Dynamics of a Stochastic Echinococcosis infection Model

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Abstract This paper is concerned a novel spreading dynamical model for Echinococcosis with stochastic parameter perturbation. we show that there exist a unique positive solution of the stochastic model. Sufficient conditions for the stationary distribution which is ergodic is established by appropriate Lyapunov functions. Furthermore, we obtain the conditions on which the system will extinct. Finally, we illustrate our results by numerical simulation.

Key words Stochastic Echinococcosis model; Lyapunov function; Ergodicity; Extinction.

1 Introduction

Echinococcosis, which is often referred to as a hydatid disease, is a parasitic disease that affects both humans and other mammals, such as sheep, dogs rodents and horses[1]. There are three different forms of Echinococcosis found in humans, where the most common form found in humans is cystic Echinococcosis. Thus, we focus on cystic Echinococcosis model in this paper.

China is one of the countries in the world which have the most serious Echinococcosis. A national survey on the prevalence of Echinococcosis in 2012 showed that China's 350 counties had Echinococcosis case reports that mainly distributed in Northwestern pastoral regions. At least 50,000,000 people in China are threatened by Echinococcosis [2].

Mathematical models can be used to how an infectious disease spreads in the real world, and how various complexities affect the dynamics. There have been some modeling studies on different aspects of Echinococcosis, and most of them are statistical models, for example, [3]-[5]. Recently, dynamic models of Echinococcosis have been discussed in [6]-[8]. The work [6] proposed a deterministic model to study the transmission dynamics of Echinococcosis in Xinjiang, China. The authors showed that Echinococcosis was endemic in Xinjiang, China with the current control measures. In [7] two mathematical models, the baseline model and the intervention model, were proposed to study the transmission dynamics of Echinococcosis, the authors showed that the infection of Echinococcosis was in an endemic state.

Of particular interest to us is a model in[8], which is given by the following dynamical model of Echinococcosis with distributed time delays. The variables in the model are defined in Table 1, and the parameters are defined in Table 2.

$$\begin{cases} dS_D &= [A_1 - \beta_1 S_D \int_0^{h_1} f_1(\tau) I_L(t-\tau)d\tau - d_1 S_D + \sigma I_D]dt, \\ dI_D &= [\beta_1 S_D \int_0^{h_1} f_1(\tau) I_L(t-\tau)d\tau - (d_1+\sigma) I_D]dt, \\ dS_L &= [A_2 - \beta_2 S_L \int_0^{h_2} f_2(\tau) I_D(t-\tau)d\tau - d_2 S_L]dt, \\ dI_L &= [\beta_2 S_L \int_0^{h_2} f_2(\tau) I_D(t-\tau)d\tau - d_2 I_L]dt, \\ dS_H &= [A_3 - \beta_3 S_H \int_0^{h_3} f_3(\tau) I_D(t-\tau)d\tau - d_3 S_H + \gamma I_H]dt, \\ dE_H &= [\beta_3 S_H \int_0^{h_3} f_3(\tau) I_D(t-\tau)d\tau - (d_3+\omega) E_H]dt, \\ dI_H &= [\omega E_H - (d_3+\mu+\gamma) I_H]dt. \end{cases} \tag{1.1}$$

Where τ be the random variable that describes the time between infection offal thrown in the environment and eaten by dogs with a probability $f_1(\tau)$. h_1 denotes the average survival time of larval cysts into the infection offal thrown in the environment. the number of infected dogs at time t is given by

$$\beta_1 S_D \int_0^{h_1} f_1(\tau) I_L(t-\tau)d\tau.$$

Analogously, the numbers of infected livestock and infected human at time t are given by $\beta_2 S_L \int_0^{h_2} f_2(\tau) I_D(t-\tau)d\tau$ and $\beta_3 S_H \int_0^{h_3} f_3(\tau) I_D(t-\tau)d\tau$, respectively.

The results showed that the dynamical properties of the model is completely determined by the basic reproduction number $R_0(R_0^2 = \frac{\beta_1\beta_2 A_1 A_2}{d_1 d_2^2(d_1+\sigma)})$. That is, if $R_0 < 1$, the disease-free equilibrium is globally asymptotically, and if $R_0 > 1$, the model is permanent and the endemic equilibrium is globally asymptotically stable.

However, in the natural world, epidemic models are inevitably subject to the environmental noise, which is an important component in an ecosystem, see [9]-[12]. In this paper, we will consider a type of environmental noise, namely white noise. Then the stochastic version corresponding to system (1.1) can be described by the following equations:

$$\begin{cases} dS_D &= [A_1 - \beta_1 S_D I_L - d_1 S_D + \sigma I_D]dt + \sigma_1 S_D dB_1(t), \\ dI_D &= [\beta_1 S_D I_L - (d_1+\sigma) I_D]dt + \sigma_2 I_D dB_2(t), \\ dS_L &= [A_2 - \beta_2 S_L I_D - d_2 S_L]dt + \sigma_3 S_L dB_3(t), \\ dI_L &= [\beta_2 S_L I_D - d_2 I_L]dt + \sigma_4 I_L dB_4(t), \\ dS_H &= [A_3 - \beta_3 S_H I_D - d_3 S_H + \gamma I_H]dt + \sigma_5 S_H dB_5(t), \\ dE_H &= [\beta_3 S_H I_D - (d_3+\omega) E_H]dt + \sigma_6 E_H dB_6(t), \\ dI_H &= [\omega E_H - (d_3+\mu+\gamma) I_H]dt + \sigma_7 I_H dB_7(t). \end{cases} \tag{1.2}$$

where $B_i(t)$ are mutually independent Brownian motions with $B_i(0) = 0$, $\sigma_i^2 > 0$ are the intensities of the white noise, $i = 1, 2, 3, 4, 5$.

Table 1: Variables in the model(adapted from [8])

Population	Definition
$S_D(t)$	susceptible dogs at time t
$I_D(t)$	infectious dogs at time t
$S_L(t)$	susceptible livestock at time t
$I_L(t)$	infectious livestock at time t
$S_H(t)$	susceptible human at time t
$E_H(t)$	exposed human at time t
$I_H(t)$	infectious human at time t

The objective of this paper is to investigate dynamics behavior of system (1.2). The remaining part of this paper is as follows. In the next section we show the existence and uniqueness of a global positive solution of model (1.2) by using Lyapunov function method as mentioned in [13]-[15]. We prove that the system has ergodic property on certain conditions in Section 3. In Section 4, we present sufficient conditions for the extinction of the disease. We illustrate Some numerical simulations in Section 5.

2 Existence and uniqueness of the positive solution

Throughout this paper, unless otherwise specified, let $(\Omega, f, \{f_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{f_t\}_{t\geq 0}$ satisfying the usual conditions(i.e. it is right continuous and f_0 contains all P -null sets). Denote

$$R^n_+ = \{x \in R^n : x_i > 0 \, for \, all \, 1 \leq i \leq n\}, \; \bar{R}^n_+ = \{x \in R^n : x_i \geq 0 \, for \, all \, 1 \leq i \leq n\}.$$

We consider the general d-dimensional stochastic differential equation

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t), \quad for \, t \geq t_0 \tag{2.1}$$

with initial value $x(t_0) = x_0 \in R^n$, where $B(t)$ denotes d-dimensional standard Brownian motions defined on the above probability space.

Define the differential operator L associated with Eq.(2.1) by

$$L = \frac{\partial}{\partial t} + \Sigma f_i(x,t)\frac{\partial}{\partial x_i} + \frac{1}{2}\Sigma[g^T(x,t)g(x,t)]_{ij}\frac{\partial^2}{\partial x_i \partial x_j}.$$

If L acts on a function $V \in C^{2,1}(R^n \times \bar{R}_+; \bar{R}_+)$, then

$$LV(x,t) = V_t(x,t) + V_x(x,t) + \frac{1}{2}trac[g^T(x,t)V_{xx}(x,t)g(x,t)]$$

Table 2: Parameters in the model(adapted from [8])

Parameter	Definition
A_1	annual recruitment rate of the dog population
d_1	natural death rate of the dog population
σ	recovery rate of transition from infected to non-infected dogs
A_2	annual recruitment rate of the livestock population
d_2	natural death rate of the livestock population
A_3	annual recruitment rate of the human population
d_3	natural death rate of the human population
μ	disease-related death rate of the human population
$\frac{1}{\omega}$	incubation period of infected individuals of the human population
γ	recovery rate of the human population
β_1	infection rate of susceptible dogs by infectious livestock
β_2	infection rate of susceptible livestock by infectious dogs
β_3	infection rate of susceptible human by infectious infectious dogs

where $V_t = \frac{\partial V}{\partial t}, V_x = (\frac{\partial V}{\partial x_1}, \cdots, \frac{\partial V}{\partial x_d})$ and $V_{xx} = (\frac{\partial^2 V}{\partial x_i \partial x_j})_{d\times d}$.

In this section we first show that the solution of system (1.2) is positive and global. To get a unique global(i.e. no explosion in a finite time) solution for any initial value, the coefficients of the equation are required to satisfy the local lipschitz condition and the linear growth condition. However, the coefficients of system (1.2) do not satisfy the linear growth condition, so the solution of system (1.2) may explore in finite time. In this section, we use the Lyapunov analysis method, to show that the solution of system (1.2) is positive and global.

Theorem 2.1 *There is a unique solution $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t))$ of system (1.2) on $t \geq 0$ for any initial value $(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) \in R^7_+$, and the solution will remain in R^7_+ with probability 1, namely, $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t)) \in R^7_+$ for all $t \geq 0$ almost surely.*

Proof. Since the coefficients of the equation are locally Lipschitz continuous for given initial value $(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) \in R^7_+$, there is a unique local solution $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t))$ on $t \in [0, \tau_e)$, where τ_e is the explosion time. To show this solution is global, we need to proof that $\tau_e = \infty$ a.s. Let $n_0 \geq 0$ be sufficiently large so that $S_D(0)$, $I_D(0)$, $S_L(0)$, $I_L(0)$, $S_H(0)$, $E_H(0)$ and $W(0)$ all lie within the interval $[1/n_0, n_0]$. For each $n \geq n_0$, define the stopping time

$$\begin{aligned}\tau_n = &\inf\{t \in [0, \tau_e) : \min\{(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t))\} \le \tfrac{1}{n} \\ &or \max\{(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t))\} \ge n\},\end{aligned} \tag{2.2}$$

where and in what follows, we set $\inf \phi = \infty$(as usual ϕ denotes the empty set). According to the definition, τ_n is increasing as $n \to \infty$. Set $\tau_\infty = \lim\limits_{n\to\infty} \tau_n$, where $\tau_\infty \le \tau_e$ a.s. If we can prove that $\tau_\infty = \infty$ a.s., then $\tau_e = \infty$ and $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t)) \in R^7_+$ a.s. for all $t \ge 0$. In other words, to complete the proof we need to show is that $\tau_\infty = \infty$ a.s.. If not, there exists a pair of constants $S > 0$ and $\varepsilon \in (0, 1)$ such that

$$P\{\tau_\infty \le S\} > \varepsilon.$$

Hence there is an integer $n_1 \ge n_0$ such that

$$P\{\tau_\infty \le S\} \ge \varepsilon,\ for\ all\ n \ge n_1.$$

Define a C^2-function $V : R^7_+ \to \bar{R}_+$ by

$$\begin{aligned}V = &(S_D - a - a\ln \tfrac{S_D}{a}) + (I_D - b - b\ln \tfrac{I_D}{b}) + (S_L - 1 - \ln S_L) + (I_L - 1 - \ln I_L) \\ &+(S_H - 1 - \ln S_H) + (E_H - 1 - \ln E_H) + (I_H - 1 - \ln I_H)\end{aligned} \tag{2.3}$$

where a, b are positive constants to be determined later. The non-negativity of this function can be seen from $u - 1 - \log u \ge 0$, $\forall u > 0$. Let $n \ge n_0$ and $S > 0$ be arbitrary. Using Itô′s formula, we obtain

$$\begin{aligned}dV = &LV dt + \sigma_1(S_D - a)dB_1(t) + \sigma_2(I_D - b)dB_2(t) + \sigma_3(S_L - 1)dB_3(t) \\ &+\sigma_4(I_L - 1)dB_4(t) + \sigma_5(S_H - 1)dB_5(t) + \sigma_6(E_H - 1)dB_6(t) + \sigma_7(I_H - 1)dB_7(t),\end{aligned} \tag{2.4}$$

$$\begin{aligned}LV = &A_1 - d_1S_D - a\tfrac{A_1}{S_D} + a\beta_1I_L + ad_1 - a\sigma\tfrac{I_D}{S_D} + \tfrac{1}{2}a\sigma_1^2 \\ &-bd_1I_D - b\beta_1S_D\tfrac{I_L}{I_D} + b(d_1 + \sigma + \tfrac{1}{2}\sigma_2^2) \\ &+A_2 - d_2S_L - \tfrac{A_2}{S_L} + \beta_2I_D + d_2 + \tfrac{1}{2}\sigma_3^2 \\ &-d_2I_L - \beta_2S_L\tfrac{I_D}{I_L} + d_2 + \tfrac{1}{2}\sigma_4^2 \\ &+A_3 - d_3S_H - \tfrac{A_3}{S_H} + \beta_3I_D + d_3 - \gamma\tfrac{I_H}{S_H} + \tfrac{1}{2}\sigma_5^2 \\ &-d_3E_H - \beta_3S_H\tfrac{I_D}{E_H} + d_3 + \omega + \tfrac{1}{2}\sigma_6^2 \\ &-(d_3 + \mu)I_H - \omega\tfrac{E_H}{I_H} + d_3 + \mu + \gamma + \tfrac{1}{2}\sigma_7^2 \\ &\le (a\beta_1 - d_2)I_L + (\beta_2 + \beta_3 - bd_1)I_D + A_1 + A_2 + A_3 + [(a + b)d_1 + 2d_2 + 3d_3] \\ &+\tfrac{1}{2}(a\sigma_1^2 + b\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2 + \sigma_6^2 + \sigma_7^2) + b\sigma + \omega + \mu + \gamma.\end{aligned} \tag{2.5}$$

Choose

$$a = \frac{d_2}{\beta_1},\quad b = \frac{\beta_2 + \beta_3}{d_1},$$

then we get

$$\begin{aligned} &LV \leq A_1 + A_2 + A_3 + [(a+b)d_1 + 2d_2 + 3d_3] \\ &+\tfrac{1}{2}(a\sigma_1^2 + b\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2 + \sigma_6^2 + \sigma_7^2) + b\sigma + \omega + \mu + \gamma := K. \end{aligned} \tag{2.6}$$

The remained proof follows that in Mao, Marion and Renshaw [16].

3 Existence of unique and ergodic stationary distribution

Here we present some theory about the stationary distribution.

Lemma 3.1 [17]. *the Markov process $X(t)$ has a stationary distribution $\mu(\cdot)$ if there exists a bounded domain $U \subset E_l$ with regular boundary Γ and*

(B.1) there is a positive number M such that $\sum_{i,j=1}^{l} a_{ij}(x)\xi_i\xi_j \geq M|\xi|^2,\ x \in U,\ \xi \in R^l$.

(B.2) there exists a nonnegative C^2 function V such that LV is negative for any $E_l \backslash U$. Then

$$P_x\Big\{\lim_{T\to\infty} \frac{1}{T}\int_0^T f(X(t))dt = \int_{E_l} f(x)\mu(dx)\Big\} = 1,$$

for all $x \in E_l$, where $f(\cdot)$ is a function integrable with respect to the measure μ.

Define a parameter

$$R_0^s = \frac{\beta_1\beta_2A_1A_2}{(d_1+\frac{1}{2}\sigma_1^2)(d_1+\sigma+\frac{1}{2}\sigma_2^2)(d_2+\frac{1}{2}\sigma_3^2)(d_2+\frac{1}{2}\sigma_4^2)}$$

Theorem 3.1 *Assume that $R_0^s > 1$, then there is a unique stationary distribution $\mu(\cdot)$ for system (1.2) and it has ergodic property.*

Proof. Step1: Verify that $(B.1)$ holds. Apparently, the diffusion matrix of system (1.2) is

$$\Lambda(S_D, I_D, S_L, I_L, S_H, E_H, I_H) = \begin{pmatrix} \sigma_1^2S_D^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2I_D^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2S_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2I_L^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2S_H^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_6^2E_H^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_7^2I_H^2 \end{pmatrix}.$$

Besides there is $R = \min_{(S_D,I_D,S_L,I_L,S_H,E_H,I_H)\in R_+^7}\{\sigma_1^2S_D^2, \sigma_2^2I_D^2, \sigma_3^2S_L^2, \sigma_4^2I_L^2, \sigma_5^2S_H^2, \sigma_6^2E_H^2, \sigma_7^2I_H^2\} > 0$ such that

$$\sum_{i,j=1}^{7} \lambda_{ij}(S_D, I_D, S_L, I_L, S_H, E_H, I_H)\xi_i\xi_j = \sigma_1^2S_D^2+\sigma_2^2I_D^2+\sigma_3^2S_L^2+\sigma_4^2I_L^2+\sigma_5^2S_H^2+\sigma_6^2E_H^2+\sigma_7^2I_H^2 \geq R \mid \xi \mid^2,$$

for all $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in \overline{U}, \xi \in R^7$, which implies condition $(B.1)$ is satisfied.

Step2: Verify that $(B.2)$ holds. Now we will construct a nonnegative C^2-function V and a closed set $U \in \Sigma$ (which lies in R^5_+ entirely) such that

$$\sup_{(S_D,I_D,S_L,I_L,S_H,E_H,I_H)\in R^7_+\backslash U} LV(S_D, I_D, S_L, I_L, S_H, E_H, I_H) < 0,$$

which can assure that $(B.2)$ is satisfied. For the convenience of calculation, let

$$\overline{S}_D = \frac{A_1}{d_1 + \frac{1}{2}\sigma_1^2}, \overline{S}_L = \frac{A_2}{d_2 + \frac{1}{2}\sigma_3^2}.$$

$\overline{I}_D$ and $\overline{I}_L$ satisfy the following equations:

$$\beta_3 \overline{S}_L \overline{I}_D = (d_2 + \tfrac{1}{2}\sigma_4^2)\overline{I}_L \tag{3.1}$$

Beyond that,

$$\widehat{S}_D = \frac{S_D}{\overline{S}_D}, \widehat{I}_D = \frac{I_D}{\overline{I}_D}, \widehat{S}_L = \frac{S_L}{\overline{S}_L}, \widehat{I}_L = \frac{I_L}{\overline{I}_L}.$$

Consider C^2-function $R^7_+ \to R$

$$\Phi(S_D, I_D, S_L, I_L, S_H, E_H, I_H) = MV_1 + V_2 + V_3,$$

we assume that $\tilde{\Phi}$ is the minimum value of Φ. Then we define a nonnegative C^2-function V,

$$V = \Phi - \tilde{\Phi},$$

where

$$V_1 = -\overline{I}_D \ln I_D - c_1 \overline{S}_D \ln S_D - c_2 \overline{S}_L \ln S_L - c_3 \overline{I}_L \ln I_L + cI_L,$$

$$V_2 = -\ln S_D - \ln S_L - \ln I_L - \ln S_H - \ln E_H - \ln I_H,$$

$$V_3 = \frac{1}{m+2}(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2},$$

and c, c_1, c_2, c_3 are positive constants to be determined later, $m > 0$ is a sufficiently small number such that

$$m < \frac{2(d_1 \wedge d_2 \wedge d_3)}{\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2 \vee \sigma_7^2} - 1,$$

M is a sufficiently large number such that

$$AM\overline{I}_D + E \leq -2, \tag{3.2}$$

where

$$\begin{aligned} E = \sup_{(S_D,I_D,S_L,I_L,S_H,E_H,I_H)\in R^7_+} &\big\{\beta_1 I_L + (\beta_2+\beta_3)I_D + Mc\beta_2\varepsilon(S_L^{m+2}+1) \\ &-\tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} + B + C\big\}, \end{aligned} \tag{3.3}$$

where ε is a sufficiently small number such that

$$Mc\beta_2\varepsilon - \frac{1}{2}\rho \leq 0.$$

B and C are defined in (3.12) and (3.14) respectively. then

$$L(-\ln S_D) = -\frac{A_1}{S_D} + \beta_1 I_L - \sigma\frac{I_D}{S_D} + d_1 + \frac{1}{2}\sigma_1^2,$$

by applying the inequality $x - 1 \geq \ln x (x > 0)$, we obtain

$$\begin{aligned}
&L(-\overline{S}_D \ln S_D) = -\tfrac{A_1}{\widehat{S}_D} + \beta_1\overline{S}_D I_L - \sigma\overline{S}_D\tfrac{I_D}{S_D} + \overline{S}_D(d_1 + \tfrac{1}{2}\sigma_1^2) \\
&\leq -A_1(\tfrac{1}{\widehat{S}_D} - 1) + \beta_1\overline{S}_D \leq A_1 \ln\widehat{S}_D + \beta_1\overline{S}_D I_L.
\end{aligned} \tag{3.4}$$

from (3.2) we can obtain

$$\frac{\overline{I}_L}{\overline{I}_D} = \frac{\beta_2\overline{S}_L}{d_2 + \frac{1}{2}\sigma_4^2},$$

Then

$$\begin{aligned}
&L(-\ln I_D) = -\beta_1 S_D\tfrac{I_L}{I_D} + (d_1 + \sigma + \tfrac{1}{2}\sigma_2^2) \\
&= -\beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D}\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D} + (d_1 + \sigma + \tfrac{1}{2}\sigma_2^2) \\
&= -\beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D}(\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D} - 1) - \beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D} + (d_1 + \sigma + \tfrac{1}{2}\sigma_2^2) \\
&\leq -\beta_1\overline{S}_D\tfrac{\beta_2\overline{S}_L}{d_2 + \frac{1}{2}\sigma_4^2} + (d_1 + \sigma + \tfrac{1}{2}\sigma_2^2) - \beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D}\ln\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D} \\
&= -(d_1 + \sigma + \tfrac{1}{2}\sigma_2^2)(\tfrac{\beta_1\overline{S}_D\beta_2\overline{S}_L}{(d_1 + \sigma + \frac{1}{2}\sigma_2^2)(d_2 + \frac{1}{2}\sigma_4^2)} - 1) - \beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D}\ln\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D} \\
&= -(R_0^s - 1)(d_1 + \sigma + \tfrac{1}{2}\sigma_2^2) - \beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D}\ln\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D} \\
&:= -A - \beta_1\tfrac{\overline{S}_D\overline{I}_L}{\overline{I}_D}\ln\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D}. \\
&L(-\overline{I}_D\ln I_D) \leq -A\overline{I}_D - \beta_1\overline{S}_D\overline{I}_L\ln\tfrac{\widehat{S}_D\widehat{I}_L}{\widehat{I}_D}.
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
&L(-\overline{S}_L\ln S_L) = -\tfrac{A_2}{\widehat{S}_L} + \beta_2\overline{S}_L I_D + \overline{S}_L(d_2 + \tfrac{1}{2}\sigma_3^2) \\
&\leq A_2\ln\widehat{S}_L + \beta_2\overline{S}_L I_D.
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
&L(-\overline{I}_L\ln I_L) = -\beta_2\overline{S}_L\overline{I}_D\tfrac{\widehat{S}_L\widehat{I}_D}{\widehat{I}_L} + \overline{I}_L(d_2 + \tfrac{1}{2}\sigma_4^2) \\
&= -\beta_2\overline{S}_L\overline{I}_D(\tfrac{\widehat{S}_L\widehat{I}_D}{\widehat{I}_L} - 1) \\
&\leq -\beta_2\overline{S}_L\overline{I}_D\ln\tfrac{\widehat{S}_L\widehat{I}_D}{\widehat{I}_L}.
\end{aligned} \tag{3.7}$$

Then

$$\begin{aligned}
&LV_1 \leq -A\overline{I}_D + (c_1\beta_1\overline{S}_D - cd_2)I_L + c_2\beta_2\overline{S}_L I_D + c\beta_2 S_L I_D \\
&+(c_1A_1 - \beta_1\overline{S}_D\overline{I}_L)\ln\widehat{S}_D + (\beta_1\overline{S}_D\overline{I}_L - c_3\beta_2\overline{S}_L\overline{I}_D)\ln\widehat{I}_D \\
&+(c_2A_2 - c_3\beta_2\overline{S}_L\overline{I}_D)\ln\widehat{S}_L + (-\beta_1\overline{S}_D\overline{I}_L + c_3\beta_2\overline{S}_L\overline{I}_D)\ln\widehat{I}_L.
\end{aligned} \tag{3.8}$$

Let

$$\begin{cases}
c_1A_1 - \beta_1\overline{S}_D\overline{I}_L = 0 \\
\beta_1\overline{S}_D\overline{I}_L - c_3\beta_2\overline{S}_L\overline{I}_D = 0 \\
c_2A_2 - c_3\beta_2\overline{S}_L\overline{I}_D = 0 \\
-\beta_1\overline{S}_D\overline{I}_L + c_3\beta_2\overline{S}_L\overline{I}_D = 0.
\end{cases} \tag{3.9}$$

The unique solution of above equations is given as follows:

$$c_1 = \frac{\beta_1 \overline{S}_D \overline{I}_L}{A_1}, c_2 = \frac{\beta_1 \overline{S}_D \overline{I}_L}{A_2}, c_3 = \frac{\beta_1 \overline{S}_D \overline{I}_L}{\beta_2 \overline{S}_L \overline{I}_D}.$$

Choose $c = \frac{c_1 \beta_1 \overline{S}_D}{d_2}$, then

$$L(MV_1) \leq -AM\overline{I}_D + Mc_2\beta_2\overline{S}_L I_D + Mc\beta_2 S_L I_D, \tag{3.10}$$

$$\begin{aligned} &LV_2 = L(-\ln S_D - \ln S_L - \ln I_L - \ln S_H - \ln E_H - \ln I_H) \\ &= -\tfrac{A_1}{S_D} + \beta_1 I_L - \sigma \tfrac{I_D}{S_D} + d_1 + \tfrac{1}{2}\sigma_1^2 - \tfrac{A_2}{S_L} + \beta_2 I_D + d_2 + \tfrac{1}{2}\sigma_3^2 - \beta_2 S_L \tfrac{I_D}{I_L} + (d_2 + \tfrac{1}{2}\sigma_4^2) \\ &-\tfrac{A_3}{S_H} + \beta_3 I_D - \gamma \tfrac{I_H}{S_H} + d_3 + \tfrac{1}{2}\sigma_5^2 - \beta_3 S_H \tfrac{I_D}{E_H} + (d_3 + \omega + \tfrac{1}{2}\sigma_6^2) \\ &-\omega \tfrac{E_H}{I_H} + (d_3 + \mu + \gamma + \tfrac{1}{2}\sigma_7^2) \\ &\leq -\tfrac{A_1}{S_D} - \tfrac{A_2}{S_L} - \beta_2 S_L \tfrac{I_D}{I_L} - \tfrac{A_3}{S_H} - \beta_3 S_H \tfrac{I_D}{E_H} - \omega \tfrac{E_H}{I_H} \\ &+\beta_1 I_L + (\beta_2 + \beta_3) I_D + B, \end{aligned} \tag{3.11}$$

where

$$B = d_1 + 2\textstyle\sum_{i=2}^{3} d_i + \tfrac{1}{2}(\sigma_1^2 + \sum_{i=3}^{7}\sigma_i^2) + \omega + \mu + \gamma. \tag{3.12}$$

$$\begin{aligned} &LV_3 = L(\tfrac{1}{m+2}(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2}) \\ &= (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+1}[A_1 + A_2 + A_3 \\ &-d_1(S_D + I_D) - d_2(S_L + I_L) - d_3(S_H + E_H) - (d_3 + \mu) I_H] \\ &+\tfrac{m+1}{2}(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^m(\sigma_1^2 S_D^2 + \sigma_2^2 I_D^2 + \sigma_3^2 S_L^2 + \sigma_4^2 I_L^2 + \sigma_5^2 S_H^2 + \sigma_6^2 E_H^2 + \sigma_7^2 I_H^2) \\ &\leq (A_1 + A_2 + A_3)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+1} \\ &-(d_1 \wedge d_2 \wedge d_3)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\tfrac{m+1}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2 \vee \sigma_7^2)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\leq C - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2}, \end{aligned} \tag{3.13}$$

where

$$\begin{aligned} C = &\sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7} \big\{(A_1 + A_2 + A_3)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+1} \\ &-\tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2}\big\}, \end{aligned} \tag{3.14}$$

$$\rho = (d_1 \wedge d_2 \wedge d_3) - \frac{m+1}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2 \vee \sigma_7^2) > 0$$

Then combined with (3.9), (3.10) and (3.11), we get the value of LV finally:

$$\begin{aligned} &LV \leq -AM\overline{I}_D + c_2 M\beta_2 \overline{S}_L I_D + Mc\beta_2 S_L I_D - \tfrac{A_1}{S_D} - \tfrac{A_2}{S_L} - \beta_2 S_L \tfrac{I_D}{I_L} \\ &-\tfrac{A_3}{S_H} - \beta_3 S_H \tfrac{I_D}{E_H} - \omega \tfrac{E_H}{I_H} - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (\beta_2 + \beta_3) I_D + B + C. \end{aligned} \tag{3.15}$$

Then define a closed set

$$\begin{aligned}U_\varepsilon = \{&(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : \varepsilon \le S_D \le \tfrac{1}{\varepsilon}, \varepsilon \le I_D \le \tfrac{1}{\varepsilon}, \varepsilon \le S_L \le \tfrac{1}{\varepsilon},\\ &\varepsilon^3 \le I_L \le \tfrac{1}{\varepsilon^3}, \varepsilon \le S_H \le \tfrac{1}{\varepsilon}, \varepsilon^3 \le E_H \le \tfrac{1}{\varepsilon^3}, \varepsilon^4 \le I_H \le \tfrac{1}{\varepsilon^4}\},\end{aligned} \tag{3.16}$$

where ε is a sufficiently small positive number such that

$$-\tfrac{\theta}{\varepsilon} + D \le -1, \tag{3.17}$$

$$-AM\overline{I}_D + Mc_2\beta_2\overline{S}_L\varepsilon + E \le -1, \tag{3.18}$$

$$-\tfrac{\theta}{\varepsilon} + F \le -1, \tag{3.19}$$

$$-\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{m+2}} + G \le -1, \tag{3.20}$$

$$-\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{3(m+2)}} + G \le -1, \tag{3.21}$$

$$-\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{4(m+2)}} + G \le -1, \tag{3.22}$$

where $\theta = A_1 \wedge A_2 \wedge A_3 \wedge \beta_2 \wedge \beta_3 \wedge \omega$, D, E, F, are positive constants which can be found from inequations . For the sake of convenience, we can divide $R^7_+ \backslash U_\varepsilon$ into the following fourteen domains,

$$\begin{aligned}
U^c_1 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < S_D < \varepsilon\},\\
U^c_2 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < I_D < \varepsilon\},\\
U^c_3 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < S_L < \varepsilon\},\\
U^c_4 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < I_L < \varepsilon^3, S_L > \varepsilon, I_D > \varepsilon\},\\
U^c_5 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < S_H < \varepsilon\},\\
U^c_6 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < E_H < \varepsilon^3, S_H > \varepsilon, I_D > \varepsilon\},\\
U^c_7 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : 0 < I_H < \varepsilon^4, E_H > \varepsilon^3\},\\
U^c_8 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : S_D > \tfrac{1}{\varepsilon}\},\\
U^c_9 &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : I_D > \tfrac{1}{\varepsilon}\},\\
U^c_{10} &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : S_L > \tfrac{1}{\varepsilon}\},\\
U^c_{11} &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : I_L > \tfrac{1}{\varepsilon^3}\},\\
U^c_{12} &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : S_H > \tfrac{1}{\varepsilon}\},\\
U^c_{13} &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : E_H > \tfrac{1}{\varepsilon^3}\},\\
U^c_{14} &= \{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R^7_+ : I_H > \tfrac{1}{\varepsilon^4}\},
\end{aligned} \tag{3.23}$$

Obviously, $R^7_+ \backslash U_\varepsilon = \bigcup\limits_{1\le i\le 14} U^c_i$. Next we will show that $LV \le -1$ on $R^7_+ \backslash U_\varepsilon$, which is equivalent to verifying it on the above fourteen domains.

Case 1. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_1^c$, in view of (3.15) and (3.17),thus

$$\begin{aligned} LV &\leq -\tfrac{A_1}{S_D} - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{A_1}{\varepsilon} + D \leq -\tfrac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \tag{3.24}$$

Where

$$\begin{aligned} D = \sup_{(S_D,I_D,S_L,I_L,S_H,E_H,I_H)\in R_+^7} &\big\{ -\tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ +\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C\big\}. \end{aligned} \tag{3.25}$$

Case 2. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_2^c$, in view of (3.2, (3.15) and (3.18),thus

$$\begin{aligned} LV &\leq -AM\overline{I}_D + Mc_2\beta_2\overline{S}_L I_D + \beta_1 I_L + (\beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D \\ &-\tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} + B + C \\ &\leq -AM\overline{I}_D + Mc_2\beta_2\overline{S}_L\varepsilon + E \leq -1. \end{aligned} \tag{3.26}$$

Case 3. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_3^c$, in view of (3.15) and (3.17),thus

$$\begin{aligned} LV &\leq -\tfrac{A_2}{S_L} - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{A_2}{\varepsilon} + D \leq -\tfrac{\theta}{\varepsilon} + F \leq -1, \end{aligned} \tag{3.27}$$

where

$$\begin{aligned} F = \sup_{(S_D,I_D,S_L,I_L,S_H,E_H,I_H)\in R_+^7} &\big\{\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2\varepsilon(I_D^{m+2} + 1) \\ -\tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} + B + C\big\}, \end{aligned} \tag{3.28}$$

Case 4. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_4^c$, in view of (3.15) and (3.17),thus

$$\begin{aligned} LV &\leq -\beta_2 S_L \tfrac{I_D}{I_L} - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{\beta_2}{\varepsilon} + D \leq -\tfrac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \tag{3.29}$$

Case 5. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_5^c$, in view of (3.15) and (3.17),thus

$$\begin{aligned} LV &\leq -\tfrac{A_3}{S_H} - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{A_3}{\varepsilon} + D \leq -\tfrac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \tag{3.30}$$

Case 6. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_6^c$, in view of (3.15) and (3.17),thus

$$\begin{aligned} LV &\leq -\beta_3 S_H \tfrac{I_D}{E_H} - \tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{\beta_3}{\varepsilon} + D \leq -\tfrac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \tag{3.31}$$

Case 7. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_7^c$, in view of (3.15) and (3.17),thus

$$\begin{aligned} &LV \leq -\omega \tfrac{E_H}{I_H} \\ &-\tfrac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{\omega}{\varepsilon} + D \leq -\tfrac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \tag{3.32}$$

Case 8. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_8^c$, in view of (3.15) and (3.19),thus

$$\begin{aligned} &LV \leq -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{1}{4}\rho(S_D)^{m+2} + G \leq -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{m+2}} + G \leq -1. \end{aligned} \tag{3.33}$$

Where

$$\begin{aligned} G = &\sup_{(S_D,I_D,S_L,I_L,S_H,E_H,I_H)\in R_+^7} \{ -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C\}. \end{aligned} \tag{3.34}$$

Case 9. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_9^c$, in view of (3.15) and (3.19),thus

$$\begin{aligned} &LV \leq -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{1}{4}\rho(I_D)^{m+2} + G \leq -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{m+2}} + G \leq -1. \end{aligned} \tag{3.35}$$

Case 10. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{10}^c$, in view of (3.15) and (3.19),thus

$$\begin{aligned} &LV \leq -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{1}{4}\rho(S_L)^{m+2} + G \leq -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{m+2}} + G \leq -1. \end{aligned} \tag{3.36}$$

Case 11. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{11}^c$, in view of (3.15) and (3.20),thus

$$\begin{aligned} &LV \leq -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\tfrac{1}{4}\rho(I_L)^{m+2} + G \leq -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{3(m+2)}} + G \leq -1. \end{aligned} \tag{3.37}$$

Case 12. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{12}^c$, in view of (3.15) and (3.19),thus

$$\begin{aligned}
LV &\le -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\
&-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\
&+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\
&\le -\tfrac{1}{4}\rho(S_H)^{m+2} + G \le -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{m+2}} + G \le -1.
\end{aligned} \tag{3.38}$$

Case 13. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{13}^c$, in view of (3.15) and (3.20),thus

$$\begin{aligned}
LV &\le -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\
&-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\
&+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\
&\le -\tfrac{1}{4}\rho(E_H)^{m+2} + G \le -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{3(m+2)}} + G \le -1.
\end{aligned} \tag{3.39}$$

Case 14. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{14}^c$, in view of (3.15) and (3.21),thus

$$\begin{aligned}
LV &\le -\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\
&-\tfrac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\
&+\beta_1 I_L + (Mc_2\beta_2\overline{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\
&\le -\tfrac{1}{4}\rho(I_H)^{m+2} + G \le -\tfrac{1}{4}\rho\tfrac{1}{\varepsilon^{4(m+2)}} + G \le -1.
\end{aligned} \tag{3.40}$$

Based on the discuss of the above ten kinds of cases, the condition $(B.2)$ in Lemma 3.1 is also satisfied. The proof of Theorem 3.1 is completed.

4 Extinction

In this section, we shall consider the extinction of the infection.

Define a parameter-

$$\widehat{R}_0 = \frac{\beta_1 S_D^0 + \beta_2 \int_0^\infty x\pi(x)dx + \beta_3 S_H^0 + \omega}{\frac{1}{8}(\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2)},$$

where

$$\pi(x) = Qx^{-2-\frac{2d_2}{\sigma_3^2}} e^{-\frac{2}{\sigma_3^2}\frac{A_2}{x}}, x \in (0, \infty).$$

$$\langle x\rangle_t = \frac{1}{t}\int_0^t x(r)dr$$

Theorem 4.1 *Let $(S_D, I_D, S_L, I_L, S_H, E_H, I_H)$ be the solution of system (1.2) with any initial value $(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) \in R_+^7$. If $\widehat{R}_0 < 1$, then the solution $(S_D, I_D, S_L, I_L, S_H, E_H, I_H)$ of system (1.2) satisfies*

$$\limsup_{t\to\infty} \frac{\ln(I_D + I_L + E_H + I_H)}{t} \le \beta_1 S_D^0 + \beta_2 \int_0^\infty x\pi(x)dx + \beta_3 S_H^0 + \omega - \frac{1}{8}(\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2) < 0 \quad a.s.$$

Namely,

$$\limsup_{t\to\infty} I_D = \limsup_{t\to\infty} I_L = \limsup_{t\to\infty} E_H = \limsup_{t\to\infty} I_H = 0.$$

And the distribution of $S_L(t)$ converges weakly to the measure which has the density

$$\pi(x) = Qx^{-2-\frac{2d_2}{\sigma_3^2}} e^{-\frac{2}{\sigma_3^2}\frac{A_2}{x}}, x \in (0,\infty),$$

where Q is a constant such that $\int_0^\infty \pi(x)dx = 1$.

Proof. Consider the following auxiliary logistic equation with random perturbation

$$dX(t) = (A_2 - d_2X)dt + \sigma_3 X dB_3(t), \tag{4.1}$$

with the initial value $X(0) = S_L(0) > 0$. Setting

$$b(x) = A_2 - d_2x, \qquad \sigma(x) = \sigma_3 x,\ x \in (0,\infty),$$

we compute that

$$\int \frac{b(u)}{\sigma^2(u)}du = \frac{1}{\sigma_3^2}\int (\frac{A_2}{u^2} - \frac{d_2}{u})du = \frac{1}{\sigma_3^2}(-\frac{A_2}{x} - d_2 \ln x) + Q.$$

Therefore

$$e^{\int \frac{b(u)}{\sigma^2(u)}du} = e^Q x^{-\frac{d_2}{\sigma_3^2}} e^{-\frac{1}{\sigma_3^2}\frac{A_2}{x}}.$$

Clearly, we have

$$\int_0^\infty \frac{1}{\sigma^2(x)} e^{\int_1^x \frac{2b(\tau)}{\sigma^2(\tau)}d\tau} = C\int_0^\infty x^{-2-\frac{2d_2}{\sigma_3^2}} e^{-\frac{2}{\sigma_3^2}\frac{A_2}{x}} dx < \infty.$$

Consequently, the condition of Theorem 1.16 in [18] follows clearly from above. Thus system (4.1) has the ergodic property, and the invariant density is given by

$$\pi(x) = Qx^{-2-\frac{2d_2}{\sigma_3^2}} e^{-\frac{2}{\sigma_3^2}\frac{A_2}{x}}, x \in (0,\infty),$$

where Q is a constant such that $\int_0^\infty \pi(x)dx = 1$. From the ergodic theorem it follows that

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t x(s)ds = \int_0^\infty x\pi(x)dx \quad a.s.$$

Let $X(t)$ be the solution of SDE (4.1) with the initial value $X(0) = S_L(0) > 0$, then we can see that

$$S_L(t) \le X(t) \quad a.s.$$

On the other hand, Integrating from 0 to t and then dividing by t on both sides of (1.2) lead to

$$\begin{cases} \frac{S_D(t)-S_D(0)}{t} &= A_1 - \beta_1\langle S_D I_L\rangle - d_1\langle S_D\rangle + \sigma\langle I_D\rangle + \frac{\sigma_1}{t}\int_0^t S_r dB_1(r), \\ \frac{I_D(t)-I_D(0)}{t} &= \beta_1\langle S_D I_L\rangle - (d_1+\sigma)\langle I_D\rangle + \frac{\sigma_2}{t}\int_0^t I_D(r)dB_2(r), \\ \frac{S_H(t)-S_H(0)}{t} &= A_3 - \beta_3\langle S_H I_D\rangle - d_3\langle S_H\rangle + \gamma\langle I_H\rangle + \frac{\sigma_5}{t}\int_0^t S_H(r)dB_5(r), \\ \frac{E_H(t)-E_H(0)}{t} &= \beta_3\langle S_H I_D\rangle - (d_3+\omega)\langle E_H\rangle + \frac{\sigma_6}{t}\int_0^t E_H(r)dB_6(r), \\ \frac{I_H(t)-I_H(0)}{t} &= \omega\langle E_H\rangle - (d_3+\mu+\gamma)\langle I_H\rangle + \frac{\sigma_7}{t}\int_0^t I_H(r)dB_7(r). \end{cases} \tag{4.2}$$

We can obtain that

$$\frac{S_D(t)-S_D(0)}{t}+\frac{I_D(t)-I_D(0)}{t}=A_1-d_1\langle S_D\rangle_t-d_1\langle I_D\rangle_t,$$

then

$$\langle S_D\rangle_t=\tfrac{A_1-d_1\langle I_D\rangle_t}{d_1}+\varphi_1(t)=\tfrac{A_1}{d_1}-\langle I_D\rangle_t+\varphi_1(t)=S_D^0-\langle I_D\rangle_t+\varphi_1(t) \tag{4.3}$$

where

$$\varphi_1(t)=-\frac{1}{d_1}\left(\frac{S_D(t)-S_D(0)}{t}+\frac{I_D(t)-I_D(0)}{t}\right),$$

and

$$\begin{aligned}&\tfrac{S_H(t)-S_H(0)}{t}+\tfrac{E_H(t)-E_H(0)}{t}+\tfrac{I_H(t)-I_H(0)}{t}\\&=A_3-d_3\langle S_H\rangle_t-d_3\langle E_H\rangle_t-(d_3+\mu)\langle I_H\rangle_t\end{aligned} \tag{4.4}$$

then

$$\langle S_H\rangle_t=S_H^0-\langle E_H\rangle_t-\tfrac{d_3+\mu}{d_3}+\varphi_2(t) \tag{4.5}$$

where

$$\varphi_2(t)=-\frac{1}{d_3}\left(\frac{S_H(t)-S_H(0)}{t}+\frac{E_H(t)-E_H(0)}{t}+\frac{I_H(t)-I_H(0)}{t}\right).$$

From [19], we can obtain that

$$\lim_{t\to\infty}\varphi_i(t)=0 \quad a.s. \;\; i=1,2.$$

let $P(t)=I_D+I_L+E_H+I_H$. Applying $It\hat{o}'s$ formula, we can obtain that

$$\begin{aligned}&d\ln P(t)=\big\{\tfrac{1}{P}[\beta_1S_DI_L-(d_1+\sigma)I_D+\beta_2S_LI_D-d_2I_L\\&+\beta_3S_HI_D-(d_3+\omega)E_H+\omega E_H-(d_3+\mu+\gamma)I_H]\\&-\tfrac{1}{2P^2}(\sigma_2^2I_D^2+\sigma_4^2I_L^2+\sigma_6^2E_H^2+\sigma_7^2I_H^2)\big\}dt\\&+\tfrac{1}{P}[\sigma_2I_DdB_2(t)+\sigma_4I_LdB_4(t)+\sigma_6E_HdB_6(t)+\sigma_7I_HdB_7(t)]\\&\le\big\{\beta_1S_D+\beta_2S_L+\beta_3S_H+\omega-\tfrac{1}{8}(\sigma_2^2\wedge\sigma_4^2\wedge\sigma_6^2\wedge\sigma_7^2)\big\}dt\\&+\sigma_2dB_2(t)+\sigma_4dB_4(t)+\sigma_6dB_6(t)+\sigma_7dB_7(t).\end{aligned} \tag{4.6}$$

Integrating (4.6) from 0 to t and then dividing by t on both sides, one can see that

$$\begin{aligned}&\tfrac{\ln P(t)}{t}-\tfrac{\ln P(0)}{t}\le\beta_1\langle S_D\rangle_t+\tfrac{\beta_2}{t}\int_0^tS_L(s)ds+\beta_3\langle S_H\rangle_t\\&+\omega-\tfrac{1}{8}(\sigma_2^2\wedge\sigma_4^2\wedge\sigma_6^2\wedge\sigma_7^2)+\tfrac{\sigma_2}{t}B_2(t)+\tfrac{\sigma_4}{t}B_4(t)+\tfrac{\sigma_6}{t}B_6(t)+\tfrac{\sigma_7}{t}B_7(t).\end{aligned} \tag{4.7}$$

As application of strong law of large numbers [20], one has

$$\lim_{t\to\infty}\frac{B_i(t)}{t}=0 \quad a.s. \;\; i=2,4,6,7.$$

Taking the superior limit on both sides of (4.7), and note that $\widehat{R}_0<1$ leads to

$$\limsup_{t\to\infty}\frac{\ln P(t)}{t}\le\beta_1S_D^0+\beta_2\int_0^\infty x\pi(x)dx+\beta_3S_H^0+\omega-\frac{1}{8}(\sigma_2^2\wedge\sigma_4^2\wedge\sigma_6^2\wedge\sigma_7^2)<0 \;\; a.s.$$

which implies that

$$\limsup_{t\to\infty} I_D = \limsup_{t\to\infty} I_L = \limsup_{t\to\infty} E_H = \limsup_{t\to\infty} I_H = 0 \ \ a.s.$$

Thus for any small $\varepsilon > 0$ there exist t_0 and a set $\Omega_\varepsilon \subset \Omega$ such that $\mathbf{P}(\Omega_\varepsilon) > 1-\varepsilon$ and $\beta_2 S_L I_D \le \varepsilon S_L$ for $t \ge t_0$ and $\omega \in \Omega_\varepsilon$. Now from

$$\begin{aligned} &[A_2 - \beta_2 S_L I_D - d_2 S_L]dt + \sigma_3 S_L dB_3(t) \\ &\le dS_L(t) \le [A_2 - d_2 S_L]dt + \sigma_3 S_L dB_3(t), \end{aligned} \tag{4.8}$$

it follows that the distribution of the process $S_L(t)$ converges to the measure with the density π. This finishes the proof.

5 Numerical simulation

We consider numerical simulations to illustrate our results by using the Milstein's Higher Order Method in [21]. We choose the initial values

$$(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) = (0.9, 0.5, 1.2, 0.6, 1.2, 0.3, 0.3)$$

The corresponding discretizing equations of model (1.2) are as follows:

$$\begin{cases} S_D(k+1) &= [A_1 - \beta_1 S_D(k) I_L(k) - d_1 S_D(k) + \sigma I_D(k)]\triangle t + \sigma_1 S_D(k)\sqrt{\triangle t}\xi_{1k} + \frac{\sigma_1^2 S_D(k)}{2}\triangle t(\xi_{1k}^2 - 1), \\ I_D(k+1) &= [\beta_1 S_D(k) I_L(k) - (d_1+\sigma) I_D(k)]\triangle t + \sigma_2 I_D(k)\sqrt{\triangle t}\xi_{2k} + \frac{\sigma_2^2 I_D(k)}{2}\triangle t(\xi_{2k}^2 - 1), \\ S_L(k+1) &= [A_2 - \beta_2 S_L(k) I_D(k) - d_2 S_L(k)]\triangle t + \sigma_3 S_L(k)\sqrt{\triangle t}\xi_{3k} + \frac{\sigma_3^2 S_L(k)}{2}\triangle t(\xi_{3k}^2 - 1), \\ I_L(k+1) &= [\beta_2 S_L(k) I_D(k) - d_2 I_L(k)]\triangle t + \sigma_4 I_L(k)\sqrt{\triangle t}\xi_{4k} + \frac{\sigma_4^2 I_L(k)}{2}\triangle t(\xi_{4k}^2 - 1), \\ S_H(k+1) &= [A_3 - \beta_3 S_H(k) I_D(k) - d_3 S_H(k) + \gamma I_H(k)]\triangle t + \sigma_5 S_H(k)\sqrt{\triangle t}\xi_{5k} + \frac{\sigma_5^2 S_H(k)}{2}\triangle t(\xi_{5k}^2 - 1), \\ E_H(k+1) &= [\beta_3 S_H(k) I_D(k) - (d_3+\omega) E_H(k)]\triangle t + \sigma_6 E_H(k)\sqrt{\triangle t}\xi_{6k} + \frac{\sigma_6^2 E_H(k)}{2}\triangle t(\xi_{6k}^2 - 1), \\ I_H(k+1) &= [\omega E_H(k) - (d_3+\mu+\gamma) I_H(k)]\triangle t + \sigma_7 I_H(k)\sqrt{\triangle t}\xi_{7k} + \frac{\sigma_7^2 I_H(k)}{2}\triangle t(\xi_{7k}^2 - 1). \end{cases} \tag{5.1}$$

where the time increment $\triangle t$is positive and ξ_{ik} are the Gaussian random variables which follow the distribution $N(0,1), i = 1,2,3,4,5,6,7$.

Example 5.1 *In order to check the existence of an ergodic stationary distribution,we choose the values of the system parameters as follows:* $\sigma_1^2 = 10^{-4}, \sigma_2^2 = 3.6\times 10^{-3}, \sigma_3^2 = 10^{-2}, \sigma_4^2 = 4\times 10^{-4}, \sigma_5^2 = 10^{-4}, \sigma_6^2 = 4\times 10^{-4}; \sigma_7^2 = 2.5\times 10^{-3}$. *Other values of the system parameters see Table3. Direct calculation leads to* $R_0^s = 754.928752 > 1$, *where* R_0^s *is defined before Theorem 3.1. In other words, the conditions of Theorem 3.1 hold. In view of Theorem 3.1, there is an ergodic stationary distribution* $\mu(\cdot)$ *of system (1.2). Fig.1,Fig.2 and Fig.3 illustrate this.*

Table 3: List of parameters

Parameter	Values	Parameter	Values
A_1	0.9	A_2	1.0
A_3	0.7	d_1	0.02
d_2	0.01	d_3	0.05
β_1	0.08	β_2	0.02
β_3	0.08	σ	0.6
μ	0.03	ω	0.3
γ	0.8		

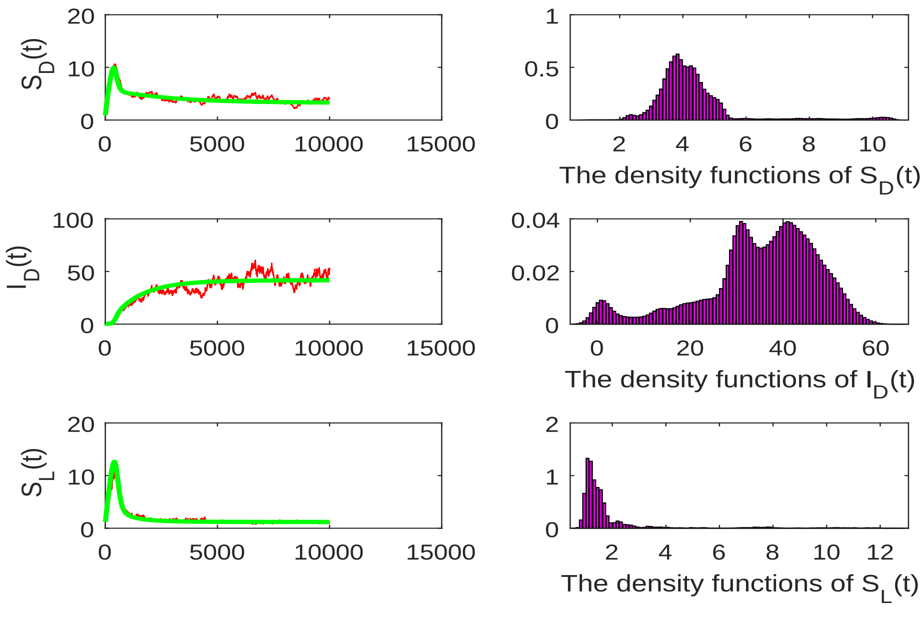

Figure 1: The solution of the stochastic system (1.2) and its histogram. The red lines represent the solution of system (1.2), and the green lines represent the solution of the corresponding undisturbed system (1.1). The pictures on the right are the histogram of the probability density function for S_D, I_D and S_L populations (Color figure online).

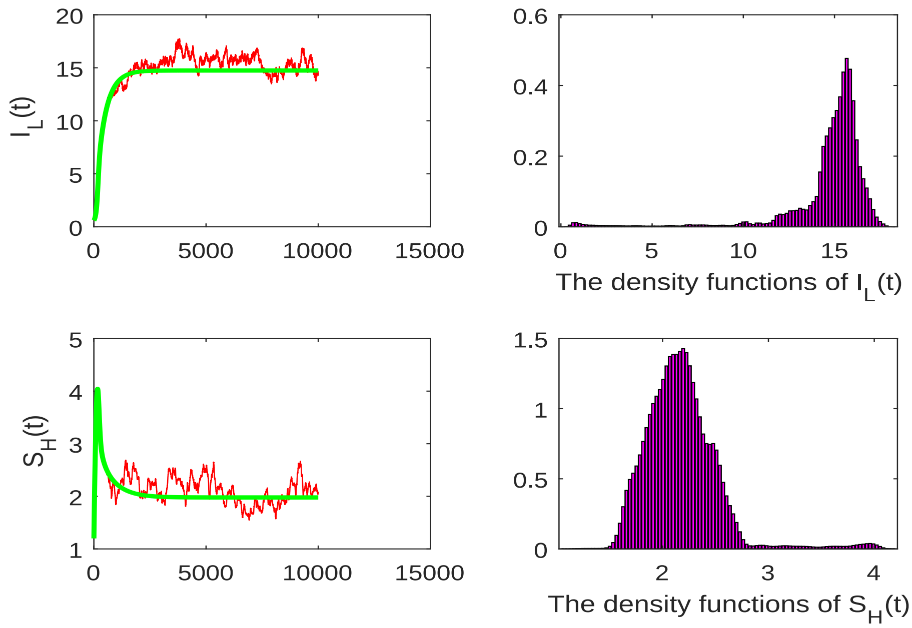

Figure 2: The solution of the stochastic system (1.2) and its histogram. The red lines represent the solution of system (1.2), and the green lines represent the solution of the corresponding undisturbed system (1.1). The pictures on the right are the histogram of the probability density function for I_L, S_H populations (Color figure online).

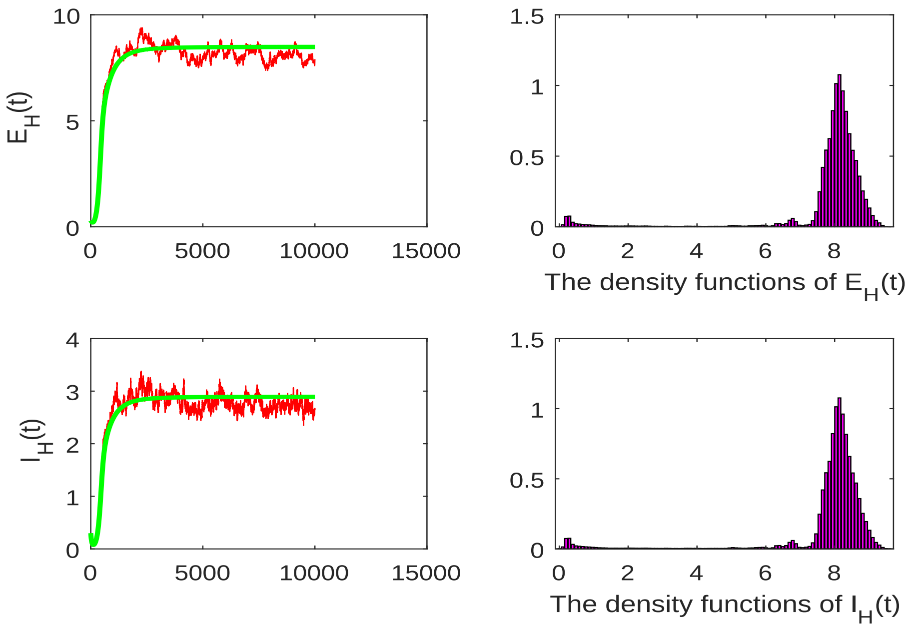

Figure 3: The solution of the stochastic system (1.2) and its histogram. The red lines represent the solution of system (1.2), and the green lines represent the solution of the corresponding undisturbed system (1.1). The pictures on the right are the histogram of the probability density function for E_H, I_H populations (Color figure online).

Table 4: List of parameters

Parameter	Values	Parameter	Values
A_1	0.5	A_2	0.6
A_3	0.5	d_1	0.1
d_2	0.08	d_3	0.05
β_1	0.02	β_2	0.02
β_3	0.02	σ	0.8
μ	0.03	ω	0.1
γ	0.8		

Example 5.2 *In order to obtain the extinction of the infection,in (1.2),we choose the values of the system parameters as follows:* $\sigma_1^2 = 10^{-2}, \sigma_2^2 = 9\times10^{-2}, \sigma_3^2 = 10^{-2}, \sigma_4^2 = 1.6\times10^{-1}, \sigma_5^2 = 4\times10^{-4}, \sigma_6^2 = 9\times10^{-2}; \sigma_7^2 = 1.6\times10^{-1}$. *Other values of the system parameters see Table 4. By Theorem* 4.1, $I_D, I_L, E_H, I_H)$ *will tends to zero exponentially with probability one. We give the simulations to support our results in Fig.4.*

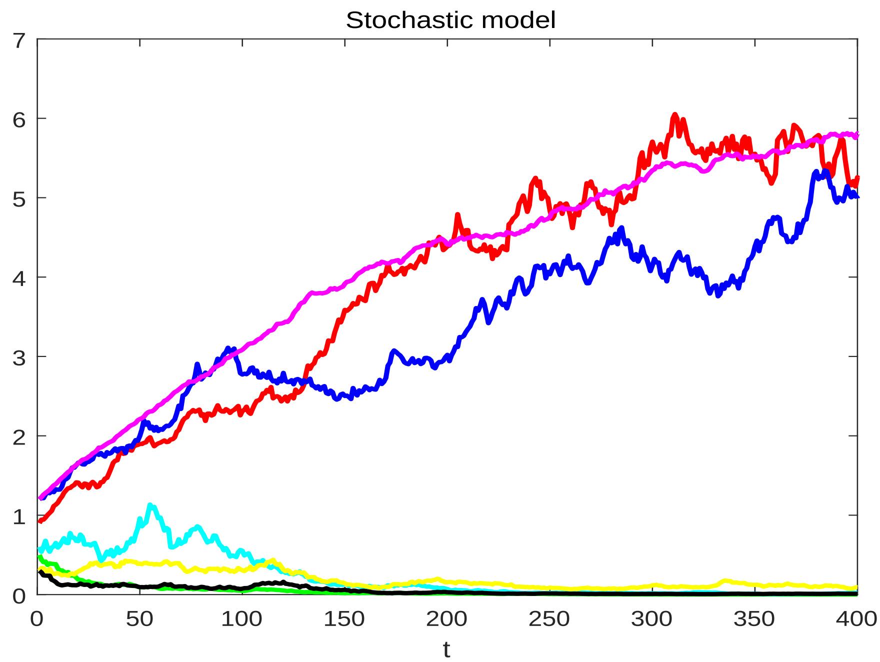

Figure 4: The solution of the stochastic system (1.2) and its histogram. The red line,blue line and magenta line represent S_D, S_L and S_H respectively. And the green line,cyan line,yellow line and black line represent I_D, I_L, E_H and I_H respectively.

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