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Output-based Event-triggered Control of Networked Systems Subject to Bilateral Packet Dropouts

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Summary

In this paper, an output-based event-triggered control problem of discrete-time networked control systems (NCSs) subject to bilateral data packet dropouts is investigated. In view of the stochastic sequences of packet dropouts in measurement channels (from sensors to controller) and control channels (from controller to actuators), the NCS is converted into a closed-loop stochastic parameter system. In the aid of a Lyapunov functional based on stochastic variables, sufficient conditions on co-design of event-triggering strategy and exponentially mean-square stability of NCSs are derived. Furthermore, an improved iterative algorithm is given to obtain the dynamic output feedback control law and event-triggering parameters from the non-convex inequalities. Finally, a numerical example and the corresponding simulation results are given to show the validity and applicability of the developed techniques.

KEYWORDS:
Networked control systems, dynamic output feedback controller, event-triggered control, bilateral packet dropouts

1 | INTRODUCTION

Networked control system (NCS) itself is a dynamic control system for data transmission through wired or wireless communication channels [1,2]. Due to its advantages of low cost, easy installation and maintenance, NCSs have been widely applied in many fields in the past decade, such as electronic information, industrial manufacturing, etc. Nevertheless, control tasks of NCSs often need to rely on unreliable networks for information transmission [3]. Therefore, solving the problem of limited communication resources and packet dropouts is one of the major contents in this field of NCSs.

To deal with the problems of communication channel congestion and limited network resources in NCSs, the event-triggered control mechanism (ETM) is investigated instead of the time-based sampling strategy. Recently, the ETM has attracted a great deal attention in the field of control theory. The idea of ETM is that the data transmission and updates are allowed only when satisfying the triggering conditions based on states and measured outputs [4,5]. This control approach essentially solves the problem of excessive resource consumption and strikes a balance between resource conservation and control performance. Notice that the ETM actively loses some unnecessary information at the expense of system performance [6], in which case the packet dropout behaviors in unreliable networks further deteriorate the performance and can even lead to instability [7,8]. The methods of handling stochastic data losses are generally classified into four categories. The first one is to model a system with data packet losses by using the Markovian jump system [9]. In the second method, the packet loss behaviors are describes by the independent Bernoulli's distribution random variables [10]. Furthermore, the dropouts are considered as a special case of time-delay. The final approach is that packet losses can be researched by methods of switched systems [11].

On the other hand, since data packet losses are caused by bandwidth limitation and network congestion, the ETM can effectively alleviate network congestion and thus reduce the probability of packet losses in realistic networked systems [12,13]. Therefore, there are existing some research results in the synthesis and analysis of ETM and packet loss. To investigate the influence of these challenges on system performance [14], the packet dropout behavior is described by a random variable with Bernoulli binary distribution. For a discrete-time linear system, an event trigger located at the sensor and the controller with one-side data packet losses is designed [15]. For multiple systems sharing a common wireless communication network, The design of event-triggered control law and one-side data packet loss behaviors are considered [16]. Building upon the aforementioned works of unilateral packet dropouts, this paper investigates stability of NCSs with ETM subject to packet dropouts in both measurement channels and control channels.

In most of the researches mentioned above, the state-based controller has a strong assumption that the state information must be available because the control strategies rely on the availability of full state information. This is in general a strong assumption as full state measurements are rarely available. However, it is difficult to accurately obtain the state information in practice [17,18]. Therefore, this paper designs a dynamic output feedback controller since the output is observable. Furthermore, the fact that the controllers and actuators can operate synchronously is often assumed in many existing results, in which the effect of network induced error is ignored. Inspired by the finite-time asynchronous output feedback control scheme [19], the ETM in this paper can admit asynchronous transmission, and transmission error is induced to detect the triggering conditions.

In this paper, the problems of stabilization and controller design for discrete-time NCSs subject to bilateral packet dropouts with ETM are investigated. The main work and contributions of this paper are summarized as follows:

(i) This paper mainly focuses on bilateral unreliable networks in which packet dropouts occur in both measurement channels and control channels. We model a closed-loop stochastic parameter system in terms of random packet dropout behaviors with Bernoulli's distribution.

(ii) Since full states of a system are generally not available, we design an output-based controller and event-triggering conditions. Some sufficient conditions and results will be obtained via a Lyapunov functional with stochastic parameters to ensure that the NCSs are exponentially mean-square stable.

(iii) It is one of the challenges to design a desired dynamic output feedback controller in this paper. Therefore, an improved iterative algorithm is proposed to solve the nonconvex optimization problem with the matrix inequality constrains. The algorithm effectively reduces the conservatism compared with the griding approach and matrix transformation.

This paper is organized as follows: Section 2 will formulate the problem. In Section 3, some stability results will be given, and an optimizationwill solved by an iterative algorithm to obtain control law and event-triggering strategy. Numerical examples and simulation results are presented in Section 4. Finally, the conclusion of this paper will be illustrated in Section 5.

Notation: The notation $\mathbb{R}^n$ refers to the n-dimensional Euclidean space. The notation $N \in \mathbb{N}_+$ refers to the one-dimensional positive integer set. Some of the symbols will be reinterpreted where appropriate in this paper. The matrix $A > 0$ (≥ 0) denotes that A is a real positive definite (semi-positive definite) matrix. The notation $diag(A)$ means that matrix A is in block-diagonal form. The notation $\|x\|$ stands for the Euclidean norm of vector x. The notation $\|A\|$ refer to the matrix norm of matrix A calculated by $\|A\| = \max_j \sum_{i=1}^{N} \left|a_{ij}\right|$, where a_{ij} is the i^{th} row and j^{th} column element of matrix A. The vector $(x, y, z) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_z}$ represents $(x^T, y^T, z^T)^T$. The notation $\mathbb{E}\{*\}$ is the expectation of the random variable "$*$". $\lambda_{min}(A)$ $(\lambda_{max}(A))$ represents the minimum (maximum) of the eigenvalue of matrix A. Furthermore, a matrix $\begin{bmatrix} M & * \\ Q & N \end{bmatrix}$ represents the symmetry matrix $\begin{bmatrix} M & Q^T \\ Q & N \end{bmatrix}$.

2 | PROBLEM FORMULATION AND PRELIMINARIES

Consider the following plant described by a discrete-time system

$$\begin{cases} x(t_{k+1}) = \Phi x(t_k) + \Gamma u_d(t_k) \\ y(t_k) = C x(t_k) \\ z(t_k) = D x(t_k) \end{cases} \tag{1}$$

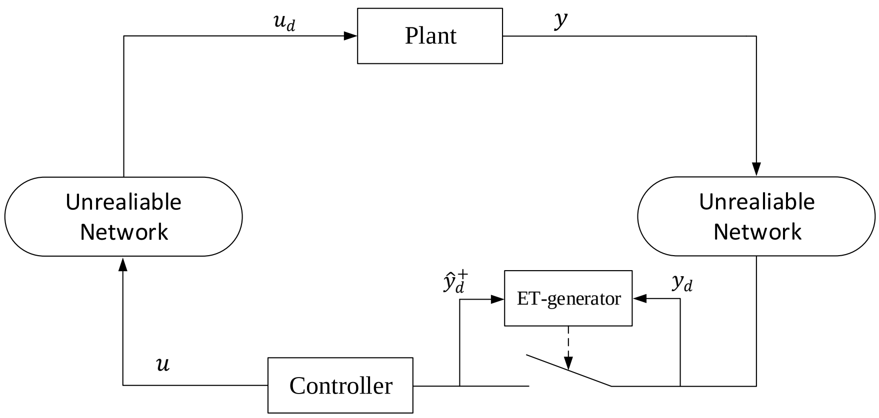

FIGURE 1 The architecture of the NCS subject to bilateral packet dropouts with ETM

where $x(t_k) \in \mathbb{R}^n$ denotes the state vector, the vector $u_d(t_k) \in \mathbb{R}^m$ represents the control input with packet dropouts, $y(t_k) \in \mathbb{R}^p$ is the measured output vector and $z(t_k) \in \mathbb{R}^q$ is the system output vector. The matrices $\Phi \in \mathbb{R}^{n\times n}$, $\Gamma \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{q\times n}$ are appropriate coefficient matrices. We define the sampling instant t_k and the transmission instant t_l.

The output-based controller of the NCS is considered by

$$\begin{cases} x_c(t_{k+1}) = \Phi_c x_c(t_k) + \Gamma_c \hat{y}_d^+(t_k) \\ u(t_k) = C_c x_c(t_k) \end{cases} \tag{2}$$

where x_c represents the controller states, $\hat{y}_d^+$ denotes the most recently available measured transmission signal to the controller, u means the output of controller and Φ_c, Γ_c, C_c are corresponding matrices. The transmission signals $y_d(t_k)$ of the measured outputs via the unreliable networks is described by

$$y_d(t_k) = (1-\delta)y(t_k) + \delta y_d(t_{k-1}) \tag{3}$$

where the output vector $y_d(t_k) \in \mathbb{R}^p$ also represents the available input signals to trigger. The stochastic variable $\delta \in \{0, 1\}$ is an element of a random process obeying the Bernoulli's distribution with

$$\mathbb{E}\{\delta\} := \bar{\delta}, \tag{4}$$

$$\mathbb{E}\{(\delta - \bar{\delta})^2\} := \bar{\delta}(1-\bar{\delta}). \tag{5}$$

Similarly, the transmission signals u_d via the control channel can be described as

$$u_d(t_k) = (1-\beta)u(t_k) + \beta u_d(t_{k-1}), \tag{6}$$

with

$$\mathbb{E}\{\beta\} := \bar{\beta}, \tag{7}$$

$$\mathbb{E}\{(\beta - \bar{\beta})^2\} := \bar{\beta}(1-\bar{\beta}). \tag{8}$$

Remark 1. In the packet dropouts models (3) and (6) of these communication channels, δ and β are random variables of Bernoulli's distributed white sequences. Take the measurement channels for example, δ takes a value between '0' and '1', where $\delta = 1$ means that data losses occur at the k^{th} step, and $\delta = 0$ represents that data packets are successfully received. If current dropout of data $y(t_k)$ occurs, i.e. $\delta = 1$, then the input of controller $y_d(t_k)$ takes the value of the data at the last value $y_d(t_{k-1})$. For the case of the successive packet dropouts, $y_d(t_k)$ takes the previous value $y_d(t_{k-N})$ without data packet dropouts, where $N \in \mathbb{N}_+$ is the number of consecutive packet losses. It can be illustrated as $y_d(t_k) = (1-\delta)y(t_k) + \delta(1-\delta)y(t_{k-1}) + ... + \delta^N(1-\delta)y(t_{k-N})$. Similarly, β has the same conclusion.

In view of (1), (2), (3) and (6), the augmented form of the networked system with stochastic parameters can be illustrated by

$$\begin{cases} \vec{x}(t_{k+1}) = \vec{\Phi}\vec{x}(t_k) + \vec{\Gamma}u(t_k) \\ y_d(t_k) = \vec{C}\vec{x}(t_k) \\ z(t_k) = \bar{D}\vec{x}(t_k) \end{cases} \tag{9}$$

where $\vec{x}(t_k) = \big(x(t_k), u_d(t_{k-1}), y_d(t_{k-1})\big)$,

$$\vec{\Phi} = \begin{bmatrix} \Phi & \beta\Gamma & 0 \\ 0 & \beta I & 0 \\ (1-\delta)C & 0 & \delta I \end{bmatrix}, \vec{\Gamma} = \begin{bmatrix} (1-\beta)\Gamma \\ (1-\beta)I \\ 0 \end{bmatrix}$$

$$\vec{C} = \begin{bmatrix} (1-\delta)C & 0 & \delta I \end{bmatrix},\ \bar{D} = \begin{bmatrix} D & 0 & 0 \end{bmatrix}$$

Then we consider the following event-triggering condition:

$$t_l \in \{t = t_k | \left\| \hat{y}_d^+(t_{k-1}) - y_d(t_k) \right\| > \sigma \left\| y_d(t_k) \right\| \} \tag{10}$$

where $\sigma \in \mathbb{R}^+$. The transmission instant t_l satisfies $t_{l+1} = t_l + nT$, $n \in \mathbb{N}_{>0}$, where $T \in [\epsilon, \infty)$ ($\epsilon > 0$) denotes the fixed sampling interval, i.e., $T = t_{k+1} - t_k$. Thus, it is obvious that the transmission time sequence $\{t_l\}$ is a subsequence of the sampling sequence $\{t_k\}$, i.e., $\{t_l\} \subseteq \{t_k\}$. To exclude the Zeno-like behavior, we prescribe $\tau_{miet} < t_{l+1} - t_l < \tau_{MATI}$, where τ_{miet} and τ_{MATI} mean the minimal inter-event time (MIET) and the maximum allowable transmission interval (MATI), respectively. Furthermore, one notices that $\hat{y}_d^+(t_k)$ depends on the following event-triggering generator.

$$\hat{y}_d^+(t_k) = \begin{cases} y_d(t_k) & when\ C(\hat{y}_d(t_k), y_d(t_k)) > 0 \\ \hat{y}_d^+(t_{k-1}) & when\ C(\hat{y}_d(t_k), y_d(t_k)) \le 0 \end{cases} \tag{11}$$

where $C(\hat{y}_d, y_d) = \left\| \hat{y}_d(t_k) - y_d(t_k) \right\| - \sigma \left\| y_d(t_k) \right\|$. In order to maintain the sustain action of the transmission, for the sake of exposition, we give that $\hat{y}_d(t_k) = \hat{y}_d^+(t_{k-1})$ by means of zero-order-hold devices.

For the k^{th} controller input, define the error variable as

$$e^+(t_k) = \hat{y}_d^+(t_k) - y_d(t_k), \tag{12}$$

In the case of $C(\hat{y}_d, y_d) > 0$, according to (11) and (12), the equality $y_d^+(t_k) = y_d(t_k)$ holds and thus $||e^+(t_k)|| = ||\hat{y}_d^+(t_k) - y_d(t_k)|| = 0$. In another case, one can obtain that $||e^+(t_k)|| = ||\hat{y}_d^+(t_k) - y_d(t_k)|| = ||\hat{y}_d^+(t_{k-1}) - y_d(t_k)|| \le \sigma ||y_d(t_k)||$. Hence, the inequality (13) always holds at the k^{th} step based on (11).

$$\left\| e^+(t_k) \right\| \le \sigma \left\| y_d(t_k) \right\| \tag{13}$$

Remark 2. Notice that, for continuous-time NCSs, event-triggering conditions need to be designed to avoid Zeno behavior (i.e., there exists infinite transmission in finite time interval) by a strictly positive lower bound of the inter-event times. However, for the discrete-time systems considered in this paper, Zeno behavior does not occur because transmission time instants can only occur at periodic sampling points[20,21]. Therefore, the possibility of Zeno behavior is excluded in the event-triggering conditions for the discrete-time systems.

Based on (9), (12) and (2), the dynamic event-triggered controller is formulated as

$$\begin{cases} x_c(t_{k+1}) = \Phi_c x_c(t_k) + \Gamma_c \vec{C}\vec{x}(t_k) + \Gamma_c e^+(t_k) \\ u(t_k) = C_c x_c(t_k) \end{cases} \tag{14}$$

Then, the closed-loop system in **FIGURE 1** is given by

$$\begin{cases} \xi(t_{k+1}) = \tilde{\Phi}\xi(t_k) + \tilde{E}e^+(t_k) \\ z(t_k) = \tilde{D}\xi(t_k) \end{cases} \tag{15}$$

where $\xi(t_k) = (\vec{x}(t_k), x_c(t_k))$ is the augmented form of the state, and the coefficient matrices are denoted by

$$\tilde{\Phi} = \begin{bmatrix} \vec{\Phi} & \vec{\Gamma}C_c \\ \Gamma_c\vec{C} & \Phi_c \end{bmatrix}, \tilde{E} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Gamma_c \end{bmatrix}, \tilde{D} = \begin{bmatrix} \bar{D} & 0 \end{bmatrix} = \begin{bmatrix} D & 0 & 0 & 0 \end{bmatrix}$$

Notice that the closed-loop system (15) consists stochastic parameters. In order to simplify the analysis, we consider the parameter separation for the system (15).

The coefficient matrices with random parameters in (9) can be separated to obtain the following results.

$$\begin{aligned}\vec{\Phi} &= \vec{\Phi}_1 + (\beta - \bar{\beta})\vec{\Phi}_2 + (\delta - \bar{\delta})\vec{\Phi}_3 \\ \vec{\Gamma} &= \vec{\Gamma}_1 + (\beta - \bar{\beta})\vec{\Gamma}_2 \\ \vec{C} &= \vec{C}_1 + (\delta - \bar{\delta})\vec{C}_2\end{aligned} \tag{16}$$

Thus, $\tilde{\Phi}$ in (15) can be described by

$$\tilde{\Phi} = \tilde{\Phi}_1 + (\beta - \bar{\beta})\tilde{\Phi}_2 + (\delta - \bar{\delta})\tilde{\Phi}_3 \tag{17}$$

where

$\tilde{\Phi}_1 = \begin{bmatrix} \vec{\Phi}_1 & \vec{\Gamma}_1 C_c \\ \Gamma_c \vec{C}_1 & \Phi_c \end{bmatrix}, \tilde{\Phi}_2 = \begin{bmatrix} \vec{\Phi}_2 & \vec{\Gamma}_2 C_c \\ 0 & 0 \end{bmatrix}, \tilde{\Phi}_3 = \begin{bmatrix} \vec{\Phi}_3 & 0 \\ \Gamma_c \vec{C}_2 & 0 \end{bmatrix}.$

$\vec{\Phi}_1 = \begin{bmatrix} \Phi & \bar{\beta}\Gamma & 0 \\ 0 & \bar{\beta} I & 0 \\ (1-\bar{\delta})C & 0 & \bar{\delta} I \end{bmatrix}, \vec{\Phi}_2 = \begin{bmatrix} 0 & \Gamma & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix},$

$\vec{\Phi}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C & 0 & I \end{bmatrix}, \vec{\Gamma}_1 = \begin{bmatrix} (1-\bar{\beta})\Gamma \\ (1-\bar{\beta}) I \\ 0 \end{bmatrix}, \vec{\Gamma}_2 = \begin{bmatrix} -\Gamma \\ -I \\ 0 \end{bmatrix},$

$\vec{C}_1 = \begin{bmatrix} (1-\bar{\delta})C & 0 & \bar{\delta} I \end{bmatrix}, \vec{C}_2 = \begin{bmatrix} -C & 0 & I \end{bmatrix}$

On the basis of the above analysis, the closed-loop system (15) can be further described as

$$\begin{cases} \xi(t_{k+1}) = \tilde{\Phi}_1 \xi(t_k) + (\beta - \bar{\beta})\tilde{\Phi}_2 \xi(t_k) \\ \qquad\qquad + (\delta - \bar{\delta})\tilde{\Phi}_3 \xi(t_k) + \tilde{E} e^{+}(t_k) \\ z(t_k) = \tilde{D}\xi(t_k) \end{cases} \tag{18}$$

Before proceeding the design of the dynamic output controller with ETM, the definition of exponentially mean-square stability and some lemmas are first presented in the following, which will be used throughout this paper.

Definition 1. (Exponential mean-square stability)

Given a scalar $\rho > 0$ and $\tau \in (0, 1)$, the NCS (18) is said to be exponential mean-square stable (EMSS) if the augmented state of the system satisfies the following inequality without other disturbances.

$$\mathbb{E}\left\{ \|\xi(t_k)\|^2 \right\} \le \rho \tau^k \mathbb{E}\left\{ \|\xi(t_0)\|^2 \right\} \tag{19}$$

for all $\xi(t_0) \in \mathbb{R}^{2n+m+p}$, $k \in \mathbb{Z}$.

The following lemmas are also given for later use.

3 | MAIN RESULTS

In this section, a theorem addressing stabilization of NCSs with ETM subject to bilateral packet dropouts will be presented.

Lemma 1. [22] For any matrices X and Y with suitable dimensions, there is an inequality as follows.

$$X^T Y + Y^T X \le X^T X + Y^T Y \tag{20}$$

Lemma 2. For symmetric matrix $P > 0$ and any matrices X and Y with suitable dimensions, there is an inequality as follows.

$$X^T P Y + Y^T P X \le X^T P X + Y^T P Y \tag{21}$$

Proof. Due to symmetric matrix $P > 0$, according to Cholesky Factorization in[23], there exists a lower triangular matrix L such that $P = LL^T$. Thus, by **Lemma 1**, we have

$$\begin{aligned} X^T P Y + Y^T P X &= (L^T X)^T L^T Y + (L^T Y)^T L^T X \\ &\le (L^T X)^T L^T X + (L^T Y)^T L^T Y \\ &= X^T P X + Y^T P Y \end{aligned} \tag{22}$$

□

Now, the following theorem is presented to provide an exponential stability condition in the mean-square sense for the NCS (18) with ETM subject to stochastic data losses.

Theorem 1. Consider the discrete system (18) with the dynamic event-triggered controller (14) subject to packet dropouts (3) and (6). The system (18) is EMSS, if there exist scalars $\varepsilon, \sigma \in \mathbb{R}_{\geq 0}$ and $\bar{\beta}, \bar{\delta} \in (0,1)$ and a symmetric matrix $P > 0$ such that

$$\begin{bmatrix} -(1-\varepsilon)P & * & * & * & * \\ \sqrt{2}P\tilde{\Phi}_1 & -P & * & * & * \\ \sqrt{\bar{\beta}(1-\bar{\beta})}P\tilde{\Phi}_2 & 0 & -P & * & * \\ \sqrt{\bar{\delta}(1-\bar{\delta})}P\tilde{\Phi}_3 & 0 & 0 & -P & * \\ \sqrt{2}\sigma PW & 0 & 0 & 0 & -P \end{bmatrix} < 0 \tag{23}$$

where $W = [\tilde{E}\vec{C}_1 + \sqrt{\bar{\delta}(1-\bar{\delta})}\tilde{E}\vec{C}_2, 0]$.

Proof. We now choose the quadratic Lyapunov functional as follows:

$$V(\xi(t_k)) = \xi^T(t_k)P\xi(t_k) \tag{24}$$

where $P > 0$ is a symmetric matrix. Then, based on (18) and (24), we have

$$\begin{aligned} V(\xi(t_{k+1})) = \xi^T(t_k)\tilde{\Phi}^T P\tilde{\Phi}\xi(t_k) + \xi^T(t_k)\tilde{\Phi}^T P\tilde{E}e^+(t_k) \\ +e^{+T}(t_k)\tilde{E}^T P\tilde{E}e^+(t_k) + e^{+T}(t_k)\tilde{E}^T P\tilde{\Phi}\xi(t_k) \end{aligned} \tag{25}$$

Based on (21), one can get

$$\begin{aligned} &[\tilde{\Phi}\xi(t_k)]^T P\tilde{E}e^+(t_k) + [\tilde{E}e^+(t_k)]^T P\tilde{\Phi}\xi(t_k) \\ &\leq \xi^T(t_k)\tilde{\Phi}^T P\tilde{\Phi}\xi(t_k) + e^{+T}(t_k)\tilde{E}^T P\tilde{E}e^+(t_k) \end{aligned} \tag{26}$$

Hence, (25) can be written as

$$\begin{aligned} &V(\xi(t_{k+1})) \leq \xi^T(t_k)\tilde{\Phi}^T P\tilde{\Phi}\xi(t_k) \\ &+\xi^T(t_k)\tilde{\Phi}^T P\tilde{\Phi}\xi(t_k) + 2e^{+T}(t_k)\tilde{E}^T P\tilde{E}e^+(t_k) \end{aligned} \tag{27}$$

By resorting to (13), the following inequality can be obtained.

$$\begin{aligned} \left\|e^+(t_k)\right\| &\leq \sigma\left\|y_d(t_k)\right\| = \sigma\left\|\vec{C}\vec{x}(t_k)\right\| \\ &= \sigma\left\|(\vec{C}_1 + \bar{\delta}(1-\bar{\delta})\vec{C}_2)\vec{x}(t_k)\right\| \end{aligned} \tag{28}$$

Then, we have

$$2e^{+T}(t_k)\tilde{E}^T P\tilde{E}e^+(t_k) \leq \vec{x}^T(t_k)[2\sigma^2\vec{C}^T\tilde{E}^T P\tilde{E}\vec{C}]\vec{x}(t_k) \tag{29}$$

Noting (5) and (8), we can get

$$\mathbb{E}\{V(\xi(t_{k+1}))|\xi(t_k)\} = \xi^T(t_k)\tilde{P}\xi(t_k) + \mathbb{E}\{\tilde{\Xi}\} \tag{30}$$

where $\tilde{P} := \tilde{\Phi}_1^T P\tilde{\Phi}_1 + \bar{\beta}(1-\bar{\beta})\tilde{\Phi}_2^T P\tilde{\Phi}_2 + \bar{\delta}(1-\bar{\delta})\tilde{\Phi}_3^T P\tilde{\Phi}_3$, $\tilde{\Xi} := \xi^T(t_k)\tilde{\Phi}_1^T P\tilde{E}e^+(t_k) + e^{+T}(t_k)\tilde{E}^T P\tilde{\Phi}_1\xi(t_k) + e^{+T}(t_k)\tilde{E}^T P\tilde{E}e^+(t_k)$, $W = [\tilde{E}\vec{C}_1 + \sqrt{\bar{\delta}(1-\bar{\delta})}\tilde{E}\vec{C}_2, 0]$,

$$\begin{aligned} \mathbb{E}\{\tilde{\Xi}\} &\leq \mathbb{E}\{\xi^T(k)\tilde{\Phi}_1^T P\tilde{\Phi}_1\xi(t_k) + 2e^{+T}(t_k)\tilde{E}^T P\tilde{E}e^+(t_k)\} \\ &\leq \mathbb{E}\{\xi^T(t_k)\tilde{\Phi}_1^T P\tilde{\Phi}_1\xi(t_k) + 2\sigma^2\vec{x}^T(t_k)\vec{C}^T\tilde{E}^T P\tilde{E}\vec{C}\vec{x}(t_k)\} \\ &= \xi^T(t_k)\tilde{\Phi}_1^T P\tilde{\Phi}_1\xi(t_k) + 2\sigma^2\vec{x}^T(t_k)[\vec{C}_1^T\tilde{E}^T P\tilde{E}\vec{C}_1 \\ &\quad +\bar{\delta}(1-\bar{\delta})\vec{C}_2^T\tilde{E}^T P\tilde{E}\vec{C}_2]\vec{x}(t_k) \\ &= \xi^T(t_k)(\tilde{\Phi}_1^T P\tilde{\Phi}_1 + 2\sigma^2W^T PW)\xi(t_k) \end{aligned}$$

In view of the scalar $\varepsilon > 0$, we have

$$\begin{aligned} &\mathbb{E}\{V(\xi(t_{k+1}))|\xi(t_k)\} - (1-\varepsilon)V(\xi(t_k)) \\ &\leq \xi^T(t_k)[\tilde{\Phi}_1^T P\tilde{\Phi}_1 + \tilde{P} + 2\sigma^2W^T PW - (1-\varepsilon)P]\xi(t_k) \end{aligned} \tag{31}$$

Furthermore, the condition (23) is given as follows

$$\mathbb{E}\{V(\xi(t_{k+1}))|\xi(t_k)\} - (1-\varepsilon)V(\xi(t_k)) < 0 \tag{32}$$

According to (31) and (32), it yields the following inequality

$$\mathbb{E}\{V(\xi(k+1))|\xi(t_k)\} - V(\xi(t_k)) \leq -\varepsilon V(\xi(t_k)) < 0 \tag{33}$$

Let $\Lambda := 2\tilde{\Phi}_1^T P\tilde{\Phi}_1 + \bar{\beta}(1-\bar{\beta})\tilde{\Phi}_2^T P\tilde{\Phi}_2 + \bar{\delta}(1-\bar{\delta})\tilde{\Phi}_3^T P\tilde{\Phi}_3 + 2\sigma^2 W^T PW - P$, and then it has $\Lambda \leq -\varepsilon P < 0$.

Thus, we know that

$$\begin{aligned}
&\mathbb{E}\{V(\xi(t_{k+1}))|\xi(t_k)\} - V(\xi(t_k)) \leq \xi^T(t_k)\Lambda\xi(t_k) \\
&\leq -\lambda_{min}(-\Lambda)\xi^T(t_k)\xi(t_k) \leq -\rho\xi^T(t_k)\xi(t_k) \\
&< -\tfrac{\rho}{\zeta}V(\xi(t_k)) := -\theta V(\xi(t_k))
\end{aligned} \tag{34}$$

where $\rho \in [0, min\{-\lambda_{min}(-\Lambda), \zeta\}]$, $\zeta := \lambda_{max}(P)$.

After completing the k^{th} iterative operation, it obtains

$$\mathbb{E}\{V(\xi(t_{k+1}))|\xi(t_k)\} \leq (1-\theta)^k V(\xi(t_0)) \tag{35}$$

By **Definition 1**, we can conclude that the closed-loop system (18) is EMSS.

By Schur-Complement, reorganizing condition (32) yields

$$\begin{bmatrix} 2\sigma^2 W^T PW - (1-\varepsilon)P & * & * & * \\ \sqrt{2}\tilde{\Phi}_1 & -P^{-1} & * & * \\ \sqrt{\bar{\beta}(1-\bar{\beta})}\tilde{\Phi}_2 & 0 & -P^{-1} & * \\ \sqrt{\bar{\delta}(1-\bar{\delta})}\tilde{\Phi}_3 & 0 & 0 & -P^{-1} \end{bmatrix} < 0 \tag{36}$$

Using Schur-Complement again, (36) is equivalent to

$$\begin{bmatrix} -(1-\varepsilon)P & * & * & * & * \\ \sqrt{2}\tilde{\Phi}_1 & -P^{-1} & * & * & * \\ \sqrt{\bar{\beta}(1-\bar{\beta})}\tilde{\Phi}_2 & 0 & -P^{-1} & * & * \\ \sqrt{\bar{\delta}(1-\bar{\delta})}\tilde{\Phi}_3 & 0 & 0 & -P^{-1} & * \\ \sqrt{2}\sigma W & 0 & 0 & 0 & -P^{-1} \end{bmatrix} < 0 \tag{37}$$

Pre-multilying/post-multiplying by $diag\{I, P, P, P, P\}$ and its transpose gives

$$\begin{bmatrix} -(1-\varepsilon)P & * & * & * & * \\ \sqrt{2}P\tilde{\Phi}_1 & -P & * & * & * \\ \sqrt{\bar{\beta}(1-\bar{\beta})}P\tilde{\Phi}_2 & 0 & -P & * & * \\ \sqrt{\bar{\delta}(1-\bar{\delta})}P\tilde{\Phi}_3 & 0 & 0 & -P & * \\ \sqrt{2}\sigma PW & 0 & 0 & 0 & -P \end{bmatrix} < 0 \tag{38}$$

Therefore, it is not hard to conclude that if there exists a symmetric matrix $P > 0$ such that (23) holds, then we have (35), i.e. the closed-loop system (18) is EMSS. This completes the proof. □

Remark 3. From the above proof process, when the (32) is satisfied, the system is exponentially stable. Since there are coupled variables in constraint (32), we cannot solve controller parameters directly by the linear matrix inequality (LMI) toolbox. Therefore, the transformation of the nonconvex inequality into the convex matrix inequality is firstly considered by the methods of Schur-complement and matrix operation.

Remark 4. Note that in the **Theorem 1**, $\tilde{\Phi}_i$ $(i = 1, 2, 3)$ and W contain unknown parameters of the controller from (17). In this case, the constraint (23) is a nonlinear matrix inequality with the character of bilinear terms due to the terms $\tilde{\Phi}_1^T P$, $\tilde{\Phi}_2^T P$, $\tilde{\Phi}_3^T P$, and $W^T P$. In general, we can solve the controller parameters of a linear matrix inequality via formulating the subsequent optimization problem where $q = 2n + m + p$.

$$\begin{aligned} &\min_{\lambda\in\mathbb{R},\Phi_i,P\in\mathbb{R}^{q\times q},W\in\mathbb{R}^{q\times(n+m+p)}} t \\ &s.t. \quad \Xi - \Upsilon \leq tI, \\ &\qquad\ \varepsilon > 0, P > 0 \end{aligned} \tag{39}$$

where $\Xi(\Phi_c, \Gamma_c, C_c, \varepsilon, P) = 2\tilde{\Phi}_1^T P\tilde{\Phi}_1 + \bar{\beta}(1-\bar{\beta})\tilde{\Phi}_2^T P\tilde{\Phi}_2 + \bar{\delta}(1-\bar{\delta})\tilde{\Phi}_3^T P\tilde{\Phi}_3 + 2\sigma^2 W^T PW$, $\Upsilon(\Phi_c, \Gamma_c, C_c, \varepsilon, P) = \varepsilon P + P$, and the terms $\tilde{\Phi}_3^T P$ and $W^T P$ contain coupled variables difficult to decouple variables by matrix transformations. In this case, the optimization problem (39) can be split into two optimization sub-problems with linear inequality constraints, and then the optimal solution is solved in terms of an iterative algorithm. Inspired by the Rank Minimization Problems [24] and LMI gridding approach [25], the iterative procedure is formulated in **Algorithm 1**. The optimal solution of the dynamic controller parameters can be obtained via this iterative algorithm.

Algorithm 1 The iterative matrix inequalities procedure.

Initialize: $P = P_0$, $i = 1$,
the initial scalars σ, β, δ, $\varepsilon = \varepsilon_0$,
the initial matrices $\Phi_c = \Phi_{c0}$, $\Gamma_c = \Gamma_{c0}$, $C_c = C_{c0}$.
while $t_{min} \geq 0$ and $i \leq M$ **do**
  $\Xi(\Phi_c, \Gamma_c, C_c, \varepsilon) = 2\tilde{\Phi}_1^T P\tilde{\Phi}_1 + \bar{\beta}(1-\bar{\beta})\tilde{\Phi}_2^T P\tilde{\Phi}_2 + \bar{\delta}(1-\bar{\delta})\tilde{\Phi}_3^T P\tilde{\Phi}_3 + 2\sigma^2 W^T PW$
  $\Upsilon(\Phi_c, \Gamma_c, C_c, \varepsilon) = \varepsilon P + P$
  Establish an auxiliary convex optimization problem
  min t s.t. $\Xi(\Phi_c, \Gamma_c, C_c, \varepsilon) - \Upsilon(\Phi_c, \Gamma_c, C_c, \varepsilon) \leq tI$
  Solve for the global optimal solution $(\Phi_c, \Gamma_c, C_c, \varepsilon)$ and t_{min}
  Update t_{min}, Φ_c, Γ_c, C_c, ε
  $i = i + 1$
  while $t_{min} \geq 0$ **do**
    if $\varepsilon < 0$ **then**
      $\varepsilon = -\varepsilon$
    end if
    $\tilde{\Xi}(P) = 2\tilde{\Phi}_1^T P\tilde{\Phi}_1 + \bar{\beta}(1-\bar{\beta})\tilde{\Phi}_2^T P\tilde{\Phi}_2 + \bar{\delta}(1-\bar{\delta})\tilde{\Phi}_3^T P\tilde{\Phi}_3 + 2\sigma^2 W^T PW$
    $\tilde{\Upsilon}(P) = \varepsilon P + P$
    Establish an auxiliary optimization problem
    min t s.t. $\Xi(P) - \Upsilon(P) \leq tI$
    Solve for the global optimal solution P, t_{min}
    Update t_{min}, P.
    $i = i + 1$
  end while
end while
return Φ_c, Γ_c, C_c.

4 | ILLUSTRATIVE EXAMPLES

In this section, an illustrative numerical example will be presented to demonstrate the effectiveness of the developed method.

Consider the discrete-time system

$$\begin{aligned} x(t_{k+1}) &= \begin{bmatrix} 1.0018 & 0.01 \\ 0.036 & -0.18 \end{bmatrix} x(t_k) + \begin{bmatrix} 0.8 \\ -0.5 \end{bmatrix} u_d(t_k) \\ y(t_k) &= \begin{bmatrix} 1.6 & -1.2 \end{bmatrix} x(t_k) \end{aligned} \tag{40}$$

In the simulation, we set the average packet dropout rates $\bar{\beta} = 0.05$, $\bar{\delta} = 0.05$ and the threshold parameter of event-triggering condition $\sigma = 0.1$. According to Algorithm 1, the optimization problem (39) is solved and the optimal controller parameters are obtained as $\varepsilon = 0.5375$, $\Phi_c = \begin{bmatrix} -0.2469 & 0.6091 \\ 0.1621 & -0.2640 \end{bmatrix}$, $\Gamma_c = \begin{bmatrix} -0.9374 \\ -0.8342 \end{bmatrix}$ and $C_c = \begin{bmatrix} -0.2167 & 0.6387 \end{bmatrix}$. The model of the closed-loop networked system with the ETM is set up by the Simulink, and the packet loss sequences are generated at randomly according to the given average packet loss rates. The simulation time-stepped size is set as $T = 0.01s$.

The obtained state responses for the system are shown in **FIGURE 2** . It can be seen from the curves that the system (40) is stabilized under the designed controller. The corresponding sequences of the data packet dropouts in the measurement channel and control channel are depicted in **FIGURE 3** . **FIGURE 4** shows the output responses of the closed-loop system with event-triggered controller subject to packet dropouts. The current output of ETM $\hat{y}^+(t_k)$ takes the value of the previous one $\hat{y}^+(t_{k-1})$ when there are merely subtle changes between them. The packets via the measured channels are updated according to the event-triggering condition, and the updated transmission signals serve as the inputs of the controller.

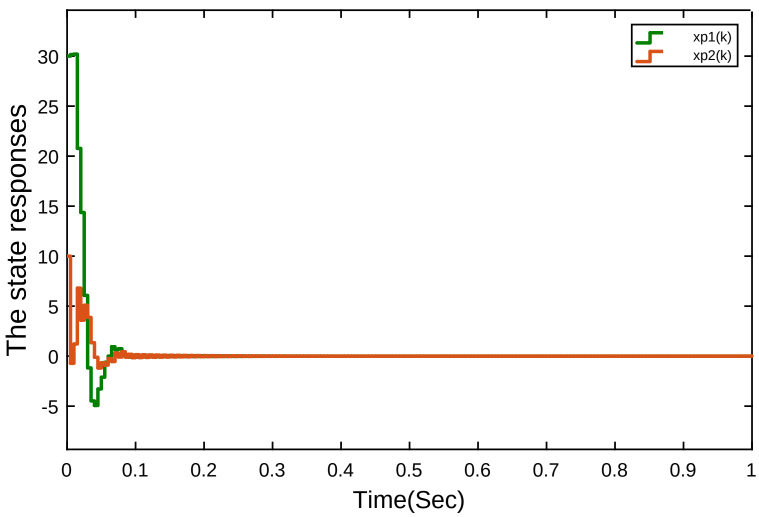

FIGURE 2 The state responses for closed-loop system with ETM and packet dropouts.

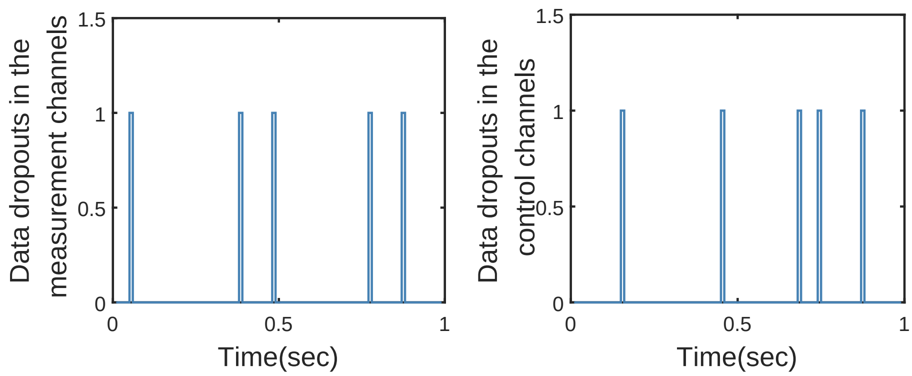

FIGURE 3 The data packet dropouts in the measurement channels and control channels with dropout rate: $\delta_1 = \delta_2 = 0.05$.

In order to illustrate the impact of packet dropouts on system stability, we consider the redesign of the controller by adjusting the packet loss rate. By solving the optimization (39) with $\bar{\beta} = 0.05, 0.1, 0.25, 0.28$, and $\bar{\delta} = 0.05, 0.1, 0.25, 0.28$, respectively,

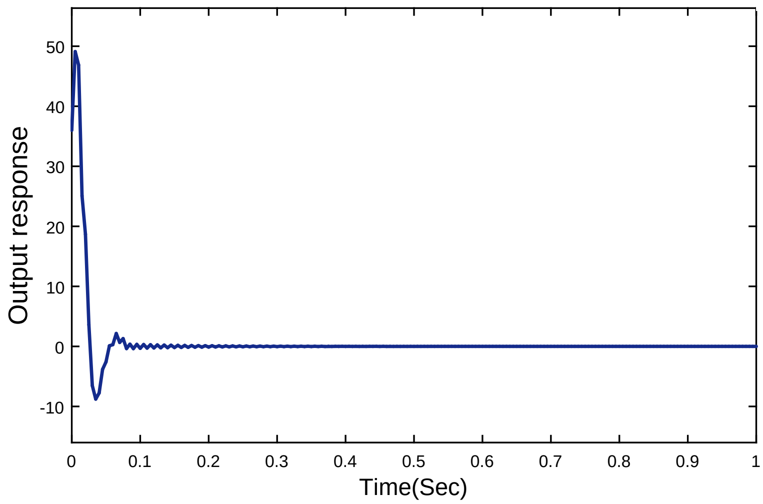

FIGURE 4 Trajectories of the output responses of the closed-loop system.

the corresponding solutions are obtained and shown in **TABLE 1** . When $\bar{\beta} = 0.28$ and $\bar{\delta} = 0.28$, the optimization (39) does not have any feasible solution. Thus, it can be seen from the results that packet dropouts phenomenon has significant impacts on the stabilization of the NCS. Specifically, the higher the probability of data losses in the network channels, the worse the stability of the system, and more difficult to find feasible solutions for the optimization of the design controller. Furthermore, the feasible controller parameters cannot be available when the packet loss rate is above a certain threshold.

TABLE 1 The results of the optimization by giving different parameters.

$\bar{\beta}$	$\bar{\delta}$	$-\varepsilon$
0.05	0.05	-0.5375
0.1	0.1	-0.0465
0.25	0.25	-0.0137
0.28	0.28	0.0013

5 | CONCLUSION

In this paper, the event-triggered control problem of NCSs subject to packet dropouts in bilateral channels is studied. By synthesizing event-triggering condition and stochastic data losses behavior, we firstly model a closed-loop stochastic parameter system. Furthermore, the optimal controller parameters are solved by means of an improved iterative algorithm such that the resulting closed-loop system is exponentially stable in the mean-square sense. In fact, the packet loss rates of the bilateral unreliable networks in this paper are required to be bounded while maintaining the desired stability and performance properties. The presented theory is illustrated by a numerical example, which showed that a dynamic output feedback controller and the ETM can be systematically designed to guarantee desired performance.

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Conflict of interest

This work does not have any conflicts of interest.

References

1. Goodwin GC, Haimovich H, Quevedo DE, Welsh JS. A moving horizon approach to Networked Control system design. *IEEE Transactions on Automatic Control.* 2004;49(9):1427–1445.

2. Zhang W, Branicky MS, Phillips SM. Stability of networked control systems. *IEEE Control Systems Magazine.* 2001;21(1):84–99.

3. Y Wang, B Niu, H Wang, N Alotaibi, E Abozinadah. Neural network-based adaptive tracking control for switched nonlinear systems with prescribed performance: an average dwell time switching approach. *Neurocomputing.* 2020;435(7):295–306.

4. Lunze J, Lehmann D. A state-feedback approach to event-based control. *Automatica.* 2010;46(1):211–215.

5. Huo X, Karimi HR, Zhao X, Wang B, Zong G. Adaptive-Critic Design for Decentralized Event-Triggered Control of Constrained Nonlinear Interconnected Systems Within an Identifier-Critic Framework.. *IEEE Transactions on Cybernetics.* 2020;:303–321.

6. Aström K. Event based control.. *Analysis and Design of Nonlinear Control Systems.* 2008;:127–147.

7. Wu J, Chen T. Design of Networked Control Systems With Packet Dropouts. *IEEE Transactions on Automatic Control.* 2007;52(7):1314–1319.

8. Gao H, Wang C. Delay-dependent robust H_∞ and L_2-L_∞ filtering for a class of uncertain nonlinear time-delay systems. *IEEE Transactions on Automatic Control.* 2003;48(9):1661–1666.

9. Li F, Shi P, Wang X, Karimi HR. Robust H_∞ Filtering for Networked Control Systems with Markovian Jumps and Packet Dropouts. *Modeling, Identification and Control.* 2014;35(3):159–168.

10. Wang L, Wang Z, Han Q, Wei G. Synchronization Control for a Class of Discrete-Time Dynamical Networks With Packet Dropouts: A Coding¨CDecoding-Based Approach. *IEEE Transactions on Cybernetics.* 2018;48(8):2437–2448.

11. Li J, Wu CY, Su Q. Robust fault detection filter design for interconnected systems subject to packet dropouts and structure changes. *IET Control Theory and Applications.* 2018;12(3):368–376.

12. Dolk VS, Heemels WPMH (Maurice). Event-triggered control systems under packet losses. *Automatica.* 2017;80:143–155.

13. Dolk VS, Borgers DP, Heemels WPMH. Output-based and Decentralized Dynamic Event-Triggered Control with Guaranteed L_p-Gain Performance and Zeno-Freeness. *IEEE Transactions on Automatic Control.* 2016;62(1):34–49.

14. Shen M, Yan S, Sun Y, Zhang G. Nonfragile H_∞ output feedback control of linear systems with an event-triggered scheme against unreliable communication links. *ISA transactions.* 2019;(84):96–103.

15. Molin A, Hirche S. Suboptimal event-triggered control for networked control systems. *Journal of Applied Mathematics and Mechanics.* 2014;94(4):277–289.

16. Rabi M, Johansson KH. Scheduling packets for Event-triggered control. *Control Conference. IEEE.* 2012;:3779–3784.

17. Ma L, Xu N, Zhao X, Zong G, Huo X. Small-Gain Technique-Based Adaptive Neural Output-Feedback Fault-Tolerant Control of Switched Nonlinear Systems With Unmodeled Dynamics. *IEEE Transactions on Systems, Man, and Cybernetics: Systems.* 2020;PP(99):1–12.

18. Ma L, Zong G, Zhao X, Huo X. Observed-based Adaptive Finite-Time Tracking Control for a Class of Nonstrict-Feedback Nonlinear Systems with Input Saturation. *Journal of the Franklin Institute.* 2020;357(16):11518–11544.

19. Cheng P, He S, Cheng J, Luan X, Liu F. Asynchronous Output Feedback Control for a Class of Conic-Type Nonlinear Hidden Markov Jump Systems Within a Finite-Time Interval. *IEEE Transactions on Systems, Man, and Cybernetics: Systems.* 2020;(99):1–8.

20. Hu S, Yue D, Yin X, Xie X, Ma Y. Adaptive event-triggered control for nonlinear discrete-time systems. *IEEE Transactions on Cybernetics.* 2016;(26):4104–4125.

21. Kooshkbaghi M, Marquez HJ. Event-Triggered Discrete-Time Cubature Kalman Filter for Nonlinear Dynamical Systems With Packet Dropout. *IEEE Transactions on Automatic Control.* 2020;65(5):2278–2285.

22. Xia C, Fei H. Observer-based event-triggered control for certain and uncertain linear systems. *Ima Journal of Mathematical Control and Information.* 2013;30(4):527–542.

23. Tsyganova JV., Kulikova MV. SVD-based Kalman Filter Derivative Computation. *IEEE Transactions on Automatic Control.* 2017;62(9):4869–4875.

24. Sun C, Dai R. An iterative approach to Rank Minimization Problems. *2015 IEEE Control and Decision Conference.* 2016;.

25. Sala A. Computer control under time-varying sampling period: An LMI gridding approach. *Automatica.* 2005;41(12):2077–2082.

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