Model Predictive Control of Hydraulic Drive Unit Considering Input Delay

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Abstract Hydraulic drive unit (HDU) is a typical actuator, but the characteristic of input delay hinders the application of many advanced control methods in HDU. Firstly, in this paper, a mathematical model of HDU with input delay is established and the parameters are identified. Then, aiming at the input delay problem in HDU, a Smith estimated compensation model predictive control (SECMPC) strategy is proposed. On the one hand, the input delay state equation is employed to be a mathematical pattern for the state observation and predictive model. However, the combination between model predictive control (MPC) and Smith estimated compensation (SEC) is realized, the system state at $k+d$ (d is the time delay coefficient) time is estimated in advance at k time to compensate the delay of the state. And then the prediction model based on input delay state equation is used for model prediction and rolling optimization. Thus the delay system which is unstable is promoted to a stable system without delay, and the effectiveness of SECMPC is proved with the HDU experiment and simulation. The SECMPC have some guiding significance for the control of systems with input delay.

Keywords: hydraulic drive unit, model predictive control, Smith estimated compensation, time delay.

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1. Introduction

Hydraulic drive unit (HDU) has the characteristics of high power density, high precision and high output power, which is suitable for working under heavy load. However, it has a series of common problems in the hydraulic system, for instance, time-varying parameters, strong nonlinearity, input delay and constraints, which make it difficult to achieve satisfactory control effect [1,2]. Some scholars have investigated the time delay problem of hydraulic system. In [3], a kind of compensator that can adapt to the time series is designed to compensate the delay for the actuator systems of hydraulic servo, which improves the hydraulic actuator control accuracy in the simulations of real-time hybrid. In [4], a robust controller of H∞ based on an observer is designed for the uncertain nonlinear discrete time-delay systems. The asymptotic stabilization of uncertain nonlinear systems with time delays is guaranteed by establishing delay parameters given upper and lower boundaries into the system matrix. And it is verified by the electro-hydraulic servo system. In [5], the control systems of electro-hydraulic servo valve with the limit cycle transmission delay induced by friction is investigated, a model is designed by considering the delay time of the accelerated limit cycle generation in transmission lines, and the limit cycle caused by friction can be predicted and eliminated effectively by designing compensator.

MPC has been extensively explored and employed in the industrial process control area for the first time [6]. The computer technology developed rapidly, under this background, MPC is gradually exploited into the servo systems, such as hydraulic servo systems. Marusak et al. [7] proposed and designed a kind of analytic controller of MPC with output and input constraints for the hydraulic drive units force control. The results of experiment revealed that this controller is effective. Peng et al. [8] put forward a controller of MPC on the basis of the neural network optimization in order to control the injection molding machine hydraulic system. The results of simulation suggested that this controller possesses a high control accuracy. Tao et al. [9] put forward an uniquely designed controller of MPC to ensure that the output of the pump-controlled asymmetric cylinder systems had no overshooting. Wang et al. [10] presented a robust MPC for the HDU of the active suspension system, the handling stability together with ride comfort of this system were greatly ameliorated. Gu et al. [11] proposed a kind of output feedback MPC, which has the hydraulic system state observer, and greatly ameliorates the anti-interference ability of this system. Yuan et al. [12,13] applied the hybrid controller constructed from of MPC and PIC for a hydraulic system, and had better performance compared with PIC and MPC. Lin et al. [14] presented a one-step strategy of MPC on the basis of neural network for hydraulic systems of die forging hydraulic press machine. In [15] the rational engine power distribution between hydraulic and mechanical paths was realized with the design of steadystate feed forward and dynamic MPC feedback controller (FMPC). Mohamed et al. [16,17] presented the employment of the MPC in the actual industrial electro-hydraulic servo system to realize the force control with high-precision. In [18], a control program having the multi-scale online estimator of electro-hydraulic actuator for the legged robot and based on a linear time-varying MPC is presented. The outcomes of experiment indicate that this control scheme reveals adaptive robustness and strong trajectory tracking property.

At present, there are some researches on MPC for state delay systems. In [19], an improved MPC algorithm is proposed for the system having input constraints and uncertain state delay through supposing the uncertain delay parameters uncertainty given upper and lower boundaries as polytopic, the algorithm property is verified via a numerical example. In [20], iterative distributed model predictive control (DMPC) for the nonlinear large-scale systems with time-delay and asynchronous state feedback has always been the focus of scholars. Based on the hypothesis that the maximum determination delay and the time interval of two continuous state determinations have upper bounds, the iterative DMPC program was designed with Lyapunov control technique for the nonlinear systems. In [21], a networked automatic generation control (AGC) system is proposed based on delay MPC for random communication delay problem, and the power system delay dynamic state equation that considered communication delay is used as the MPC predictive model. There are few researches on MPC for input delay systems. In [22], the predicted ship motion is input to the shipborne stabilized platform driven by the electro hydraulic to compensate the time delay of the stabilized platform, which improves the improves the stabilization effect of the stabilized platform. In [23], the fuzzy pattern predictive control synthesis of the power buffer controlled by network utilized for the dc microgrid (MG) dynamic stabilization is explored. The delay of network between actuator and controller (input delay) and between controller and sensor (output delay) are calculated and compensated by the state equation without delay and the network delay compensator (NDC), which improves the effectiveness of the controller and robustness.

For the input delay system, since the system has d-step delay, the control increment $\Delta u(k-d)$ at *k*-d time corresponds to state $\hat{x}(k)$ of *k* time. If the traditional MPC is used to control the input delay system, during state observation and feedback, the input increment $\Delta u(k-d)$ and state $\hat{x}(k)$ are employed directly to acquire state $\hat{x}(k+1)$, which makes the state observation inaccurate. Moreover, as the system has d-step delay, the reference trajectory *r*(*k*) corresponds to the state $\hat{x}(k+d)$ in the future. If traditional MPC is adopted to control the input delay system, the model prediction and rolling optimization will be conducted according to state $\hat{x}(k+1)$, which makes the increment of control $\Delta u(k)$ obtained by rolling optimization inaccurate and ultimately leads to the control system unstable. Therefore, the problem can be solved after presenting the Smith estimated compensation MPC (SECMPC) strategy, in this paper. On the one hand, the input delay state equation is used as the system mathematical model for the predictive model and state observation. On the other hand, the combination between MPC and SEC is achieved, the system state at *k*+*d* (*d* is the time delay coefficient) time is estimated in advance at *k* time, to compensate the delay of the state. Then the prediction model based on input delay state equation is used for model prediction and rolling optimization. So that the input delay system which is unstable is promoted to a stable system without delay.

The arrangement of major content for this work is revealed as below. In the Section 2, the HDU mathematical pattern is constructed and the system parameters are identified. In the Section 3, the conventional MPC is presented. While in the Section 4, the SECMPC mentioned in our work is presented, and it is verified through the simulation and experiment of HDU. The Section 5 is the conclusion.

2. Mathematical Simulation Model of HDU

2.1 The Theoretical Model

Hydraulic driving unit (HDU) is a module including the system of proportional valve and the asymmetric system of hydraulic cylinder controlled via valve. The system of proportional valve is generally classified to be the proportional module, as in Eq. (1) below.

$$u=K_u x_v \tag{1}$$

in which u represents the control signal of proportional valve; x_v is spool displacement; K_u is proportional valve gain.

Without considering external interference and elastic load, schematic design of the asymmetric system of hydraulic cylinder controlled by valve is reflected in the Fig. 1.

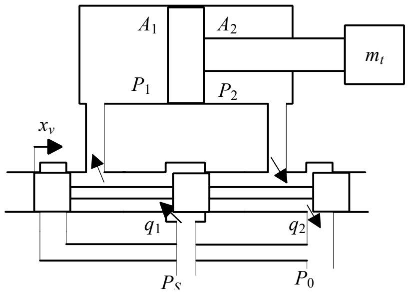

Fig. 1. Schematic design of the asymmetric hydraulic cylinder controlled by valve

The mathematical pattern of the asymmetric hydraulic cylinder controlled via valve [27] is established, as shown in Eq. (2) and the corresponding variables are revealed in the Table 1.

$$\begin{cases} q_L = A_p \dfrac{\mathrm{d}x_p}{\mathrm{d}t} + C_{tp} p_L + \dfrac{V_t}{4\beta_e} \dfrac{\mathrm{d}p_L}{\mathrm{d}t} \\ q_L = K_q x_v + K_c p_L \\ A_p p_L = m_t \dfrac{\mathrm{d}^2 x_p}{\mathrm{d}^2 t} + B_p \dfrac{\mathrm{d}x_p}{\mathrm{d}t} \end{cases} \tag{2}$$

where $q_L=(q_1+q_2)/2$; $p_L=p_1-n\cdot p_2$; $n=A_2/A_1$.

Table 1 The variables of HDU

Symbol	Meaning
A_1	The nonrod cavity area
A_2	The rod cavity area
q_1	The flow of left chamber
q_2	The flow of rigth chamber
p_1	The pressure of left chamber
p_2	The pressure of rigth chamber
x_p	The piston displacement of cylinder
K_q	The flow gain of spool valve
K_c	The coefficient between flow and pressure
C_{tp}	The leakage coefficient of cylinder
V_t	The volume of cylinder chamber
β_e	The effective bulk modulus
B_p	The damping coefficient

The HDU transfer function is got by combining Eq. (1) and Eq. (2) and carrying out Laplace transform

$$\frac{X_p}{U} = \frac{K_v}{s\left(\dfrac{s^2}{\omega_h^2} + \dfrac{2\xi_h}{\omega_h}s + 1\right)} \tag{3}$$

where K_v is the open-loop amplification factor, ω_h represents the hydraulic inherent frequency, while ζ_h represents hydraulic damping.

In a electro-hydraulic servosystem, due to the influence of the actuator, controller and sensor, there must be time delay in the system. Therefore, Eq. (3) can be further rewritten into the transfer function with time delay, as shown in Eq. (4)

$$\frac{X_p}{U} = \frac{K_v}{s\left(\dfrac{s^2}{\omega_h^2} + \dfrac{2\xi_h}{\omega_h}s + 1\right)} e^{-\tau s} \tag{4}$$

in which τ represents a delay time.

Set $x_1=x_p$, $x_2=\dot{x}_1$, $x_3=\dot{x}_2$, and convert the transfer Eq. (4) into the state equation, as shown in Eq. (5)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_h^2 & -2\xi_h\omega_h \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_v\omega_h^2 \end{bmatrix} u(t-\tau) \tag{5}$$

According to the principle of the conventional spool valve control system, it can be found that the metering flow of out orifice Q_b and in orifice Q_a are controlled through the sliding spool s at the same time, and the Q_b value and Q_a value are decided via the sliding spool s displacement. Nevertheless, the speed v of the piston rod of cylinder is decided via the metering flow of orifice Q_a, as a result the reduction of metering out orifice caused via the motion of sliding spool is unnecessary on account of it resulted in the repeated losses of throttling.

2.2 Model Parameters Identification

In the HDU system, there are some nonlinear factors, such as valve pressure and flow characteristics and oil compression characteristics. Therefore, it is necessary to identify the model parameters and the experimental system is shown in Fig. 2. In the system of HDU, there exist some nonlinear elements, for instance, the compression features of oil, the flow features along with the valve pressure. As a result, the identification of the pattern parameters is essential and the system of experiment is illustrated in the Fig. 2.

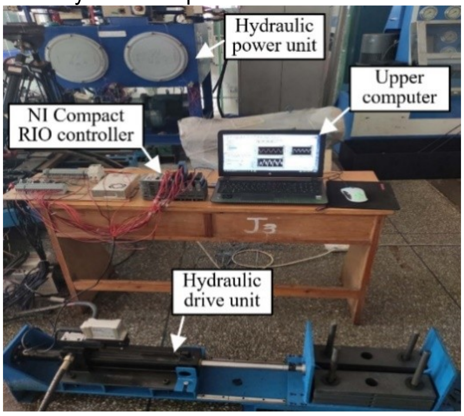

Fig. 2. The experiment system of HDU

The parameters for the system of experiment are illustrated in the Table 2.

Table 2. The parameters associated with the system of research

Parameter	Value
The oil pressure P_s/bar	100
The total load mass m_t/Kg	40
The diameter of piston /mm	40
The diameter of rod /mm	25
The stroke of cylinder /mm	250
The type of displacement sensor	MTS EHM0290M
The type of proportional valve	Atos DLHZ0-TEB
The signal of proportional valve /V	-10~+10
The type of controller	NI Compact RIO 9033
The sampling period /ms	2

A frequency sweep signal with frequency range of 0-2Hz and amplitude of 2.0V is selected as the control voltage The sweep signal with 2.0V amplitude and the frequency ranged from 0 to 2Hz is chose to be a control voltage signal. The voltage of spool feedback and input voltage are reflected in the Figure 3. The system identification toolbox of Matlab is employed for the identification of the pattern parameters. The displacement of experiment and identification pattern are presented in the Fig. 4.

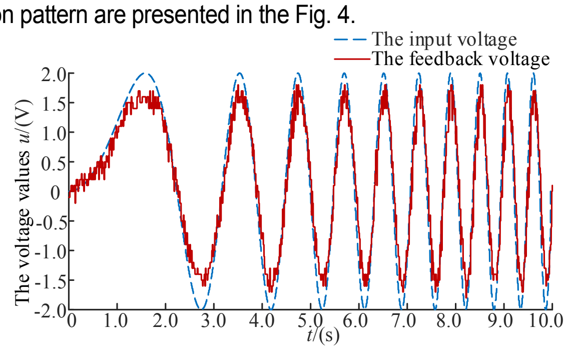

Fig.3 The frequency sweep control signal of HDU

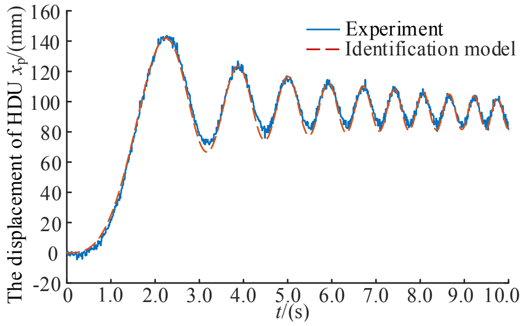

Fig.4 The displacement of identification pattern and the HDU in experiment

The error of identification is 1.51 percent based on Eq. (6).

$$v = \frac{\int_{t_0}^{t_s} \left|\hat{\eta}(t) - \eta(t)\right|}{\int_{t_0}^{t_s} \left|\eta(t)\right|} \tag{6}$$

in which, v represents an error of identification, t_0 represents the start time of identification, t_s represents the end time of identification, $\eta(t)$ represents the displacement of experiment at t time, $\hat{\eta}(t)$ represents the pattern displacement of identification at t time.

The identification result is shown in Eq. (7).

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -10810.81 & -102.75 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 64.25 \end{bmatrix} u(t-0.002\times 5) \tag{7}$$

The time delay coefficient d=5 and delay time τ=10ms can be acquired via Eq. (7). The equation of Eq. (7) in discrete state, as shown in Eq. (8), can be used in the state equation MPC of the HDU.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.002 & 0 \\ 0 & 0.9799 & 0.0018 \\ 0 & -19.4042 & 0.7954 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0001 \\ 0.1153 \end{bmatrix} u(k-5) \tag{8}$$

3. Traditional MPC

The discrete state equation for the system of single-input single-output (SISO) in the absence of delay is as follows:

$$\begin{cases} \boldsymbol{x}_m(k+1) = \boldsymbol{A}_m \boldsymbol{x}_m(k) + \boldsymbol{B}_m u(k) \\ y(k) = \boldsymbol{C}_m \boldsymbol{x}_m(k) \end{cases} \tag{9}$$

At the aim of eliminating the error of the actual pattern and prediction model, the space pattern in augmented state is acquired with the introduction of the integral behavior, as shown below

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\Delta u(k) \\ y(k) = \boldsymbol{C}\boldsymbol{x}(k) \end{cases} \tag{10}$$

where $\boldsymbol{x}(k) = \left[\Delta \boldsymbol{x}_m(k)^{\mathrm{T}} \; y(k)\right]^{\mathrm{T}}$, $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_m & 0 \\ \boldsymbol{C}_m \boldsymbol{A}_m & \boldsymbol{I} \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_m \\ \boldsymbol{C}_m \boldsymbol{B}_m \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} 0 & \boldsymbol{I} \end{bmatrix}$.

The MPC of systems without delay on the basis of state feedback is illustrated in the Fig. 5, which principally contained rolling optimization, prediction model, feedback correction as well as state observation. At each k sampling time, the observed state of $\hat{x}(k)$ is substituted into the prediction model to obtain the predicted value Y. According to the reference trajectory $r(k)$ and the predicted value Y, rolling optimization is carried out to acquire the optimal increment of control $\Delta u(k)$. However, the increment of control $\Delta u(k)$ is numerically integrated to to gain the quantity of control $u(k)$ and applied to the controlled object HDU; however, the increment of control $\Delta u(k)$ and observed state $\hat{x}(k)$ both are used for the state observation and feedback to obtain the observed state $\hat{x}(k+1)$.The z^{-1} exhibits a step backward in time, which means to redefine the next time as k, and the whole process is repeated.

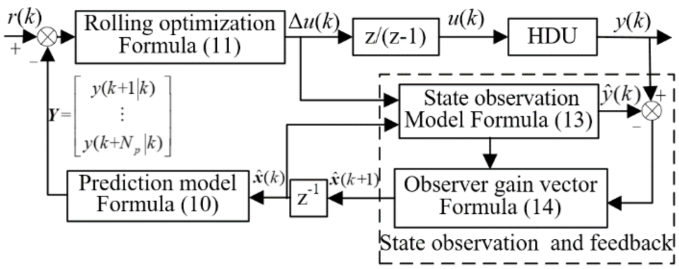

Fig.5 The schematic diagram of traditional MPC

(1) Prediction model

For the SISO system, the prediction pattern of control horizon Nc and prediction horizon Np can be acquired via the Eq. (10).

$$Y=F\hat{x}(k)+\boldsymbol{\Phi}\Delta U \tag{11}$$

in which $\hat{x}(k)$ represents the observed state, $Y=\left[y(k+1|k)\;\; y(k+2|k)\;\; y(k+3|k)\;\; \cdots\;\; y(k+N_p|k)\right]^{T}$,

$F=\begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}$, $\boldsymbol{\Phi}=\begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_c}B \end{bmatrix}$, $\Delta U=\left[\Delta u(k)\;\; \Delta u(k+1)\;\; \Delta u(k+2) \cdots \;\; \Delta u(k+N_c-1)\right]^{T}$

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(2). Rolling optimization

The target of HDU system is to keep the control signal from changing too fast, so that the output can track the high-precision reference signal. So, the objective function is set as follows

$$J=\left(R-Y\right)^{T}\left(R-Y\right)+\Delta U^{T}Q\Delta U \tag{12}$$

in which $R=R_c r(k)$,the size of the matrix R_c is 1 row and N_p columns, and the elements of matrix R_c are 1, Q represents the weight matrix.

When the input constraints are not considered, the minimum value of J can be calculated by the analytic method to gain the optimal sequence of control increment ΔU , shown as blow

$$\Delta U=\left(Q+\boldsymbol{\Phi}^{T}\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^{T}\left(R-Fx(k)\right) \tag{13}$$

ΔU is acquired through minimizing J at each k sampling time. And the first element $\Delta u(k)$ of ΔU is employed for controlled object, as exhibited in the Fig. 6. At next k+1 sampling time, the above-mentioned process is conducted repeatedly.

(3). State observation and feedback

State feedback can improve the performance of the system more effectively. But state variables (such as the velocity and acceleration of the hydraulic cylinder) are often not directly measured. However, state observers can estimate state variables that cannot be measured. The closed loop system state is assessed via the pole assignment method.

The state observation model is obtained by Eq. (11).

$$\begin{cases} \hat{x}(k+1)=A\hat{x}(k)+B\Delta u(k) \\ \hat{y}(k)=C\hat{x}(k) \end{cases} \tag{14}$$

And the observed state can be got by Eq. (15).

$$\hat{x}(k+1)=A\hat{x}(k)+B\Delta u(k)+K_{ob}\left(y(k)-\hat{y}(k)\right) \tag{15}$$

in which K_{ob} represents the gain vector of observer, which can be acquired through the 'place' command of MATLAB software.

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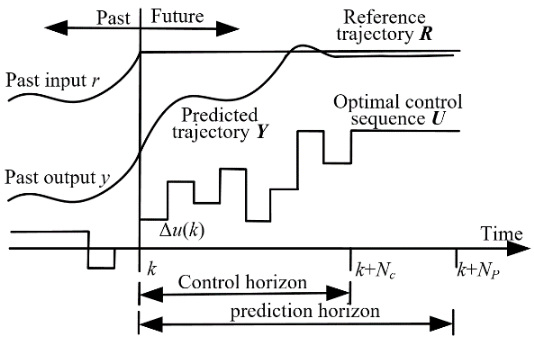

Fig.6 The conventional MPC rolling optimization

(4). Stability analysis

The first element of ΔU is revealed as below

$$\begin{aligned}\Delta u(k) &= [1\ 0\ \cdots\ 0]\left(\boldsymbol{Q}+\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\left(\boldsymbol{R}_r r(k)-\boldsymbol{F}\hat{\boldsymbol{x}}(k)\right)\\ &= K_y r(k)-\boldsymbol{K}_{mpc}\hat{\boldsymbol{x}}(k)\end{aligned} \tag{16}$$

in which K_y represents his first element of $\left(\boldsymbol{Q}+\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{R}_c$, $\boldsymbol{K}_{mpc}$ is the row of $\left(\boldsymbol{Q}+\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{F}$.

Combination of Eq. (16) with Eq. (9) to acquire

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}K_y r(k)-\boldsymbol{B}\boldsymbol{K}_{mpc}\hat{\boldsymbol{x}}(k) \tag{17}$$

Substitute Eq. (10) into Eq. (15) to lead to

$$\tilde{\boldsymbol{x}}(k+1)=\left(\boldsymbol{A}-K_{ob}\boldsymbol{C}\right)\tilde{\boldsymbol{x}}(k) \tag{18}$$

where

$$\tilde{\boldsymbol{x}}(k)=\boldsymbol{x}(k)-\hat{\boldsymbol{x}}(k) \tag{19}$$

Combination of Eq. (19) with Eq. (17) to acquire

$$\boldsymbol{x}(k+1)=\left(\boldsymbol{A}-\boldsymbol{B}K_{mpc}\right)\boldsymbol{x}(k)-\boldsymbol{B}\boldsymbol{K}_{mpc}\tilde{\boldsymbol{x}}(k)+\boldsymbol{B}K_y r(k) \tag{20}$$

The combination between Eq. (18) and Eq. (20) to acquire the equation in closed-loop state

$$\begin{bmatrix}\tilde{\boldsymbol{x}}(k+1)\\ \boldsymbol{x}(k+1)\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}-\boldsymbol{K}_{ob}\boldsymbol{C} & \boldsymbol{0}\\ -\boldsymbol{B}\boldsymbol{K}_{mpc} & \boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}_{mpc}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{x}}(k)\\ \boldsymbol{x}(k)\end{bmatrix}+\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{B}K_y\end{bmatrix}r(k) \tag{21}$$

It can be seen from Eq. (21) that the eigenvalues for the system of closed-loop MPC are constructed from the eigenvalues of predictive control loop along with the observer loop, and they are independent of each other. Therefore, the observer and predictive control laws can be designed separately at the aim of guaranteeing the stability for the system of the closed-loop MPC.

4. Smith Estimated Compensation Model Predictive Control (SECMPC)

The state model of the input delay systems is exhibited as below

$$\begin{cases}\boldsymbol{x}_d(k+1)=\boldsymbol{A}_d\boldsymbol{x}_d(k)+\boldsymbol{B}_d u(k-d)\\ y(k)=\boldsymbol{C}_d\boldsymbol{x}_d(k)\end{cases} \tag{22}$$

in which d represents the coefficient of time delay.

The augmented state pattern of Eq. (22) is established to eliminate the error, shown as below

$$\begin{cases}\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}\Delta u(k-d)\\ y(k)=\boldsymbol{C}\boldsymbol{x}(k)\end{cases} \tag{23}$$

in which, $\boldsymbol{x}(k)=\left[\Delta\boldsymbol{x}_d(k)^{\mathrm{T}}\ y(k)\right]^{\mathrm{T}}$, $\boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_d & 0\\ \boldsymbol{C}_d\boldsymbol{A}_d & \boldsymbol{I}\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}\boldsymbol{B}_d\\ \boldsymbol{C}_d\boldsymbol{B}_d\end{bmatrix}$, $\boldsymbol{C}=\left[0\ \ \boldsymbol{I}\right]$.

If the traditional MPC is used to control the time-delay system, the following problems will occur:

(1). For the input delay system, since the system has d-step delay, the control increment $\Delta u(k-d)$ at time k-d corresponds to the state $x(k)$ of k time. If the conventional MPC is employed for the control of input time-delay system, the state $\hat{x}(k)$ and increment of control $\Delta u(k\text{-}d)$ are applied directly to acquire the state $\hat{x}(k+1)$ in the process of state observation and feedback, which makes the state observation inaccurate.

(2) For the input delay system, since the system has d-step delay, the reference trajectory $r(k)$ corresponds to the state $x(k+d)$ in the future. If traditional MPC is adopted to control the input delay system, the model prediction and rolling optimization will be conducted according to the state $x(k+1)$, which makes the increment of control $\Delta u(k)$ obtained by optimization inaccurate and ultimately leads to the control system unstable.

4.1 The Description of SECMPC

Aiming at the above two problems, the SECMPC strategy is proposed in this paper.

(1). The input delay state equation (23) is used as the mathematical model for prediction model and state observation. The control increment $\Delta u(k-d)$ at k-d time and the state $x(k)$ at k time are applied to obtain the state $x(k+1)$ at k+1 time, which makes the observation of state more accurate.

(2). The combination between MPC and SEC is achieved. The system state at $k+d$ (d is the time delay coefficient) time is estimated in advance at k time by a Smith estimator to compensate the delay of the state. Then the prediction model based on input delay state equation is used for model prediction and rolling optimization. Thus the control effect of the system is similar to the system without input delay.

At each k sampling time, the Smith estimated state $\hat{x}(k+d)$ is substituted into the prediction model based on input delay state equation to obtain the predicted value $\boldsymbol{Y}_d$. According to the reference trajectory $r(k)$ and the predicted value $\boldsymbol{Y}_d$, the rolling optimization is carried out to gain the optimal increment of control $\Delta u(k)$. On the one hand, the increment of control $\Delta u(k)$ is numerically integrated in order to acquire the control quantity $u(k)$ and applied to the controlled object HDU; however, the observed state $\hat{x}(k)$ and increment of control $\Delta u(k-d)$ are used for the state observation and feedback to obtain the observed state $\hat{x}(k+1)$; however, the state $\hat{x}(k+1)$ and the increment of control $\left[\Delta u(k-(d-1)) \cdots \Delta u(k)\right]^{\mathrm{T}}$ are applied for the Smith estimator to obtain the estimated state $\hat{x}(k+d+1)$. The z^{-1} represents a step backward in time, which means to redefine the next time as k, and the whole process is repeated. The specific process of SECMPC is illustrated in the Fig. 7.

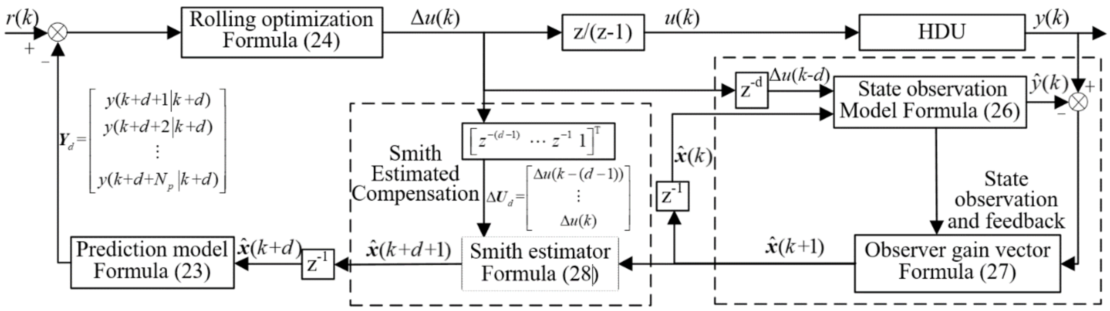

Fig.7 The schematic diagram of SECMPC

(1) Prediction model

The system prediction model of control horizon N_c and prediction horizon N_p is obtained according to the input delay state Eq. (23) and the state $\hat{x}(k+d)$ estimated by Smith estimator.

$$\boldsymbol{Y}_d = \boldsymbol{F}\hat{\boldsymbol{x}}(k+d) + \boldsymbol{\Phi}\Delta \boldsymbol{U} \tag{24}$$

in which $\hat{x}(k+d)$ represents the state estimated by Smith estimator,.

$$\boldsymbol{Y}_d = \left[y(k+d+1|k+d) \;\; y(k+d+2|k+d) \;\; y(k+d+3|k+d) \cdots \;\; y(k+d+N_p|k+d) \right]^{\mathrm{T}}$$

$$\Delta \boldsymbol{U} = \left[\Delta u(k) \;\; \Delta u(k+1) \;\; \Delta u(k+2) \cdots \;\; \Delta u(k+N_c-1) \right]^{\mathrm{T}},$$

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{CA} \\ \boldsymbol{CA}^2 \\ \vdots \\ \boldsymbol{CA}^{N_p} \end{bmatrix}, \quad \boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{CB} & 0 & \cdots & 0 \\ \boldsymbol{CAB} & \boldsymbol{CB} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \boldsymbol{CA}^{N_p-1}\boldsymbol{B} & \boldsymbol{CA}^{N_p-2}\boldsymbol{B} & \cdots & \boldsymbol{CA}^{N_p-N_c}\boldsymbol{B} \end{bmatrix}.$$

(2). Rolling optimization

According to the reference trajectory $r(k)$ and the predicted value $\boldsymbol{Y}_d$, the objective function is set as below

$$J_d = (R - Y_d)^{\mathrm{T}}(R - Y_d) + \Delta U^{\mathrm{T}} Q \Delta U \tag{25}$$

in which $R = R_c r(k)$, the size of the matrix $\boldsymbol{R_c}$ is 1 row and Np columns, and the elements of matrix $\boldsymbol{R_c}$ are 1,$\boldsymbol{Q}$ represents the weight matrix.

When the input constraints are not considered, the minimum value of J_d can be calculated by the analytic method in order to get the optimal sequence of control increment ΔU, as below

$$\Delta U = (Q + \Phi^{\mathrm{T}}\Phi)^{-1}\Phi^{\mathrm{T}}(R - F\hat{x}(k+d)) \tag{26}$$

ΔU is acquired via minimizing J at each k sampling time. And the first element $\Delta u(k)$ of ΔU is exploited for the controlled target, as illustrated in the Fig. 8. At the next k+1 sampling time, the above-mentioned process is implemented repeatedly.

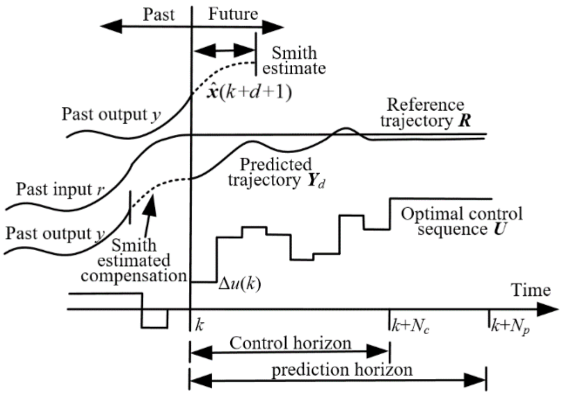

Fig.8 The rolling optimization of SECMPC

(3). State observation and feedback

The state observation model is obtained according to Eq. (23)

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + B\Delta u(k-d) \\ \hat{y}(k) = C\hat{x}(k) \end{cases} \tag{27}$$

The feedback of difference between the output of original system and state observation is conducted to get the observed state.

$$\hat{x}(k+1) = A\hat{x}(k) + B\Delta u(k-d) + K_{ob}(y(k) - \hat{y}(k)) \tag{28}$$

in which $\boldsymbol{K_{ob}}$ represents the gain vector of observer, and which can be gained via the 'place' command of MATLAB software.

(4). Smith estimated compensation

The system state at k+d time is estimated in advance at k time via the Smith estimator, the estimated state is as follows

$$\hat{x}(k+d+1) = G\hat{x}(k+1) + \Psi\Delta U_d \tag{29}$$

where, $G = A^{d-1}$, $\Psi = \left[A^{d-1}B \;\; A^{d-2}B \;\; A^{d-3}B \cdots B\right]$, $\Delta U_d = \left[\Delta u(k-(d-1)) \cdots \Delta u(k-1) \;\; \Delta u(k)\right]^{\mathrm{T}}$.

(5). Stability analysis

The state delay is compensated by the Smith estimator, which is correspond to moving the system's delay part out of the closed-loop control loop. The system's delay features only makes the original output signal (the output signal of system without time delay) to have delay time τ, and does not change the performance of the output signal. Therefore, the Smith estimated compensation makes the unstable system with delay to be upgraded to the stable system without delay. For the stability analysis of the system without delay, refer to Section 3.

The higher the accuracy of Smith estimator and the shorter the delay time are, the higher the estimated accuracy of the state will be, and the closer to the system without delay the control performance will be.

4.2 The Simulation Analysis

For the sake of confirming the SECMPC effectiveness, simulation system is carried out. The block diagram of simulation is exhibited in the Fig. 9. At the same time, three groups of models are simulated, as shown in Table 3. Group *A* adopts the traditional MPC to control the mathematical model of HDU without delay; Group *B* adopts the traditional MPC to control the mathematical model of HDU with delay; Group *C* adopts the SECMPC to control the mathematical model of HDU with delay. The system initial state is set to $\hat{x}(1)=[0\ 0\ 0\ 0]$.

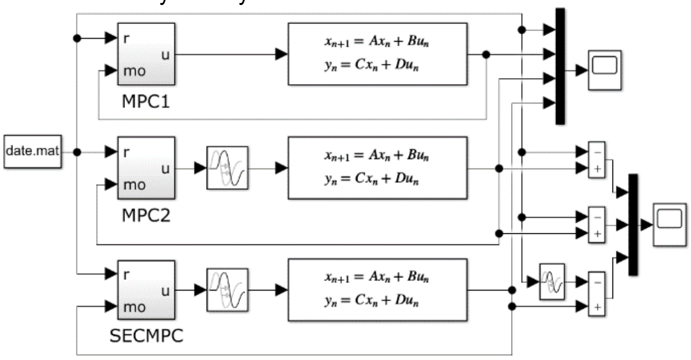

Fig.9 The block diagram of HDU simulation of distinct control methods

Table 3. The three groups of models are simulated

Groups	Model	Control method
A	System without delay	Traditional MPC
B	System with delay	Traditional MPC
C	System with delay	SECMPC

The parameters of SECMPC are shown in Table 4. The weight matrix $\boldsymbol{Q}$, the control horizon N_c as well as prediction horizon N_p of traditional MPC and SECMPC are the same.

Table 4. The SECMPC parameters

Parameter	Value
The control horizon	6
The prediction horizon	10
The weight matrix	0.5
The sampling period /s	0.002
The delay time T_d/ms	10
The delay coefficient d	5

A sinusoidal wave having 50mm amplitude and 0.5Hz frequency is taken as the input displacement r. The simulation output displacement of each group is shown in Fig. 10. The control effect of group *A* and Group *C* is better and the displacement error is smaller. The control effect of group *B* is poor, the displacement error is large, and there is obvious oscillation. Compared with group *B*, the control performance of group *C* is obviously improved, which verifies the effectiveness of SECMPC.

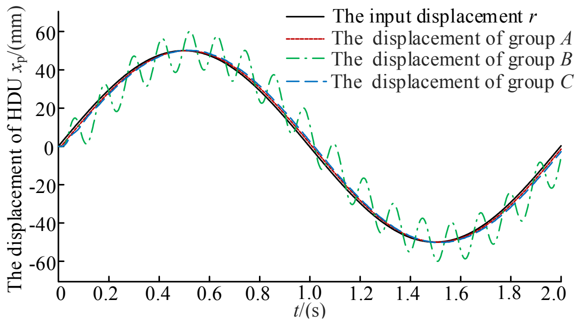

Fig.10 The HDU simulation displacement of different control methods

The displacement error e_A between the displacement of group *A* and input r, the displacement error e_C between the displacement of group *C* and input r, the displacement error e_{Cd} between the displacement of group *C* and input r with delay time 10ms, are shown in Fig. 11. The displacement error e_A and e_{Cd} are basically superimposable. It indicates that the output displacement of group *C* is only delayed 10ms compared with that of group *A*, which verifies the correctness of the SECMPC.

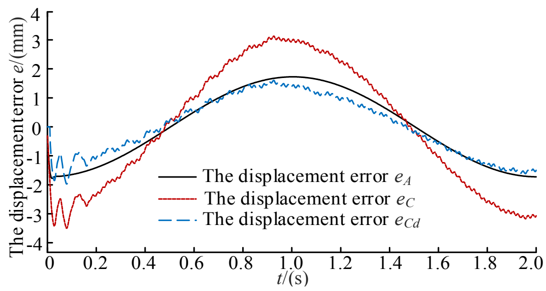

Fig.11 The HDU simulation displacement error of different control methods

The delay time τ of the HDU mathematical model is changed to 4ms, and the output displacement and displacement error are shown in Fig. 12-13. The e_{Cd} in Fig. 13 is the displacement error between the displacement of group C and input r with delay time 4ms. The delay time T_d of the HDU mathematical model is changed to 20ms, and the output displacement and displacement error are reflected in the Fig. 14-15. The e_{Cd} in the Fig. 15 represents the error of displacement between the displacement of group C and input r with delay time 20ms. According to Fig. 10-15, for the traditional MPC, the longer the delay time is, the more obvious the output displacement vibration is and the more unstable the control is; for the SECMPC, the longer the delay time is, the lower the state estimated accuracy of Smith estimator is, and the worse the control performance is, but compared with the traditional MPC, the control performance of SECMPC is obviously improved.

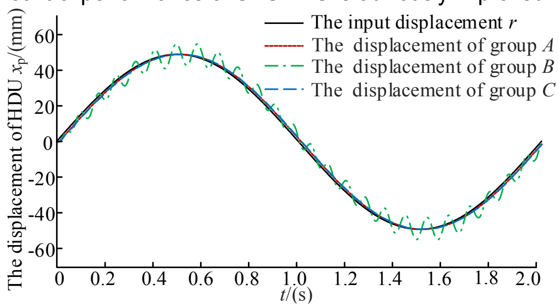

Fig.12 Simulation displacement of the system with delay time 4ms

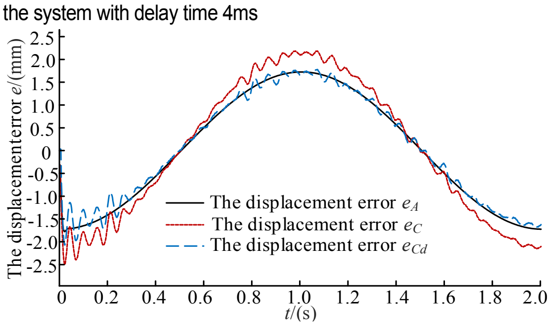

Fig.13 The displacement error of the system with delay time 4ms

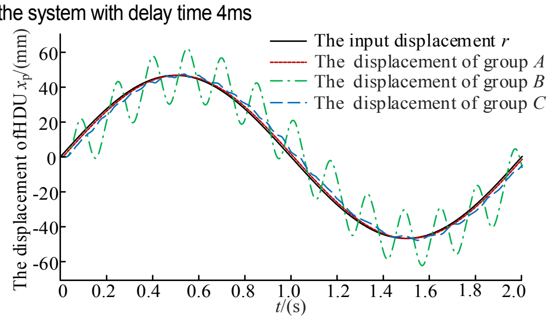

Fig.14 Simulation displacement of the system with delay time 20ms

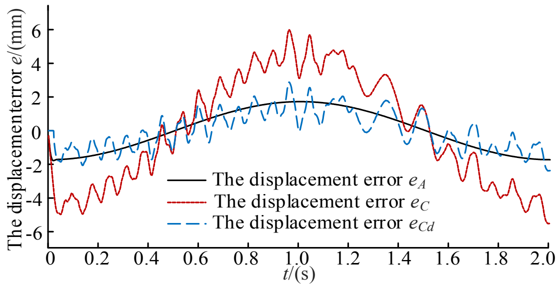

Fig.15 The displacement error of the system with delay time 20ms

4.3 The Experiment

For the sake of confirming the SECMPC effectiveness, the traditional SECMPC and MPC proposed in this paper are used to conduct control experiments on HDU.

The experiment system is the same as that for model parameters identification, as shown in Fig. 2. The parameters of MPC and SECMPC are shown in Table 4. The sinusoidal wave with frequency of 0.5Hz and amplitude of 50mm is still taken as input displacement *r*. The initial displacement of the HDU is 0, that is, the original state is $x(1)=[0\ 0\ 0\ 0]$ .The experimental displacement of HDU is reflected in the Fig. 16, and the displacement error of experiment is illustrated in the Fig. 17. The displacement of traditional MPC control has obvious vibration, and the displacement error of SECMPC control is significantly lower than that of traditional MPC, which verifies the effectiveness of SECMPC.

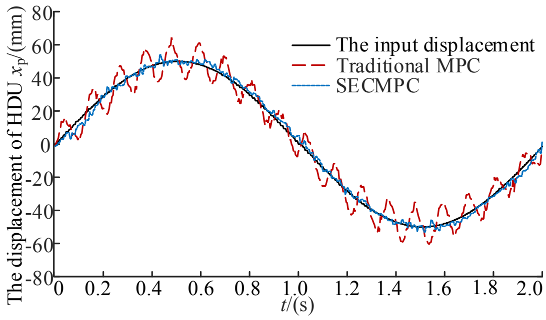

Fig.16 The experimental displacement of HDU of different control methods

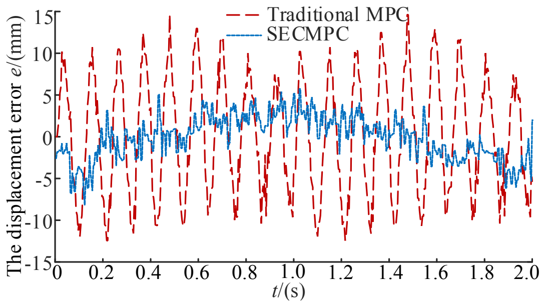

Fig.17 The experimental displacement error of different control methods

5. Conclusions

Aiming at the input delay of HDU, the strategy of Smith estimated compensation model predictive control (SECMPC) is proposed. On the one hand, the input delay state equation is employed as a mathematical pattern for the predictive model and state observation. On the other hand, the combination between MPC and SEC is realized, the system state at *k*+*d* (*d* is the time delay coefficient) time is estimated in advance at the *k* time to compensate the delay of the state. Then the prediction model based on input delay state equation is used for model prediction and rolling optimization. So that the delay system which is unstable is promoted to a stable system without delay.

The SECMPC are applicable not only to HDU with input delay, but also to other systems having constraints and input delay.

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