

A century ago the Stern-Gerlach experiment ruled unequivocally in favor of Quantum Mechanics

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Dedicated to Helmut Schwarz on the occasion of his 80th birthday.

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In 1921, Otto Stern conceived the idea for an experiment that would decide between a classical and a quantum description of atomic behavior, as epitomized by the Bohr-Sommerfeld-Debye model of the atom. This model entailed not only the quantization of the magnitude of the orbital electronic angular momentum but also of the projection of the angular momentum on an external magnetic field – the so-called space quantization. Stern recognized that space quantization would have observable consequences: namely, that the magnetic dipole moment due to the orbital angular momentum would be space quantized as well, taking two opposite values for atoms whose only unpaired electron has just one quantum of orbital angular momentum. When acted upon by a suitable inhomogeneous magnetic field, a beam of such atoms would be split into two beams consisting of deflected atoms with opposite projections of the orbital angular momentum on the magnetic field. In contradistinction, if atoms behaved classically, the atomic beam would only broaden along the field gradient and have maximum intensity at zero deflection, i.e., where there would be a minimum or no intensity for a beam split due to space quantization. Stern anticipated that, although simple in principle, the experiment would be difficult to carry out – and invited Walther Gerlach to team up with him. Gerlach’s realism and experimental skills together with his sometimes stubborn determination to make things work proved invaluable for the success of the Stern-Gerlach experiment (SGE). After a long struggle, Gerlach finally saw, on 8 February 1922, the splitting of a beam of silver atoms in a magnetic field. The absence of the concept of electron spin confused and confounded the interpretation of the SGE, as the silver atoms were, in fact, in a ^2S state, with zero orbital and $\frac{1}{2}$ spin angular momentum. However, a key quantum feature whose existence the SGE was designed to test – namely space quantization of electronic angular momentum – was robust enough to transpire independent of whether the electronic angular momentum was orbital or due to spin. The SGE entails other key aspects of quantum mechanics such as quantum measurement, state preparation, coherence, and entanglement. Confronted with the outcome of the SGE, Stern noted: “I still have objections to the idea of beauty of quantum mechanics. But she is correct.”

I. INTRODUCTION

Although by 1922, experimental evidence for the emerging quantum mechanics was both compelling and diverse (black-body radiation, optical and X-ray spectra, the photo-effect, heat capacity, the Franck-Hertz experiment, see, e.g., Ref. [1]), the molecular beam experiment of Stern and Gerlach (concluded on 8 February 1922) amounted to a much-needed confidence boost for quantum theory. Devised as a question posed to nature to decide between a classical and a quantum description of atomic behavior, the Stern-Gerlach experiment (SGE) ruled unequivocally in favor of the latter.

In 1920-1921, when Otto Stern, FIG. 1, conceived the idea for the SGE [6, 7], atomic behavior was epitomized by the 1916 Bohr-Sommerfeld-Debye quantum model of the atom [8–10]. Stern expected that the SGE would prove this model wrong. After all, shortly after Niels Bohr published in 1913 the first sequel of his atomic model trilogy [8], Stern – and his close colleague and friend Max von Laue – took a vow [11], p. 74: “If this

nonsense of Bohr should, in the end, prove to be right, we will quit physics.”

Stern had invoked the molecular beam method once before, likewise with the aim to test the Old Quantum Theory (1900-1925). In his first beam experiment [12–14], Stern examined whether atoms possessed zero-point translational energy, whose existence had been previously hypothesized by Einstein and Stern as part of their attempt to explain the residual heat capacity of molecular hydrogen at low temperatures [15]. Stern’s 1920 experiment confirmed that thermal gaseous atoms in fact obey the classical Maxwell-Boltzmann distribution of velocities and yielded no evidence for translational zero-point energy. Only in 1927, it had been recognized that the residual heat capacity is due to nuclear spin which gives rise to the ortho and para allotropic modifications of molecular hydrogen [16, 17].

The absence of the concept of spin, albeit electronic, is what would confuse and confound the interpretation of the SGE as well. However, a key quantum feature of the Bohr-Sommerfeld-Debye atom whose existence the SGE was designed to test – namely space quantization of electronic angular momentum – was robust enough to transpire independent of whether the electronic angular

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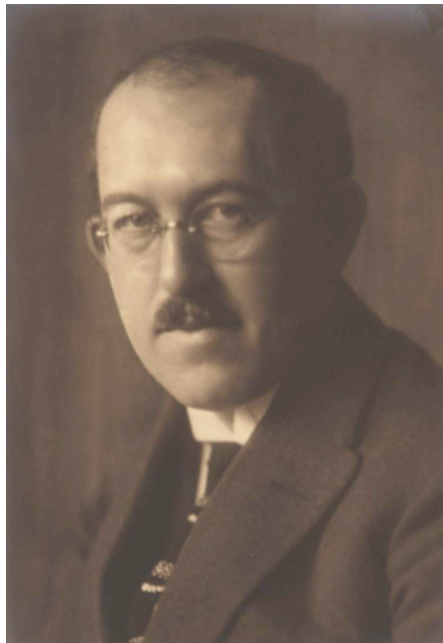


FIG. 1. Otto Stern (1888-1969) was trained in Physical Chemistry by Otto Sackur [2] at the University of Breslau and in Theoretical Physics by Albert Einstein^a at the German University in Prague and at the ETH Zurich [3]. In 1913, Stern became *Privatdozent* for Theoretical Physics at Zurich and in 1914, under Max von Laue’s auspices, at the University of Frankfurt. In 1919, within Frankfurt’s Institute for Theoretical Physics headed by Max Born, Stern launched his molecular beam method to examine the fundamental assumptions of theory that transpire in atomic, molecular, optical, and nuclear physics. Stern’s experimental endeavors at Frankfurt (1919-1922), Hamburg (1923-1933), and, upon his forced emigration, in Pittsburgh (1933-1945) provided insights into the quantum world that were independent of spectroscopy and that concerned well-defined isolated systems, hitherto accessible only to *Gedanken* experiments. Apart from the SGE, Stern’s seminal experiments include the three-stage Stern-Gerlach experiment; experimental evidence for de Broglie’s matter waves; measurements of the magnetic dipole moment of the proton and the deuteron; experimental demonstration of momentum transfer upon absorption or emission of a photon; the experimental verification of the Maxwell-Boltzmann velocity distribution via deflection of a molecular beam by gravity. In 1944, Otto Stern was awarded the 1943 Nobel prize in Physics (unshared) “for his contribution to the development of the molecular ray [beam] method and his discovery of the magnetic moment of the proton.” The official number of nominations provided by the Nobel Archives for Otto Stern is eighty-two, more than any other Physics Nobel laureate on public record. Thirty nominations were for the Stern-Gerlach experiment, fifty-two for Stern’s other molecular beam work [4]. For more on Stern, see [5].

^a Stern’s contact to Einstein was mediated by Sackur via Sackur’s and Einstein’s common colleague and friend Fritz Haber.

momentum was orbital or due to spin.

II. STERN’S QUESTION TO NATURE

But what was Stern’s idea for an experiment that was supposed to “decide unequivocally between quantum-theoretical and classical views” [6, 7]? The Bohr-Sommerfeld-Debye model of the atom entailed not only the quantization of the magnitude $|\mathbf{L}|$ of the electronic orbital angular momentum \mathbf{L} ,

$$|\mathbf{L}| = L\hbar \text{ with } L = 0, 1, 2, \dots \quad (1)$$

but also the quantization of the projection, $L_Z = |\mathbf{L}| \cos \theta$, of \mathbf{L} on the spatial direction \mathbf{Z} defined, for instance, by an external magnetic field vector \mathfrak{H} , see FIG. 4:

$$L_Z = M\hbar \text{ with } M = L, L-1, \dots, -L \quad (2)$$

Stern realized that this so-called space quantization (*Richtungsquantelung*) of angular momentum – that only allows for discrete values of the angle θ subtended by \mathbf{L} and \mathbf{Z} such that $\cos \theta = M/L$ – would have observable consequences: the magnetic moment

$$\boldsymbol{\mu} = -\frac{e}{2m_e}\mathbf{L} \quad (3)$$

due to the orbital motion of the electron will then be space-quantized in a magnetic field as well, i.e., only take a discrete set of values given by the projection quantum number M

$$\mu_Z = -\frac{e}{2m_e}L_Z = -\frac{e}{2m_e}M\hbar \quad (4)$$

In particular, for a “one-quantum” atom, i.e., an atom whose only unpaired electron possesses just one quantum of angular momentum, $L = 1$, and thus has $M = \pm 1$, cf.



FIG. 2. Walther Gerlach (1889-1979) became a major player in experimental physics already when he was a PhD student in Friedrich Paschen's laboratory at the University of Tübingen. Among Gerlach's many achievements is the first quantitative measurement of the pressure of light (with absolutely measured radiation energy), done jointly with his PhD student Alice Golsen in 1924. Gerlach's wide-ranging research programs at the Universities of Tübingen, Frankfurt, and Munich entailed spectroscopy and spectral analysis, the study of the magnetic properties of matter, and radioactivity. Gerlach stayed in Germany during the Nazi era but never joined the NSDAP. However, during the last sixteen months of the existence of the Third Reich, Gerlach held the high-ranking position of the Plenipotentiary for Nuclear Research (a.k.a. *Uranprojekt*). He supported the effort of the German physicists to achieve a controlled chain reaction in a uranium reactor until the last moments before the effort was halted by the Allied Alsos Mission. His behavior during the Third Reich remains controversial. After World War Two, Gerlach dedicated his boundless elan to reconstructing German academia. He held the presidency of the University of Munich (1948-1951) and of the Fraunhofer Society (1948-1951) as well as the vice-presidency of the German Science Foundation (1949-1961) and the German Physical Society (1956-1957). As a member of *Göttinger Achtzehn*, he signed the Göttingen Declaration (1957) against arming the Bundeswehr with nuclear weapons. Gerlach was co-nominated, with Otto Stern, thirty times for the Nobel Prize in Physics for the Stern-Gerlach experiment.

FIG. 5, the only possible values of the magnetic dipole moment will be

$$\mu_Z = \pm \frac{e}{2m_e} \hbar \equiv \pm \mu_B \quad (5)$$

with e and m_e the electron charge and mass and μ_B the elementary magnetic quantum that Stern referred to as the *Bohr magneton*. As Stern put it [6, 7]:

Now, whether the quantum theoretical or classical description is appropriate can be decided by an essentially very simple experiment. One only needs to investigate the deflection that a beam of atoms experiences in a suitable inhomogeneous magnetic field. ... [For a one-quantum atom], the spot on the collection plate [where the beam is to be collected upon its passage through the inhomogeneous magnetic field] will be split in two, each part having the same size and half the intensity of the original spot. If one drops the assumption that all atoms have the same

velocity, then the Maxwell velocity distribution would lead to the result that both spots would be broader and more washed out. In any case, if the deflection of the atoms with the most probable velocity is greater than the radius of the cross section of the atom beam, there must be a minimum intensity at the position of the original spot. Exactly the opposite follows from the classical theory. ... Now, the number of atoms with a given value of θ [where $\theta \equiv \arccos(\mathbf{Z}, \mathbf{L})$, see FIG. 4] is proportional to $\sin \theta$. The number of these atoms thus has a maximum for $\theta = \pi/2$, i.e., for $M = 0$ and a zero deflection. Thus according to the classical theory, for each velocity all possible deflections between zero and the calculated quantum theoretical value arise and the number of atoms with a given deflection is the greater the smaller the deflection. In the magnetic field, the spot on the collection plate would only be broadened but its maximum intensity would always remain at the lo-

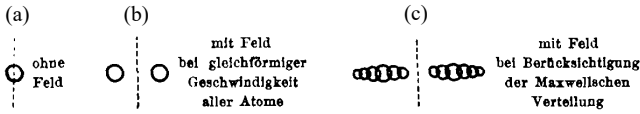


FIG. 3. Schematic view of the spots (beam deposits) on the collection plate as anticipated by Stern in Ref. [6] for (a) a zero magnetic field and (b) and (c) for a horizontal magnetic field whose gradient is likewise horizontal. Panel (b) pertains to a single beam velocity and panel (c) to a Maxwellian distribution of beam velocities. The dashed vertical line marks the position of the undeflected beam. Reproduced from Ref. [20].

cation of the original spot. In this way the experiment, if successful, would unequivocally decide between the quantum theoretical and classical description.

The contrast between a classical and a quantum mechanical outcome of the experiment as expected by Stern is illustrated in FIG. 3. In his letter to Stern from 24 November 1921, Wolfgang Pauli pointed out that in the classical case, same-velocity atoms would not result in an intensity maximum but rather a sharp-edged image of the source, see Ref. [18], p. 113. The method of reading the images became the subject of a controversy with Nikolay Semyonov that was finally resolved by Stern in 1927 [19].

In any case, Stern expected that in an inhomogeneous magnetic field of gradient $\partial\mathfrak{H}/\partial Z$, the silver atoms will be subject to a force \mathfrak{F} whose components

$$\mathfrak{F} = \mu_Z \frac{\partial\mathfrak{H}}{\partial Z} = \pm\mu_B \frac{\partial\mathfrak{H}}{\partial Z} \quad (6)$$

cf. Eq. (5), will impart to the atoms equal but opposite deflections – along and against the direction of the magnetic field gradient. Stern calculated that the deflection s on a path of length ℓ through the magnetic field will be

$$s = \pm \frac{\mu_B}{2m} \frac{\partial\mathfrak{H}}{\partial Z} \frac{\ell^2}{v^2} \quad (7)$$

where m is the mass of the atoms and v their velocity, taken as $v = \sqrt{(3kT/m)}$, with temperature $T = 1300$ K. For a beam of ground-state silver atoms (assumed to have $L = 1$ and $M = \pm 1$), feasible magnetic field gradients (on the order of 10^4 Gauss/cm), and dimensions of the molecular beam apparatus (whose core wasn't much bigger than a fountain pen), Stern estimated the separation, $2s$, of the spots corresponding to $\mu_Z = +\mu_B$ and $\mu_Z = -\mu_B$ to be on the order of $10 \mu\text{m}$ [6]. At which point Stern realized that he would need to team up with “a real experimentalist” in order to get this experiment done. Stern [21]:

I was attuned to molecular beams through the measurement of molecular velocities and so I tried the experiment. I did it jointly with

Gerlach, because it was a difficult matter, and so I wanted to have a real experimental physicist working with me. It went quite nicely ... for instance, I would build a little torsional balance to measure the [magnetic] field that worked but not very well. Then Gerlach would build a very fine one that worked much better. Incidentally, I'd like to emphasize one thing on this occasion, [namely] that we did not [acknowledge] sufficiently at the time the help that we received from [Erwin] Madelung. Born was already gone then [moved to his new post at Göttingen] and his successor was Madelung. Madelung essentially suggested to us the [realization of the inhomogeneous] magnetic field [by making use] of an edge [and groove combination].

This is how Walther Gerlach, FIG. 2, reminisced about his recruitment for the SGE by Stern [22]:

One day Stern would come to me and say: ‘Do you know what space quantization is?’ I would say: ‘No, I have no idea.’ ‘But you should actually know that. Recently Debye and Sommerfeld published [papers] suggesting that the [anomalous] Zeeman effect can be explained by a quantum effect, by the so-called space quantization. That is, [the magnetic dipole of] a silver or sodium atom can only have two settings [orientations] in a magnetic field, it cannot adjust itself at will or precess, but can only have two very specific settings [orientations], or actually even three, namely perpendicular to the magnetic field or in ... the direction or against the direction [of the magnetic field] ...

Repeated discussions with Stern during our daily visits at *Café Rühl* finally led to a plan to make the experiment in such a way that there was hope of seeing space quantization.

Gerlach perhaps thought that he would just have to modify his ongoing experiment on the magnetic properties of bismuth. Finally he agreed: “Yes, I want to try it” [22]. But then, Gerlach continued,

[Stern] would come back again: “It isn't worth it, I've miscalculated, power of ten too little.” And then, it went back and forth a couple of times for a week or a fortnight and one day he would come back and say: “Yes, now I've done [the calculations] properly and the thing only works if you get fields with an inhomogeneity of about ten or fifty thousand Oersted per centimeter – and that's not possible.” And then I said to him: “Yes, I am almost there, I already have ten thousand [Oersted per centimeter], namely for my planned

bismuth experiment.” “So,” he said, “let’s try it.”

And they did. The collaboration between Stern and Gerlach was a stroke of luck not only for the Stern-Gerlach experiment (SGE) but for physics at large. It brought together Stern’s “out-of-the-box” thinking (*Querdenken*) with Gerlach’s skills and tenacity in the laboratory.

Stern credited for his way of thinking his apprenticeship in theoretical physics with Albert Einstein [21]:

I learned the *Querdenken* from him [Einstein] ... I also learned from Einstein to talk nonsense every now and then. Einstein registered with pleasure when he had made a mistake. He would admit his mistake and remark: “It’s not my fault that *der liebe Gott* [the dear Lord] didn’t make things the way I had imagined.”

To which Immanuel Estermann, a close co-worker and friend of Stern’s, later added [23]:

From his collaboration with Einstein, the real benefit was to learn how to distinguish which problems of contemporary physics were important and which were not so important; which questions to ask and which experiments to undertake in order to answer the questions. Thus from a brief scientific collaboration evolved a close, life-long friendship, which would be the basis for Stern’s great achievements.

Gerlach’s time at Paschen’s institute in Tübingen proved formative for both his personality and his experimental abilities. Either became a key prerequisite for the success of the Stern-Gerlach experiment and other precision measurements where Gerlach pushed the limits of the possible. Gerlach provided the following definition of a precision measurement [24]:

By “precision measurement” we mean an investigation in which all sources of error are taken into account and all observed phenomena are clarified: It is also characteristic of [a precision] measurement that each individual step is theoretically and numerically justified, its influence on the course of the experiments thoroughly tested, spelled out, and presented in all detail; in short, the reader must be able to form a judgment from the description of the experiments about the evidential value and the certainty of the results.

Gerlach’s own work had set a standard of precision physics.

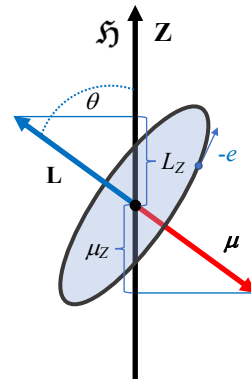


FIG. 4. Angular momentum \mathbf{L} (blue arrow) of an orbiting electron and the magnetic dipole moment $\boldsymbol{\mu} = \frac{e}{2m_e}\mathbf{L}$ (red arrow) it generates. Note that in the schematic, we set $\frac{e}{2m_e} = 1$. Also shown are the projections L_Z and μ_Z of the angular momentum and the magnetic dipole moment, respectively, on the space-fixed axis \mathbf{Z} as defined by the magnetic field vector \mathfrak{H} . With θ the angle subtended by the vectors \mathbf{Z} and \mathbf{L} , $L_Z = |\mathbf{L}| \cos \theta$.

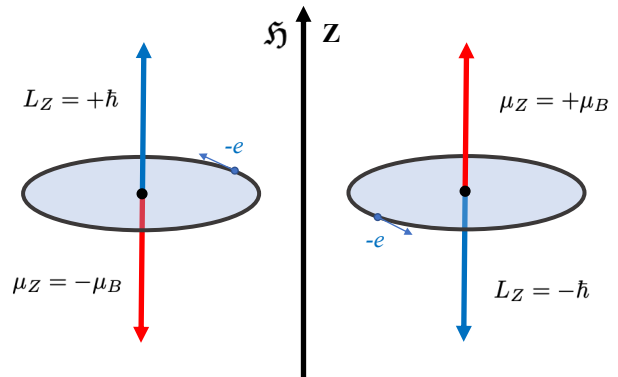


FIG. 5. Possible values of the projections of the electronic angular momentum L_Z and the magnetic dipole moment μ_Z of a one-quantum atom on the direction of a magnetic field \mathfrak{H} as inferred from the Bohr-Sommerfeld-Debye model of the atom by Otto Stern in Ref. [6, 7]. See also FIG. 4.

III. TEASING OUT NATURE’S ANSWER

Luckily, Stern and Gerlach chose for their experiment silver atoms ($^2S_{1/2}$), favored by Stern, rather than bismuth atoms ($^4S_{3/2}$), whose magnetic properties Gerlach was eager to investigate in a separate experiment. However, there could have been even less fortunate choices than Bi, see Sec. VII.

A schematic of the Stern-Gerlach apparatus is shown in FIG. 6. The silver beam was produced by effusion of silver vapor from a 1 mm diameter orifice of an oven into the vacuum. As shown in Stern’s previous beam experiment [12, 13], the silver atoms obeyed the Maxwell-Boltzmann velocity distribution and so the scenario depicted in FIG. 3c was expected to come into force if

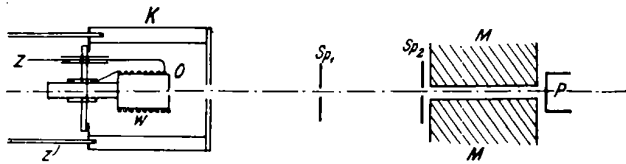


FIG. 6. Schematic of the Stern-Gerlach apparatus. The silver beam effuses from an oven (O), passes through a pinhole (Sp_1) and a rectangular slit (Sp_2) before it enters the magnetic field generated by the pole pieces (M) and finally reaches the collector plate (P). The distances between the components of the 3rd generation apparatus (that made it possible to see the splitting of the silver beam for the first time) were as follows: O to exit pinhole from cooler, 2–3 cm; exit pinhole from cooler to rectangular collimation slit Sp_2 , 7–12 cm; path through the magnetic field, 3 cm. The measured maximum inhomogeneity of the magnetic field in the beam region was about 23 kG/cm. Reproduced from Ref. [20].

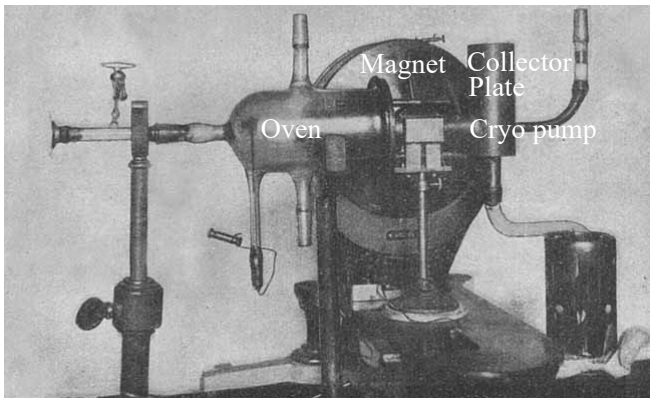


FIG. 7. Photograph of the Stern-Gerlach apparatus with improvements of 1922–1924 (4th generation). See also FIG. 6. Adapted from Ref. [25].

space quantization existed. The electrically heated oven was placed within a water-cooled capped double-walled brass cylinder to whose inner wall it was attached via a quartz capillary. The whole contraption (oven and water cooler) was placed within a glass differentially-pumped vacuum chamber equipped with feed-throughs and attached to the probe chamber. The differentially-pumped probe chamber housed the pole pieces of the electromagnet. The edge piece held the collimation pinholes (later slits), which facilitated proper alignment. This had to be accurate within $5 \mu\text{m}$ for slits with horizontal (i.e., along the direction of the magnetic field and its gradient, both perpendicular to the beam velocity) dimension of 30 to $60 \mu\text{m}$. At the end of the probe chamber was the glass collector plate (with a surface area of just a few mm^2) that was attached to another double-walled cylinder serving as a liquid-air or dry-ice cryo pump.

The Stern-Gerlach apparatus evolved over three generations of improvements before the SGE came to a successful conclusion. These entailed different designs of the oven as well as implementations of the magnetic field, the

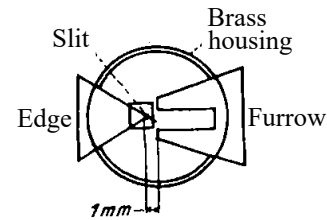


FIG. 8. The center-piece of the Stern-Gerlach apparatus viewed along the direction of the silver beam. The pole pieces that generated the inhomogeneous magnetic field were placed inside a brass tube (for mechanical stability) sealed to the source chamber on one end and the detection region with the collector plate on the other. The pole pieces (edge and furrow) were energized by an external water-cooled electromagnet. The 90° edge was slightly flattened; the furrow was just 1.2 mm wide, mounted at a distance of 1 mm from the edge. Adapted from Ref. [20].

collimation elements, and the handling of the vacuum. FIG. 7 shows a photo of the 4th generation apparatus that Gerlach built for his later (1924–1925) investigations of the magnetic properties of atoms (no photographs of the earlier versions of the apparatus are available). We note that in all generations of the apparatus, the magnetic field and its gradient were oriented horizontally, with the edge on the left and the furrow on the right with respect to the beam velocity, cf. FIG. 8.

The effort needed in order to make the experiment work was tremendous. Moreover, it was mostly the result of Gerlach's lonely toil as Stern, who did not believe in the reality of space quantization to begin with, left on 1 October 1921 to assume a professorship in Theoretical Physics at the University of Rostock.

During the night of 4 November 1921, Gerlach observed for the first time a broadening of the silver beam in an inhomogeneous magnetic field. This provided evidence that silver atoms carried a magnetic dipole moment – but the spatial resolution did not suffice to demonstrate the existence of space quantization. During the Christmas recess, Gerlach and Stern reconfigured their apparatus again, but Gerlach's subsequent attempts to see space quantization failed. At their mid-way meeting in Göttingen in early February 1922, Gerlach and Stern decided to try the experiment one more time. On the train back to Frankfurt, Gerlach recollected a modification he had made earlier when examining crystals by X rays using the Debye-Scherrer method, namely to use a slit instead of a pinhole to boost both flux and spatial resolution. Gerlach had even reported on the improvement he achieved with a slit as opposed to a pinhole at the German Physics Day in Jena in September 1921 [26]. Upon his arrival in Frankfurt, Gerlach replaced the pinhole (of $50 \mu\text{m}$ diameter) defining the silver beam at the entrance into the inhomogeneous magnetic field by a rectangular $30 \times 800 \mu\text{m}^2$ slit with its narrower side along the magnetic field direction [27]. During the night from the 7th to the 8th of February 1922, Gerlach achieved

the ultimate success.

Wilhelm Schütz (1900-1972), who was in 1922 Gerlach's PhD student, described the difficulties of the SGE as well as the final triumph on 8 February 1922 as follows [28]:

The old apparatus had only yielded a broadening of the silver beam [deposit on the glass plate] of the expected magnitude ... due to the inhomogeneous magnetic field. A major improvement of the apparatus with the aim to further increase its resolution was [therefore] necessary. During this rebuilding period, Stern moved to Rostock to assume a Professorship for Theoretical Physics there. He would show up in Frankfurt every now and then (during Christmas 1921 and Easter 1922) for discussions and to measure the inhomogeneity of the magnetic field ... Soon came the time when I was able to enter the holy premises of the laboratory and take a look at the pumps, when [the technician Mr.] Schmidt was not on duty and Prof. Gerlach had to sleep once in a while ... Anyone who has not been through it cannot at all imagine how great were the difficulties with an oven to heat the silver up to about 1300° K within an apparatus which could not be heated in its entirety [the seals would melt] and where a vacuum of 10^{-5} Torr had to be produced and maintained for several hours. The cooling was done with solid carbon dioxide and acetone or with liquid air. The pumping speed of the Gaede mercury backing pumps and the Volmer mercury diffusion pumps was ridiculously low compared with the performance of modern pumps. And then their fragility; the pumps were made of glass and quite often they broke, either from the thrust of boiling mercury ... or from the dripping of condensed water vapor. In that case the effort of several days of pumping, required during the warming up and heating of the oven, was lost. Also, one could be by no means certain that the oven would not burn through during the four- to eight-hour exposure time. Then both the pumping and the heating of the oven had to be started from scratch. It was a Sisyphus-like labor and the main load of responsibility lay on the broad shoulders of Prof. Gerlach. In particular, W. Gerlach would take over the night shifts. He would get in at about 9 p.m. equipped with a pile of reprints and books. During the night he then read the proofs and reviews, wrote papers, prepared lectures, drank plenty of cocoa or tea and smoked a lot. When I arrived the next day at the institute, heard the intimately familiar noise of the running pumps,

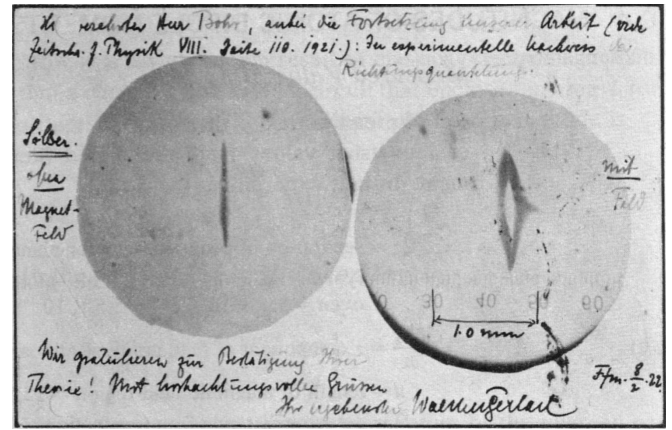


FIG. 9. Postcard dispatched by Gerlach to Niels Bohr on the day of the triumph, 8 February 1922 [18], p. 116. The microphotographs show the silver beam deposits obtained in the absence (left) and presence (right) of the magnetic field. In the absence of the magnetic field, the deposit corresponds to an image of the second collimation slit (Sp_2 in FIG. 3). See text.

and found Gerlach still in the lab, it was a good sign: nothing broke during the night.

Then I arrived at the institute one morning in February 1922; it was a wonderful morning: with cool air and fresh snow! W. Gerlach was once again at it, developing the deposit of an atomic beam that had been passing through an inhomogeneous magnetic field for eight hours. Full of expectation, we applied the development process, whereupon we experienced the success of several months of effort: The first splitting of a silver beam in an inhomogeneous magnetic field. After Master Schmidt and, if I remember correctly, E. Madelung had seen the splitting, we went to Mr Nacken to the Mineralogical Institute to have the finding recorded on a microphotograph. Then I was tasked with sending a telegram to Professor Stern in Rostock, with the text: “Bohr is right after all!”

On the day of the triumph, Gerlach also sent a postcard to Niels Bohr, which showed reproductions of the microphotographs of the silver beam deposits obtained with and without the inhomogeneous magnetic field, see FIG. 9 and FIG. 10. The accompanying text read, in translation: “Attached is the experimental proof of space quantization (silver without and with field). We congratulate you on the confirmation of your theory. With best regards, yours Walther Gerlach.”

IV. FUNDING FOR THE SGE

The apparatus was constructed and operated during the hyperinflation period that beset Germany in the aftermath of World War One. Support for the experiment came from several sources, most notably, the *Physikalischer Verein Frankfurt*. The *Verein's* long-time chairman was Wilhelm Eugen Hartmann (1853-1915), founder of the *Hartmann & Braun* company. Einstein, then director of the Kaiser Wilhelm Institute for Physics in Berlin, provided 10,000 Marks for “the construction of an apparatus to investigate the band spectra of monoatomic metal vapors” [29], p. 476 that were in the end used for the purchase of an electromagnet made by *Hartmann & Braun* [30], pp. 802, 813. The *Messer* company donated some liquid air [31]. Silver of high purity was acquired from *Heraeus* [20]. Additional funding came from the *Association of Friends and Sponsors of the University of Frankfurt* as well as from Max Born’s unstinting effort to raise funds to support the SGE [21]. He took advantage of the great interest in Einstein and relativity theory by presenting a series of public lectures

in the biggest lecture-hall of the University ... and charged an entrance fee ... The money thus earned helped us for some months, but as inflation got worse ... new means had to be found [32], p.78.

Born mentioned the dire situation “jokingly” to a friend who was departing on a trip to New York; a few weeks later, Born was incredulous when a postcard arrived simply saying that he should write to Henry Goldman and giving the address [32], p.78:

At first I took it for another joke, but on reflection, I decided that an attempt should be made ... [A] nice letter was composed and dispatched, and soon a most charming reply arrived and a cheque for some hundreds of dollars ... After Goldman’s cheque had saved our experiments, the work [on the Stern–Gerlach experiment] went on successfully.

Goldman, a founder of the investment firm Goldman Sachs and progenitor of Woolworth Co stores, had family roots in Frankfurt.

V. OUTCOME OF THE SGE

Gerlach and Stern published what they saw as the main results of the SGE in two installments: on 1 March 1922, they submitted a paper entitled, in translation, “The experimental proof of space quantization in a magnetic field” [27] and on 1 April 1922 a paper entitled, in translation, “The magnetic moment of the silver atom” [33]. These publications were followed by a review where Gerlach and Stern gave technical details of the SGE and

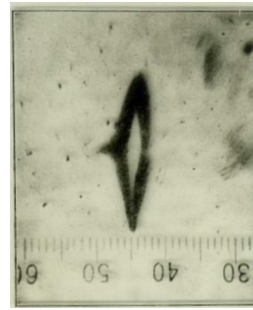


FIG. 10. Microphotograph of the silver beam deposit obtained on 8 February 1922 after passing a beam of silver atoms through the inhomogeneous magnetic field for eight hours. The deposit was about 1.1 mm tall and the splitting amounted to only about 60 to 100 μm , corresponding to an angular deflection of the beam of just a few mrad. Adapted from Ref. [4] by rotating the double-image published therein by 180° so as to make the orientation of the image consistent with the positions of the pole pieces generating the magnetic field (edge on the left, furrow on the right with respect to the propagation direction of the silver atoms, cf. FIG. 8).

provided the following summary of the experiment’s outcome [20]:

The experiments reported herein provide

1. The experimental proof of the Debye-Sommerfeld space quantization in a magnetic field
2. The experimental determination of the Bohr magneton.

While the first statement summarized Stern’s and Gerlach’s epochal discovery, the second was true only approximately and, moreover, on account of what could be called “an uncanny conspiracy of Nature” [34, 35]: As we know today, the silver atoms were in their electronic ground state $^2S_{\frac{1}{2}}$, with spin, orbital, and total angular momentum quantum numbers $S = \frac{1}{2}$, $L = 0$, and $J = \frac{1}{2}$, respectively, and possible values of the projection quantum number $M_S = \pm\frac{1}{2}$, see Refs. [34–36]. Thus their magnetic moment $\boldsymbol{\mu} = -g_S\mathbf{S}\mu_B$ was due to electron spin \mathbf{S} and had a magnitude $|\boldsymbol{\mu}| = \sqrt{S(S+1)}g_S\mu_B = \frac{\sqrt{3}}{2}g_S\mu_B$ and components $\mu_Z = \mp\frac{1}{2}g_S\mu_B$ for the projections $S_Z = \pm\frac{1}{2}\hbar$ of the spin angular momentum on the Z axis, see FIG. 11.

VI. RECEPTION OF THE SGE

The reception of the outcome of the SGE was that of quiet astonishment, as illustrated by the individual reactions below. Let’s begin with that of **Otto Stern** himself [21]:

But the way the experiment turned out, I didn’t understand at all. [How could there

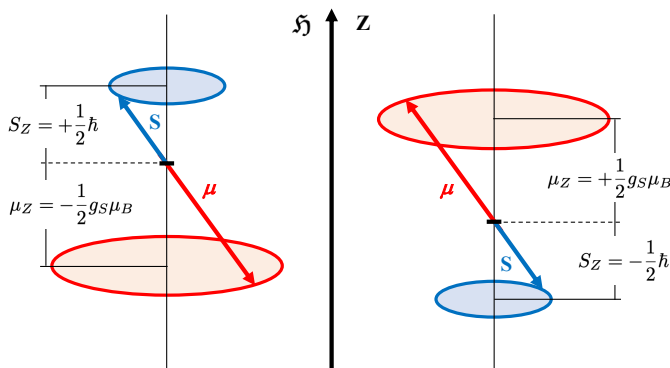


FIG. 11. Quantum-mechanical vector model of a spin- $\frac{1}{2}$ particle – such as a ground-state silver atom – in a magnetic field \mathfrak{H} whose Z -component defines the space-fixed axis \mathbf{Z} (note that the magnetic field must have at least one nonvanishing component orthogonal to Z in order to satisfy Maxwell's equation $\nabla \cdot \mathfrak{H} = 0$). The magnetic moment due to spin (in red) is $\boldsymbol{\mu} = -g_S \mathbf{S} \mu_B$ with g_S the gyromagnetic ratio of the electron and μ_B the Bohr magneton. The Larmor precession of frequency $\nu_L = g_S \mu_B \mathfrak{H} / h$ about Z of the spin angular momentum \mathbf{S} (in blue) of magnitude $\sqrt{S(S+1)}\hbar$ and constant projection $S_Z = M_S \hbar$ with $M_S = \pm \frac{1}{2}$ averages out the X - and Y -components S_X and S_Y of \mathbf{S} in compliance with the uncertainty principle. See text and FIG. 5.

be] the discrete beams—and yet, [there was] no birefringence. We [even] made some additional experiments about it [at Rostock]. It was absolutely impossible to understand. This is also quite clear, one needed not only the new quantum theory, but also the magnetic electron. These two things weren't there yet at the time. ... I still have objections to the idea of beauty of quantum mechanics. But she is correct.

Walther Gerlach would emphasize that [18], p. 121:

[The proof] of space quantization was the first experiment that made an atomic state predicted by quantum theory accessible to a direct measurement.

The first published reaction came from Stern's mentor **Albert Einstein** who, in the wake of the SGE, teamed up with **Paul Ehrenfest** only to express more puzzlement about the workings of the SGE [37]:

The difficulties spelled out [herein] show how unsatisfactory [our] attempts at interpreting the results found by Stern and Gerlach are.

We add that although Gerlach ended up doing the experiment mostly by himself, Einstein and Ehrenfest coined the term Stern-Gerlach experiment rather than Gerlach-Stern experiment, in recognition of the fact that the experiment was Stern's idea. In his letter to Max Born, Einstein underscored once more his frustration at being

out of his depth in the face of the SGE, while expressing confidence in the experiment's outcome [38], p. 103:

The most interesting achievement at this point is the experiment of Stern and Gerlach. The alignment of the atoms without collisions via radiative [exchange] is not comprehensible based on the current [theoretical] methods; it should take more than 100 years for the atoms to align. I have done a little calculation about this with [Paul] Ehrenfest. [Heinrich] Rubens considers the experimental result to be absolutely certain.

In his letter to Gerlach [39], **Friedrich Paschen** extolled the significance of the SGE for quantum theory:

Your experiment proves for the first time the reality of Bohr's [atomic] states.

For where there is space quantization of angular momentum, there must also be quantization of angular momentum. In keeping with this implication of the SGE, **Wolfgang Pauli** quipped:

This should convert even the nonbeliever Stern.

One may wonder why the value of the projection quantum number $M = 0$ was left out of Stern's original considerations. There doesn't seem to be a clear answer. However, after the completion of the experiment, **Niels Bohr** wrote to Gerlach [40]:

I would be very grateful if you or Stern could let me know, in a few lines, whether you interpret your experimental results in this way that the atoms are oriented only parallel or opposed, but not normal to the field, as one could provide theoretical reasons for the latter assertion.

We note that the $L = 0$ value of the orbital angular momentum was excluded from the Bohr model as it would lead to the collapse of the atom.

In 1927, **Isidor Rabi** came to Europe as a Barnard Fellow (later Rockefeller Fellow) and worked intermittently with Sommerfeld, Heisenberg, Bohr, and Pauli – the last in Hamburg, where Rabi succumbed to the lures of Otto Stern's molecular beam laboratory. Rabi [41]:

As a beginning graduate student back in 1923, I ... hoped with ingenuity and inventiveness I could find ways to fit the atomic phenomena into some kind of mechanical system ... My hope to [do that] died when I read about the Stern-Gerlach experiment. ... The results were astounding, although they were hinted at by quantum theory ... This convinced me once and for all that an ingenious classical mechanism was out and that we had to face the fact that the quantum phenomena required a completely new orientation.

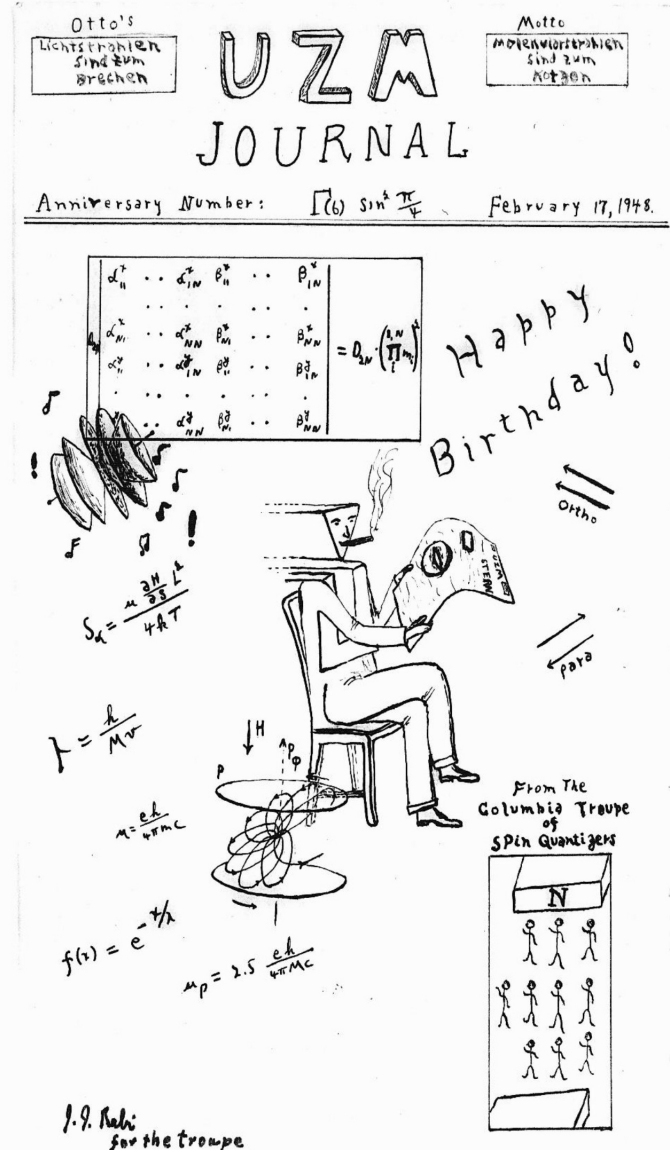


FIG. 12. A drawing by Isidor Rabi presented to Otto Stern on his 60th birthday, on 17 February 1948. The drawing features a smorgasbord of milestone achievements of Stern's Hamburg group in the format of a page from an "Anniversary Number" of a fictitious journal dedicated to publishing Stern's molecular beam research. The name of the journal, "UZM," is an allusion to the series entitled *Untersuchungen zur Molekularstrahlmethode* (UzM) [Investigations by the Molecular Beam Method] of thirty numbered papers published between 1926 and 1933 by Stern's Hamburg institute in *Zeitschrift für Physik*. Otto's Motto "Lichtstrahlen sind zum brechen, Molekularstrahlen sind zum kotzen" is an affectionate "secret code" between Stern and Gerlach from their Frankfurt time – a pun expressing their occasional disgust with the difficult atomic/molecular beam experiments [*Brechen* means refraction as well as vomiting; *Kotzen* is a vulgar word for vomiting. A free translation, without the pun, would be: Light beams refract, atomic beams disgust.] To which anniversary the journal's "Number" [issue] refers is expressed in terms of a product of the gamma function $\Gamma(6) = 120$ with $\sin^2(\pi/4) = 1/2$. The central cigar-smoking figure (Stern himself?) whose head and body took the shape of the edge and groove of the Stern-Gerlach magnet reads another issue of the UZM Journal showcasing an image of a split molecular beam. Also included are references to the magnetic dipole moments of the electron and the proton and to the de Broglie wavelength of matter waves, whose experimental verification, within 1% accuracy, Stern considered his greatest contribution to physics. The "Happy Birthday" wish comes from the "Columbia Troupe of Spin Quantizers," on whose behalf it is signed by the troupe's leader and the drawing's author, I.I. Rabi. Reproduced from Ref. [18], p. 238.

As Rabi’s student Norman Ramsey noted [42],

Rabi’s work in Stern’s laboratory was decisive in turning his interest toward molecular beam research.

This is what Rabi said about Hamburg during the Pauli-Stern era:

When I was at Hamburg University, it was one of the leading centers of physics in the world. There was a close collaboration between Stern and Pauli, between experiment and theory ... Further, Stern’s and Pauli’s presence attracted many illustrious visitors to Hamburg. Bohr and Paul Ehrenfest were frequent visitors ... From Stern and from Pauli I learned what physics should be. For me it was not a matter of more knowledge – I learned a lot of physics as a graduate student. Rather, it was the development of taste and insight; it was the development of standards to guide research, a feeling for what is good and what is not so good. Stern had this quality of taste in physics and he had it to the highest degree. As far as I know, Stern never devoted himself to a minor question.

Rabi became Stern’s principal correspondent on the topic of molecular beams and, upon Stern’s forced emigration in 1933, it was Rabi’s laboratory at Columbia University that took over from Stern in spearheading pace-setting molecular beam research [43].

Not everyone was unfazed by the outcome of the SGE. Stern’s and Gerlach’s Frankfurt colleague, **Alfred Landé**, who had worked on unriddling the anomalous Zeeman effect since 1919, provided a prescient interpretation of the SGE based on his understanding of the electronic structure of atoms [44, 45]. Upon his entry into the fray, Landé gradually modified the sets of quantum numbers introduced by Bohr, Sommerfeld, and Debye to characterize atomic states. By adapting the concept of vector addition of angular momenta to the case of quantized electronic angular momenta of atoms, Landé came up with an organizing principle that made it possible to capture both the patterns and the subtleties of atomic Zeeman spectra amassed on the eve of the discovery of electron spin in 1925. This organizing principle was based on the g -factor, whose preliminary form Landé introduced in 1921 and kept refining until 1923. In the process, Landé attributed both integer and half-integer values to the quantum number R characterizing the angular momentum of the atomic core, with projections $m_R = -R, -R + 1, \dots, +R$ on a magnetic field and concluded that the g -factor of a doublet with $R = \frac{1}{2}$ was equal to 2. The accuracy of Landé’s g -factor (within 1%) served as a reliable guide to Pauli on his path to reassigning Landé’s half-integer quantum number of the core to the *outer* electron – and henceforth to the *exclusion*

principle. Whereupon Samuel Goudsmit and George Uhlenbeck realized that the half-integer quantum number of the electron must correspond to the electron’s “additional degree of freedom” [46], i.e., to its inner angular momentum – spin – characterized by the quantum number $S = \frac{1}{2}$ and the projection quantum number $m_S = \pm \frac{1}{2}$. In relation to Landé’s g -factor, Uhlenbeck’s and Goudsmit’s discovery of electron spin required replacing R with S .

When the SGE came about, Landé tackled it as a manifestation of the anomalous Zeeman effect (AZE). After all, it was the “number mystery” [47] of the AZE that led Sommerfeld and Debye to introduce the notion of space quantization of angular momentum in the first place ... Thus, unlike Bohr, Sommerfeld, and pretty much everybody else, Landé, with his theory of the AZE, would not be fooled: he noted that had the silver atoms been in a one-quantum state (i.e., $L = 1$), the silver beam would be split into three beams, corresponding to $m_L = -1, 0, +1$. However, since splitting into only two beams was observed in the SGE, Landé concluded that the silver atoms must have been in a doublet state, with $L = 0$ and $R = \frac{1}{2}$ and thus $m_R = \pm \frac{1}{2}$. The deflection would then correspond to $1 \mu_B$ on account of the doublet’s gyromagnetic ratio $g = 2$, $\mu = 2\mu_B m_R = \pm \mu_B$, as observed in the SGE, cf. Sec. V.

Despite his daily contact with Stern and Gerlach during the period 1920-1922 when they labored on the SGE at Frankfurt’s Institute for Theoretical Physics, Landé’s interpretation of the SGE was barely noticed by anyone within the Institute or without.

Strangely enough, it was as late as 1937 when **Ronald Fraser** determined that the ground-state orbital angular momentum and the associated magnetic moments of silver, hydrogen, and sodium were zero [48] and thus the doublet splitting seen in the SGE had to be attributed to spin. Electron spin had not been mentioned or the term symbols [49] used in Stern’s and Gerlach’s subsequent work on magnetic deflection of atoms and molecules, see Sec. VII, with only one exception: in the 1933 paper on magnetic deflection of oxygen molecules [50], it is noted that the ground state of O_2 is a $^3\Sigma$ state.

VII. ADDITIONAL SGE-TYPE EXPLORATIONS BY GERLACH AND STERN

Upon submitting for publication their full-length, 28-page paper on space quantization in the Spring of 1924 [20], Gerlach continued exploring the magnetic properties of atoms and their space quantization in an SGE-type experiment. TABLE I gives a summary of the atomic species investigated, their term symbols and magnetic properties as we understand them today, and Gerlach’s inferences from his experimental findings presented in his 37-page 1925 paper [25]. For this exploratory study, Gerlach built a 4th-generation SGE apparatus capable of producing quality data (beam images) reliably and in half the time needed using the 3rd-generation instrument.

TABLE I. Magnetic properties of Group 10 to 15 atoms explored by Gerlach with a 4th-generation SGE apparatus [25]. In modern notation, the ground state of each atom is characterized by the term symbol $^{2S+1}L_J$. Note that the maximum expected deflection is proportional to the magnetic dipole moment $\mu_Z = -Jg\mu_B$, where g is the g -factor [36, 44, 45], J the total angular momentum quantum number, and μ_B the Bohr magneton. See text.

Atom	Term Symbol	Number of M states	g -factor	Maximum μ_Z/μ_B	Gerlach's inference
Fe	5D_4	9	3/2	6	Appears diamagnetic
Bi, Sb	$^4S_{3/2}$	4	2	3	No deflection due to molecule formation?
Cu, Au	$^2S_{1/2}$	2	2	1	“Regular” deflection
Ni	3F_4	9	5/4	5	Zero deflection for $M = 0$ and significant deflection for $M \neq 0$
Pb, Sn	3P_0	1	Undefined	0	No deflection
Tl	$^2P_{1/2}$	2	2/3	1/3	“Tiny” deflection

TABLE II. Magnetic and electric deflection experiments conducted at Otto Stern’s institute in Hamburg and Pittsburgh. The magnetic deflections of the isotopologs of the hydrogen molecule were related to the magnetic dipole moments μ_P and μ_D of the proton and the deuteron. The nuclear magneton $\mu_n \approx \mu_B/1836$. See text.

Deflected atom or molecule	Magnetic dipole moment inferred	Electric dipole moment inferred	Reference
H ₂ O	$\approx 1/1000 \mu_B$	–	[51]
Hg	inconclusive	–	[51]
Na, K	$1 (\pm 4\%) \mu_B$	–	[52]
Tl	$0.3 (\pm 4\%) \mu_B$	–	[52]
H	1	–	[53]
NaI, KI, TlI	–	≈ 10 Debye	[54]
CsCl, RbBr	–	$\ll 10$ Debye	[54]
Bi	$0.75\text{--}0.85 \mu_B$	–	[55]
Li	$0.96\text{--}1.04 \mu_B$	–	[56]
K	$1 (\pm 5\%) \mu_B$	–	[57]
O ₂	$0.3\text{--}2.0 \mu_B$	–	[50]
H ₂	$\mu_P \approx 2.5\mu_n$	–	[58, 59]
D ₂ , HD	$\mu_D \approx 0.7\mu_n$	–	[60, 61]

Perhaps the most striking result Gerlach obtained with his advanced SGE-type apparatus was the deflection pattern for a beam of nickel atoms [25], see FIG. 13. Apart from the image of deflected atoms, Gerlach also saw undeflected atoms that had nevertheless passed through the same inhomogeneous field as the deflected ones. As

the current understanding suggests, the deflected Ni(3F) atoms were in the $M = \pm 1$ state and the undeflected ones in the $M = 0$ state. However, an open question remains: the ground state of nickel, with $S = 1$ and $L = 3$, admits $J = 4, 3, 2$ and thus $|M| = 4, 3, 2, 1, 0$. Where are the atoms with $|M| = 4, 3, 2$?

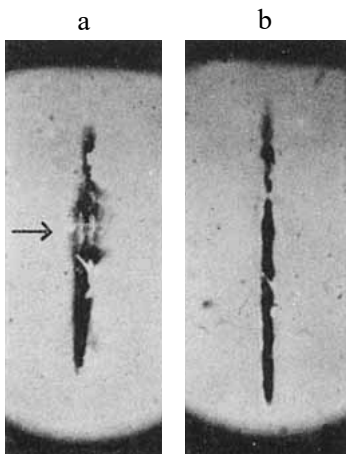


FIG. 13. Images of the deflection patterns for nickel (a) in an inhomogeneous magnetic field of 200 kG/cm and (b) without a field. The maximum deflection is about $180\ \mu\text{m}$ as compared with $120\ \mu\text{m}$ for silver in the 4th generation apparatus. This is the only image published by Gerlach that shows both deflected and undeflected atoms that passed through the same magnetic field. The undeflected atoms have $M = 0$. The arrow marks the maximum inhomogeneity of the magnetic field. See also text and TABLE I. Reproduced from Ref. [25].



FIG. 14. Image of the deflection pattern for iron atoms in an inhomogeneous magnetic field of 200 kG/cm. No deflected Fe atoms were detected. The arrow marks the maximum inhomogeneity of the magnetic field. See also text and TABLE I. Reproduced from Ref. [25].

A similar question arises for the deflection pattern Gerlach observed for iron atoms, whose ground state, 5D , has $S = 2$ and $L = 2$ and thus admits $J = 4, 3, 2, 1, 0$. However, Gerlach observed no deflected atoms at all, see FIG. 14, as if all the iron atoms $\text{Fe}(^5D)$ were in an $M = 0$ state. Assuming that the Fe atoms formed Fe_2 molecules would not help explain the observed lack of magnetic deflection as the ground state of Fe_2 is the highly paramagnetic $^9\Sigma_g$ state [62–64].

Otto Stern, too, undertook additional deflection experiments with a much-improved apparatus, see TABLE II.

Between 1926 and 1928, Stern and his co-workers at Stern’s *Institut für physikalische Chemie der Hamburgischen Universität* investigated magnetic deflection of water molecules (and concluded that the magnetic moment involved was on the order of the nuclear magneton) and Hg atoms [51], K, Na, and Tl atoms [52], hydrogen atoms [53], Bi atoms [55], Li atoms (with an estimate of the nuclear moment) [56], and, again, K atoms [57]. Interestingly, the magnetic deflection pattern of Bi atoms found in Stern’s laboratory was more or less consistent with the behavior of a 4S state, unlike Gerlach’s experiment that showed no deflection, cf. TABLE I. Stern’s group also carried out analogous experiments in an inhomogeneous electric field to determine the deflection patterns due to the electric dipole moments of KCl, KI, TlI, NaI, CsCl, and RbBr molecules [54]. In none of these papers, the spin, orbital, and total angular momentum quantum numbers had been assigned (or even mentioned).

In 1933, “with the sword of Nazism hanging over their heads” [65], Stern et al. resumed magnetic deflection experiments to examine the magnetic properties of oxygen molecules [50] and, more importantly, of hydrogen [58, 59] and deuterium molecules [60], with the goal of determining the magnetic dipole moments of the proton and the deuteron. These experiments could not be properly concluded because of “external circumstances” [60], i.e., the Nazi racial laws that led to the dismissal of Stern’s Jewish coworkers and Stern’s own emigration. One of the shortcuts taken was that the molecular beams were not velocity-selected. This contributed to the deviation of the values obtained by Stern et al. for the magnetic moment of the proton and deuteron by about 10% from today’s values of $2.793\ \mu_n$ and $0.855\ \mu_n$, respectively, cf. TABLE II and CODATA.

Otto Stern and his co-workers had thus provided unequivocal evidence that the proton has an internal structure and, unlike the electron, is not a point-like particle. Moreover, Stern’s finding that the deuteron has a smaller magnetic dipole moment than the proton [61] indicated that the neutron possessed a magnetic dipole moment as well, one oriented oppositely to that of the proton. Today we know that the magnetic dipole moment of the neutron is $-1.913\ \mu_n$, which implies that the neutron has an internal electric charge distribution that, however, perfectly “neutralizes itself” on the outside, as a neutron consists of one up quark and two down quarks.

Stern’s Nobel citation singles out this eleventh-hour work, cf. FIG. 1.

VIII. QUANTUM THEORY OF THE STERN-GERLACH EXPERIMENT

FIG. 11 provides a glimpse of the quantum theory of the internal atomic states involved in the SGE. The entanglement of the internal degrees of freedom with the translational ones was touched upon already in Werner Heisenberg’s 1927 “uncertainty principle” paper

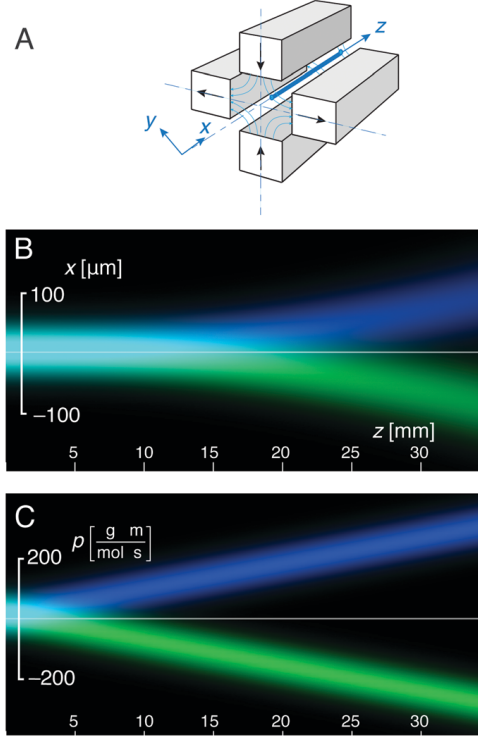


FIG. 15. (A) Magnetic field gradient generated by a quadrupole arrangement of permanent magnets. The beam path indicated (solid blue line) passes through a region where the magnetic field is parallel to the y-axis, but varies in magnitude along the x-axis (uniaxial field gradient whose field lines are all parallel but vary in density in the direction perpendicular to the magnetic field). This magnetic field satisfies Maxwell's equation, cf. FIG. 11. (B) Projections of the Wigner matrix elements onto the spatial axis as a function of position along the beam path through the Stern-Gerlach apparatus. (C) Projections of the Wigner matrix elements onto the transverse momentum dimension along the beam path through the apparatus. Note that in this figure, the space-fixed axes x, y, and z are defined such that y is the quantization axis and z is the beam propagation direction. Reproduced from Ref. [66] with permission from the authors.

[67] (however, without calling it entanglement). David Bohm continued in the same vein in his 1951 book where he described the translation of the spin-carrying atoms in the SGE as a wavepacket. He noted that the spreading of the wavepacket on the way to the detector must be less than the atoms' spin-dependent deflection and that the minimum spreading of the wavepacket was given by the uncertainty principle [68]. Bohm and subsequently Eugene Wigner elevated the SGE to the ultimate exemplar of quantum measurement, where he characterized the SGE as an illustration of "the statistical correlation between the state of the 'apparatus' (the position coordinate) and the state of the object (the spin) ..." [69], Wigner wrote:

This shows that the state of the system – object-plus-apparatus (spin and positional coordinates of the particle, i.e., the whole state of the particle) – shows characteristics which neither of the separated beams alone would have [had]. If the two beams [were] brought together by [a] magnetic field ..., the two beams [would] interfere and the spin [would] be vertical again [like it was before

the splitting]. This could be verified by letting the [re]united beam pass through a second magnet ...

In their 1987 analysis of the SGE, Marlan Scully, Willis Lamb, and Asim Barut derived approximate analytic expressions for the expectation values of the atoms' spatial coordinates [70]. The possibility of recombining the two beams split by the Stern-Gerlach magnet and their subsequent interference was revisited in 1988 by Julian Schwinger, Marlan Scully, and Berthold-Georg Englert [71]. Likening the passage of the atoms through the Stern-Gerlach magnet to the "great fall" of Humpty-Dumpty, they concluded, like Wigner did before them, that there were "technical and fundamental limitations on the realizability of the [Stern-Gerlach interferometer], i.e., that one "couldn't put Humpty-Dumpty together again" [72].

Although in the air throughout, Erwin Schrödinger's notion of entanglement [73, 74] appears to have been explicitly used in connection with the SGE only as late as 1999 by Gilbert Reinisch [75]. Most recently, John Briggs applied the 1937 imaging theorem of Edwin Kemble [76] to the motion of atoms over macroscopic dis-

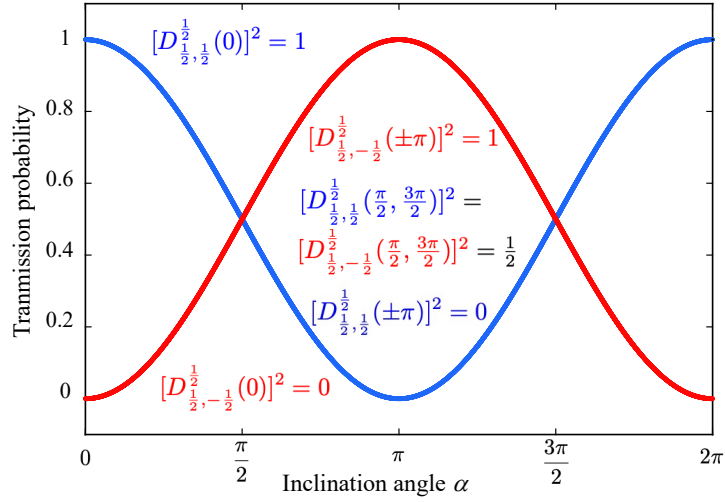


FIG. 16. Probabilities that the $|S = \frac{1}{2}, M_S = \frac{1}{2}\rangle$ state prepared by the polarizer will be transmitted by the analyzer as either the $|S = \frac{1}{2}, M_S = \frac{1}{2}\rangle$ (blue) or $|S = \frac{1}{2}, M_S = -\frac{1}{2}\rangle$ (red) spin state. The polarizer and analyzer are Stern-Gerlach magnets that are rotated with respect to one another about the direction of the undeflected beam by an inclination angle α . See text.

tances such as those encountered in the SGE and concluded that the perception of classical (trajectories) or quantal (wavepackets) behavior depends on the accuracy of detecting the atoms' motion. Briggs' treatment is based on the assumption that the wavefunction always describes a statistical ensemble of identically-prepared particles and that no meaning can be ascribed to the wavefunction of a single particle [77, 78]; it leads to the conclusion that the quantum-to-classical transition occurs due to a unitary evolution of the wavefunction. In contrast, decoherence theory [79–82] assumes a single-particle wavefunction whose evolution is non-unitary due to environmental effects and whose collapse leads to a particular outcome of a measurement. See also Ref. [83].

Hendrik Ulbricht and his coworkers made use of the extended Wigner probability density function [84, 85] to represent the propagation of the total wavefunction through the Stern-Gerlach apparatus [66]. The matrix elements of the Wigner probability density function depend, apart from the spin variables, on the spatial variables and their associated momenta. Ulbricht et al. have found that the dephasing of the off-diagonal elements of the Wigner probability density function is entirely due to the quantum dynamics and thus does not require spin relaxation or any other type of dissipation in order to reproduce the outcome of the SGE. The results of the quantum simulations based on the Wigner function for the parameters of the original SGE [20] are summarized in FIG. 15 reproduced from Ref. [66]. Note that due to the constant, equal and opposite forces acting upon the two polarization states, $|\frac{1}{2}, \frac{1}{2}\rangle$ and $|\frac{1}{2}, -\frac{1}{2}\rangle$ (blue and green), their spatial separation increases quadratically whereas their transverse momenta grow linearly along the beam path. The final separation of the states at 3.5 cm along their path through the magnetic field amounts to be consistent with Stern and Gerlach's observation.

What about multiple Stern-Gerlach experiments? Their analysis (as well as that of a single SGE) has become textbook material, see, e.g., [86, 87]. For a pair of Stern-Gerlach magnets (termed polarizer and analyzer) rotated about the direction of the undeflected beam by an inclination angle α with respect to one another, the transmission probability of an $|S, M_S\rangle$ state through the combined contraption is given by

$$|\langle S, M_S | S M'_S \rangle|^2 = [D_{M'_S, M_S}^S(\alpha)]^2 \quad (8)$$

where $D_{M'_S, M_S}^S(\alpha)$ is the Wigner rotation matrix [86, 87]. Eq. (8) gives the dependence on α of the probabilities $|\langle \frac{1}{2}, \frac{1}{2} | \frac{1}{2}, \frac{1}{2} \rangle|^2 = \cos^2 \frac{\alpha}{2}$ (blue) and $|\langle \frac{1}{2}, \frac{1}{2} | \frac{1}{2}, -\frac{1}{2} \rangle|^2 = \sin^2 \frac{\alpha}{2}$ (red) that the $|S = \frac{1}{2}, M_S = \frac{1}{2}\rangle$ state prepared by the polarizer will pass the analyzer as either the $|S = \frac{1}{2}, M_S = \frac{1}{2}\rangle$ or $|S = \frac{1}{2}, M_S = -\frac{1}{2}\rangle$ state. For $\alpha = 0$, the $|\frac{1}{2}, \frac{1}{2}\rangle$ state is just “remeasured” by the analyzer, yielding the same $|\frac{1}{2}, \frac{1}{2}\rangle$ state upon transmission, whereas the $|\frac{1}{2}, -\frac{1}{2}\rangle$ state (red) is not transmitted at all. As α increases, $|\frac{1}{2}, \frac{1}{2}\rangle$ is transmitted less and $|\frac{1}{2}, -\frac{1}{2}\rangle$ more until $\alpha = \frac{\pi}{2}$ is reached where the two states are transmitted with the same probability $\langle \frac{1}{2}, \frac{1}{2} | \frac{1}{2}, \frac{1}{2} \rangle^2 = \langle \frac{1}{2}, \frac{1}{2} | \frac{1}{2}, -\frac{1}{2} \rangle^2 = \frac{1}{2}$. At $\alpha = \pi$, the polarizer and analyzer are said to be crossed, in which case the blue state which was transmitted with certainty at $\alpha = 0$ is blocked altogether while it is the turn of the red state to be “remeasured” and transmitted with certainty through the analyzer.

It was none other than Otto Stern's mentor, Albert Einstein, who pointed out to Stern in 1928 [88, pp. 128–131] that, say, the blue state will be transmitted even if the analyzer-polarizer system were crossed ($\alpha = \pi$) provided the state underwent a spin-flip on its way from the polarizer to the analyzer. Apparently, Einstein had continued mulling over the SGE, in keeping with his quip that “On quantum theory I use up more of my brains

Aspect ... Dalibard→**Cohen-Tannoudji**→**Kastler ... Stern/Rabi**
Bloch ... Stern/Rabi
Bloembergen→**Purcell**→**Stern/Rabi**
Chu→**Commins**→**Stern/Rabi**
Clauser→**Thaddeus**→**Townes ... Stern/Rabi**
Cohen-Tannoudji→**Kastler ... Stern/Rabi**
Cornell→**Pritchard**→**Kleppner**→**Ramsey**→**Stern/Rabi**
Glauber→**Schwinger ... Stern/Rabi**
Hänsch→**Schawlow**→**Townes ... Stern/Rabi**
Haroche→**Cohen-Tannoudji**→**Kastler**
Cornell→**Pritchard**→**Kleppner/Ramsey**→**Stern/Rabi**
Herschbach ... Ramsey→**Stern/Rabi**
Kastler ... Stern/Rabi
Ketterle→**Pritchard**→**Kleppner**→**Ramsey**→**Stern/Rabi**
Kusch→**Stern/Rabi**
Lamb ... Stern/Rabi
Phillips→**Kleppner**→**Ramsey**→**Stern/Rabi**
Purcell ... Stern/Rabi
Ramsey→**Stern/Rabi**
Schawlow→**Townes ... Stern/Rabi**
Schwinger ... Stern/Rabi
Townes ... Stern/Rabi
Weiss→**Zacharias**→**Stern/Rabi**
Wieman→**Hänsch**→**Schawlow**→**Townes ... Stern/Rabi**
Wineland→**Ramsey**→**Stern/Rabi**
Zeilinger→**Rauch**→**Bonse**→**Kappler**→**Gerlach**

FIG. 17. Summary of links between Physics Nobel Laureates and Stern/Rabi and Gerlach. The arrow (→) indicates “a student or post-doc of” and dots (...) indicate “some other association.” Nobel laureates are in boldface. Adapted from a table by Daniel Kleppner, Ref. [43].

[Hirnschmalz] than on relativity” [89]. This idea, whose variant was implemented by Stern and his coworkers [90, 91], would be later developed by Isidor Rabi [92, 93] into his molecular beam resonance method, see Refs. [4] for a full-length historical account and [94, 95] for ongoing work.

Over the past decade, Ron Folman and his group, have demonstrated experimentally that the Stern-Gerlach splitting of a beam of freely propagating atoms subject to macroscopic magnets is a “fully coherent quantum process” [96, 97] and implemented a full-loop Stern-Gerlach matter-wave interferometer. A key element of their setup is an atom-chip beam splitter that makes use of a minimum-uncertainty atomic sample derived from a Bose-Einstein condensate of atoms (^{87}Rb) that are state-prepared in an equal superposition of two hyperfine Zeeman states. Upon subjecting this superposition to an accurately controllable magnetic field pulse (both in magnitude and direction) generated by the atom chip wires, the state of the atoms becomes a coherent superposition of two distinct momentum states. Each is due to an internal-state-dependent Zeeman force imparted over a precisely controlled time interval during which the magnetic field gradient is on. These momentum states are then allowed to spatially separate, which concludes the

momentum and spatial splitting of the original state-prepared atomic sample.

In the half-loop Stern-Gerlach interferometer (SGI) arrangement [98, 99], the spin of the two wavepackets is then flipped to be the same. A second magnetic field pulse, whose gradient is different for the two momentum states (due to their different distance from the chip) is timed such that the momentum difference between the two components would vanish and the space-time paths of the atoms would become parallel as a result, see Fig. 2 of Ref. [96]. In the vertical (1D, longitudinal) arrangement of their experiment, Folman et al. let the separated wavepackets freely propagate under gravity and eventually overlap as they spread. Thereby a spatial interference pattern analogous to that of a double-slit experiment is generated, see Fig. 3 of Ref. [96].

A full-loop SGI is realized by recombining the wavepackets in a fashion reminiscent of a Mach-Zehnder interferometer [100] using four magnetic gradient pulses, see Fig. 4 of Ref. [96]. Even in the absence of environmental decoherence, in order to maintain spin coherence at the recombination point, the wavepackets have to be brought together with a spatial, ΔZ , and momentum, ΔP_Z , precision such that $\Delta Z \ll \sigma_Z$ and $\Delta P_Z \ll \sigma_P$, where σ_Z and σ_P are the corresponding uncertainties of



FIG. 18. Reenactment of the “cigar episode” by Dudley Herschbach (right) and the author in 2003 [35]. While Friedrich holds a glass slide silver-coated under vacuum and just taken out of the vacuum chamber vented with dry nitrogen, Herschbach blows sulfurous cigar breath onto the slide to test his hearing (or Otto Stern’s telling) of the story. The reenactment revealed that the silver deposit requires exposure to cigar smoke (not simply sulfurous breath) to form any visible contrast between the masked (light) part of the slide – shaped in the form of the magnet pole pieces – and the outer (dark) part of the slide exposed to the smoke (see inset). Merely exhaling sulfurous breath on a slide turned out to have no discernible effect. But exposure to cigar smoke quickly blackened the regions of the slide outside the mask, within a few seconds to a few minutes depending on whether the dose of smoke was profuse or mild. We think it likely that Stern did have a cigar in hand while Gerlach, busy venting the apparatus and removing the plate, was without his typical cigar. Photo credit: Doo Soo Chung and Sunil Sheth.

the original wavepackets before splitting that fulfill the uncertainty relation $\sigma_Z \sigma_P \geq \hbar/2$. Thus the recombination of the macroscopic positions and momenta has to be implemented with microscopic precision. In the experiments of Folman et al., the maximal achieved splittings in position and momentum were $4\sigma_Z$ and $60\sigma_P$, respectively [97]. Thus, Humpty-Dumpty *can* be put together again if all its [Humpty-Dumpty was an egg] pieces are matched accurately enough.

We note that the Stern-Gerlach-type macroscopic-object interferometry has been explored at least since 1990s [101, 102] but the ultimate success was only achieved in 2019 [103].

IX. CONCLUSIONS

Much of quantum mechanics as we know it is embodied in the Stern-Gerlach experiment. Whether quantum measurement, state preparation, coherence, or entanglement – apart from the quantization of angular momentum and its spatial projections – the SGE has it all. Its conceptual clarity has made the SGE a quintessential prototype for our thinking about quantum systems and the quantum-classical correspondence.

The legacy of Stern’s and Gerlach’s molecular beam work in general and of the Stern-Gerlach experiment in particular is aptly reflected in the Physics Nobel prizes awarded so far for work in atomic, molecular, optical, and chemical physics. This is illustrated in a list shown in FIG. 17 of Nobel laureates scientifically related to Rabi,

Stern, and Gerlach.

The execution of the SGE also contributed to the treasure trove of stories on how physics experiments work: More than 60 years ago, Otto Stern described to Dudley Herschbach the first sighting of the silver beam deposit on the collector plate of the Stern-Gerlach apparatus. Stern’s explanation, in paraphrase, was [35]:

After venting to release the vacuum, Gerlach removed the collector plate. But he could see no trace of the silver atom beam and handed the plate to me. With Gerlach looking over my shoulder as I peered closely at the plate, we were surprised to see gradually emerge the trace of the beam ... Finally we realized what [had happened]. I was then the equivalent of an assistant professor. My salary was too low to afford good cigars, so I smoked bad cigars. These had a lot of sulfur in them, so my breath on the plate turned the silver into silver sulfide, which is jet black, so easily visible. It was like developing a photographic film.

The cigar episode was reenacted 20 years ago by Dudley Herschbach and the author, see FIG. 18.

From the SGE unfolded novel perspectives as well as wide-ranging and far-reaching applications. Among them are the prototypes for nuclear magnetic resonance, optical pumping, the laser, and atomic clocks, as well as incisive discoveries such as the magnetic moment of the proton and deuteron that ushered in nuclear physics or

the Lamb shift and the anomalous increment in the magnetic moment of the electron, which launched quantum electrodynamics [104].

In the 1960s, the molecular beam technique made inroads into chemistry as well, by enabling the study of elementary chemical reactions as single binary collisions of chemically well-defined reagents in the gas phase [105], elementary reactions on solid surfaces [106], as well as time-resolved studies of chemical bond making & breaking [107]. The ensuing field of chemical reaction dynamics has remained one of the chief preoccupations of chemical/molecular physics to date [108, 109]. While the refinement of mass-spectrometric techniques enabled conclusive elucidations of complex reaction mechanisms in the gas phase [110, 111], the development of soft desorption ionization beam methods (electrospray) [112] led to biological and medical applications of molecular beams [113, 114]. Spectroscopy of molecular species loaded into or produced within superfluid helium nanodroplets [115–117] provided new incisive means to elucidate their structure, reactivity, and solvation [118–120].

In the 1990s, a renaissance began in atomic physics, nurtured by the development of techniques to cool and trap atoms [121–123]. Based on a combination of molecular beams with laser cooling, these techniques enabled the realization of quantum degeneracy in atomic gases [124–128], optical manipulation of quantum systems [129–131], addressing foundational questions [132], examination and quantum simulation of condensed-matter systems [133–140], prototype quantum computers [141, 142] and, last but not least, they transformed metrology [143], including the testing of aspects of gravity [96, 144].

Over the past decade, also laser cooling of molecules has been demonstrated [145], refined [146–152], and is poised for use in tests of fundamental symmetries and searches for dark matter [153–155].

Quantum mechanics surely owes more to the Stern-Gerlach experiment than the Stern-Gerlach experiment to quantum mechanics.

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