# A considerable study about the DNA dynamics arising in oscillator-chain of Peyrard-Bishop model

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**Abstract**

In this work, the study of the Peyrard-Bishop DNA dynamic model equation analytically and numerically will present. The Kudryashov method and modified Kudryashov method are used to find the solution of the Peyrard-Bishop DNA dynamic model equation analytically. A cubic B-spline collocation method is used to obtain a numerical solution of the Peyrard-Bishop DNA dynamic model equation. A comparison between the results obtained by the analytical methods and the numerical method is investigated. We give some figures to show how accurate the solutions will be obtained from analytical and numerical methods.

**Keywords:** The DNA dynamics model; The Kudryashov method; The modified Kudryashov method; A cubic B-spline collocation method.

## 1 Introduction

Everyone can know that finding analytical and numerical solutions for mathematical models contributes greatly to the interpretation of the physical properties of these models from all previous studies in this field. There are also many mathematical models that have applications in various fields such as engineering, physics, chemistry and fluid mechanics to the last of these sciences. Many researchers have recently addressed many of these phenomena through an analytical or numerical aspect [1–7].

In this paper, I am going to ponder how to obtain solitary wave solutions in the scientific model in DNA flow [8]. The importance of this paper lies in finding different analytical solutions for a model in addition to confirming these solutions numerically and showing the extent of convergence between the numerical solution and the analytical solution by finding the absolute error between them. This show is first described by Peyrard-Bishop, which takes into consideration the incorporation of a non-linear interaction between adjoining relocation with hydrogen bonds [9].

For exploring the appearance of solitonic structures of the oscillator-chain of Peyrard-Bishop demonstrate has been analyzed by [9, 10]. The adjust between powerless nonlinearity and scattering within the DNA energetic demonstrate with straight scattering and nonlinear scattering emerges within the works of Dusuel et al. [11] and Alvarez et al. [12]. The treatment of scientific and physical modeling of conditions of DNA elements appears that those can be decreased to a critical nonlinear arrangement. The nonlinearity of the DNA energetic model arises in localized waves in which have many impressive highlights, as for an illustration in transporting vitality without scattering.

This article is organized as follows: In the second section, we present an analysis of the model under study. In the third section, we introduce the analytical solutions for this model. In the fourth section, we present the numerical solution of the proposed model. In the fifth section, we show some figures for some analytical and numerical solutions. Finally, in the last section, we present the conclusion and a summary of what we will do in this work.

# 2 Peyrard-Bishop DNA dynamic model equation

It is common for a DNA molecule to be a double helix. This means that it consists of two complementary polymeric chains wrapped around each other [13]. B-shaped DNA in the Watson Crick model is a double helix, containing two strings. The masses of nucleotides are not significantly different which means that one can assume a homogeneous crystal structure. Strings are joined together by hydrogen bonds so that these bonds are weak while the harmonic longitudinal length is strong, and the PB model ignores all displacements alongside the transverse [14]. The Hamiltonian model of Berrard and Bishop [14], and the equations in literature, are designed by Morse's potential as

$$F_m(f_n - g_n) = D[e^{-a(f_n-g_n)} - 1]^2, \tag{1}$$

in which $f_n$ and $g_n$ are the displacements of the nucleotides. Also, the Hamiltonian for the DNA chain was described by Zdravkovíc [14]. Moreover, the improved version of the PB model, introduced by Dauxois [15]. The Hamiltonian for describing the strand aperture the hydrogen bonds can be stated as [16]

$$G(f) = \frac{1}{2m}q_n^2 + \frac{k_1}{2}\Delta^2 f_n + \frac{k_2}{4}\Delta^4 f_n + \delta(e^{-a\sqrt{2}f_n})^2, \quad \Delta f_n = f_{n+1} - f_n \tag{2}$$

in which $k_1$ and $k_2$ denote the strength for the linear and nonlinear couplings respectively and $q_n = m\dot{f_n}$ is the momentum for the displacement $f_n$. Searching starting with the hamiltonian (2) the equation of motion in the continuum limit can be stated by the following form

$$f_{tt} - (l_1 + 3l_2 f_{xx}) - 2\sqrt{2}aDe^{-af}(e^{-af} - 1) = 0, \tag{3}$$

with $l_1 = \frac{k_1}{m}d^2, l_2 = \frac{k_2}{m}d^4, D = \frac{\delta}{m}, \alpha \equiv \sqrt{2}a$ and being $d$ the inter-site nucleotide distance in the DNA ladder [17–19]. In this paper, consider the Peyrard-Bishop DNA dynamic model equation as follows

$$f_{tt} - (l_1 + 3l_2 f_x^2)f_{xx} - 2\alpha\omega e^{-\alpha f}(e^{-\alpha f} - 1) = 0, \tag{4}$$

where $l_1, l_2, \alpha$ and $\omega = D$ are constants.

# 3 Analytical solutions

In this section, we give a detailed view of the Kudryashov and the modified Kudryashov methods are used to find the solution of the Peyrard-Bishop DNA dynamic model equation analytically.

### 3.1 The Kudryashov method

The partial differential equation (4) with the following transformation:

$$f(x,t) = h(\xi), \qquad \xi = x - \beta t, \tag{5}$$

can be reduced to the ordinary differential equation as:

$$\beta^2 h'' - (l_1 + 3l_2 h'^2)h'' - 2\alpha\Omega e^{-\alpha h}(e^{-\alpha h} - 1) = 0, \tag{6}$$

By multiplying (6) by $h'$ and integrating once with respect to $\xi$, we get

$$\frac{(\beta^2 - l_1)}{2}(h')^2 - \frac{3}{4}l_2(h')^4 + \Omega e^{-\alpha h}(e^{-\alpha h} - 2) + R = 0, \tag{7}$$

By starting hypothesis is taken to be

$$\phi(\xi) = e^{-\alpha h(\xi)}, \tag{8}$$

By appending (8) into (7), the nonlinear equation is achieved as follows:

$$\frac{(\beta^2 - l_1)}{2\alpha^2}\phi^2(\phi')^2 - \frac{3}{4\alpha^4}l_2(\phi')^4 + \Omega\phi^5(\phi - 2) + R\phi^4 = 0, \tag{9}$$

Now, we can express for the Kudryashov method in a finite series as follows:

$$\phi(\xi) = A_0 + \sum_{i=1}^{N} A_i\Omega^i(\xi), \tag{10}$$

where, $A_0, A_1, A_2, ..., A_N$ are constants and $N$ is a positive integer that can be determined by usind homogeneous balancing method. The function $\Omega(\xi)$ can be expressed in this form:

$$\Omega(\xi) = \frac{1}{1 + de^{\xi}}, \tag{11}$$

where (11) achieve the ordinary differential equation

$$\Omega'(\xi) = \Omega(\xi)(\Omega(\xi) - 1). \tag{12}$$

We can compensate by (10) with some derivatives that we need into (9) then, we get from that of a polynomial as a function in $\Omega(\xi)$

$$P(\Omega(\xi)) = 0. \tag{13}$$

Thus equating the coefficient of each power of $\Omega(\xi)$ in the above equation to zero gives a set of nonlinear algebraic equations with the aid of symbolic computation using Mathematica which will be used to yield the exact solutions for (9).

Now, if we make balancing between $\phi^6$ and $\phi^2(\phi')^2$ or $(\phi')^4$ in (9) we get $6N = 4N + 4$, thus $N = 2$. This offers a truncated series form (10) of the form

$$\phi(\xi) = A_0 + A_1\Omega(\xi) + A_2\Omega^2(\xi). \tag{14}$$

Then, substituting (14) into (9), we get the following system of algebraic equations:

$$\omega A_0^6 - 2\omega A_0^5 + RA_0^4 = 0,$$

$$6\omega A_1 A_0^5 - 10\omega A_1 A_0^4 + 4RA_1 A_0^3 = 0,$$

$$6\omega A_2 A_0^5 + 15\omega A_1^2 A_0^4 - 10\omega A_2 A_0^4 - 20\omega A_1^2 A_0^3 + 4RA_2 A_0^3 + \frac{\beta^2 A_1^2 A_0^2}{2\alpha^2} + 6RA_1^2 A_0^2 - \frac{A_1^2 l_1 A_0^2}{2\alpha^2} = 0,$$

$$30\omega A_1 A_2 A_0^4 + 20\omega A_1^3 A_0^3 - 40\omega A_1 A_2 A_0^3 - 20\omega A_1^3 A_0^2 + \frac{2\beta^2 A_1 A_2 A_0^2}{\alpha^2} + 12RA_1 A_2 A_0^2 + \frac{A_1^2 l_1 A_0^2}{\alpha^2}$$
$$-\frac{\beta^2 A_1^2 A_0^2}{\alpha^2} - \frac{2A_1 A_2 l_1 A_0^2}{\alpha^2} + \frac{\beta^2 A_1^3 A_0}{\alpha^2} + 4RA_1^3 A_0 - \frac{A_1^3 l_1 A_0}{\alpha^2} = 0,$$

$$15\omega A_2^2 A_0^4 - 20\omega A_2^2 A_0^3 + 60\omega A_1^2 A_2 A_0^3 + 15\omega A_1^4 A_0^2 + \frac{\beta^2 A_1^2 A_0^2}{2\alpha^2} + \frac{2\beta^2 A_2^2 A_0^2}{\alpha^2} + 6RA_2^2 A_0^2 - 60\omega A_1^2 A_2 A_0^2$$
$$+\frac{4A_1 A_2 l_1 A_0^2}{\alpha^2} - \frac{4\beta^2 A_1 A_2 A_0^2}{\alpha^2} - \frac{2A_2^2 l_1 A_0^2}{\alpha^2} - \frac{A_1^2 l_1 A_0^2}{2\alpha^2} - 10\omega A_1^4 A_0 + \frac{5\beta^2 A_1^2 A_2 A_0}{\alpha^2} + 12RA_1^2 A_2 A_0$$
$$+\frac{2A_1^3 l_1 A_0}{\alpha^2} - \frac{2\beta^2 A_1^3 A_0}{\alpha^2} - \frac{5A_1^2 A_2 l_1 A_0}{\alpha^2} + \frac{\beta^2 A_1^4}{2\alpha^2} + RA_1^4 - \frac{A_1^4 l_1}{2\alpha^2} - \frac{3A_1^4 l_2}{4\alpha^4} = 0,$$

$$-2\Omega A_1^5 + 6\Omega A_0 A_1^5 + \frac{l_1 A_1^4}{\alpha^2} + \frac{3l_2 A_1^4}{\alpha^4} - \frac{\beta^2 A_1^4}{\alpha^2} + \frac{\beta^2 A_0 A_1^3}{\alpha^2} + \frac{3\beta^2 A_2 A_1^3}{\alpha^2} + 60\Omega A_0^2 A_2 A_1^3 + 4RA_2 A_1^3$$
$$-40\Omega A_0 A_2 A_1^3 - \frac{A_0 l_1 A_1^3}{\alpha^2} - \frac{3A_2 l_1 A_1^3}{\alpha^2} - \frac{6A_2 l_2 A_1^3}{\alpha^4} + \frac{10A_0 A_2 l_1 A_1^2}{\alpha^2} - \frac{10\beta^2 A_0 A_2 A_1^2}{\alpha^2}$$
$$+60\omega A_0^3 A_2^2 A_1 - 60\Omega A_0^2 A_2^2 A_1 + \frac{8\beta^2 A_0 A_2^2 A_1}{\alpha^2} + 12RA_0 A_2^2 A_1 + \frac{2\beta^2 A_0^2 A_2 A_1}{\alpha^2}$$
$$-\frac{8A_0 A_2^2 l_1 A_1}{\alpha^2} - \frac{2A_0^2 A_2 l_1 A_1}{\alpha^2} + \frac{4A_0^2 A_2^2 l_1}{\alpha^2} - \frac{4\beta^2 A_0^2 A_2^2}{\alpha^2} = 0,$$

$$\omega A_1^6 + \frac{\beta^2 A_1^4}{2\alpha^2} - 10\omega A_2 A_1^4 + 30\omega A_0 A_2 A_1^4 - \frac{l_1 A_1^4}{2\alpha^2} - \frac{9l_2 A_1^4}{2\alpha^4} + \frac{6A_2 l_1 A_1^3}{\alpha^2} + \frac{24A_2 l_2 A_1^3}{\alpha^4} - \frac{6\beta^2 A_2 A_1^3}{\alpha^2}$$
$$+\frac{13\beta^2 A_2^2 A_1^2}{2\alpha^2} + 90\omega A_0^2 A_2^2 A_1^2 + 6RA_2^2 A_1^2 - 60\omega A_0 A_2^2 A_1^2 + \frac{5\beta^2 A_0 A_2 A_1^2}{\alpha^2} - \frac{5A_0 A_2 l_1 A_1^2}{\alpha^2}$$
$$-\frac{13A_2^2 l_1 A_1^2}{2\alpha^2} - \frac{18A_2^2 l_2 A_1^2}{\alpha^4} + \frac{16A_0 A_2^2 l_1 A_1}{\alpha^2} - \frac{16\beta^2 A_0 A_2^2 A_1}{\alpha^2} + 20\omega A_0^3 A_2^3 - 20\omega A_0^2 A_2^3$$
$$+\frac{4\beta^2 A_0 A_2^3}{\alpha^2} + 4RA_0 A_2^3 + \frac{2\beta^2 A_0^2 A_2^2}{\alpha^2} - \frac{4A_0 A_2^3 l_1}{\alpha^2} - \frac{2A_0^2 A_2^2 l_1}{\alpha^2} = 0,$$

$$6\omega A_2 A_1^5 + \frac{3l_2 A_1^4}{\alpha^4} - 20\omega A_2^2 A_1^3 + 60\omega A_0 A_2^2 A_1^3 + \frac{3\beta^2 A_2 A_1^3}{\alpha^2} - \frac{3A_2 l_1 A_1^3}{\alpha^2} - \frac{36A_2 l_2 A_1^3}{\alpha^4}$$
$$+\frac{13A_2^2 l_1 A_1^2}{\alpha^2} + \frac{72A_2^2 l_2 A_1^2}{\alpha^4} - \frac{13\beta^2 A_2^2 A_1^2}{\alpha^2} + \frac{6\beta^2 A_2^3 A_1}{\alpha^2} + 60\omega A_0^2 A_2^3 A_1 + 4RA_2^3 A_1 - 40\omega A_0 A_2^3 A_1$$
$$+\frac{8\beta^2 A_0 A_2^2 A_1}{\alpha^2} - \frac{6A_2^3 l_1 A_1}{\alpha^2} - \frac{8A_0 A_2^2 l_1 A_1}{\alpha^2} - \frac{24A_2^3 l_2 A_1}{\alpha^4} + \frac{8A_0 A_2^3 l_1}{\alpha^2} - \frac{8\beta^2 A_0 A_2^3}{\alpha^2} = 0,$$

$$15\omega A_2^2 A_1^4 - \frac{3l_2 A_1^4}{4\alpha^4} + \frac{24A_2 l_2 A_1^3}{\alpha^4} - 20\omega A_2^3 A_1^2 + 60\omega A_0 A_2^3 A_1^2 + \frac{13\beta^2 A_2^2 A_1^2}{2\alpha^2} - \frac{13A_2^2 l_1 A_1^2}{2\alpha^2}$$
$$-\frac{108A_2^2 l_2 A_1^2}{\alpha^4} + \frac{12A_2^3 l_1 A_1}{\alpha^2} + \frac{96A_2^3 l_2 A_1}{\alpha^4} - \frac{12\beta^2 A_2^3 A_1}{\alpha^2} + \frac{2\beta^2 A_2^4}{\alpha^2} + 15\omega A_0^2 A_2^4 + RA_2^4$$

$$-10\omega A_0 A_2^4 + \frac{4\beta^2 A_0 A_2^3}{\alpha^2} - \frac{2A_2^4 l_1}{\alpha^2} - \frac{4A_0 A_2^3 l_1}{\alpha^2} - \frac{12A_2^4 l_2}{\alpha^4} = 0,$$

$$-10\omega A_1 A_2^4 + 30\omega A_0 A_1 A_2^4 + \frac{4l_1 A_2^4}{\alpha^2} + \frac{48 l_2 A_2^4}{\alpha^4} - \frac{4\beta^2 A_2^4}{\alpha^2} + 20\omega A_1^3 A_2^3 + \frac{6\beta^2 A_1 A_2^3}{\alpha^2} - \frac{6A_1 l_1 A_2^3}{\alpha^2}$$
$$-\frac{144 A_1 l_2 A_2^3}{\alpha^4} + \frac{72 A_1^2 l_2 A_2^2}{\alpha^4} - \frac{6A_1^3 l_2 A_2}{\alpha^4} = 0,$$

$$-2\omega A_2^5 + 6\omega A_0 A_2^5 + \frac{2\beta^2 A_2^4}{\alpha^2} + 15\omega A_1^2 A_2^4 - \frac{2l_1 A_2^4}{\alpha^2} - \frac{72 l_2 A_2^4}{\alpha^4} + \frac{96 A_1 l_2 A_2^3}{\alpha^4} - \frac{18 A_1^2 l_2 A_2^2}{\alpha^4} = 0,$$

$$6\omega A_1 A_2^5 + \frac{48 l_2 A_2^4}{\alpha^4} - \frac{24 A_1 l_2 A_2^3}{\alpha^4} = 0,$$

$$\omega A_2^6 - \frac{12 A_2^4 l_2}{\alpha^4} = 0.$$

Thus, solving the above system gives
**Case 1:**

$$A_0 = 0,\ \ A_1 = -\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},\ \ A_2 = \frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},\ \ \beta = \mp\frac{\sqrt{2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega} + \alpha^2 l_1 + 3l_2}}{\alpha},$$

$$R = \frac{-4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega} - 3l_2}{4\alpha^4}.$$

By using (5), (8), (11) and (14) yields the following bright soliton solution for (4)

$$f_{1,2}(x,t) = -\frac{1}{\alpha} ln\Bigg(-\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1 + de^{(x-\beta t)})} + \frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1 + de^{(x-\beta t)})^2}\Bigg). \tag{15}$$

**Case 2:**

$$A_0 = 0,\ \ A_1 = \frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},\ \ A_2 = -\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}},\ \ \beta = \mp\frac{\sqrt{-2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\Omega} + \alpha^2 l_1 + 3l_2}}{\alpha},$$

$$R = \frac{4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega} - 3l_2}{4\alpha^4}.$$

By using (5), (8), (11) and (14) yields the following bright soliton solution for (4)

$$f_{3,4}(x,t) = -\frac{1}{\alpha} ln\Bigg(\frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1 + de^{(x-\beta t)})} - \frac{2\sqrt{3}\sqrt{l_2}}{\alpha^2\sqrt{\omega}(1 + de^{(x-\beta t)})^2}\Bigg). \tag{16}$$

## 3.2 The modified Kudryashov method

We illustrate the modified Kudryashov method in this section by suppose a solution of (9) given in a series take shape

$$\phi(\xi) = \sum_{n=0}^{N} A_n Q^n(\xi), \tag{17}$$

where, $A_0, A_1, A_2, ..., A_N$ are constants to be calculate; $N$ is a whole number to be obtained, while $Q(\xi)$ is introduce by:

$$Q(\xi) = \frac{1}{1+bm^{\xi}}, \tag{18}$$

which accept the below differential equation:

$$Q'(\xi) = (Q^2(\xi) - Q(\xi))\ln(m). \tag{19}$$

Putting (17) and its possible derivatives like:

$$\begin{aligned} \phi'(\xi) &= \sum_{n=0}^{N} A_n n Q^n (Q-1)\ln(m), \\ \phi''(\xi) &= \sum_{n=0}^{N} A_n n Q^n (Q-1)((1+n)(Q-n)\ln(m))^2, \end{aligned} \tag{20}$$

in (9) yields a polynomial in $Q(\xi)$;

$$P(Q(\xi)) = 0. \tag{21}$$

Thus equating the coefficient of each power of $Q(\xi)$ in the above equation to zero gives a set of nonlinear algebraic equations which will be used to yield the exact solutions for (9). This offers a truncated series form (17) of the form

$$\phi(\xi) = A_0 + A_1 Q(\xi) + A_2 Q^2(\xi). \tag{22}$$

Then, substituting (22) into (9), we get the following system of algebraic equations:

$$A_0^4 R + A_0^6\omega - 2A_0^5\omega = 0,$$

$$4A_1A_0^3R + 6A_1A_0^5\omega - 10A_1A_0^4\omega = 0,$$

$$\begin{gathered} -\frac{A_1^2A_0^2l_1\log^2(m)}{2\alpha^2} + \frac{A_1^2A_0^2\beta^2\log^2(m)}{2\alpha^2} + 4A_2A_0^3R + 6A_1^2A_0^2R \\ +6A_2A_0^5\omega + 15A_1^2A_0^4\omega - 10A_2A_0^4\omega - 20A_1^2A_0^3\omega = 0, \end{gathered}$$

$$\begin{gathered} \frac{A_1^2A_0^2l_1\log^2(m)}{\alpha^2} - \frac{2A_1A_2A_0^2l_1\log^2(m)}{\alpha^2} - \frac{A_1^3A_0l_1\log^2(m)}{\alpha^2} + \frac{2A_1A_2A_0^2\beta^2\log^2(m)}{\alpha^2} \\ -\frac{A_1^2A_0^2\beta^2\log^2(m)}{\alpha^2} + \frac{A_1^3A_0\beta^2\log^2(m)}{\alpha^2} + 12A_1A_2A_0^2R + 4A_1^3A_0R \\ +30A_1A_2A_0^4\omega + 20A_1^3A_0^3\omega - 40A_1A_2A_0^3\omega - 20A_1^3A_0^2\omega = 0, \end{gathered}$$

$$\begin{gathered} \frac{4A_1A_2A_0^2l_1\log^2(m)}{\alpha^2} - \frac{2A_2^2A_0^2l_1\log^2(m)}{\alpha^2} - \frac{A_1^2A_0^2l_1\log^2(m)}{2\alpha^2} + \frac{2A_1^3A_0l_1\log^2(m)}{\alpha^2} \\ -\frac{5A_1^2A_2A_0l_1\log^2(m)}{\alpha^2} - \frac{A_1^4l_1\log^2(m)}{2\alpha^2} - \frac{3A_1^4l_2\log^4(m)}{4\alpha^4} + \frac{A_1^2A_0^2\beta^2\log^2(m)}{2\alpha^2} \\ +\frac{2A_2^2A_0^2\beta^2\log^2(m)}{\alpha^2} - \frac{4A_1A_2A_0^2\beta^2\log^2(m)}{\alpha^2} + \frac{5A_1^2A_2A_0\beta^2\log^2(m)}{\alpha^2} - \frac{2A_1^3A_0\beta^2\log^2(m)}{\alpha^2} \\ +\frac{A_1^4\beta^2\log^2(m)}{2\alpha^2} + 6A_2^2A_0^2R + 12A_1^2A_2A_0R + A_1^4R + 15A_2^2A_0^4\omega - 20A_2^2A_0^3\omega + 60A_1^2A_2A_0^3\omega \end{gathered}$$

$$+15A_1^4A_0^2\omega-60A_1^2A_2A_0^2\omega-10A_1^4A_0\omega=0,$$

$$\frac{A_1^4l_1\log^2(m)}{\alpha^2}+\frac{3A_1^4l_2\log^4(m)}{\alpha^4}-\frac{A_0A_1^3l_1\log^2(m)}{\alpha^2}-\frac{3A_2A_1^3l_1\log^2(m)}{\alpha^2}-\frac{6A_2A_1^3l_2\log^4(m)}{\alpha^4}$$
$$+\frac{10A_0A_2A_1^2l_1\log^2(m)}{\alpha^2}-\frac{8A_0A_2^2A_1l_1\log^2(m)}{\alpha^2}-\frac{2A_0^2A_2A_1l_1\log^2(m)}{\alpha^2}+\frac{4A_0^2A_2^2l_1\log^2(m)}{\alpha^2}$$
$$-\frac{A_1^4\beta^2\log^2(m)}{\alpha^2}+\frac{A_0A_1^3\beta^2\log^2(m)}{\alpha^2}+\frac{3A_2A_1^3\beta^2\log^2(m)}{\alpha^2}-\frac{10A_0A_2A_1^2\beta^2\log^2(m)}{\alpha^2}$$
$$+\frac{8A_0A_2^2A_1\beta^2\log^2(m)}{\alpha^2}+\frac{2A_0^2A_2A_1\beta^2\log^2(m)}{\alpha^2}-\frac{4A_0^2A_2^2\beta^2\log^2(m)}{\alpha^2}+4A_2A_1^3R$$
$$+12A_0A_2^2A_1R-2A_1^5\omega+6A_0A_1^5\omega+60A_0^2A_2A_1^3\omega-40A_0A_2A_1^3\omega$$
$$+60A_0^3A_2^2A_1\omega-60A_0^2A_2^2A_1\omega=0,$$

$$-\frac{A_1^4l_1\log^2(m)}{2\alpha^2}-\frac{9A_1^4l_2\log^4(m)}{2\alpha^4}+\frac{6A_2A_1^3l_1\log^2(m)}{\alpha^2}+\frac{24A_2A_1^3l_2\log^4(m)}{\alpha^4}$$
$$-\frac{5A_0A_2A_1^2l_1\log^2(m)}{\alpha^2}-\frac{13A_2^2A_1^2l_1\log^2(m)}{2\alpha^2}-\frac{18A_2^2A_1^2l_2\log^4(m)}{\alpha^4}$$
$$+\frac{16A_0A_2^2A_1l_1\log^2(m)}{\alpha^2}-\frac{4A_0A_2^3l_1\log^2(m)}{\alpha^2}-\frac{2A_0^2A_2^2l_1\log^2(m)}{\alpha^2}$$
$$+\frac{A_1^4\beta^2\log^2(m)}{2\alpha^2}-\frac{6A_2A_1^3\beta^2\log^2(m)}{\alpha^2}+\frac{13A_2^2A_1^2\beta^2\log^2(m)}{2\alpha^2}$$
$$+\frac{5A_0A_2A_1^2\beta^2\log^2(m)}{\alpha^2}-\frac{16A_0A_2^2A_1\beta^2\log^2(m)}{\alpha^2}+\frac{4A_0A_2^3\beta^2\log^2(m)}{\alpha^2}+\frac{2A_0^2A_2^2\beta^2\log^2(m)}{\alpha^2}$$
$$+6A_2^2A_1^2R+4A_0A_2^3R+A_1^6\omega-10A_2A_1^4\omega+30A_0A_2A_1^4\omega+90A_0^2A_2^2A_1^2\omega$$
$$-60A_0A_2^2A_1^2\omega+20A_0^3A_2^3\omega-20A_0^2A_2^3\omega=0,$$

$$\frac{3A_1^4l_2\log^4(m)}{\alpha^4}-\frac{3A_2A_1^3l_1\log^2(m)}{\alpha^2}-\frac{36A_2A_1^3l_2\log^4(m)}{\alpha^4}+\frac{13A_2^2A_1^2l_1\log^2(m)}{\alpha^2}$$
$$+\frac{72A_2^2A_1^2l_2\log^4(m)}{\alpha^4}-\frac{6A_2^3A_1l_1\log^2(m)}{\alpha^2}-\frac{8A_0A_2^2A_1l_1\log^2(m)}{\alpha^2}-\frac{24A_2^3A_1l_2\log^4(m)}{\alpha^4}$$
$$+\frac{8A_0A_2^3l_1\log^2(m)}{\alpha^2}+\frac{3A_2A_1^3\beta^2\log^2(m)}{\alpha^2}-\frac{13A_2^2A_1^2\beta^2\log^2(m)}{\alpha^2}$$
$$+\frac{6A_2^3A_1\beta^2\log^2(m)}{\alpha^2}+\frac{8A_0A_2^2A_1\beta^2\log^2(m)}{\alpha^2}-\frac{8A_0A_2^3\beta^2\log^2(m)}{\alpha^2}+4A_2^3A_1R$$
$$+6A_2A_1^5\omega-20A_2^2A_1^3\omega+60A_0A_2^2A_1^3\omega+60A_0^2A_2^3A_1\omega-40A_0A_2^3A_1\omega=0,$$

$$\frac{96A_1A_2^3l_2\log^4(m)}{\alpha^4}+\frac{24A_1^3A_2l_2\log^4(m)}{\alpha^4}-\frac{12A_2^4l_2\log^4(m)}{\alpha^4}-\frac{108A_1^2A_2^2l_2\log^4(m)}{\alpha^4}$$
$$-\frac{3A_1^4l_2\log^4(m)}{4\alpha^4}+\frac{12A_1A_2^3l_1\log^2(m)}{\alpha^2}-\frac{2A_2^4l_1\log^2(m)}{\alpha^2}-\frac{4A_0A_2^3l_1\log^2(m)}{\alpha^2}$$
$$-\frac{13A_1^2A_2^2l_1\log^2(m)}{2\alpha^2}+\frac{2A_2^4\beta^2\log^2(m)}{\alpha^2}+\frac{4A_0A_2^3\beta^2\log^2(m)}{\alpha^2}+\frac{13A_1^2A_2^2\beta^2\log^2(m)}{2\alpha^2}$$

$$-\frac{12A_1A_2^3\beta^2\log^2(m)}{\alpha^2}+A_2^4R+15A_0^2A_2^4\omega-10A_0A_2^4\omega-20A_1^2A_2^3\omega+60A_0A_1^2A_2^3\omega+15A_1^4A_2^2\omega=0,$$

$$\frac{48A_2^4l_2\log^4(m)}{\alpha^4}+\frac{72A_1^2A_2^2l_2\log^4(m)}{\alpha^4}-\frac{144A_1A_2^3l_2\log^4(m)}{\alpha^4}-\frac{6A_1^3A_2l_2\log^4(m)}{\alpha^4}$$

$$+\frac{4A_2^4l_1\log^2(m)}{\alpha^2}-\frac{6A_1A_2^3l_1\log^2(m)}{\alpha^2}+\frac{6A_1A_2^3\beta^2\log^2(m)}{\alpha^2}$$

$$-\frac{4A_2^4\beta^2\log^2(m)}{\alpha^2}-10A_1A_2^4\omega+30A_0A_1A_2^4\omega+20A_1^3A_2^3\omega=0,$$

$$-\frac{2A_2^4l_1\log^2(m)}{\alpha^2}-\frac{72A_2^4l_2\log^4(m)}{\alpha^4}+\frac{96A_1A_2^3l_2\log^4(m)}{\alpha^4}-\frac{18A_1^2A_2^2l_2\log^4(m)}{\alpha^4}$$

$$+\frac{2A_2^4\beta^2\log^2(m)}{\alpha^2}-2A_2^5\omega+6A_0A_2^5\omega+15A_1^2A_2^4\omega=0,$$

$$\frac{48A_2^4l_2\log^4(m)}{\alpha^4}-\frac{24A_1A_2^3l_2\log^4(m)}{\alpha^4}+6A_1A_2^5\omega=0,$$

$$A_2^6\omega-\frac{12A_2^4l_2\log^4(m)}{\alpha^4}=0.$$

Thus, solving the above system gives

**Case 1:**

$$A_0=0,\ \ A_1=-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}},\ \ A_2=\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}},\ \ \beta=\mp\frac{\sqrt{2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}+\alpha^2l_1+3l_2\log^2(m)}}{\alpha},$$

$$R=\frac{-4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}\log^2(m)-3l_2\log^4(m)}{4\alpha^4}.$$

By using (5), (8), (18) and (22) yields the following bright soliton solution for (4)

$$f_{1,2}(x,t)=-\frac{1}{\alpha}ln\left(-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})}+\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})^2}\right). \tag{23}$$

**Case 2:**

$$A_0=0,\ \ A_1=\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}},\ \ A_2=-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}},\ \ \beta=\mp\frac{\sqrt{2\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}+\alpha^2l_1+3l_2\log^2(m)}}{\alpha},$$

$$R=\frac{4\sqrt{3}\alpha^2\sqrt{l_2}\sqrt{\omega}\log^2(m)-3l_2\log^4(m)}{4\alpha^4}.$$

By using (5), (8), (18) and (22) yields the following bright soliton solution for (4)

$$f_{3,4}(x,t)=-\frac{1}{\alpha}ln\left(\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})}-\frac{2\sqrt{3}\sqrt{l_2}\log^2(m)}{\alpha^2\sqrt{\omega}(1+bm^{(x-\beta t)})^2}\right). \tag{24}$$

# 4 Numerical solutions using a cubic B-spline collocation method

In this section, we take approximations for spaces $x$ and $t$ derivatives as [22]. Now, we assume that $f(x,t)$ the exact solution at the grid point $(x_i, t_j)$ and $f_{i,j}$ is the numerical solution at the same point. The required values of $f_i$ and its first and the second derivatives, $f_i'$ and $f_i''$, at nodal points $x_i$ are identified in terms of $c_i$ as

$$\begin{aligned} f_{i,j} &= c_{i,j_1} + 4c_{i,j} + c_{i,j-1}, \\ f_x = f_{i,j}' &= \frac{3}{h}(c_{i+1,j} - c_{i-1,j}), \\ f_{xx} = f_{i,j}'' &= \frac{6}{h^2}(c_{i,j-1} + c_{i,j+1} - 2c_{i,j}), \end{aligned} \tag{25}$$

and if the time derivative is discretized using finite differences, we have where

$$f_{tt} = \frac{c_{i,j-1} + c_{i,j+1} - 2c_{i,j}}{k^2}. \tag{26}$$

Substituting (26) into (4) a we get

$$\begin{aligned} \frac{c_{i,j-1} + c_{i,j+1} - 2c_{i,j}}{k^2} &- (l_1 + 3l_2(\frac{3}{h}(c_{i+1,j} - c_{i-1,j}))^2)(\frac{6}{h^2}(c_{i,j-1} + c_{i,j+1} - 2c_{i,j})) \\ &- 2\alpha\omega e^{-\alpha(c_{i,j_1} + 4c_{i,j} + c_{i,j-1})}(e^{-\alpha(c_{i,j_1} + 4c_{i,j} + c_{i,j-1})} - 1) = 0, \end{aligned} \tag{27}$$

Now we can be solved the (27) by many methods.

## 4.1 The numerical results

Now, we will intreduce some numerical resultes for the Peyrard-Bishop DNA dynamic model equation. In Table 1 we introduce comparison between the absolute value of numerical results with the absolute value of analytical solution (15) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, t = 1, k = 0.01, h = 0.1, d = 1$. In Figure 5 we introduce the absolute value of analytical and the absolute value of numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$.

Table 1: Comparison between numerical results and analytical solution

| $x$ | Numerical solution | Exact solution | Absolute error |
|---|---|---|---|
| -5 | 5.44008 | 5.44008 | 8.94098 E-7 |
| -4 | 4.68701 | 4.68702 | 1.92824 E-6 |
| -3 | 4.06242 | 4.06242 | 3.65362 E-6 |
| -2 | 3.62989 | 3.62989 | 3.12923 E-6 |
| -1 | 3.42318 | 3.42318 | 4.82165 E-6 |
| 0 | 3.42113 | 3.42112 | 9.77842 E-6 |
| 1 | 3.62319 | 3.62319 | 1.48434 E-6 |
| 2 | 4.05108 | 4.05108 | 3.93886 E-6 |
| 3 | 4.67227 | 4.67227 | 3.08816 E-6 |
| 4 | 5.42322 | 5.42322 | 1.46944 E-6 |
| 5 | 6.25290 | 6.25290 | 6.63292 E-7 |

In Table 2 we introduce comparison between the numerical results with the analytical solution (16) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, t = 1, k = 0.01, h = 0.1, d = 1$. In Figure 6 we introduce analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$.

Table 2: Comparison between the numerical results with the analytical solution

| $x$ | Numerical solution | Exact solution | Absolute error |
|---|---|---|---|
| -5 | 5.06351 | 5.06351 | 7.31236 E-7 |
| -4 | 4.08336 | 4.08336 | 1.60977 E-6 |
| -3 | 3.13635 | 3.13636 | 3.29208 E-6 |
| -2 | 2.27369 | 2.27369 | 3.79312 E-6 |
| -1 | 1.60539 | 1.60539 | 2.62002 E-6 |
| 0 | 1.30025 | 1.30024 | 1.01515 E-5 |
| 1 | 1.47338 | 1.47338 | 3.62099 E-6 |
| 2 | 2.05599 | 2.05590 | 3.56673 E-6 |
| 3 | 2.87752 | 2.87752 | 3.46209 E-6 |
| 4 | 3.80764 | 3.80764 | 1.74354 E-6 |
| 5 | 4.78129 | 4.78129 | 7.97996 E-7 |

In Table 3 we introduce comparison between the absolute value of numerical results with the absolute value of analytical solution (23) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, t = 1, k = 0.01, h = 0.1, b = 1, m = 0.1$. In Figure 7 we introduce the absolute value of analytical and the absolute value of numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, b = 1, m = 0.1$.

Table 3: Comparison between numerical results and analytical solution

| $x$ | Numerical solution | Exact solution | Absolute error |
|---|---|---|---|
| -5 | 8.92689 | 8.92689 | 1.48864 E-7 |
| -4 | 6.82056 | 6.82056 | 1.11557 E-6 |
| -3 | 4.89867 | 4.89868 | 1.09047 E-5 |
| -2 | 3.49315 | 3.49318 | 8.73023 E-5 |
| -1 | 3.14647 | 3.14646 | 1.04599 E-4 |
| 0 | 3.14263 | 3.14262 | 1.93872 E-4 |
| 1 | 3.71882 | 3.71889 | 1.67069 E-4 |
| 2 | 5.28412 | 5.28415 | 4.02388 E-5 |
| 3 | 7.26172 | 7.26172 | 4.45683 E-6 |
| 4 | 9.39024 | 9.39024 | 4.50439 E-7 |
| 5 | 11.5857 | 11.5857 | 6.41029 E-8 |

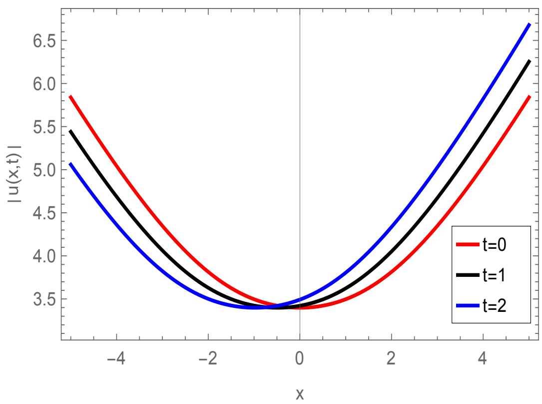
In Table 4 we introduce comparison between the numerical results with the analytical solution (24) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, t = 1, k = 0.01, h = 0.1, b = 1, m = 0.1$. In Figure 2 we introduce analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, b = 1, m = 0.1$.

Table 4: Comparison between the numerical results with the analytical solution

| $x$ | Numerical solution | Exact solution | Absolute error |
|---|---|---|---|
| -5 | 8.86341 | 8.86341 | 1.23149 E-7 |
| -4 | 6.56126 | 6.56126 | 9.12388 E-7 |
| -3 | 4.26305 | 4.26306 | 8.96286 E-6 |
| -2 | 2.00367 | 2.00374 | 7.52585 E-5 |
| -1 | 0.08889 | 0.08903 | 1.44369 E-4 |
| 0 | 0.18073 | 0.18097 | 2.37149 E-4 |
| 1 | 1.51398 | 1.51416 | 1.75912 E-4 |
| 2 | 3.74443 | 3.74447 | 4.55415 E-5 |
| 3 | 6.03968 | 6.03969 | 5.07657 E-6 |
| 4 | 8.34153 | 8.34153 | 5.13391 E-7 |
| 5 | 10.6439 | 10.6439 | 7.18443 E-8 |

# 5 Graphical results and discussion

Now that we have completed the analytical and numerical calculations of the model under study using the analytical and numerical methods described above, we present in this section some forms in the two-dimensional and three-dimensional to show the accuracy of the solutions obtained by the analytical methods and this shows us clearly in the forms 1-4. Also in the following figures we show how accurate the numerical method used is and its great agreement with the analytical solutions as shown in Figure 5-8.



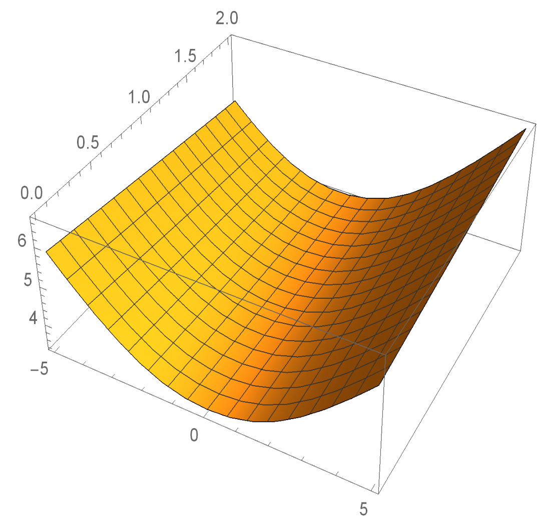




Figure 1: Analytical solution (15) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$.

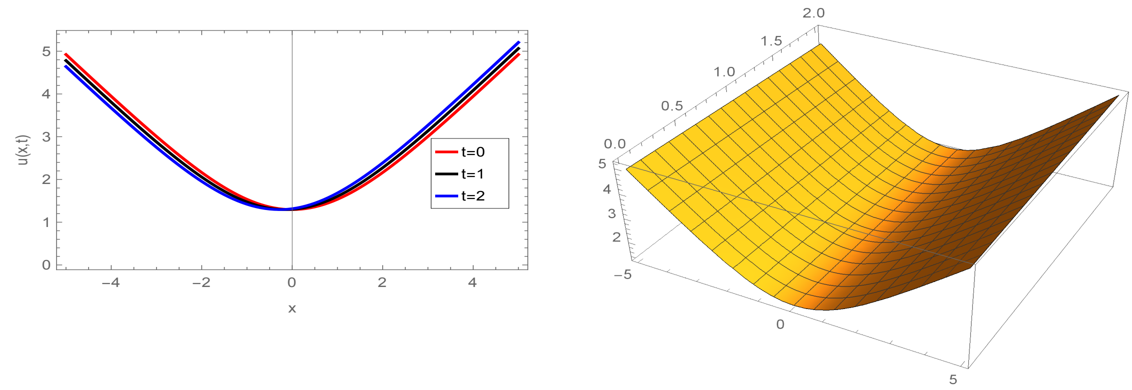



Figure 2: Analytical solution (16) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$

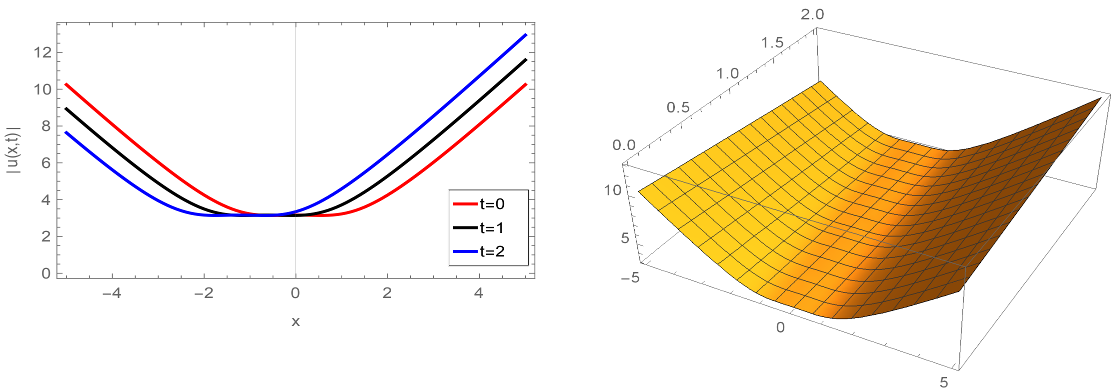



Figure 3: Analytical solution (23) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1.$

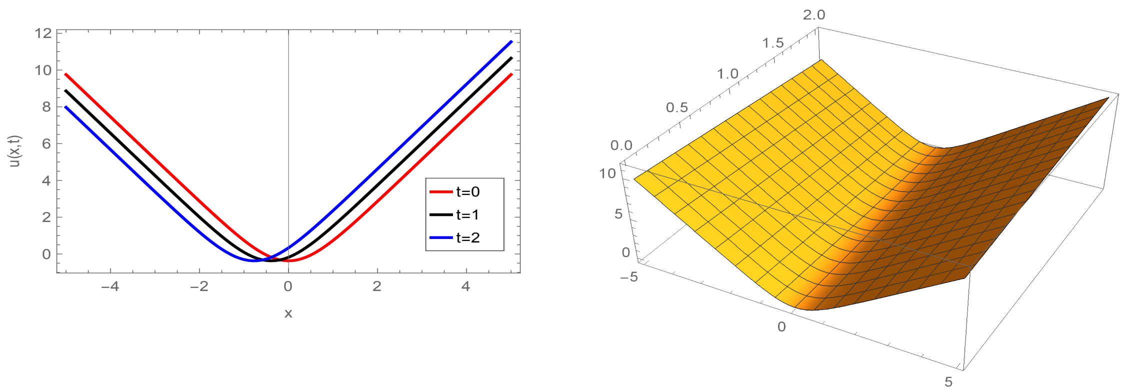



Figure 4: Analytical solution (24) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$

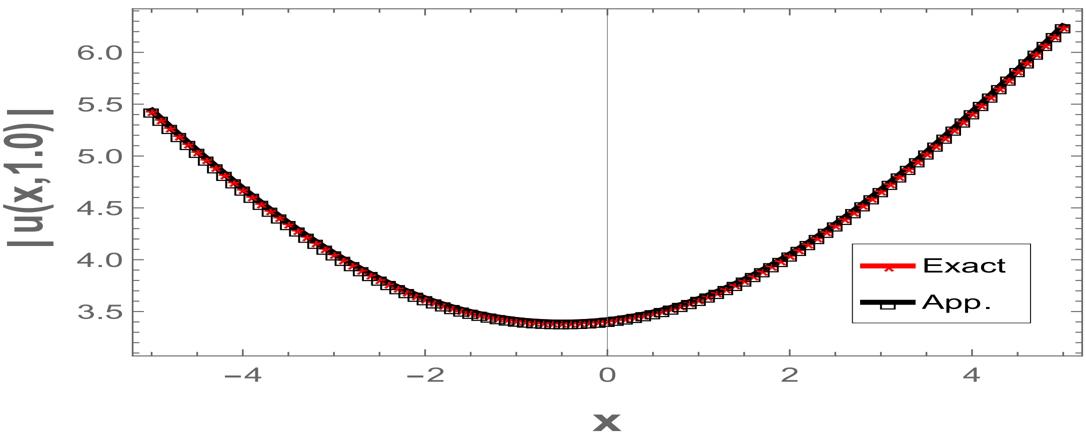



Figure 5: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$.

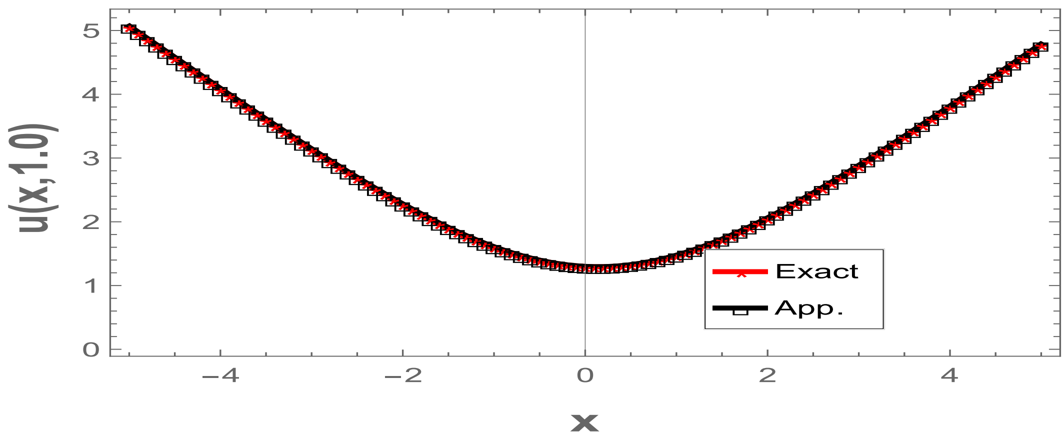



Figure 6: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, d = 1$

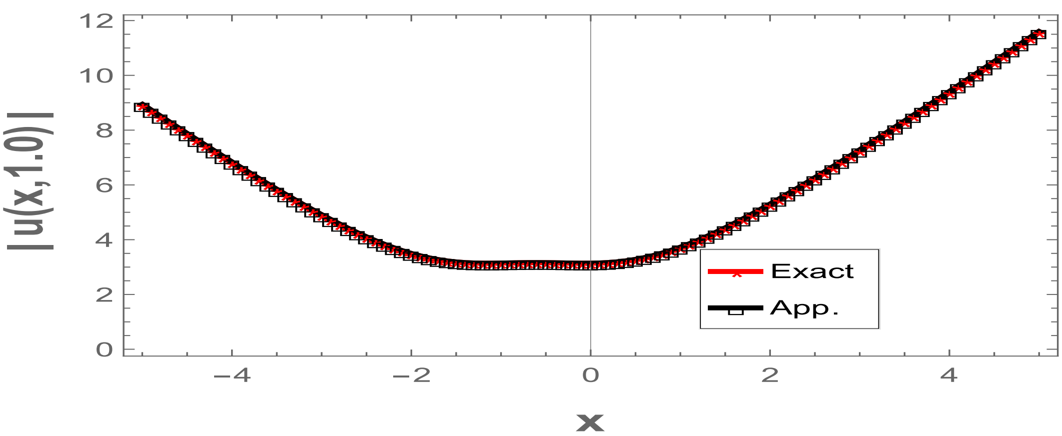



Figure 7: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$.

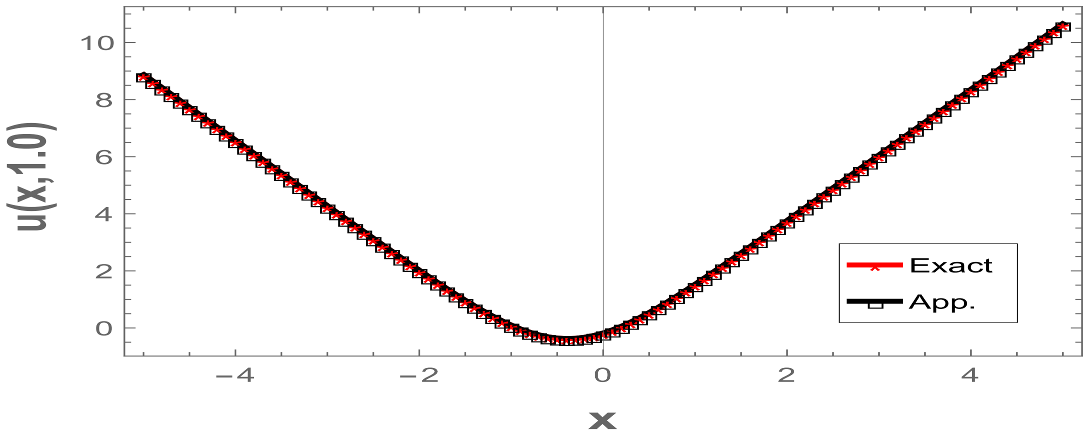



Figure 8: Analytical and numerical solutions for (4) at $\alpha = 1, \Omega = 0.1, l_2 = 0.01, l_1 = 0.1, m = 0.1, b = 1$.

# 6 Conclusion

In this paper, we have studied the Peyrard-Bishop DNA dynamic model equation analytically and numerically. The Kudryashov method and modified Kudryashov method have been used to find the solution of the Peyrard-Bishop DNA dynamic model equation analytically. Also, a cubic B-spline collocation method has been used to obtain a numerical solution of the Peyrard-Bishop DNA dynamic model equation. Various solutions to the equation have been realized in this study. Finally, we depict some of the obtained solutions graphically and conclude that The results we obtained, whether numerical or analytical, are accurate, efficient, and versatile in mathematical physics to solve other NLEEs.

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