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Synchronization analysis chaos of fractional derivatives chaotic satellite systems via feedback active control methods

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Abstract In this research article, a new fractional derivative chaotic satellite system is presented. Nature of different fractional derivative (order) satellite systems with phase portrait analysis versus parameters are analysed through utilization of the fractional calculus in computational simulation. Phase portrait analysis of fractional derivatives of the different satellite systems is drawn and tabled with various parameters values. In new fractional derivative satellite systems, chaos is existed in less than 3D (dimensional) satellite systems. The results are validated by the different tools:- equilibrium points, dissipative, Lyapunov exponents and bifurcation diagrams. Feedback and active control techniques for controlling chaos synchronization of new fractional derivative satellite systems are achieved.

Keywords Fractional derivative calculus, synchronization of chaos, satellite systems.

1 Introduction

Since many decades, fractional calculus has been one of the most effective methods for describing dynamical systems. It is found in a variety of scientific and engineering domains, as well as in interdisciplinary fields. Many physical systems, such as the fractional order Chen system [1], the fractional-order hyper-chaotic Chen system, the fractional order Lorenz system [2], the fractional order Rossler system [4], the fractional derivative Duffing system [3], and the fractional derivative financial system [5], have been well defined through fractional-order differential equations that behave chaotically or hyper-chaotically.In several domains of science and engineering, such as medicine, biological tissues, bioengineering, ECG testing, cardiac tissue, photoelasticity, fluid mechanics, material science, and so on, the notion of fractional derivative is utilised to simulate the behaviour of real systems([7], [8], [9], [10], [11], [12]). There are numerous distinctions between fractional-order and corresponding ordinary differential equation systems [13]. Kumar et al. used sliding mode approaches to discuss fractional derivative hyper chaotic financial systems as well as fractional derivative Rabinovich-Fabrikant systems in their research articles ([14], [15]).

In the realm of research, chaos management and synchronisation for fractional order systems have gotten a lot of interest. Due to its wide-ranging potential used in various disciplines such as chemical reaction, weather forecasting, power converters, aerospace, signal process, radar technology, physics of lasers, secure communication, global positioning systems and biological systems over the last several decades, chaotic synchronisation has become an interesting subject in the field of nonlinear sciences([16], [17]). Deng and Li, who created the synchronisation of fractional-order Lu systems [18], were the first to introduce fractional-order chaotic system synchronisation. A few researchers and scientists have since established synchronisation of various fractional-order chaotic systems. Gao et al. investigated fractional order chaotic systems' masterslave synchronisation [3]. Applying a scalar drive signal, the basic synchronisation methods were used to synchronise fractional-derivatives chaotic Arneodo systems [19]. Hegazi et al.havepresented extended projective synchronisation of two chaotic or hyper-chaotic non-integer systems [20].

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Satellite systems play a vital part in space technological advancements, scientific activities, tele-communication, civil and military applications, and so on, thanks chevaliers chevaliers chevaliers chevaliers chevaliers chevaliers chevaliers chevaliers chevaliers Predictive control, adaptive control, and sliding mode control, among other techniques and approaches, were utilised to synchronise and control the satellite systems([21], [22]). Modern space mission designs that include multiplying satellites flying information have used fractional-order satellite synchronisation. This is addressed by the synchronisation control method, which regulates the relative inaccuracies between satellite systems([23], [24]). The chaotic satellite system has received a lot of attention. Many scholars and scientists (Kuang and Tan, [25]; Tsui and Jones [26] ; Kong and Zhoul ([27], [28]); Kuang, Tan, Arichandran and Leung [29] etc.)have focused on this topic. Various strategies have been used by Hamidzadeh and Esmaelzadeh [30] to control and synchronise chaotic satellites. The generalised projective synchronisation of chaotic satellites problem using linear matrix inequality was studied by Farid and Moghaddam [31]. In their articles on sliding mode attitude control of a small satellite for ground tracking operations, Goeree and Fasse [32] have discussed their findings. Kumar and Khan in their research works, ([23], [24], [33], [34], [35])measured the chaos in satellite systems using many tools and established the synchronisation of chaotic satellite systems using numerous methodologies. Furthermore, much study is required to investigate the systems.

In this research paper, we examine the fractional-order chaotic behaviour of satellite systems with various orders, as a result of the previous debate. Equilibrium points, dissipativity, bifurcation diagrams, and Lyapunov exponents are used to investigate the nature of fractional order satellite systems. The occurrence of chaos is thereby justified in the fractional order satellite system's lowest dimension, which is less than 3. We also use feedback and active mode control approaches to produce chaos control and synchronisation of fractional-order satellite systems. These researches add to our knowledge of fractional order satellite systems' behaviour. Telecommunications, weather forecasting, GPS systems, and earth observation can all benefit from these measures. These studies will prove that our study paper is unique.

This paper is organized as follows: section 1 is introduction; section 2 describes the basic definitions of fractional-order chaotic system; in section 3, we discuss the system description and assumption of satellite systems; in section 4, we address the controlling chaos in the fractional-order chaotic satellite system; in section 5, we present the synchronization of fractional-order identical satellite systems via active control technique; we add the numerical simulation in section 6; conclusion is given in section 7; finally, statements and declarations are added in section 8.

2 Basic concepts of fractional derivatives

The non-integer-order integro-differential operator ${}_aD_t^{\alpha_1}$.in fractional calculus is a generalisation of integration and differentiation notions. It's written in the following manner:

$$ {}_aD_t^{\alpha_1} = \begin{cases} \frac{d^{\alpha_1}}{dt^{\alpha_1}}, & \text{if } \mathscr{R}(\alpha_1) > 0 \\ 1, & \text{if } \mathscr{R}(\alpha_1) = 0 \\ \int_a^t (d\tau)^{-\alpha_1}, & \text{otherwise}, i.e. \mathscr{R}(\alpha_1) < 0. \end{cases} \tag{1} $$

The definition of the generalised Riemann-Liouville definition [8] is as follows:

$$ D^{\alpha_1} f(t) = \frac{d^{\alpha_1}}{dt^{\alpha_1}} J^{n-\alpha_1} f(t), \quad \alpha_1 > 0, \tag{2} $$

where $n = [\alpha 1]$and n is the first integer greater than or equal to $\alpha 1$,$J^{\beta 1}$ is the Riemann-Liouville integral operator of $\beta_1$order, which is defined as follows:

$$ J^{\beta_1} f(t) = \frac{1}{\Gamma(\beta_1)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\beta_1}} d\tau \tag{3} $$

for $0 < \beta_1 \leq 1$, where $\Gamma(.)$ is the gamma function.

The definitions below are utilised as

$$ D^{\alpha_1} f(t) = J^{n-\alpha_1} f^n(t), \quad \alpha_1 > 0, \tag{4} $$

where $n = [\alpha_1]$. The operator D^{α_1} is commonly referred to as the Caputo differential operator of order $\alpha_1$since it was initially employed for the solution of practical problems by Caputo ([8], [36]).

2.1 **Fractional derivatives and integrals: Basic definitions and properties** ([8], [36], [37])

Definition 1: A real function $f(t), t > 0$ is indeed in Caputo space $\mathscr{C}\alpha 1, \alpha 1 \in \mathscr{R}$if there is a real integer $p(> \alpha 1)$,such that $f(t) = t^p f1(t)$ with $f1(t) \in \mathscr{C}[0, \infty)$.

Definition 2: A real function $f(t), t > 0$ is indeed in Caputo space $\mathscr{C}^m_{\alpha_1}, m \in IN \cup 0$ if $f^{(m)} \in \mathscr{C}_{\alpha_1}$.

Definition 3: If $f \in \mathscr{C}_{\alpha_1}$and $\alpha_1 \geq -1$, then the Riemann-Liouville integral of order $\alpha_1, (\alpha_1 > 0)$ are obtained by

$$I^{\alpha_1} f(t) = \frac{1}{\Gamma(\alpha_1)} \int_0^t (t - \alpha_1)^{\alpha_1 - 1} f(\tau) d\tau, \quad t > 0. \tag{5}$$

Definition 4: The fractional derivative of $f, f \in \mathscr{C}^m_{-1}, m \in IN \cup 0$,described by Caputo is:

$$D^{\alpha_1} f(t) = \frac{d^m}{dt^m} f(t), \quad \alpha_1 = m = I^{m-\alpha_1} \frac{d^m f(t)}{dt^m}, m-1 < \alpha_1 < m, \quad m \in IN. \tag{6}$$

Note that for $m-1 < \alpha_1 \leq m, \quad m \in IN$,

$$I^{\alpha_1} D^{\alpha_1} f(t) = f(t) - \sum_{k=0}^{m-1} \frac{d^k f}{dt^k}(0) \frac{t^k}{k!}, I^{\alpha_1} t^{\alpha_1} = \frac{\Gamma(v+1)}{\Gamma(\alpha_1 + v + 1)} t^{\alpha_1 + v}. \tag{7}$$

3 System description and assumption of satellite system

The attitude dynamics of the satellite are described in the inertial coordinate system as ([40], [41]):

$$\dot{\mathrm{T}} = \mathrm{T}_a + \mathrm{T}_b + \mathrm{T}_c, \tag{8}$$

The total momentum acting on the satellite is T. The flywheel torques, gravitational torques, and disturbance torques are represented by T_a,T_b and T_c, respectively. The overall momentum T is calculated as follows:

$$\mathrm{T} = \mathbf{I}\omega, \tag{9}$$

where ω is the angular velocity and $\mathbf{I}$ is the inertia matrix. The total momentum derivatives T are represented as

$$\dot{\mathrm{T}} = \mathbf{I}\dot{\omega} + \omega \times \mathbf{I}\omega. \tag{10}$$

The cross-product of the vectors is denoted by the symbol $\times$. When we combine these equations, we get

$$\mathbf{I}\dot{\omega} + \omega \times \mathbf{I}\omega = \mathrm{T}_a + \mathrm{T}_b + \mathrm{T}_c. \tag{11}$$

We have made our decision, $\mathbf{I} = diag(\mathbf{I}_x, \mathbf{I}_y, \mathbf{I}_z)$.

$$\mathrm{T}_a = \begin{bmatrix} \mathrm{T}_{ax} \\ \mathrm{T}_{ay} \\ \mathrm{T}_{az} \end{bmatrix}; \mathrm{T}_b = \begin{bmatrix} \mathrm{T}_{bx} \\ \mathrm{T}_{by} \\ \mathrm{T}_{bz} \end{bmatrix}; \mathrm{T}_c = \begin{bmatrix} \mathrm{T}_{cx} \\ \mathrm{T}_{cy} \\ \mathrm{T}_{cz} \end{bmatrix}.$$

The satellite system ([21], [40], [41]) is written as.

$$\begin{aligned} \mathbf{I}_x \dot{\omega}_x &= \omega_y \omega_z (\mathbf{I}_y - \mathbf{I}_z) + h_x + u_x, \\ \mathbf{I}_y \dot{\omega}_y &= \omega_x \omega_z (\mathbf{I}_z - \mathbf{I}_x) + h_y + u_y, \\ \mathbf{I}_z \dot{\omega}_z &= \omega_x \omega_y (\mathbf{I}_x - \mathbf{I}_y) + h_z + u_z, \end{aligned} \tag{12}$$

where

$$h_x = \left[\mathrm{T}_{ax} + \mathrm{T}_{bx} + \mathrm{T}_{cx}\right]; h_y = \left[\mathrm{T}_{ay} + \mathrm{T}_{by} + \mathrm{T}_{cy}\right];$$
$$h_z = \left[\mathrm{T}_{az} + \mathrm{T}_{bz} + \mathrm{T}_{cz}\right].$$

u_x, u_y and $u_z$are three control torques, while h_x, h_y and $h_z$are perturbing disturbance torques. We suppose that $\mathbf{I}_x > \mathbf{I}_y > \mathbf{I}_z = 1.\mathbf{I}_x = 3$, $\mathbf{I}_y = 2$ and $\mathbf{I}_z = 1$. The perturbing torques [26] is defined in the form as

$$\begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} = \begin{pmatrix} -1.2 & 0 & \frac{\sqrt{6}}{2} \\ 0 & 0.35 & 0 \\ -\sqrt{6} & 0 & -0.4 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}. \tag{13}$$

The following is the formula for a three-dimensional chaotic satellite system:

$$\begin{aligned}
\dot{x} &= \tfrac{d}{dt}(x(t)) = \sigma_x yz - \tfrac{1.2}{\mathbf{I}_x}x + \tfrac{\sqrt{6}}{2\mathbf{I}_x}z, \\
\dot{y} &= \tfrac{d}{dt}(y(t)) = \sigma_y xz + \tfrac{0.35}{\mathbf{I}_y}y, \\
\dot{z} &= \tfrac{d}{dt}(z(t)) = \sigma_z xy - \tfrac{\sqrt{6}}{\mathbf{I}_z}x - \tfrac{0.4}{\mathbf{I}_z}z,
\end{aligned} \tag{14}$$

where $\sigma_x = \frac{\mathbf{I}_y - \mathbf{I}_z}{\mathbf{I}_x}$; $\sigma_y = \frac{\mathbf{I}_z - \mathbf{I}_x}{\mathbf{I}_y}$; $\sigma_z = \frac{\mathbf{I}_x - \mathbf{I}_y}{\mathbf{I}_z}$ and $\sigma_x = \frac{1}{3}$, $\sigma_y = -1$ and $\sigma_z = 1$. The 3D chaotic satellite system becomes:

$$\begin{aligned}
\dot{x} &= \tfrac{d}{dt}(x(t)) = \tfrac{1}{3}yz - ax + \tfrac{1}{\sqrt{6}}z, \\
\dot{y} &= \tfrac{d}{dt}(y(t)) = -xz + by, \\
\dot{z} &= \tfrac{d}{dt}(z(t)) = xy - \sqrt{6}x - cz,
\end{aligned} \tag{15}$$

a, b, and c are known parameters values. $a = 0.4$, $b = 0.175$, and $c = 0.4$ are the values we have. The order $\alpha_1 = 0.95$ of the 3D (dimensional) fractional derivative satellite system is represented as:

$$\begin{aligned}
\tfrac{d^{\alpha_1}x(t)}{dt^{\alpha_1}} &= \tfrac{1}{3}yz - ax + \tfrac{1}{\sqrt{6}}z, \\
\tfrac{d^{\alpha_1}y(t)}{dt^{\alpha_1}} &= -xz + by, \\
\tfrac{d^{\alpha_1}z(t)}{dt^{\alpha_1}} &= xy - \sqrt{6}x - cz,
\end{aligned} \tag{16}$$

Equilibrium Points The equilibrium point of a fractional derivative dynamical system is the same as the equilibrium point of an integer-order dynamical system [37]. By solving the following system of equations, the equilibrium points of the satellite system (16) can be found. $\dot{X}(t) = 0$

$$F(x) = \begin{bmatrix} \frac{1}{3}yz - ax + \frac{1}{\sqrt{6}}z = 0 \\ -xz + by = 0 \\ xy - \sqrt{6}x - cz = 0. \end{bmatrix}$$

Equilibrium points are

$$\aleph_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \aleph_1 = \begin{bmatrix} 1.1910 \\ 2.5766 \\ 0.3785 \end{bmatrix}, \aleph_2 = \begin{bmatrix} 0.1582 \\ -1.3641 \\ -1.5086 \end{bmatrix}, \aleph_3 = \begin{bmatrix} -0.1582 \\ -1.3641 \\ 1.5086 \end{bmatrix}, \aleph_4 = \begin{bmatrix} -1.1910 \\ 2.5766 \\ -0.3785 \end{bmatrix}. \tag{17}$$

The satellite system's Jacobian matrix, (16)is calculated as follows:

$$J(X) = \begin{bmatrix} -a & 0.33 * z & (0.33 * y + \frac{1}{\sqrt{6}}) \\ -z & b & -x \\ (y - \sqrt{6}) & x & -c. \end{bmatrix} \tag{18}$$

At each equilibrium point, one of the eigenvalues of Jacobian matrix (18) has a positive real portion. This demonstrates that the equilibrium points $\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_3$, and $\aleph_4$are saddle-focus, which is inherently unstable. As a result, all five of the satellite system's equilibrium points (16)are unstable equilibrium points [24]. **Invariant: y-axis** If $x(0) = 0$ and $z(0) = 0$, then for all t x and z retain zero , according to equations (16). As a result, the y-axis is an orbit.

$$\frac{d^{\alpha_1}y(t)}{dt^{\alpha_1}} = by(t), \text{hence} \quad y(t) = y(0)e^{bt}; \text{for} \quad x, z = 0. \tag{19}$$

As a result, for the equilibrium, the y-axis illustrated the section of the unstable manifold at the origin.

Dissipative system We can write the system (16) as follows in vector notation:

$$\frac{d^{\alpha_1}X(t)}{dt^{\alpha_1}} = F(x) = \begin{bmatrix} F_1(x,y,z) \\ F_2(x,y,z) \\ F_3(x,y,z), \end{bmatrix} \tag{20}$$

where, $X(t) = (x, y, z)$ and

$$F(x) = \begin{bmatrix} F_1(x,y,z) = \frac{1}{3}yz - ax + \frac{1}{\sqrt{6}}z, \\ F_2(x,y,z) = -xz + by \\ F_3(x,y,z) = xy - \sqrt{6}x - cz \end{bmatrix}$$

where, $a = 0.4, b = 0.175$ and $c = 0.4.$. We assume any smooth boundary area $\Lambda(t) \in \mathscr{R}^3$with $\Lambda(t) = \Theta_t(\Lambda)$, where $\Theta_t$displayed the flow of f.

The volume of $\Lambda(t)$ is $\mathbf{V}(t)$.

We get through Liouville's theorem

$$\dot{\mathbf{V}}(t) = \int_{\Lambda(t)} (\nabla.F) dxdydz \tag{21}$$

The divergence of the satellite system (21)is calculated as:

$$\nabla.f = \left[\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right] = -a + b - c = -0.625 \tag{22}$$

From (21) and (22), the first derivatives ordinary differential equation can be written as follows:

$$\dot{\mathbf{V}}(t) = -0.625\mathbf{V}(t) \tag{23}$$

We derive the following solution by integrating the equation (23):

$$\mathbf{V}(t) = e^{-0.625t}\mathbf{V}(0) \tag{24}$$

That is, the volumes with initial points decline by a factor of e as time t passes. The equation (24) $\mathbf{V}(t) \to 0$ when $t \to \infty$.The system's limit settings are constrained to a single limit set with zero volume. A weird attractor is determined by the asymptotic motion of fractional derivative satellite system (16). This demonstrates the dissipative character (behaviours)of the satellite system (24) [23].

4 The fractional derivative unpredicted satellite system: chaos control

The fractional derivative highly nonlinear system in the form is written as

$$\frac{d^{\alpha_1} x}{dt^{\alpha_1}} = f(x) + \mathbf{B}u. \tag{25}$$

where $x \in \mathbf{R}^n$ is a system's state vector,$u \in \mathbf{R}^m$ is a system's input vector, $f : \mathbf{R}^n \to \mathbf{R}^n$ is a nonlinear map from $\mathbf{R}^n$ to $\mathbf{R}^n$. $\in \mathbf{R}^{nm}$ is a constant matrix. Assume $x^* \in \mathbf{R}^n$ is the controlled chaotic system's equilibrium point. That means, $f(x^*) = 0$.

Remark 1 The goal of this study is to determine and analyse the stability of x^*. Under the sake of simplicity, we state all formulations and conclusions for the scenario where the equilibrium point of the controlled dynamic system (25)is at the origin of $\mathbf{R}^n$. That is, $x^* = 0$. By adjusting variables, It is possible to move the every equilibrium point to the origin, hence there is no loss of generality.

Remark 2 Consider the change of variables $y = x - x^*$ and assume $x^* = 0$.is an equilibrium point of the controlled chaotic system (25). Then, y derivative is equal to-

$$\frac{d^{\alpha_1} y}{dt^{\alpha_1}} = \frac{d^{\alpha_1} x^*}{dt^{\alpha_1}} = f(x) + \mathbf{B}u = f(y + x^*) = s(y). \tag{26}$$

where $s(0) = 0$. The system is in equilibrium at the origin with the new variable y. As a result, we will always assume that $f(x)$ fulfils $f(0) = 0$ and analyse the stability of the origin $x = 0$. without losing generality. Using Taylor series for the nonlinear function $f(x)$ of system (25)

$$f(x) = f(0) + \frac{\partial f(x)}{\partial x} + h(x). \tag{27}$$

where $h(x)$ fulfils $\lim_{\|x\| \to 0} \frac{\|h(x)\|}{\|x\|} = 0$.

We notice that when $f(0) = 0$ is substituted into the controlled chaotic system (25) and combined with (27), we get

$$\frac{d^{\alpha_1} x}{dt^{\alpha_1}} = Ax + h(x) + Bu. \tag{28}$$

where $A = \frac{\partial f(x)}{\partial x}$ at $x = 0$.

4.1 Controllers with linear state feedback

Theorem: (A;B) is entirely state controllable for the linearization of dynamical system (28) of controlled chaotic system (25). Control law of the linear state feedback is therefore specified as $u = B^T P\phi$, where P is a unique positive definite symmetry matrix, and the controlled chaotic system (25) is asymptotically stable in origin. The equation for Riccati algebra matrices is $PA + ATP - PBB^T P + Q = 0$, where Q is an arbitrary positive definite matrix.

4.2 Simulations and control of a fractional derivative chaotic satellite system

The eigenvalues of system (18) at equilibrium point $\aleph_0 = (0,0,0)$ is
$\lambda_{01} = -0.4 + 0.99\iota$, $\lambda_{02} = -0.4 + 0.99\iota$ and $\lambda_{03} = 0.175$ are obtained, where $\iota as\sqrt{-1}$. Because one of the eigenvalues is positive, both the linearized and original forms of the fractional order satellite system are unstable.

The linear state feedback law of unpredictable system is provided by after controllers arithmetic.

$$u = -\mathbf{B}^T P \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \tag{29}$$

At equilibrium points $\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_3$ and $\aleph_4$ the corresponding positive definite symmetric matrices are denoted as;

$$p_0 = \begin{bmatrix} 0.0977 & 0 & 0.0160 \\ 0 & 0 & 0 \\ 0.0160 & 0 & 0.0350 \end{bmatrix}; p_1 = \begin{bmatrix} 0.2803 & 0.2246 & 0.2132 \\ 0.2246 & 0.1799 & -0.1708 \\ -0.2132 & -0.1708 & 0.1622 \end{bmatrix}; p_2 = \begin{bmatrix} 0.0.0278 & 0.0129 & 0.0080 \\ 0.0129 & 0.0060 & 0.0037 \\ 0.0080 & 0.0037 & 0.0023 \end{bmatrix};$$
$$p_3 = \begin{bmatrix} 0.3607 & -0.1673 & 0.1040 \\ -0.1673 & 0.0776 & -0.0482 \\ 0.1040 & -0.0482 & 0.0300 \end{bmatrix}; p_4 = \begin{bmatrix} 0.0068 & -0.0070 & -0.0214 \\ -0.0070 & 0.0349 & 0.0220 \\ -0.0214 & 0.0220 & 0.0673 \end{bmatrix}.$$

5 Using active control techniques, synchronization of identical fractional derivative satellite systems

For the satellite system Equation, (16), two identical master (drive) and slave (response) systems are rebuilt in the form of x and y subscripts, respectively:
Drive (master) system:

$$\begin{aligned} \frac{d^{\alpha_1} x_1(t)}{dt^{\alpha_1}} &= \tfrac{1}{3}x_2(t)x_3(t) - ax_1(t) + \tfrac{1}{\sqrt{6}}x_3(t), \\ \frac{d^{\alpha_1} x_1(t)}{dt^{\alpha_1}} &= -x_1(t)x_3(t) + bx_2(t), \\ \frac{d^{\alpha_1} x_1(t)}{dt^{\alpha_1}} &= x_1(t)x_2(t) - \sqrt{6}x_1(t) - cx_3(t). \end{aligned} \tag{30}$$

Response (slave) System:

$$\begin{aligned} \frac{d^{\alpha_1} y_1(t)}{dt^{\alpha_1}} &= \tfrac{1}{3}y_2(t)y_3(t) - ay_1(t) + \tfrac{1}{\sqrt{6}}y_3(t) + u_1(t), \\ \frac{d^{\alpha_1} y_2(t)}{dt^{\alpha_1}} &= -y_1(t)y_3(t) + by_2(t) + u_2(t), \\ \frac{d^{\alpha_1} y_3(t)}{dt^{\alpha_1}} &= y_1(t)y_2(t) - \sqrt{6}y_1(t) - cy_3(t) + u_3(t), \end{aligned} \tag{31}$$

where the three controller torques are $u_1(t)$, $u_2(t)$ and $u_3(t)$. The synchronization of error signal is defined as

$$\begin{aligned} e_1(t) &= y_1(t) - x_1(t), \\ e_2(t) &= y_2(t) - x_2(t), \\ e_3(t) &= y_3(t) - x_3(t). \end{aligned} \tag{32}$$

The error dynamics is written as:

$$\begin{aligned} \frac{d^{\alpha_1} e_1(t)}{dt^{\alpha_1}} &= \frac{d^{\alpha_1} y_1(t)}{dt^{\alpha_1}} - \frac{d^{\alpha_1} x_1(t)}{dt^{\alpha_1}} = \frac{1}{3}(y_2(t)y_3(t) - x_2(t)x_3(t)) - a(y_1(t) - x_1(t)) + 1/\sqrt{6}(y_3(t) - x_3(t)) + u_1(t), \\ \frac{d^{\alpha_1} e_2(t)}{dt^{\alpha_1}} &= \frac{d^{\alpha_1} y_2(t)}{dt^{\alpha_1}} - \frac{d^{\alpha_1} x_2(t)}{dt^{\alpha_1}} = (x_3(t)x_1(t) - y_3(t)y_1(t)) + b(y_2(t) - x_2(t)) + u_2(t), \\ \frac{d^{\alpha_1} e_3(t)}{dt^{\alpha_1}} &= \frac{d^{\alpha_1} y_3(t)}{dt^{\alpha_1}} - \frac{d^{\alpha_1} x_3(t)}{dt^{\alpha_1}} = (y_1(t)y_2(t) - x_1(t)x_2(t)) - \sqrt{6}(y_3(t) - x_3(t)) - c(y_3(t) - x_3(t)) + u_3(t). \end{aligned} \tag{33}$$

We define the the active control function as,

$$
\begin{aligned}
u_1(t) &= -\frac{1}{3}(y_2(t)y_3(t) - x_2(t)x_3(t)) + \frac{1}{\sqrt{6}}e_3(t) + v_1(t),\\
u_2(t) &= -(x_3(t)x_1(t) - y_2(t)y_3(t)) + be_2 - e_2(t) + v_2(t),\\
u_1(t) &= -(y_1(t)y_2(t) - x_1(t)x_2(t)) + \sqrt{6}e_1(t) + v_3(t).
\end{aligned}
\tag{34}
$$

The terms $v_i(t)$ are the linear function of the error terms $e_i(t)$, $i = 1,2,3$. With the choice of $u_i(t)$ given by the previous equations, error dynamics is written as

$$
\begin{aligned}
\frac{d^{\alpha_1}e_1(t)}{dt^{\alpha_1}} &= -ae_1(t) + v_1(t),\\
\frac{d^{\alpha_1}e_2(t)}{dt^{\alpha_1}} &= -e_2(t) + v_2(t),\\
\frac{d^{\alpha_1}e_3(t)}{dt^{\alpha_1}} &= -ce_3(t) + v_3(t).
\end{aligned}
\tag{35}
$$

The control terms $v_i(t)$are selected to stabilise the error system. For such functions, there isn't a single option. We take

$$
\begin{bmatrix} v_1(t)\\ v_2(t)\\ v_3(t)\end{bmatrix} = A \begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix}; A = [a_{ij}]_{3x3}.
\tag{36}
$$

A is chosen so that the criterion $|arg(\lambda_i(J))| > \alpha_1 \frac{\pi}{2}$is satisfied for all eigenvalues λ_i of the error system. At the equilibrium point of the error system, the Jacobi matrix of the above system is J. We take

$$
A = \begin{bmatrix} a-1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & c-1\end{bmatrix}.
\tag{37}
$$

The eigen value of the linear error system are $-1,0,-1$. Fractional derivative stability theory, the error $e_i(t), i = 1,2,3$ will converge to zero when $t \to \infty$. That is,

$$
\lim_{t\to\infty} e(t) = 0
$$

This indicates that the slave and master systems are synchronized. The stability theory of fractional order systems is thus used to create synchronisation between fractional-order master and slave systems.

Table 1: Fractional-order satellite systems phase portrait analysis versus parameters $(\alpha_1; a; b; c)$.

		Nature of the system
Figure 1(a)	$\alpha_1 \leq 0.90; a = 0.4; b = 0.175; c = 0.4$	Stable
Figure 1(b)	$\alpha_1 = 0.91; a = 0.4; b = 0.175; c = 0.4$	Stable
Figure 1(c)	$\alpha_1 = 0.92; a = 0.4; b = 0.175; c = 0.4$	Periodic
Figure 1(d)	$\alpha_1 = 0.93; a = 0.4; b = 0.175; c = 0.4$	Quasi-Periodic
Figure 1(e)	$\alpha_1 = 0.94; a = 0.4; b = 0.175; c = 0.4$	Periodic
Figure 1(f)	$\alpha_1 = 0.95; a = 0.4; b = 0.175; c = 0.4$	Chaotic
Figure 1(g)	$\alpha_1 = 0.95; a = 0.30; b = 0.175; c = 0.4$	Quasi-Periodic
Figure 1(h)	$\alpha_1 = 0.95; a = 0.35; b = 0.175; c = 0.4$	Periodic
Figure 1(i)	$\alpha_1 = 0.95; a = 0.4; b = 0.100; c = 0.4$	Chaotic
Figure 1(j)	$\alpha_1 = 0.95; a = 0.4; b = 0.150; c = 0.4$	Chaotic
Figure 1(k)	$\alpha_1 = 0.95; a = 0.4; b = 0.175; c = 0.10$	Periodic
Figure 1(l)	$\alpha_1 = 0.95; a = 0.4; b = 0.175; c = 0.30$	Chaotic

6 Numerical simulation

Take $x(0) = (3.5, 0.5, 0.8)^T$ and $y(0) = (2.5, 1.5, 0.3)^T$ as initial conditions for fractional derivative master system and fractional derivative slave satellite systems. The phase portraits and time series graphs of fractional-order satellite systems with different orders are illustrated in Figures (1)(a-l). Table provides the characteristics of fractional derivative satellite systems through analysis the phase portraits diagrams and time series diagrams of the systems with different order and parameters values. Figure (2) is shown the phase portraits of three and two dimensional fractional order satellite system with order $\alpha_1 = 0.95$. Figures (3)(a-c) show the bifurcation plotting with respect to varying parameters a, b and c respectively. The Lyapunov exponents of satellite system, at $t = 200.$ is calculated. It is shown in Figure (4). Feedback control of fractional derivative unpredictable satellite is displayed in (4) when controlling is started at $t = 20$. Figures (6)(a-c) show the tracking the trajectories of slave to master fractional derivative satellite system with order $\alpha_1 = 0.95$ in x_1y_1, x_2y_2 and x_3y_3 respectively. Figure (7) is provided the synchronization of fractional derivative of error dynamic in the form of $e_1e_2e_3$ with respect to time t at the initial condition of error system, $e(0) = (-1.0, 1.0, -0.5)^T$. That is,

$$\lim_{t\to\infty} e(t) = 0$$

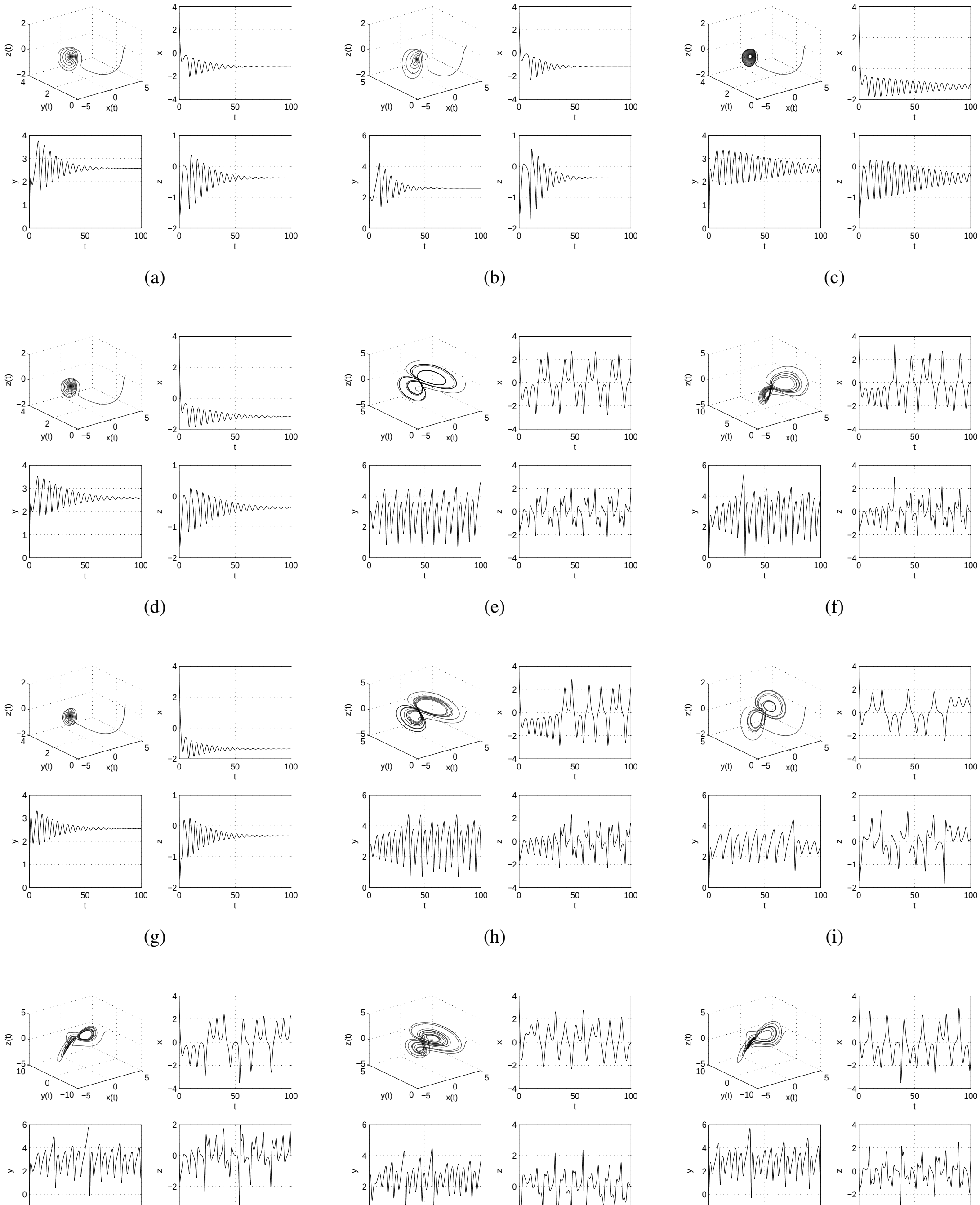


Fig. 1: 3-D Phase portrait and time series graphs of chaotic satellite systems with different fractional orders (without controller).

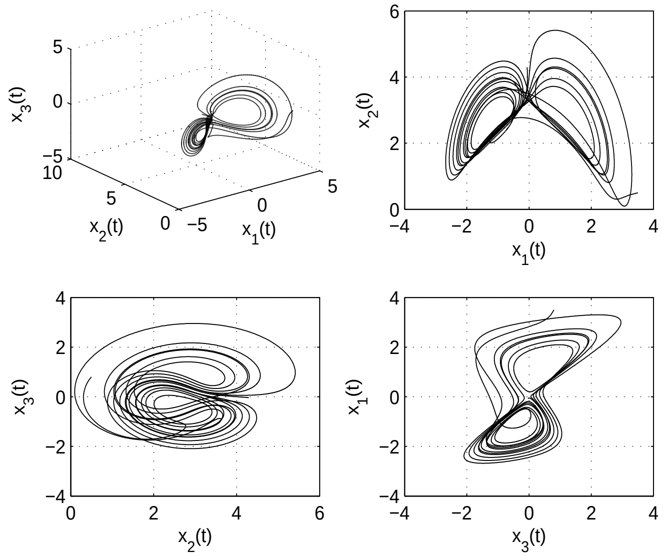


Fig. 2: 3D and 2D (dimensional) phase portraits of fractional order satellite system with $\alpha_1 = 0.95$.

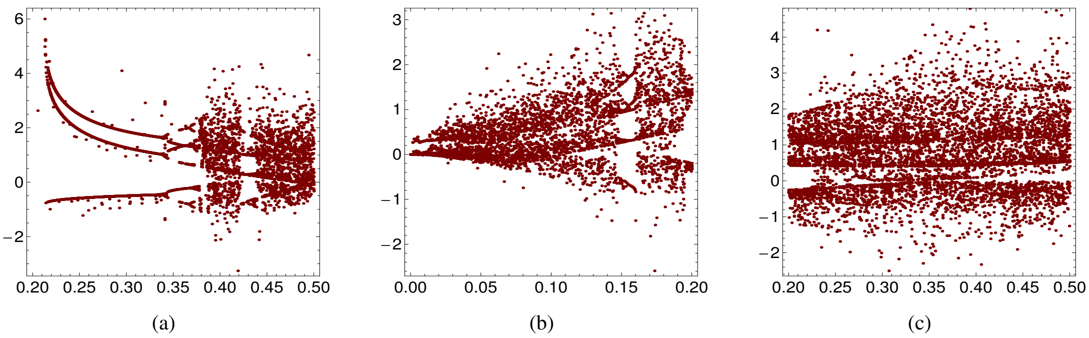


Fig. 3: Bifurcation plotting with varying the parameters *a*, *b* and *c* respectively.

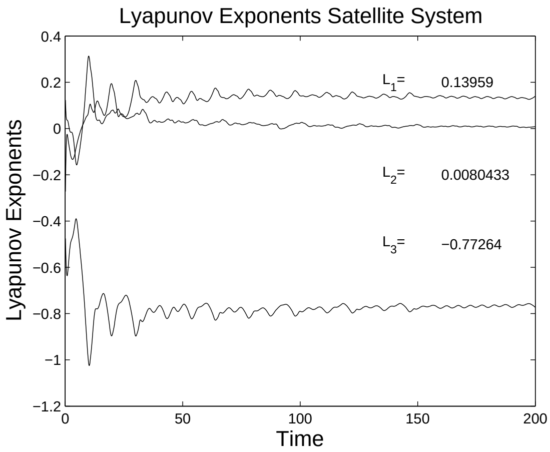


Fig. 4: Lyapunov exponent values of fractional order chaotic satellite system.

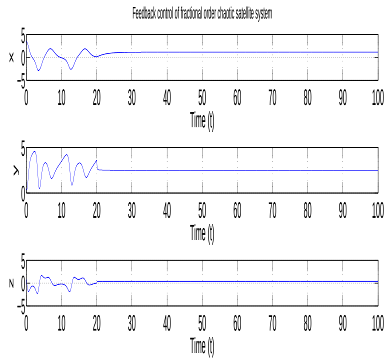
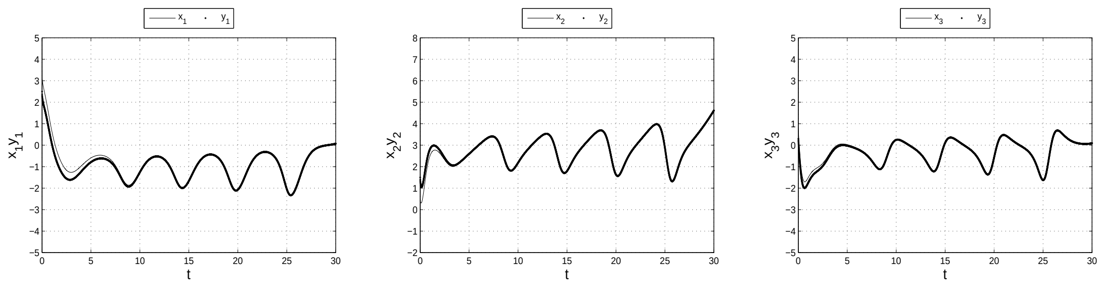


Fig. 5: Feedback controllers of fractional-order chaotic satellite system started at $t = 20$ second.



(a) (b) (c)

Fig. 6: Tracking trajectories for fractional-order master satellite to the salve system with order $\alpha_1 = 0.95$ (with controllers).

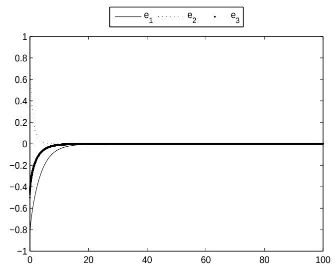


Fig. 7: Stabilization of fractional error dynamics (with controllers).

7 Conclusions

We looked at how to solve fractional-order dynamical systems in our research paper. We've established the fundamentals of fractional dynamics. The chaotic behaviour of specific fractional order satellite systems was studied using equilibrium points. Different methods, such as dissipativity, equilibrium points, bifurcation diagrams, and Lyapunov exponents, are used to analyse the chaotic behaviour of fractional-order satellite systems with phase portrait analysis vs parameters. Phase portrait analysis of fractional derivatives of the different satellite systems is drawn and tabled with various parameters values. Such tools have allowed us to rationalise the system's chaos. To verify the unstable zone, we got the equilibrium points of fractional-order satellite systems and calculated the eigenvalue of the Jacobian matrix of the satellite system at each equilibrium point. It has been realised a feedback control approach for a novel fractional-order satellite system. Using active control methodology, we were able to synchronise two identical fractional-order satellite systems. The veracity of the results is confirmed through fractional-order system synchronisation. These investigations could help with secure telecommunications, data security, radar detection, weather forecasting, GPS systems, and earth observation.

8 Statements and Declarations

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8.2 Authors Contributions

All authors contributed to the formulation and design of research article. All authors have read and approved the final manuscript.

8.3 Acknowledgements

The author is grateful to Amity University Patna for providing him with time to conduct independent research as part of his job. In addition, the author states that any data or code related to this article can be obtained by contacting the corresponding author.

8.4 Conflicts of Interest

The authors declare that they have no conflicts of interest.

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