of many micro particles, for the given conversion time t_Δ, the sum of kinetic energy of thermal drift motion of many micro particles in $\sum_{i=1}^{I} \frac{1}{2} m_i {V_{ri}}^2$, which reflects its particle characteristics. Thus it reveals exactly both wave and particle characteristics. This "quantized wave particle duality equation" is a perfect mathematical expression of the physical essence of wave particle duality. It is also a perfect link and fusion between the Planck blackbody radiation formula in the micro world (the front part of the equation) and the Boltzmann thermal kinetic energy formula in the macro world (the rear part of the equation). It also reflects the perfect physical representation

of internal interaction and energy transformation during the conversion transition time t_Δ (the middle part of the equation). At the same time, its quantization and the common eigenvalue of quantum wave, also reflects the difference between micro and macro, with the intrinsic resonance, quantization of motion orbit and quantization orbit transition in the micro world. It means the radiation propagation of energy quantization and wave particle duality, that is, the physical essence in the micro world.

In general, the above has carried out rigorous mathematical derivation and physical interpretation, especially the result of the final formula reflects the perfect integration of quantum theory and classical physics, rather than conflict. The first part is the quantization effect, which is the common intrinsic resonance characteristic of all electrons. It is the collective choice specified by the physical mechanism. The integer orbit, quantized phase and quantized energy mainly reflect the quantum theoretical characteristics of the wave behaviors of the radiations and propagations. The other part reflects the collective or cumulative effect of the movement of many micro electrons or particles, which is the concrete embodiment of the statistical analysis of the macro description of the classical theory, that is, the classical physical characteristics of the particle behaviors of matters. From the above derivation process, it can be seen that from the perspective of methodology, the common factor and sum in mathematics correspond to the fusion of the intrinsic quantum effect and the collective cumulative effect in physics. That is to say, from the method of theoretical analysis, the complex nonlinear system is cleverly resolved into a decomposition analysis method for a common quantization factor and the sum of linear systems. Therefore, quantum theory has successfully obtained a reasonable and correct theoretical interpretation. All this is also a complete interpretation of the wave particle duality of matter, the perfect integration of classical physics and quantum theory, with no conflict and coherent theoretical system.

V. Summary and Conclusion

In summary, basing on the mature theory and rigorous mathematical proof of classical physics, starting from the measured and verified electron orbital model in the atom, the equation of motion of the electron spiral orbit is reasonably constructed, and the eigen-solution satisfying the resonance condition is obtained. It is proved that the quantization conclusion about the radiation and propagation energy is generally suitable for all atomic models or similar physical related conditions. Any previous experimental verification is selective or incomplete. The classical theoretical proof in this paper is that the exhaustion method is of course complete, ending the history that the quantization hypothesis proposed by quantum theory based on experimental conclusions has never been directly proved in the theoretical way consistent with the physical essence. It essentially reflects the fusion law of the collective statistical effect and the eigen quantization effect of the existing classical theory in the micro world, that is, from the disordered thermal motion disturbance to the orderly eigen quantum motion law, which is mainly manifested in the eigen harmonic resonance, the quantization of phase and energy, and the perfect quantized natural law of the propagations of the quantum waves. For the first time with physical mathematical formulas, "quantized wave particle duality equation" has been achieved in this paper, and reveal the real physical meaning of energy quantization and wave particle duality in a unified equation. It is completely proved from pure theory that these two hypotheses are no longer the premise of quantum theory, but the inevitable conclusion, with linking up the micro world and the macro world and their relationship, and revealing the physical essence of quantum theory. Therefore, the long-term conflict between quantum theory and classical physics is finally resolved, and this is actually a self-consistent and coherent theoretical system.

For a long time, quantum theory is self-contained, systematic and practical under the guidance of Planck's law, wave particle duality, uncertainty principle, superposition state, statistical probability theory, Schrodinger equation and other theoretical systems, and has solved many theoretical and technical problems. However, it is obviously a pity to have a convincing and complete interpretation of its physical mechanism, especially the part

that has serious conflict with classical physics. As Einstein called it, quantum theory is a statistical approximation of the decisive theory. Of course, it is also necessary to promote the follow-up research work. Using the theories and methods established, in addition to the research on the "atomic quantization model", it is also necessary to further explore and update the research on other physical mechanisms such as other quantum theoretical hypotheses, for example, the investigation on "photoelectron double slit experiment" is under taking. Completely resolve the conflict between quantum theory and classical physics, and establish and improve a self-consistent and coherent theoretical system.

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