

Three Coupled Waveguide Sagnac Loop Reflectors for High Performance Integrated Optical Filters

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Abstract—We theoretically investigate advanced multi-functional integrated photonic filters formed by three waveguide coupled Sagnac loop reflectors (3WC-SLRs). By tailoring the coherent mode interference, the spectral response of the 3WC-SLR resonators is engineered to achieve diverse filtering functions with high performance. These include optical analogues of Fano resonances that yield ultrahigh spectral extinction ratios (ERs) and slope rates, resonance mode splitting with high ERs and low free spectral ranges, and classical Butterworth, Bessel, Chebyshev, and elliptic filters. A detailed analysis of the impact of the structural parameters and fabrication tolerances is provided to facilitate device design and optimization. The requirements for practical applications are also considered. These results theoretically verify the effectiveness of using 3WC-SLR resonators as multi-functional integrated photonic filters for flexible spectral engineering in diverse applications.

Keywords—Integrated optics, resonators, Fano resonance, mode splitting, classical filters.

I. INTRODUCTION

IN concert with the advance of complementary metal-oxide-semiconductor (CMOS) fabrication technologies, particularly as applied to photonic integrated circuits, integrated photonic resonators have become key building blocks [1, 2]. With narrowband wavelength selectivity, strong resonance field enhancement, and versatile filter shapes, these micro/nano-scale devices have been widely used for diverse applications including filters, lasers, modulators, buffers, switches, sensors, and signal processors [2-10]. and have also been applied to phase-sensitive all-pass filters [11-15]. Integrated ring resonators, particularly in CMOS compatible platforms, have also formed the basis for Kerr microcombs, [16-31] that have experienced wide applications to microwave photonics, [32-55] data communications [56-58] and neural networks [59, 60], and quantum optics [61- 69]. Photonic crystal cavities form a key class of integrated resonator that have been widely investigated [70-76]. For all of these devices, CMOS compatible platforms have proven to be indispensable [77 - 88] for their low loss, reproducibility and manufacturability, particularly for nonlinear optical applications where they have out-performed silicon [89-103] and even chalcogenide glass [104-117] and even other materials [118-120] because of their low nonlinear absorption

[121-123]. This has been a motivation for integrating novel 2D materials onto CMOS platforms, to increase their nonlinear performance [124 - 130].

Fano resonances are a fundamental physical phenomenon featuring an asymmetric resonant lineshape profile induced by interference between a discrete localized state and a continuum state [131-134]. It was first discovered in the absorption spectra of noble gases and later on has been extended to a much wider scope [135, 136]. In recent years, various types of photonic resonators have been designed to realize optical analogues of Fano resonances, which have attracted great interest and found wide applications in optical switching, sensing, light focusing beyond the diffraction limit, topological optics, data storage, and many others [[131, 133, 137-139].

Resonance mode splitting is a fundamental phenomenon in photonic resonators that occurs when two or more mutually coupled modes co-exist in the same resonant cavity [140, 141]. It can achieve a reduced free spectral range (FSR) and an increased quality (Q) factor while maintaining a small physical cavity length, thus yielding a compact device footprint, low power consumption, and versatile filter shapes for dense-wavelength division-multiplexing (DWDM) and microwave photonics applications [142, 143]. Recently, many applications based on mode-split resonators have been demonstrated, such as optical buffering [144], dispersion compensation [145,146], signal multicasting [147, 148], differential equation solving [149], microwave signal generation [150], and sensing [151].

Butterworth, Bessel, Chebyshev, and elliptic filters are classical filters that model and govern signal filtering and processing in communications and computing systems [152-154]. These fundamental filters are broadly used in applications such as noise reduction, spectral analysis, and signal synthesis [155-158]. Photonic resonators offer a powerful solution to realize these filters in optical domain [159-161]. With much broader operation bandwidth than their electronic counterparts, these optical filters play an important role in high-speed optical communications and information processing [162].

To realize Fano-resonances, resonance mode splitting, and classical filters based on integrated photonic resonators could offer competitive advantages including compact footprint, high scalability, high stability, and mass producibility for practical applications. Recently, we investigated multi-functional integrated photonic filters based on cascaded

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Sagnac loop reflectors (SLRs) [163] and two waveguide coupled Sagnac loop reflectors (2WC-SLRs) [164] formed by self-coupled silicon-on-insulator (SOI) nanowires. Here, we theoretically investigate more complex device structures including three waveguide coupled Sagnac loop reflectors (3WC-SLRs) that yield greatly enhanced performance together with more versatile filtering functions. We tailor coherent mode interference in the 3WC-SLR resonators to achieve versatile filter shapes with high performance, including optical analogues of Fano resonances with ultrahigh extinction ratios (ERs) and slope rates (SRs), resonance mode splitting with high ERs and low FSRs, and classical Butterworth, Bessel, Chebyshev, and elliptic filters. A detailed analysis of the impact of the structural parameters and fabrication tolerances is provided. The requirements for practical applications are also considered in our design and achieved for the proposed devices. These results highlight the strong potential of 3WC-SLR resonators for flexible spectral engineering in diverse applications such as optical filtering, switching, and sensing.

II. DEVICE CONFIGURATION

Figure 1 illustrates schematic configurations of the 3WC-SLR resonators. Two types of 3WC-SLR resonators are

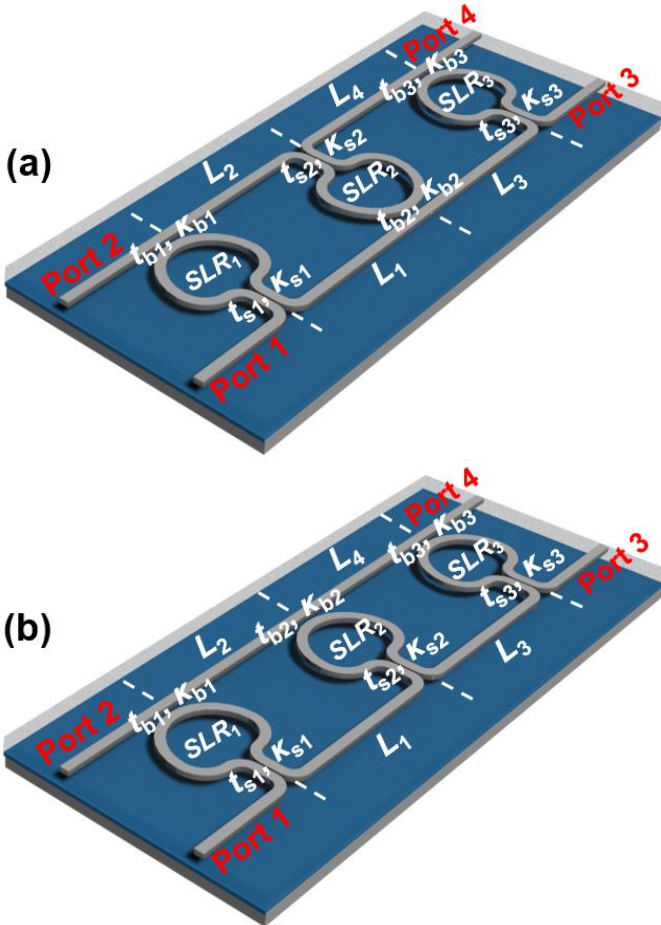


TABLE I
DEFINITIONS OF STRUCTURAL PARAMETERS OF THE 3WC-SLR RESONATORS

Waveguides	Length	Transmission factor ^a	Phase shift ^b
Bus waveguides between SLRs ($i = 1, 2, 3, 4$)	L_i	a_i	φ_i
Sagnac loop in SLR_i ($i = 1, 2, 3$)	L_{SLRi}	a_{si}	φ_{si}
Directional couplers	Field transmission coefficient ^c	Field cross-coupling coefficient ^c	
Coupler in SLR_i ($i = 1, 2, 3$)	t_{si}	k_{si}	
Coupler between SLR_i and bus waveguide ($i = 1, 2, 3$)	t_{bi}	k_{bi}	

^a $a_i = \exp(-\alpha L_i / 2)$, $a_{si} = \exp(-\alpha L_{SLRi} / 2)$, α is the power propagation loss factor.

^b $\varphi_i = 2\pi n_g L_i / \lambda$, $\varphi_{si} = 2\pi n_g L_{SLRi} / \lambda$, n_g is the group index and λ is the wavelength.

^c $t_{si}^2 + \kappa_{si}^2 = 1$ and $t_{bi}^2 + \kappa_{bi}^2 = 1$ for lossless coupling are assumed for all the directional couplers.

III. ULTRA-SHARP FANO RESONANCES

In this section, we tailor the spectral response of the zig-zag 3WC-SLR resonator to realize optical analogues of Fano resonances with high ERs and SRs. The power transmission spectrum from Port 2 to Port 4 of the zig-zag 3WC-SLR resonator is depicted in Fig. 2(a). The device structural

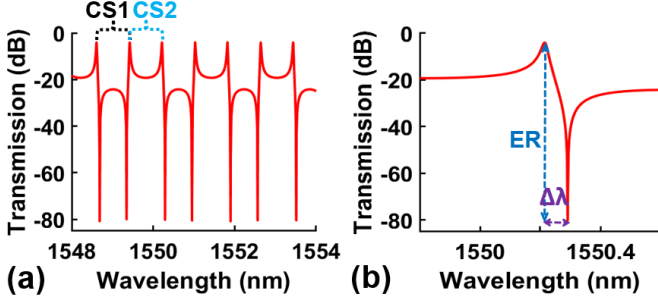


Fig. 2. (a) Power transmission spectrum of the zig-zag 3WC-SLR resonator from Port 2 to Port 4 when $L_{SLR} = L_{1,2,3,4} = 115 \mu\text{m}$, $t_s = 0.743$, and $t_b = 0.994$. (b) Zoom-in view of (a) in the wavelength range of 1549.8 nm–1550.65 nm. CS: channel spacing. ER: extinction ratio. $\Delta\lambda$: wavelength difference between the resonance peak and notch.

parameters are $L_{SLR} = L_{1,2,3,4} = 115 \mu\text{m}$, $t_s = 0.743$, and $t_b = 0.994$. One can see that there are periodical Fano resonances with identical asymmetric resonant line-shapes in each period. The FSR is about 200 GHz, which equals the sum of the two channel spacings (CS1 and CS2). The two CSs are very close to each other (CS1 = 101.71 GHz and CS2 = 98.88 GHz), reflecting the high SR of the Fano resonances.

Figure 2(b) shows a zoom-in view of Fig. 2(a) in the wavelength range of 1549.8 nm – 1550.65 nm, which shows a Fano resonance with an ultra-high ER of 76.32 dB and an ultra-high SR of 997.66 dB/nm. The ER is defined as the difference between the maximum and the minimum transmission, and the SR is defined as the ratio of the ER to

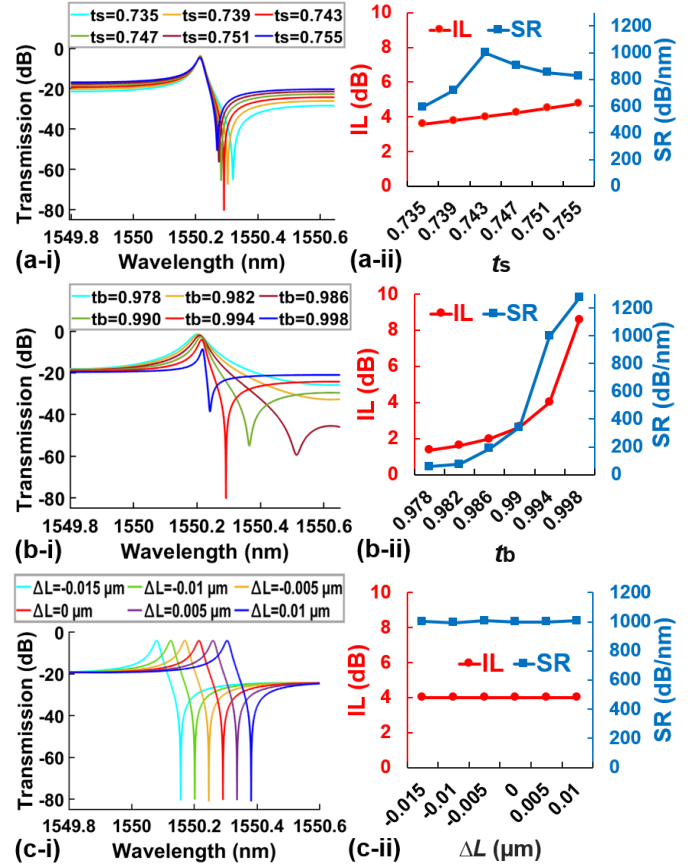


Fig. 3. (a-i) Power transmission spectra and (a-ii) the corresponding SR and IL for various t_s when $t_b = 0.994$ and $L_{SLR} = L_{1,2,3,4} = 115 \mu\text{m}$, respectively. (b-i) Power transmission spectra and (b-ii) the corresponding IL and SR for various t_b when $t_s = 0.743$ and $L_{SLR} = L = 115 \mu\text{m}$, respectively. (c-i) Power transmission spectra and (c-ii) the corresponding IL and SR for different $L = L_{1,2,3,4}$ when $t_s = 0.743$, $t_b = 0.994$, and $L_{SLR} = 115 \mu\text{m}$, respectively.

the wavelength difference between the resonance peak and notch (i.e., $\Delta\lambda$ in Fig. 2(b)). The high ER and SR reflect the high performance of the Fano resonances resulting from strong coherent optical mode interference in the compact resonator with only three SLRs. Table II compares the performance of the Fano resonances generated by the parallel 2WC-SLR resonator [164] and the zig-zag 3WC-SLR resonator. As compared with the parallel 2WC-SLR resonator, the zig-zag 3WC-SLR resonator can generate Fano resonances with increased ER and SR as well as decreased insertion loss (IL), all of which are highly desirable in practical applications. Moreover, the periodical filter shape of the zig-zag 3WC-SLR resonator is also useful for applications in wavelength division multiplexed (WDM) systems.

TABLE II
PERFORMANCE COMPARISON OF PARALLEL 2WC-SLR AND ZIG-ZAG 3WC-SLR RESONATORS IN ACHIEVING FANO RESONANCES

Device	IL (dB)	ER (dB)	SR (dB/nm)	FSR (GHz)	Ref.
Parallel 2WC-SLR resonator ^a	6.15	13.76	416.96	601.8	[164]
Zig-zag 3WC-SLR resonator	4	76.32	997.66	200.59	This work

^a The structural parameters are $L_{SLRi} = L_i = 115 \mu\text{m}$ ($i = 1, 2$), $t_s = 0.743$, and $t_b = 0.94$.

In Figs. 3(a) – (c), we further investigate the impact of the device structural parameters including t_s , t_b , and $L = L_{1,2,3,4}$ on the performance of the Fano resonances generated by the zig-zag 3WC-SLR resonator. In each figure, we changed only one structural parameter, keeping the others the same as those in Fig. 2. Figure 3(a-i) compares the power transmission spectra for various t_s . The corresponding IL and SR are depicted in Fig. 3(a-ii). The IL increases with t_s , while the SR first increases and then decreases with t_s , achieving a maximum value of 997.66 dB/nm at $t_s = 0.743$. The non-monotonic relationship between the SR and t_s is a combined result of both a decrease in $\Delta\lambda$ and a non-monotonic variation in ER. The latter mainly arises from the difference between the internal (transmission) and external (coupling) cavity loss, which is similar to that for different coupling regimes in microring resonators (MRRs) [167]. Figure 3(b-i) compares the power transmission spectra for various t_b . The IL and SR functions of t_b are depicted in Fig. 3(b-ii). Both the IL and SR increase with t_b , reflecting a trade-off between them. Note that although the ER for $t_b = 0.994$ is higher than for $t_b = 0.998$, the SR for $t_b = 0.994$ is higher than that for $t_b = 0.998$ due to a more significantly decreased $\Delta\lambda$. In Figs. 3(c-i) and (c-ii), we compare the corresponding results for various ΔL , which is the length variation of the connecting bus waveguides. To simplify the comparison, we assume the same ΔL for each connecting bus waveguides $L_{1,2,3,4}$ and keep L_{SLR} constant. As ΔL increases, the IL and SR remain unchanged while the resonance redshifts. This highlights the high fabrication tolerance and also indicates that the resonance wavelength can be tuned by introducing thermo-optic micro-heaters [149, 168, 169] or carrier-injection electrodes [142] along the connecting bus waveguides to tune the phase shift.

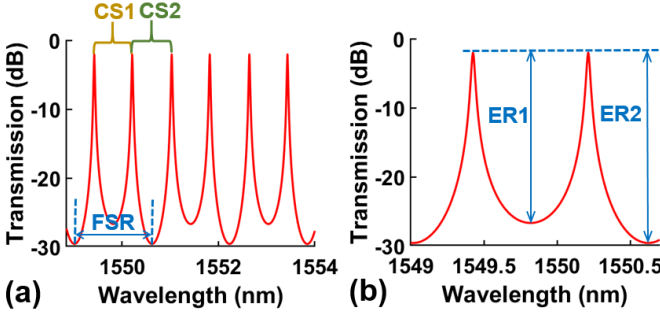


Fig. 4. (a) Power transmission spectrum of the zig-zag 3WC-SLR resonator from Port 2 to Port 4 when $L_{\text{SLR}} = L_{1,2,3,4} = 115 \mu\text{m}$, $t_s = 0.72$, and $t_b = 0.99$. (b) Zoom-in view of (a) in the wavelength range of 1549 nm – 1550.7 nm.

IV. RESONANCE MODE SPLITTING

In this section, we tailor the mode interference in the zig-zag 3WC-SLR resonator to achieve resonance mode splitting with high ERs and low FSRs. The resonance mode splitting with multiple densely spaced resonances can break the intrinsic dependence between the Q factor, FSR, and physical cavity length, thus allowing low FSRs and high Q factors in resonators with a compact footprint. Figure 4(a) shows the power transmission spectrum from Port 2 to Port 4 of the zig-zag 3WC-SLR resonator. The structural parameters are $L_{\text{SLR}} = L_{1,2,3,4} = 115 \mu\text{m}$, $t_s = 0.72$, and $t_b = 0.99$, which are designed in

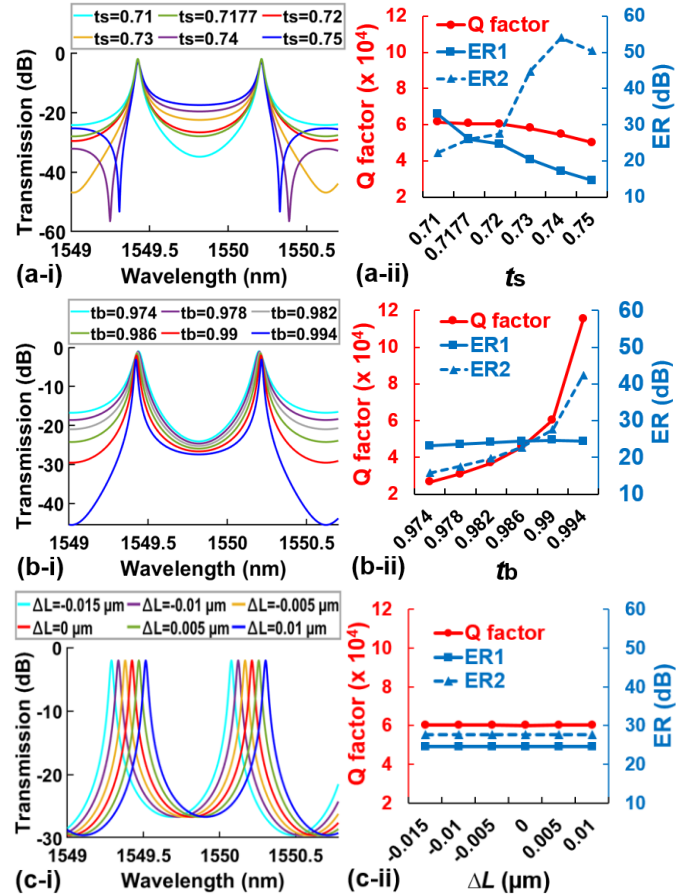


Fig. 5. (a-i) Power transmission spectra of the zig-zag 3WC-SLR resonator for various t_s for input from Port 2 to Port 4 when $t_b = 0.99$ and $L_{\text{SLR}} = L_{1,2,3,4} = 115 \mu\text{m}$. (a-ii) Calculated Q factor and ERs (ER1 and ER2) as functions of t_s for the transmission spectra in (a-i). (b-i) Power transmission spectra and (b-ii) the corresponding IL and ERs for different t_b when $t_s = 0.72$ and $L_{\text{SLR}} = L_{1,2,3,4} = 115 \mu\text{m}$, respectively. (c-i) Power transmission spectra and (c-ii) the corresponding Q factor and ERs for different L when $t_s = 0.72$, $t_b = 0.99$, and $L_{\text{SLR}} = 115 \mu\text{m}$, respectively.

order to achieve a CS of about 100 GHz between adjacent split resonances. In Fig. 4(a), CS1 = 98.33 GHz and CS2 = 102.26 GHz. There are two split resonances within a FSR of ~ 200.59 GHz. Figure 4(b) shows a zoom-in view of Fig. 4(a) in the wavelength range of 1549 nm – 1550.7 nm. The IL, Q factor, ER1, and ER2 of the two split resonances in Fig. 4(b) are ~ 2.02 dB, $\sim 6.03 \times 10^4$, ~ 24.65 dB, and ~ 27.55 dB, respectively.

We further investigate the impact of varying t_s , t_b , and $L = L_{1,2,3,4}$ on the Q factor, ER1, and ER2 of the split resonances, all of which are important parameters reflecting the degree of mode splitting. In Figs. 5(a) – (c), we only changed one structural parameter in each figure, keeping the others the same as those in Fig. 4. Figure 5(a-i) shows the spectral response for various t_s . The Q factor and ERs (ER1 and ER2) as functions of t_s are depicted in Fig. 5(a-ii). As t_s increases, the Q factor slightly decreases while the ER1 and ER2 change more dramatically, resulting in a change in the spectral response towards that of the Fano resonances in Fig. 2(a). The non-monotonic change in ER2 with t_s follows the trend of the SR in Fig. 3(a-ii) for similar reasons. In particular, ER1 equals to ER2 when $t_s = 0.7177$. Under this condition, the Q factor

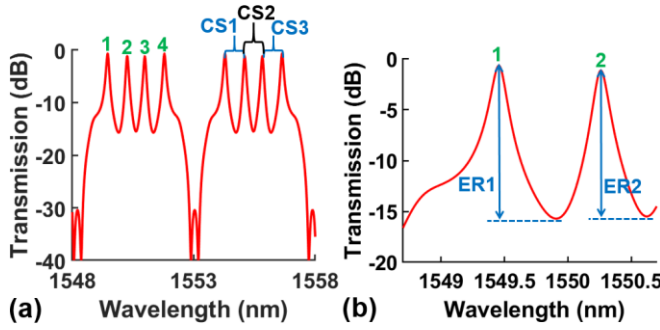


Fig. 6. (a) Power transmission spectrum of the zig-zag 3WC-SLR resonator from Port 1 to Port 3 when $L_{\text{SLR}} = 115 \mu\text{m}$, $L_{1,3} = 115 \mu\text{m}$, $L_{2,4} = 230 \mu\text{m}$, and $t_s = t_b = 0.88$. (b) Zoom-in view of (a) in the wavelength range of 1548.7 nm – 1550.7 nm.

and effective FSR are $\sim 6.06 \times 10^4$ and ~ 100.30 GHz (i.e., half of the FSR in Fig. 4(a)), respectively. To achieve the same FSR, the circumference of a comparable MRR (with the same waveguide geometry and loss) is $690 \mu\text{m}$, which is 6 times the length of the SLRs. This highlights the reduced cavity length enabled by the mode splitting in the 3WC-SLR resonator. On the other hand, the Q factor of a comparable MRR with the same FSR and ER is $\sim 6.08 \times 10^4$ – almost the same as that of the zig-zag 3WC-SLR resonator. This indicates that the reduced cavity length did not come at the expense of a significant decrease in Q factor. The spectral response for different t_b are shown in Fig. 5(b-i). The corresponding Q factor and ERs are depicted in Fig. 5(b-ii). The ER1 remains almost unchanged while both the ER2 and the Q factor increase with t_b , at the expense of a slightly increased IL. The corresponding results for different ΔL are shown in Fig. 5(c-i) and (c-ii). Following the trend in Fig. 2(c), the filter shape remains unchanged while the resonance redshifts as ΔL increases.

The number of split resonances can be changed by varying the length of the connecting bus waveguides. Figure 6(a-i) shows the power transmission spectrum from Port 1 to Port 3 of the zig-zag 3WC-SLR resonator. The structural parameters are $L_{\text{SLR}} = 115 \mu\text{m}$, $L_{1,3} = 115 \mu\text{m}$, $L_{2,4} = 230 \mu\text{m}$, and $t_s = t_b = 0.88$. Clearly, there are four split resonances in each FSR. The CSs between the split resonances are $\text{CS1} = \text{CS3} = 100.46$ GHz and $\text{CS2} = 90.37$ GHz. Figure 6(b) shows a zoom-in view of Fig. 6(a) in the wavelength range of 1548.7 nm – 1550.7 nm. To quantitatively analyze the improvement in the performance of the multiple split resonances, in Table III we further compare the resonance mode splitting in the zig-zag 3WC-SLR resonator with that of five cascaded SLRs [163] which can also generate four split resonances. As compared with the five cascaded SLRs that only include standing-wave (SW) filter elements, the mode interference between the SW and travelling-wave (TW) filter elements in the zig-zag 3WC-SLR resonator yields higher ERs for the split resonances, a smaller difference between the ERs of the split resonances, and fewer required SLRs.

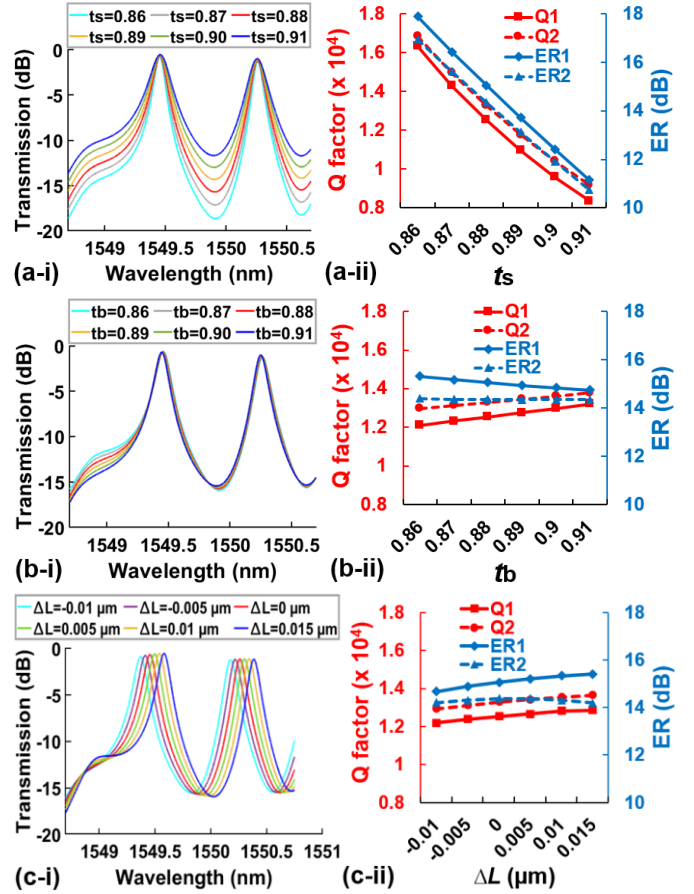


Fig. 7. (a-i) Power transmission spectra of the zig-zag 3WC-SLR resonator for various t_s for input from Port 1 to Port 3 when $t_b = 0.88$, $L_{\text{SLR}} = 115 \mu\text{m}$, $L_{1,3} = 115 \mu\text{m}$, and $L_{2,4} = 230 \mu\text{m}$. (a-ii) Calculated Q factors (Q1 and Q2) and ERs (ER1 and ER2) as functions of t_s for the transmission spectra in (a-i). (b-i) Power transmission spectra and (b-ii) the corresponding Q factors and ERs for different t_b when $t_s = 0.88$, $L_{\text{SLR}} = 115 \mu\text{m}$, $L_{1,3} = 115 \mu\text{m}$, and $L_{2,4} = 230 \mu\text{m}$. (c-i) Power transmission spectra and (c-ii) the corresponding Q factors and ERs versus connecting bus waveguides length variation ΔL when $L_{\text{SLR}} = 115 \mu\text{m}$ and $t_s = t_b = 0.88$, respectively.

TABLE III
PERFORMANCE COMPARISON OF ZIG-ZAG 3WC-SLR AND FIVE CASCADED SLR RESONATORS IN ACHIEVING MODE SPLITTING

		Five cascaded SLR resonator ^a	Zig-zag 3WC-SLR resonator
Number of split resonances		4	4
Number of SLRs		5	3
IL (dB)	IL1, IL4 ^b	1.88	0.65
	IL2, IL3	0.91	1.13
ER (dB)	ER1, ER4	5.03	15.05
	ER2, ER3	4.55	14.35
CS (GHz)	CS1	13.8	100.46
	CS2	16.24	90.37
	CS3	13.8	100.46
Ref.		[163]	This work

^a The structural parameters of the five cascaded SLR resonator are $L_{\text{SLR}i}$ ($i = 1-5$) = $115 \mu\text{m}$, L_i ($i = 1-4$) = $115 \mu\text{m}$, and t_{si} ($i = 1-5$) = 0.88 .

^b ILs of the four split resonances are labelled as IL1–IL4 from left to right. Due to symmetry of the split resonances, there are $\text{IL1} = \text{IL4}$, $\text{IL2} = \text{IL3}$, $\text{ER1} = \text{ER4}$, and $\text{ER2} = \text{ER3}$.

TABLE IV
STRUCTURAL PARAMETERS OF CLASSICAL FILTERS BASED ON PARALLEL 3WC-SLR RESONATORS

	Butterworth	Bessel	Chebyshev Type I	Chebyshev Type II	Elliptic
t_s	0.89	0.83	0.89	0.85	0.96
t_b	0.94	0.94	0.97	0.94	0.795
L_{SLR} (μm)	173	173	173	173	692
L_i (μm)	173 ($i = 1, 3$) 346 ($i = 2, 4$)	173 ($i = 1, 3$) 346 ($i = 2, 4$)	173 ($i = 1, 3$) 346 ($i = 2, 4$)	173 ($i = 1, 3$) 346 ($i = 2, 4$)	346 ($i = 1-4$)
Input / output ports	Port 2 / Port 3	Port 2 / Port 3	Port 2 / Port 3	Port 2 / Port 4	Port 2 / Port 4
FSR (GHz) ^a	100.004	100.004	100.004	100.004	100.004

^a The structural parameters are designed in order to achieve a FSR of ~ 100.004 GHz in the C band to meet the ITU-T spectral grid standard G694.1 [170].

In Figs. 7(a) – (c), we investigate the impact of t_s , t_b , and ΔL on the performance of the resonance mode splitting based on the zig-zag 3WC-SLR resonator in Fig. 6. We only changed one structural parameter in each figure, keeping the others the same as those in Fig. 6 (a). The power transmission spectra for different t_s and t_b are shown in Figs. 7(a-i) and (b-i), respectively. The corresponding Q factors (Q1 and Q2) and ERs (ER1 and ER2) for the first two resonances from the left side are shown in Figs. 7(a-ii) and (b-ii) respectively. In Fig. 7(a), all the Q factors and ERs decrease with t_s , along with slightly decreased ILs. In Fig. 7(b), as t_b increases, the difference between the two Q factors as well as the difference between the two ERs gradually decrease, resulting in a more symmetric resonance line-shape, which is desirable for reducing filtering distortions. Figures 7(c-i) and (c-ii) compares the corresponding results for various ΔL . As ΔL increases, the ER1 slightly increase and ER2 slightly decrease while both Q factors slightly increase, which make the filter shapes more asymmetric.

V. HIGH PERFORMANCE CLASSICAL FILTERS

In this section, we tailor the mode interference in the parallel 3WC-SLR resonator to realize classical filters including Butterworth, Bessel, Chebyshev, and elliptic filters, that all exhibit broad filtering bandwidths and high ERs. The spectral response of these practical filters (solid lines) together with the ideal passband filter (dashed line) are shown in Fig. 8. The Butterworth filter has a flat passband response, while the Bessel filter has a linear phase response over the passband. The Chebyshev filters have either passband ripples (Type I) or stopband ripples (Type II) together with a flat response in the opposite band, resulting in a steeper roll-off than the Butterworth filter. The elliptic filter has both passband and stopband ripples that yields the steepest roll-off among the four types of filters [153].

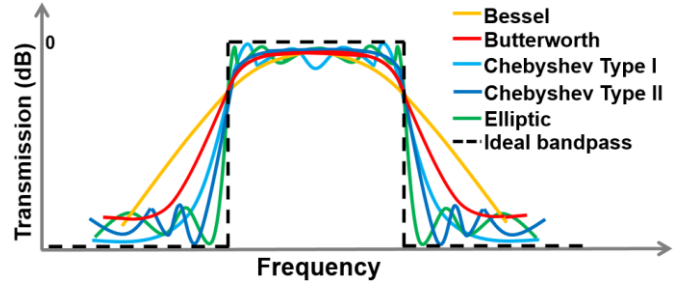


Fig. 8. Spectral responses of classical bandpass filters (solid lines) including Butterworth, Bessel, Chebyshev (Type I and Type II), and elliptic filters. The spectral response of an ideal bandpass filter (dashed lines) is also shown for comparison.

Figure 9(a) shows the power transmission spectrum and corresponding group delay response of the parallel 3WC-SLR resonator from Port 2 to Port 3. As can be seen, there is Butterworth filter shape with flat-top passbands arising from coherent mode interference within the parallel 3WC-SLR resonator, which can be used for low-distortion signal filtering in optical communication systems [171, 172]. The structural parameters are provided in Table IV. The IL, ER, and 3-dB bandwidth (BW) of the Butterworth filter are ~ 1.71 dB, ~ 7.12 dB, and ~ 28.08 GHz, respectively. We then investigate the impact of t_s , t_b , and ΔL on the performance of the Butterworth filter. The results are shown in Figs. 9(b) – (d), respectively. In Fig. 9(b), the bandwidth of the passband increases with t_s , together with slightly degraded filtering flatness. In Fig. 9(c), the resonance is split, with the spectral range between the split resonances increasing with t_b . This indicates that the Butterworth filter shape gradually transitions to a Chebyshev Type I filter shape with improved roll-off and degraded flatness. In Fig. 9(d), as ΔL increases, the filter shape remains unchanged while the resonance redshifts, showing similar trends to the Fano resonances in Fig. 3(c-i) and the resonance mode splitting in Fig. 5(c-i). This reflects the high fabrication tolerance and the feasibility to realize tunable Butterworth filters.

The spectral and group delay responses of the Bessel filter based on the parallel 3WC-SLR resonator are shown in Fig. 10(a). The input and output ports are Port 2 and Port 3, respectively, the same as those for the Butterworth filter in Fig. 9. The structural parameters are provided in Table IV. As can be seen, the Bessel filter with a flat-top group delay response is achieved, which is useful for applications such as optical buffering and delay lines [173, 174]. The group delay response versus t_s , t_b , and ΔL are shown in Figs. 10(b) – (d),

respectively. In Fig. 10(b), the bandwidth of the group delay response increases with t_s , at the expense of a decreased maximum group delay value and degraded flatness. In Fig. 10(c), the maximum group delay on both sides increases with t_b , while the group delay at the center wavelength shows the opposite trend, resulting in higher unevenness in the group delay response. In Fig. 10(d), the group delay response remains unchanged but redshifts as ΔL increases.

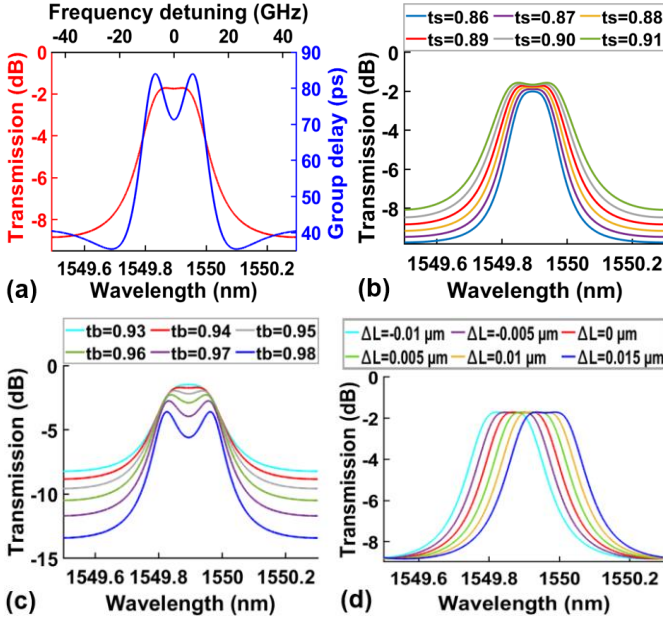


Fig. 9. Butterworth filter based on the parallel 3WC-SLR resonator. (a) Power transmission spectra and group delay response from Port 2 to Port 3. The structural parameters are $L_{\text{SLR}} = 173 \mu\text{m}$, $L_{1,3} = 173 \mu\text{m}$, $L_{2,4} = 346 \mu\text{m}$, $t_s = 0.89$, and $t_b = 0.94$. (b) – (d) Power transmission spectra versus t_s , t_b , and ΔL , respectively. The structural parameters are kept the same as those in (a) except for the varied parameters.

Figure 11(a) shows the power transmission spectrum and group delay response of the Chebyshev Type II filter based on the parallel 3WC-SLR resonator. The input port is Port 2, which is the same as those for the Butterworth and Bessel filters, while the output port is changed from Port 3 to Port 4. The structural parameters are provided in Table IV. Clearly, there is a Chebyshev Type II filter shape with equal stopband ripples and a flat response in the passband. The IL, maximum stopband ripple, ER, and 3-dB BW are ~ 1.26 dB, ~ 0.037 dB, 27.33 dB, and ~ 18.77 GHz, respectively. This filter function with a very flat passband and strongly rejected band can be

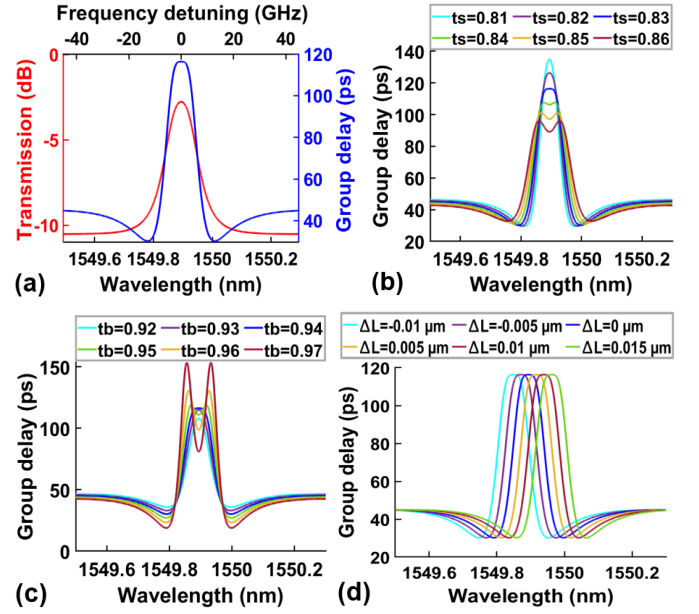


Fig. 10. Bessel filter based on the parallel 3WC-SLR resonator. (a) Power transmission spectra and group delay response from Port 2 to Port 3 when the structural parameters are $L_{\text{SLR}} = 173 \mu\text{m}$, $L_{1,3} = 173 \mu\text{m}$, $L_{2,4} = 346 \mu\text{m}$, $t_s = 0.83$, and $t_b = 0.94$. (b) – (d) Group delay responses versus t_s , t_b , and ΔL , respectively. The structural parameters are kept the same as those in (a) except for the varied parameters.

useful for cleaning and extracting channels from crosstalk in a WDM optical communications system, respectively [175]. The power transmission spectra versus t_s , t_b , and ΔL are shown in Figs. 11(b) – (d), respectively. As shown in Figs. 11(b) and (c), by increasing t_s and keeping constant t_b or vice versa, the notch depth of the single resonance first increases and then the single resonance is split with an increased spectral range between the split resonances. This is a typical phenomenon for resonance mode splitting, which has also been observed in Refs. [149, 176]. In Fig. 11(d), the filter shape remains unchanged but redshifts as ΔL increases, following the trends for previous filters.

Finally, we tailor the mode interference in the parallel 3WC-SLR resonator to realize elliptic filters. As compared with the Butterworth and Chebyshev type I filters that have all-pole transfer functions, elliptic filters include both poles and zeros in the transfer function, which can provide higher stopband extinction levels and better roll-off [160]. Figure 12(a) shows the power transmission spectrum and group delay of the parallel 3WC-SLR resonator from Port 2 to Port 4. There is an elliptic filter shape with ripples in both the passband and stopband. The structural parameters are also provided in Table IV. The IL and notch depth of filter are ~ 0.59 dB and ~ 31.5 dB, respectively. The maximum passband and stopband ripples are ~ 1.56 dB and ~ 18.25 dB, respectively. The ripples in the passband and stopband make the filtering roll-off steeper. These ripples together with the steep roll-off result in a reasonable trade-off between the minimum signal degradation and the maximum noise/interference rejection [177]. The power transmission spectra versus t_s , t_b , and ΔL are shown in Figs. 12(b) – (d), respectively. In Figs. 12(b) and (c), the evolution of the split

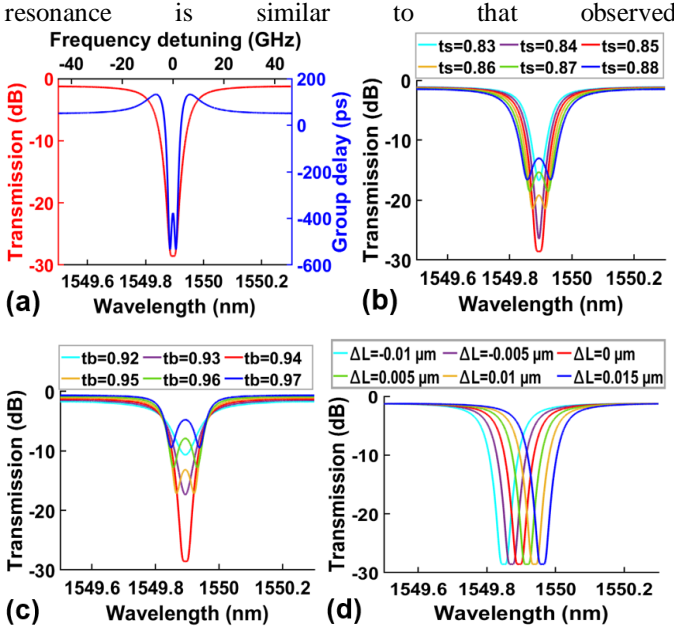


Fig. 11. Chebyshev Type II filter based on the parallel 3WC-SLR resonator. (a) Power transmission spectra and corresponding group delay response from Port 2 to Port 4 when $L_{\text{SLR}} = 173 \mu\text{m}$, $L_{1,3} = 173 \mu\text{m}$, $L_{2,4} = 346 \mu\text{m}$, $t_s = 0.85$, and $t_b = 0.94$. (b) – (d) Power transmission spectra versus t_s , t_b , and ΔL , respectively. The structural parameters are kept the same as those in (a) except for the varied parameters.

in Figs. 11 (b) and (c). In Fig. 12(d), unlike the trends for previous filters that exhibit an unchanged filter shape when ΔL is varied, the filter shape shows a slight asymmetry in the stop-band when ΔL is away from 0. This is mainly due to the asynchronous feedback in the elliptic filter, which results in asymmetrically located zeros around the center frequency [178].

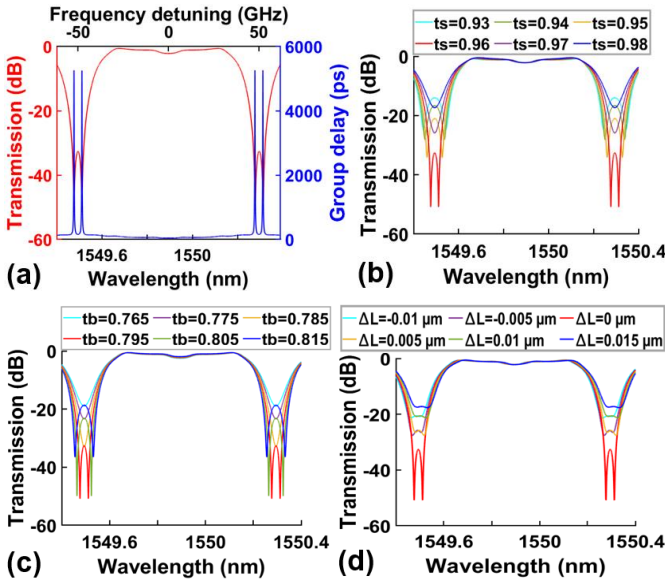


Fig. 12. Elliptic filter based on the parallel 3WC-SLR resonator. (a) Power transmission spectrum and corresponding group delay response from Port 2 to Port 4. The structural parameters are $L_{\text{SLR}} = 692 \mu\text{m}$, $L_{1,2,3,4} = 346 \mu\text{m}$, $t_s = 0.96$, and $t_b = 0.795$. (b) – (d) Power transmission spectra versus t_s , t_b , and ΔL , respectively. The structural parameters are kept the same as those in (a) except for the varied parameters.

VI. CONCLUSIONS

We have theoretically investigated advanced multi-functional integrated photonic filters based on 3WC-SLR resonators. Mode interference in the 3WC-SLR resonators is tailored to achieve high performance filtering functions including optical analogues of Fano resonances with ultrahigh ERs and SRs, resonance mode splitting with high ERs and low FSRs, and classical Butterworth, Bessel, Chebyshev, and elliptic filters. The requirements for practical applications are considered in our designs, together with detailed analysis of the impact of structural parameters and fabrication tolerances. This work highlights the 3WC-SLR resonators as a powerful and versatile approach to flexible spectral engineering for a diverse range of applications.

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