Combo optical soliton and rogue wave solutions of the time fractional perturbed Radhakrishnan-Kundu-Lakshmanan model

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Abstract

In this paper, we apply the tan(⊖/2) expansion and the Kudryashov general approaches to the time fractional perturbed Radhakrishnan-Kundu -Lakshmanan (RKL) equation. These integration schemes provide a number of optical soliton solutions of the model. The solutions registered with constraint conditions on the parameters that follow their existence criteria. To the constraint conditions, the solutions offer various transmission signals through optical fibres, such as double periodic optical solitons, combo optical periodic and rogue waves, combo periodic and shock waves, combo periodic and solitons, and combo double singular solitons. Moreover, after interaction of rogue and periodic waves, it is shown that the rogue waves are going to diminish after a certain time keeping periodic nature of the interaction. In fact, interaction of periodic and rogue waves produces periodic rogue type breather waves, that indicates the amplitude of the rogue waves gradually decreases, and vanishes after a certain time. Some dynamical signals are plotted in the graphs by picking suitable values on the parameters.

Keywords The time fractional perturbed Radhakrishnan-Kundu -Lakshmanan (RKL) equation; the tan(⊖/2) expansion method; the Kudryashov general method; optical soliton; Rogue waves.

1 Introduction

An exciting subclass of nonlinear evolution equations (NLEEs) can be expressed complex nonlinear dynamical phenomena, and provide deep quantitative insight into the dynamical systems under examination. The nonlinear Schrodinger type NLEEs are famous examples of such special subclass that facilitate deeper conceptual understanding of complex nonlinear physical aspects such as quantum electronics, photonics, electromagnetism, plasma physics and fluid dynamics [1, 2, 3, 4]. In fact, they are exactly solvable dynamical equations, they offer an interesting area of research in the theory of optical solitons to investigate soliton transmission through optical fibres, and in multidimensional applications in mathematical physics [5, 6].

Recently, a lot of efforts have been devoted to relate the topic of optical solitons in view point of nonlinear signals in communication sciences [7, 8, 9, 10, 11, 12, 13, 14]. The models: Lakshmanan-Porsezian-Daniel model, complex Ginzburg-Landau equation, Kundu-Eckhaus equation, Fokas-Ienells equation, Kaup-Newll model, Radhkrishnan-Kundu-Lakshmanan equation, resonant nonlinear Schrodinger's equation and the references therein, were successfully addressed dynamical transmission solitons solutions [10, 11, 12, 15, 16, 17, 18].

A systematic approach for obtaining the solutions of various type of NLEEs was introduced in [2, 14, 19, 20, 21, 22, 23, 24]. However, our aim is to examine combo optical soliton and rogue wave solutions of the dimensionless form of the conformable time fractional perturbed Radhkrishnan-Kundu-Lakshmanan equation [25]. It is based on the $\tan(\ominus/2)$ expansion and the proposed Kudryashov general approaches [13, 24]. The nonlinear combo optical solitons often lead to combo double singular solitons, combo rogue and periodic solitons, double periodic optical solitons and optical double solitons. The nature of these solitons is still unknown to the time fractional perturbed RKL model.

2 The description of the $\tan(\ominus/2)$ expansion and the Kudryashov general methods

In the section, we briefly review the $\tan(\ominus/2)$ expansion approach and the proposed Kudrayashov general approaches to solve the nonlinear evoluton equations (NLEEs) [13, 14, 24, 26]. These methods are highly effective and algebraic schemes to drive the combo optical solitons, periodic double solitons and rogue waves, and to deep study the nonlinear properties of the NLEEs. ,

2.1 The $\tan(\ominus/2)$ expansion approach

This method has been extensively used to obtain a number of exact solutions for a class of NLEEs [13, 14] . The main procedure of the approach is as follows:
Step 1. Let us consider the general form of a nonlinear evolution equation as:

$$\Gamma(u_{xx}, u_{xt}, u_{tt}, u_t, u^n, \dots.) = 0, \tag{2.1}$$

where $u(x,t)$ is an unknown function, Γ is a polynomial of $u(x,t)$ and its derivatives. The NLEE (2.1) can be converted to the following ordinary differential equation via the travelling variable transformation form $\zeta = kx - \omega t$ as

$$\mathcal{H}(\Psi, \Psi', \Psi'', \dots) = 0. \tag{2.2}$$

The constants k and ω indicate the wave number and speed of the wave respectively.
Step 2. One considers the general form of a trail solution [13] of (2.2) is

$$\Psi(\ominus) = \sum_{r=0}^{n} \mathcal{L}_r \tan^r(\frac{\ominus}{2}), \tag{2.3}$$

where $\mathcal{L}_r$ are free constants to be later calculated, such that $\mathcal{L}_n \neq 0$, and the $\ominus$ is a function of ζ, which satisfies the condition,

$$\frac{d\ominus}{d\zeta} = \lambda \sin(\ominus(\zeta)) + \mu \cos(\ominus(\zeta)) + \nu. \tag{2.4}$$

The condition (2.4) leads to the following nineteen solutions:

Set 1: If $\Lambda = \lambda^2+\mu^2-\nu^2 < 0$, and $\mu-\nu \neq 0$, then $\ominus(\zeta) = 2\tan^{-1}\left[\frac{\lambda}{\mu-\nu} - \frac{\sqrt{-\Lambda}}{\mu-\nu}\tan(\frac{\sqrt{-\Lambda}}{2}\bar{\zeta})\right]$.

Set 2: If $\Lambda = \lambda^2+\mu^2-\nu^2 > 0$, and $\mu-\nu \neq 0$, then $\ominus(\zeta) = 2\tan^{-1}\left[\frac{\lambda}{\mu-\nu} + \frac{\sqrt{\Lambda}}{\mu-\nu}\tanh(\frac{\sqrt{\Lambda}}{2}\bar{\zeta})\right]$.

Set 3: If $\Lambda = \lambda^2 + \mu^2 - \nu^2 > 0$, and $\mu \neq 0$ and $\nu = 0$, then
$\ominus(\zeta) = 2\tan^{-1}\left[\frac{\lambda}{\mu} + \frac{\sqrt{\lambda^2+\mu^2}}{\mu}\tanh(\frac{\sqrt{\lambda^2+\mu^2}}{2}\bar{\zeta})\right]$.

Set 4: If $\Lambda = \lambda^2 + \mu^2 - \nu^2 < 0$, and $\mu = 0$ and $\nu \neq 0$, then
$\ominus(\zeta) = 2\tan^{-1}\left[-\frac{\lambda}{\nu} + \frac{\sqrt{\nu^2-\lambda^2}}{\nu}\tan(\frac{\sqrt{\nu^2-\lambda^2}}{2}\bar{\zeta})\right]$.

Set 5: If $\Lambda = \lambda^2 + \mu^2 - \nu^2 > 0$, and $\mu - \nu \neq 0$ and $\lambda = 0$, then
$\ominus(\zeta) = 2\tan^{-1}\left[\sqrt{\frac{\mu+\nu}{\mu-\nu}}\tanh(\frac{\sqrt{\mu^2-\nu^2}}{2}\bar{\zeta})\right]$.

Set 6: If $\lambda = 0$, and $\nu = 0$, then $\ominus(\zeta) = \tan^{-1}\left[\frac{e^{2\mu\bar{\zeta}}-1}{e^{2\mu\bar{\zeta}}+1}, \frac{2e^{\mu\bar{\zeta}}}{e^{2\mu\bar{\zeta}}+1}\right]$.

Set 7: If $\lambda = 0$, and $\nu = 0$, then $\circleddash(\zeta) = \tan^{-1}\left[\frac{e^{2\mu\bar{\zeta}}}{e^{2\mu\bar{\zeta}}+1}, \frac{2e^{\mu\bar{\zeta}}-1}{e^{2\mu\bar{\zeta}}+1}\right]$.

Set 8: If $\lambda^2 + \mu^2 = \nu^2$, then $\circleddash(\zeta) = 2\tan^{-1}\left[\frac{\lambda\bar{\zeta}+2}{(\mu-\nu)\bar{\zeta}}\right]$.

Set 9: If $\lambda = \mu = \nu = k\lambda$, then $\circleddash(\zeta) = 2\tan^{-1}\left[e^{k\lambda\bar{\zeta}} - 1\right]$.

Set 10: If $\lambda = \nu = k\lambda$ and $\mu = -\lambda k$, then $\circleddash(\zeta) = -2\tan^{-1}\left[\frac{e^{k\lambda\bar{\zeta}}}{e^{k\lambda\bar{\zeta}}-1}\right]$.

Set 11: If $\nu = \lambda$, then $\circleddash(\zeta) = -2\tan^{-1}\left[\frac{(\lambda+\mu)e^{\mu\bar{\zeta}}-1}{(\lambda-\mu)e^{\mu\bar{\zeta}}-1}\right]$.

Set 12: If $\lambda = \nu$, then $\circleddash(\zeta) = 2\tan^{-1}\left[\frac{(\mu+\nu)e^{\mu\bar{\zeta}}+1}{(\mu-\nu)e^{\mu\bar{\zeta}}-1}\right]$.

Set 13: If $\nu = -\lambda$, then $\circleddash(\zeta) = 2\tan^{-1}\left[\frac{e^{\mu\bar{\zeta}}+\mu-\lambda}{e^{\mu\bar{\zeta}}-\mu-\lambda}\right]$.

Set 14: If $\mu = -\nu$, then $\circleddash(\zeta) = 2\tan^{-1}\left[\frac{\lambda e^{\lambda\bar{\zeta}}}{1-\nu e^{\lambda\bar{\zeta}}}\right]$.

Set 15: If $\mu = 0$ and $\lambda = \nu$, then $\circleddash(\zeta) = -2\tan^{-1}\left[\frac{\nu\bar{\zeta}+2}{\nu\bar{\zeta}}\right]$.

Set 16: If $\lambda = 0$ and $\mu = \nu$, then $\circleddash(\zeta) = 2\tan^{-1}\left[\nu\bar{\zeta}\right]$.

Set 17: If $\lambda = 0$ and $\mu = -\nu$, then $\circleddash(\zeta) = -2\tan^{-1}\left[\frac{1}{\nu\bar{\zeta}}\right]$.

Set 18: If $\lambda = 0$ and $\mu = 0$, then $\circleddash(\zeta) = \nu\zeta + \mathcal{K}$.

Set 19: If $\mu = \nu$, then $\circleddash(\zeta) = 2\tan^{-1}\left[\frac{e^{\lambda\bar{\zeta}}-\nu}{\lambda}\right]$,

where $\bar{\zeta} = \zeta + \mathcal{K}$.
Step 3. Inserting the above solutions into (2.3) together with (2.4) and substituting into (2.2), then we obtain a system of algebraic equations. After a direct computation of such system of algebraic equations, the calculation provides the constraints $\mathcal{L}_0, \mathcal{L}_r$ and ω. Finally, putting the constraints into (2.3) with the above set of solutions, they lead to the exact solutions of the NLEEs (2.1).

2.2 The Kudryashov general approach

The Kudryashov approach is a unique method to obtain the generalized periodic double solitons and rogue waves of the NLEEs [24, 26]. We extend this method to a general quadratic nonlinear Riccati equation. The algorithm of the KG method is as follows:

We first construct the ODE (2.2) from the NLEEs (2.1) to similar algorithm of the Step 1 in the section 2.1. Consider a series solutions of (2.2) [24, 26],

$$\Psi(\zeta) = \sum_{r=0}^{n} \mathcal{L}_r (F(\zeta))^r, \tag{2.5}$$

where $\mathcal{L}_r$ are constants to be later calculated with $\mathcal{L}_n \neq 0$. The function $F(\zeta)$ satisfies a general form of the Ricatti equation,

$$F'(\zeta) = A + BF(\zeta) + CF^2(\zeta), \tag{2.6}$$

which leads to twenty seven solutions of four different clusters as follows:
Cluster 1. For $\Lambda = B^2 - 4AC > 0$ and $BC \neq 0$ or $AC \neq 0$, the solutions of (2.6) leads to :

$$F_1 = \frac{-1}{2C}\left\{B + \sqrt{\Lambda}\tanh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)\right\},$$
$$F_2 = \frac{-1}{2C}\left\{B + \sqrt{\Lambda}\coth\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)\right\},$$
$$F_3 = \frac{-1}{2C}\left\{B + \sqrt{\Lambda}\left[\tanh\left(\sqrt{\Lambda}\zeta\right) \pm i\sec h(\sqrt{\Lambda}\zeta)\right]\right\},$$
$$F_4 = \frac{-1}{2C}\left\{B + \sqrt{\Lambda}\left[\coth\left(\sqrt{\Lambda}\zeta\right) \pm \csc h(\sqrt{\Lambda}\zeta)\right]\right\},$$
$$F_5 = \frac{-1}{4C}\left\{2B + \sqrt{\Lambda}\left[\tanh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right) + \coth\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)\right]\right\},$$
$$F_6 = \frac{1}{2C}\left\{-B + \frac{\sqrt{(M^2+N^2)(\Lambda)} - M\sqrt{\Lambda}\cosh(\sqrt{\Lambda}\zeta)}{M\sinh(\sqrt{\Lambda}\zeta) + N}\right\},$$

$$F_7 = \frac{1}{2C}\left\{-B - \frac{\sqrt{(N^2-M^2)(\Lambda)} + M\sqrt{\Lambda}\sinh(\sqrt{\Lambda}\zeta)}{M\cosh(\sqrt{\Lambda}\zeta) + N}\right\},$$

where M and N are two non-zero constants with $N^2 - M^2 > 0$.

$$F_8 = \frac{2A\cosh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)}{\sqrt{\Lambda}\sinh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right) - B\cosh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)},$$

$$F_9 = \frac{-2A\sinh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)}{B\sinh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right) - \sqrt{\Lambda}\cosh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)},$$

$$F_{10} = \frac{2A\cosh\left(\sqrt{\Lambda}\zeta\right)}{\sqrt{\Lambda}\sinh\left(\sqrt{\Lambda}\zeta\right) - B\cosh\left(\sqrt{\Lambda}\zeta\right) \pm i\sqrt{\Lambda}},$$

$$F_{11} = \frac{2A\sinh\left(\sqrt{\Lambda}\zeta\right)}{\sqrt{\Lambda}\cosh\left(\sqrt{\Lambda}\zeta\right) - B\sinh\left(\sqrt{\Lambda}\zeta\right) \pm \sqrt{\Lambda}},$$

$$F_{12} = \frac{4A\sinh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)\cosh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)}{-2B\sinh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)\cosh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right) + 2\sqrt{\Lambda}\cosh^2\left(\frac{\sqrt{\Lambda}}{4}\zeta\right) - \sqrt{\Lambda}}.$$

Cluster 2. For $\Lambda = B^2 - 4AC < 0$ and $BC \neq 0$ or $AC \neq 0$, the solutions of (2.6) leads to :

$$F_{13} = \frac{1}{2C}\left\{-B + \sqrt{-\Lambda}\tan\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)\right\},$$

$$F_{14} = \frac{-1}{2C}\left\{B + \sqrt{-\Lambda}\cot\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)\right\},$$

$$F_{15} = \frac{1}{2C}\left\{-B + \sqrt{-\Lambda}\left[\tanh\left(\sqrt{-\Lambda}\zeta\right) \pm \sec(\sqrt{-\Lambda}\zeta)\right]\right\},$$

$$F_{16} = \frac{-1}{2C}\left\{B + \sqrt{-\Lambda}\left[\cot\left(\sqrt{-\Lambda}\zeta\right) \pm \csc(\sqrt{-\Lambda}\zeta)\right]\right\},$$

$$F_{17} = \frac{1}{4C}\left\{-2B + \sqrt{-\Lambda}\left[\tan\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right) + \cot\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)\right]\right\},$$

$$F_{18} = \frac{1}{2C}\left\{-B + \frac{\pm\sqrt{(M^2+N^2)(-\Lambda)} - M\sqrt{-\Lambda}\cos(\sqrt{-\Lambda}\zeta)}{M\sin(\sqrt{-\Lambda}\zeta) + N}\right\},$$

$$F_{19} = \frac{1}{2C}\left\{-B - \frac{\pm\sqrt{(N^2-M^2)(-\Lambda)} + M\sqrt{-\Lambda}\sin(\sqrt{-\Lambda}\zeta)}{M\cos(\sqrt{-\Lambda}\zeta) + N}\right\},$$

where M and N are two non-zero constants with $N^2 - M^2 > 0$.

$$F_{20} = \frac{-2A\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{\sqrt{-\Lambda}\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right) + B\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)},$$
$$F_{21} = \frac{2A\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{-B\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right) + \sqrt{-\Lambda}\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)},$$
$$F_{22} = \frac{-2A\cos\left(\sqrt{-\Lambda}\zeta\right)}{\sqrt{-\Lambda}\sin\left(\sqrt{-\Lambda}\zeta\right) + B\cos\left(\sqrt{-\Lambda}\zeta\right) \pm i\sqrt{-\Lambda}},$$
$$F_{23} = \frac{2A\sin\left(\sqrt{-\Lambda}\zeta\right)}{\sqrt{-\Lambda}\cos\left(\sqrt{-\Lambda}\zeta\right) - B\sin\left(\sqrt{-\Lambda}\zeta\right) \pm \sqrt{-\Lambda}},$$
$$F_{24} = \frac{4A\sin\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)\cos\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)}{-2B\sin\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)\cos\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right) + 2\sqrt{-\Lambda}\cos^2\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right) - \sqrt{-\Lambda}},$$

Cluster 3. For $A = 0$ and $BC \neq 0$, the solutions of (2.6) leads to :

$$F_{25} = \frac{-BM_1}{C\{M_1 + \cosh(B\zeta) - sinh(B\zeta)\}},$$
$$F_{26} = \frac{-B\{\cosh(B\zeta) + sinh(B\zeta)\}}{C\{M_1 + \cosh(B\zeta) + sinh(B\zeta)\}},$$

where M_1 is an arbitrary constant.

Cluster 4. For $C \neq 0$, $B = 0$ and $A = 0$, the solutions of (2.6) leads to :

$$F_{27} = \frac{-1}{C\zeta + N_1},$$

where N_1 is an arbitrary constant. In addition, for the particular cases on the auxiliary equation (2.6), gives the results:

Case 1. For $A = 0$, $B = -1$ and $C = 1$,

$$F(\zeta) = \frac{1}{1 + he^{\zeta}},$$

Case 2. For $B = 0$ and $C = 1$,

$$F(\zeta) = -\sqrt{-A}\tanh(\sqrt{-A}\zeta), \quad A < 0,$$
$$F(\zeta) = -\sqrt{-A}\coth(\sqrt{-A}\zeta), \quad A < 0,$$
$$F(\zeta) = \sqrt{A}\tan(\sqrt{A}\zeta), \quad A > 0,$$
$$F(\zeta) = -\sqrt{A}\cot(\sqrt{A}\zeta), \quad A > 0,$$
$$F(\zeta) = \frac{-1}{\zeta}, \quad A = 0.$$

3 The time fractional perturbed Radhakrishnan-Kundu-Lakshmanan equation

In this article, to apply the $\tan(\ominus/2)$ expansion and the Kudryashov general methods, we consider the dimensionless time fractional perturbed Radhakrishnan-Kundu-Lakshmanan equation (RKLE) [25]. The RKL model is as follows:

$$iD_t^q\mathcal{V} + \alpha\mathcal{V}_{xx} + \beta|\mathcal{V}|^2\mathcal{V} - i\gamma\mathcal{V}_x - i\delta(|\mathcal{V}|^2\mathcal{V})_x - i\varepsilon(|\mathcal{V}|^2)_x\mathcal{V} - i\rho\mathcal{V}_{xxx} = 0, 0 < q \leq 1. \quad (3.1)$$

The complex valued function $\mathcal{V}(x,t)$ in the model (3.1) represents the wave profile of solitons. The fractional time-based progration can be formed in terms of $D_t^q\mathcal{V}$. The coefficients α is the GVD, β the nonlinearity, δ the spatio-temporal third order dispersion, and γ the effect of self-steepening, and ε, ρ the effect of dispersions [16]. The time fractional perturbed RLK equation coincides to the original RKL model for the value $q = 1$ [7, 8, 9, 16]. The conformable fractional derivative $\mathcal{T}_q(\mathcal{G})$ of a function $\mathcal{G} : (0,\infty) \to \mathbb{R}$ of order q is defined by [27],

$$T_q(\mathcal{G})(t) = \lim_{\epsilon\to 0} \frac{\mathcal{G}(t + \epsilon t^{1-q}) - \mathcal{G}(t)}{\epsilon}, \quad t > 0, \quad 0 < q < 1. \quad (3.2)$$

The fractional derivative $\mathcal{T}_q(\mathcal{G})$ satisfies the following properties [27]:

i. $\mathcal{T}_q(l_1\mathcal{G}_1 + l_2\mathcal{G}_2) = l_1\mathcal{T}_q(\mathcal{G}_1) + l_2\mathcal{T}_q(\mathcal{G}_2), \quad l_1, l_2 \in \mathbb{R}$,

ii. $\mathcal{T}_q(t^p) = pt^{p-q}, \quad p \in \mathbb{R}$,

iii. $\mathcal{T}_q(h\mathcal{G}) = \mathcal{G}\mathcal{T}_q(h) + h\mathcal{T}_q(\mathcal{G})$,

iv. $\mathcal{T}_q(\frac{\mathcal{G}}{h}) = \frac{h\mathcal{T}_q(\mathcal{G}) - \mathcal{G}\mathcal{T}_q(h)}{h^2}$,

v. If $\mathcal{G}$ is differentiable, then $\mathcal{T}_q(\mathcal{G})(t) = t^{1-q}\frac{d\mathcal{G}}{dt}$.

This model considered here could have wider applicability and other aspects as optical double solitons, periodic solitons and rogue type wave solutions to scrutinise via the $\tan(\ominus/2)$ expansion and the Kudryashov general methods. These methods make the RKL equation to be highly interesting. In this section, we apply the complex transformation variable with fractional time to (2.1).

3.1 The ODE form of the time fractional perturbed RKL equation

One considers the transformation complex variable with time fraction [25] as

$$\mathcal{V}(x,t) = \mathcal{H}(\zeta)e^{i\phi(x,t)}, \tag{3.3}$$

where $\mathcal{H}(\zeta)$ is the amplitude portion with $\zeta = \sigma(x - \tau\frac{t^q}{q})$, the phase component $\phi(x,t) = -\kappa x + w\frac{t^q}{q} + \varsigma$, and the constants $\tau, \varepsilon, w, \kappa$ are respectively, the soliton velocity, the phase constant, the wave number and the frequency of the soliton. Inserting (3.3) into (2.1), the calculations lead to the following ordinary differential equations,

$$\sigma^2(\alpha + 3\kappa\rho)\mathcal{H}'' + (\beta - \kappa\delta)\mathcal{H}^3 - (w + \gamma\kappa + \alpha\kappa^2 + \rho\kappa^3)\mathcal{H} = 0, \tag{3.4}$$

$$\sigma^2\rho\mathcal{H}''' - (\tau + 2\alpha\kappa + \gamma + 3\kappa^2\rho)\mathcal{H}' - (3\delta + 2\varepsilon)\mathcal{H}^2\mathcal{H}' = 0, \tag{3.5}$$

from real and imaginary parts. Now integrating (3.5) once, it gives [25],

$$3\sigma^2\rho\mathcal{H}'' - 3(\tau + 2\alpha\kappa + \gamma + 3\kappa^2\rho)\mathcal{H} - (3\delta + 2\varepsilon)\mathcal{H}^3 = 0. \tag{3.6}$$

As both (3.6) and (3.4) are satisfied by $\mathcal{H}$, so it provides the following relations,

$$\kappa = -\frac{3\beta\rho + 2\alpha\varepsilon + 3\alpha\delta}{6\rho(\delta + \varepsilon)}, \tag{3.7}$$

$$\tau = \frac{\rho(w + \gamma\kappa + \alpha\kappa^2 + \rho\kappa^3)}{\alpha + 3\kappa\rho} - (3\rho\kappa^2 + 2\alpha\kappa + \gamma). \tag{3.8}$$

4 Applications

In this section, we examine the nature of the soliton solutions and the dynamical phenomena of the time fractional perturbed RKL equation (3.1). To construct the significant soliton solutions, we apply the $\tan(\ominus/2)$ expansion and the Kudryashov general methods to the corresponding differential equation (3.6) of (3.1).

4.1 Application of the $\tan(\ominus/2)$ method to the time fractional perturbed RKL equation

We now compute the balance number of (3.6), which leads to $n = 1$. Then the trail solution (2.3) for the $\tan(\ominus/2)$ expansion approach takes the form,

$$\Psi(\ominus) = \mathcal{L}_0 + \mathcal{L}_1 \tan(\frac{\ominus}{2}). \tag{4.1}$$

Putting (4.1) into (3.6) along with (2.4), we attain a polynomial of $\sin(\zeta)$ and $\cos(\zeta)$ functions, and then after equating coefficients, lead to a system of equations. The solutions of the system of equations via symbolic computation, yield the following constraints:

$$\sigma = \pm\sqrt{-\frac{6\kappa^2\rho + 4\alpha\kappa + 2\gamma + 2\tau}{\rho(\lambda^2+\mu^2-\nu^2)}},$$
$$\mathcal{L}_0 = \mp i\lambda\sqrt{\frac{3(3\kappa^2\rho + 2\alpha\kappa + \gamma + \tau)}{(3\delta+2\varepsilon)(\lambda^2+\mu^2-\nu^2)}},$$
$$\mathcal{L}_1 = \pm i(\mu-\nu)\sqrt{\frac{3(3\kappa^2\rho + 2\alpha\kappa + \gamma + \tau)}{(3\delta+2\varepsilon)(\lambda^2+\mu^2-\nu^2)}}. \tag{4.2}$$

Now if we combine the above constraints in (4.1), and substituting into (3.3), then the solutions Set [1-19] of (2.4) provide the following thirteen valid exact soliton solutions under the conditions on the constraints to the model (3.1):

$$\mathcal{V}_{1,1}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)(\lambda^2+\mu^2-\nu^2)}}\left\{\mp\lambda \pm (\mu-\nu)\left[\tfrac{\lambda}{\mu-\nu} - \tfrac{\sqrt{-\Lambda}}{\mu-\nu}\tan(\tfrac{\sqrt{-\Lambda}}{2}\bar{\zeta})\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,2}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)(\lambda^2+\mu^2-\nu^2)}}\left\{\mp\lambda \pm (\mu-\nu)\left[\tfrac{\lambda}{\mu-\nu} + \tfrac{\sqrt{\Lambda}}{\mu-\nu}\tanh(\tfrac{\sqrt{\Lambda}}{2}\bar{\zeta})\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,3}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)(\lambda^2+\mu^2)}}\left\{\mp\lambda \pm \mu\left[\tfrac{\lambda}{\mu} + \tfrac{\sqrt{\lambda^2+\mu^2}}{\mu}\tanh(\tfrac{\sqrt{\lambda^2+\mu^2}}{2}\bar{\zeta})\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,4}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)(\lambda^2-\nu^2)}}\left\{\mp\lambda \mp \nu\left[-\tfrac{\lambda}{\nu} + \tfrac{\sqrt{\nu^2-\lambda^2}}{\nu}\tan(\tfrac{\sqrt{\nu^2-\lambda^2}}{2}\bar{\zeta})\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,5}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)(\mu^2-\nu^2)}}\left\{\pm(\mu-\nu)\left[\sqrt{\tfrac{\mu+\nu}{\mu-\nu}}\tanh(\tfrac{\sqrt{\mu^2-\nu^2}}{2}\bar{\zeta})\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,6}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\mu^2}}\left\{\pm\mu\tan\left(\tfrac{1}{2}\tan^{-1}\left[\tfrac{2e^{\mu\bar{\zeta}}}{e^{2\mu\bar{\zeta}}+1}\right]\right)\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,7}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\mu^2}}\left\{\pm\mu\tan\left(\tfrac{1}{2}\tan^{-1}\left[\tfrac{e^{2\mu\bar{\zeta}}-1}{e^{2\mu\bar{\zeta}}+1}\right]\right)\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,8}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)k^2\lambda^2}}\left\{\mp k\lambda \mp 2k\lambda\left[\tan\left(-\tan^{-1}\left[\tfrac{e^{k\lambda\zeta}}{e^{k\lambda\zeta}-1}\right]\right)\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,9}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\mu^2}}\left\{\mp\lambda \pm (\mu-\lambda)\left[\tan\left(-\tan^{-1}\tfrac{(\lambda+\mu)e^{\mu\bar{\zeta}}-1}{(\lambda-\mu)e^{\mu\bar{\zeta}}-1}\right)\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,10}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\mu^2}}\left\{\mp\nu \pm (\mu-\nu)\left[\tfrac{(\mu+\nu)e^{\mu\bar{\zeta}}+1}{(\mu-\nu)e^{\mu\bar{\zeta}}-1}\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,11}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\mu^2}}\left\{\mp\lambda \pm (\mu+\lambda)\left[\tfrac{e^{\mu\bar{\zeta}}+\mu-\lambda}{e^{\mu\bar{\zeta}}-\mu-\lambda}\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$
$$\mathcal{V}_{1,12}(x,t) = i\sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\lambda^2}}\left\{\mp\lambda \mp 2\nu\left[\tfrac{\lambda e^{\lambda\bar{\zeta}}}{1-\nu e^{\lambda\bar{\zeta}}}\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)}$$
$$\mathcal{V}_{1,13}(x,t) = \sqrt{\tfrac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)\nu^2}}\left\{\mp\nu\tan\left[\tfrac{\nu\zeta+\mathcal{K}}{2}\right]\right\} e^{i(-\kappa x + w\frac{t^q}{q} + \varsigma)},$$

where $\zeta = \sqrt{\frac{2(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(4AC-B^2)\rho}}(x - \tau\frac{t^q}{q})$.

The graphical description, it is shown that $\mathcal{V}_{1,1}(x,t)$, $\mathcal{V}_{1,4}(x,t)$ and $\mathcal{V}_{1,13}(x,t)$ solutions provide double period optical solitons. In particular, we present the 3D and $2D$ graphs in Figure 1 for the solution $\mathcal{V}_{1,1}(x,t)$. The figure 1 shows that amplitude of the wave start from zero and gradually increases for time goes on. Again, for choosing suitable values of the parameters, the solution $\mathcal{V}_{1,13}(x,t)$ presents an interaction of rogue and periodic waves, that produce periodic rogue waves. Moreover, it is clear that the rogue waves are going to diminish after a certain time, keeping periodic nature of the interaction.

The solutions $\mathcal{V}_{1,2}(x,t)$, $\mathcal{V}_{1,3}(x,t)$ and $\mathcal{V}_{1,5}(x,t)$, represent interaction of periodic and shock waves, which produce an optical double solitons. We present graphs of the solution $\mathcal{V}_{1,2}(x,t)$ in Figure 3

The rest of the solutions $\mathcal{V}_{1,6}(x,t)$, $\mathcal{V}_{1,7}(x,t)$, $\mathcal{V}_{1,8}(x,t)$, $\mathcal{V}_{1,9}(x,t)$, $\mathcal{V}_{1,10}(x,t)$, $\mathcal{V}_{1,11}(x,t)$ and $\mathcal{V}_{1,12}(x,t)$ provide to similar interaction of periodic and rogue waves that produces periodic rogue type breather waves. We plot the graphs in Figure 4 for solution $\mathcal{V}_{1,6}(x,t)$ and it shows that the amplitude of the rogue waves gradually decreases along the parallax and after a certain time it will be vanished. In addition, the 3D and contour plots of $\mathcal{V}_{1,10}(x,t)$ also display orthogonally interacted with periodic waves and periodic rogue waves in Figure 5. Thus all such signal type optical solitons are interested to use in optical fibre communications.

4.2 Application of the Kudryashov general method to the time fractional perturbed RKL equation

After computing the balance number $n = 1$ of (3.6), the series solution (2.5) takes the form,

$$\Psi(\zeta) = \mathcal{L}_0 + \mathcal{L}_1 F(\zeta). \tag{4.3}$$

Insetting (4.3) into (3.6) along with (2.6), we obtain a polynomial functions of $F(\zeta)$. After equating the coefficients of $F(\zeta)$, it leads to a system of equations, and their algebraic solutions provide the following constraints:

$$\begin{aligned}
\sigma &= \sqrt{\frac{2(3\kappa^2\rho + 2\alpha\kappa + \gamma + \tau)}{(4AC - B^2)\rho}},\\
\mathcal{L}_0 &= \pm B\sqrt{\frac{3(3\kappa^2\rho + 2\alpha\kappa + \gamma + \tau)}{(3\delta + 2\varepsilon)(4AC - B^2)}},\\
\mathcal{L}_1 &= \pm C\sqrt{\frac{12(3\kappa^2\rho + 2\alpha\kappa + \gamma + \tau)}{(3\delta + 2\varepsilon)(4AC - B^2)}}.
\end{aligned} \tag{4.4}$$

Combining the constraints (4.3), (4.4), (3.3) and the Cluster 1, Cluster 2 and Cluster 3, the calculations provide the following valid exact soliton solutions of the NLEEs (2.1):

Cluster 1. For $\Lambda = B^2 - 4AC > 0$ and $BC \neq 0$ or $AC \neq 0$, the solutions of (2.6) leads to :

$$\mathcal{V}_{2,1}(x,t) = \left[\pm B \mp \left\{B + \sqrt{\Lambda}\tanh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,2}(x,t) = \left[\pm B \mp \left\{B + \sqrt{\Lambda}\coth\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,3}(x,t) = \left[\pm B \mp \left\{B + \sqrt{\Lambda}\left[\tanh\left(\sqrt{\Lambda}\zeta\right) \pm i\sec h(\sqrt{\Lambda}\zeta)\right]\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,4}(x,t) = \left[\pm B \mp \left\{B + \sqrt{\Lambda}\left[\coth\left(\sqrt{\Lambda}\zeta\right) \pm \csc h(\sqrt{\Lambda}\zeta)\right]\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,5}(x,t) = \left[\pm B \mp \frac{1}{2}\left\{2B + \sqrt{\Lambda}\left[\tanh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right) + \coth\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)\right]\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,6}(x,t) = \left[\pm B \pm \left\{-B + \frac{\sqrt{(M^2+N^2)(\Lambda)} - M\sqrt{\Lambda}\cosh(\sqrt{\Lambda}\zeta)}{M\sinh(\sqrt{\Lambda}\zeta) + N}\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,7}(x,t) = \left[\pm B \pm \left\{-B - \frac{\sqrt{(N^2-M^2)(\Lambda)} + M\sqrt{\Lambda}\sinh(\sqrt{\Lambda}\zeta)}{M\cosh(\sqrt{\Lambda}\zeta) + N}\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

where M and N are two non-zero constants with $N^2 - M^2 > 0$.

$$\mathcal{V}_{2,8}(x,t) = \left[\pm B \pm \frac{4AC\cosh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)}{\sqrt{\Lambda}\sinh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right) - B\cosh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,9}(x,t) = \left[\pm B \mp \frac{4AC\sinh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)}{B\sinh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right) - \sqrt{\Lambda}\cosh\left(\frac{\sqrt{\Lambda}}{2}\zeta\right)}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,10}(x,t) = \left[\pm B \pm \frac{4AC\cosh\left(\sqrt{\Lambda}\zeta\right)}{\sqrt{\Lambda}\sinh\left(\sqrt{\Lambda}\zeta\right) - B\cosh\left(\sqrt{\Lambda}\zeta\right) \pm i\sqrt{\Lambda}}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,11}(x,t) = \left[\pm B \pm \frac{4AC\sinh\left(\sqrt{\Lambda}\zeta\right)}{\sqrt{\Lambda}\cosh\left(\sqrt{\Lambda}\zeta\right) - B\sinh\left(\sqrt{\Lambda}\zeta\right) \pm \sqrt{\Lambda}}\right]\Gamma_1 e^{i\phi(x,t)},$$
$$\mathcal{V}_{2,12}(x,t) = \left[\pm B \pm \frac{8AC\sinh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)\cosh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)}{-2B\sinh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right)\cosh\left(\frac{\sqrt{\Lambda}}{4}\zeta\right) + 2\sqrt{\Lambda}\cosh^2\left(\frac{\sqrt{\Lambda}}{4}\zeta\right) - \sqrt{\Lambda}}\right]\Gamma_1 e^{i\phi(x,t)}.$$

Cluster 2. For $\Lambda = B^2 - 4AC < 0$ and $BC \neq 0$ or $AC \neq 0$, the solutions of (2.6) leads to :

$$\mathcal{V}_{2,13}(x,t) = \left[\pm B \pm \left\{-B + \sqrt{-\Lambda}\tan\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,14}(x,t) = \left[\pm B \mp \left\{B + \sqrt{-\Lambda}\cot\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,15}(x,t) = \left[\pm B \pm \left\{-B + \sqrt{-\Lambda}\left[\tan\left(\sqrt{-\Lambda}\zeta\right) \pm \sec(\sqrt{-\Lambda}\zeta)\right]\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,16}(x,t) = \left[\pm B \mp \left\{B + \sqrt{-\Lambda}\left[\cot\left(\sqrt{-\Lambda}\zeta\right) \pm \csc(\sqrt{-\Lambda}\zeta)\right]\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,17}(x,t) = \left[\pm B \pm \frac{1}{2}\left\{-2B + \sqrt{-\Lambda}\left[\tan\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right) + \cot\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)\right]\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,18}(x,t) = \left[\pm B \pm \left\{-B + \frac{\pm\sqrt{(M^2+N^2)(-\Lambda)} - M\sqrt{-\Lambda}\cos(\sqrt{-\Lambda}\zeta)}{M\sin(\sqrt{-\Lambda}\zeta) + N}\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,19}(x,t) = \left[\pm B \pm \left\{-B - \frac{\pm\sqrt{(N^2-M^2)(-\Lambda)} + M\sqrt{-\Lambda}\sin(\sqrt{-\Lambda}\zeta)}{M\cos(\sqrt{-\Lambda}\zeta) + N}\right\}\right]\Gamma_1 e^{i\phi(x,t)},$$

where M and N are two non-zero constants with $N^2 - M^2 > 0$.

$$\mathcal{V}_{2,20}(x,t) = \left[\pm B \mp \frac{4AC\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{\sqrt{-\Lambda}\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right) + B\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,21}(x,t) = \left[\pm B \pm \frac{4AC\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{-B\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right) + \sqrt{-\Lambda}\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,22}(x,t) = \left[\pm B \mp \frac{4AC\cos\left(\sqrt{-\Lambda}\zeta\right)}{\sqrt{-\Lambda}\sin\left(\sqrt{-\Lambda}\zeta\right) + B\cos\left(\sqrt{-\Lambda}\zeta\right) \pm i\sqrt{-\Lambda}}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,23}(x,t) = \left[\pm B \pm \frac{4AC\sin\left(\sqrt{-\Lambda}\zeta\right)}{\sqrt{-\Lambda}\cos\left(\sqrt{-\Lambda}\zeta\right) - B\sin\left(\sqrt{-\Lambda}\zeta\right) \pm \sqrt{-\Lambda}}\right]\Gamma_1 e^{i\phi(x,t)},$$

$$\mathcal{V}_{2,24}(x,t) = \left[\pm B \pm \frac{8AC\sin\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)\cos\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)}{-2B\sin\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right)\cos\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right) + 2\sqrt{-\Lambda}\cos^2\left(\frac{\sqrt{-\Lambda}}{4}\zeta\right) - \sqrt{-\Lambda}}\right]\Gamma_1 e^{i\phi(x,t)}.$$

Cluster 3. For $A = 0$ and $BC \neq 0$, the solutions of (2.6) leads to :

$$\mathcal{V}_{2,25}(x,t) = \left[\pm B \mp \frac{2BM_1}{M_1 + \cosh(B\zeta) - sinh(B\zeta)}\right]\Gamma_1 e^{i\phi(x,t)}.$$

$$\mathcal{V}_{2,26}(x,t) = \left[\pm B \mp \frac{2B\{\cosh(B\zeta) + sinh(B\zeta)\}}{M_1 + \cosh(B\zeta) + sinh(B\zeta)}\right]\Gamma_1 e^{i\phi(x,t)},$$

where M_1 is an arbitrary constant, $\Gamma_1 = \sqrt{\frac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)(4AC-B^2)}}$, $\zeta = \sqrt{\frac{2(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(4AC-B^2)\rho}}(x-\tau\frac{t^q}{q})$ and $\phi(x,t) = -\kappa x+w\frac{t^q}{q}+\varsigma$. The additional solutions for the particular values of A, B, C are:

Case 1. For $A = 0$, $B = -1$ and $C = 1$,

$$\mathcal{V}_{3,1}(x,t) = \pm i\sqrt{\frac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)}}\frac{1}{1+he^{\zeta}}e^{i(-\kappa x+w\frac{t^q}{q}+\varsigma)},$$

Case 2. For $B = 0$ and $C = 1$,

$$\mathcal{V}_{3,2}(x,t) = \pm\sqrt{\frac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)A}}\sqrt{-A}\tanh(\sqrt{-A}\zeta)e^{i(-\kappa x+w\frac{t^q}{q}+\varsigma)}, \quad A<0,$$

$$\mathcal{V}_{3,3}(x,t) = \pm\sqrt{\frac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)A}}\sqrt{-A}\coth(\sqrt{-A}\zeta)e^{i(-\kappa x+w\frac{t^q}{q}+\varsigma)}, \quad A<0,$$

$$\mathcal{V}_{3,4}(x,t) = \pm\sqrt{\frac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)A}}\sqrt{A}\tan(\sqrt{A}\zeta)e^{i(-\kappa x+w\frac{t^q}{q}+\varsigma)}, \quad A>0,$$

$$\mathcal{V}_{3,5}(x,t) = \pm\sqrt{\frac{3(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(3\delta+2\varepsilon)A}}\sqrt{A}\cot(\sqrt{A}\zeta)e^{i(-\kappa x+w\frac{t^q}{q}+\varsigma)}, \quad A>0,$$

where $\zeta = \sqrt{\frac{2(3\kappa^2\rho+2\alpha\kappa+\gamma+\tau)}{(4AC-B^2)\rho}}(x-\tau\frac{t^q}{q})$.
The method presents mainly double periodic soliton, combo periodic and solitons, combo double singular solitons and combo rogue and periodic soliton solutions. We graphically present some graphs of the solutions for particular cases only in Figures 6, 7 and 8.

5 Conclusion

The main results of this paper is the determination of families of optical solitons through the optical fibres of the time fractional perturbed RKL equation. To obtain the optical soliton solutions, we apply the integration schemes $\tan(\ominus/2)$ expansion and Kudryashov general methods. The soliton solutions registered with constraint conditions on parameters that follow their existence criteria. The families of solutions display various transmission signals such as double periodic optical solitons, combo optical periodic and rogue waves, combo periodic and shock waves, combo periodic and solitons, and combo double singular solitons. One interesting feature of these solitons that interaction of rogue and periodic waves produces the rogue waves which diminish after a certain time keeping periodic nature of the interaction. Moreover, interaction of periodic and rogue waves displays periodic rogue type breather waves, that indicates the amplitude of the rogue waves gradually decreases and vanishes after a certain time. The dynamical signals are plotted in the figures by setting suitable values on the parameters. Let us point out the model could be investigated to get multi-solitons and multi-rogue waves by other existing schemes, in particular, Hirota bilinear approach[19, 20] and Darboux transformation [21].

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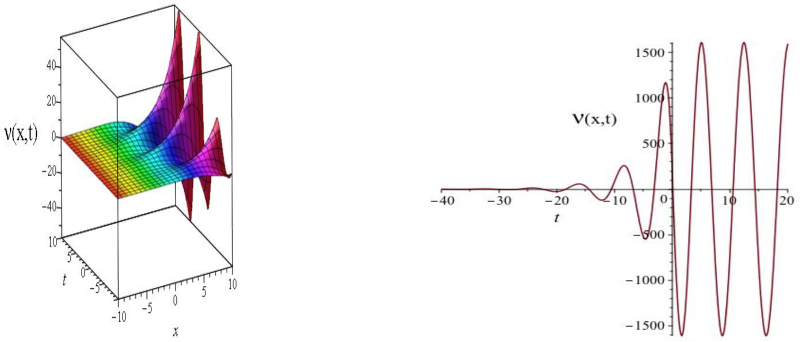


Figure 1: The 3D and 2D double periodic solitons of $\mathcal{V}_{1,1}(x,t)$, from the real part with $\alpha=\beta=w=1$, $\varepsilon=1+i$, $\lambda=3, \mu=2, \nu=4$, $\rho=\gamma=\delta=\vartheta=-1$, $\varsigma=2$, $q=\frac{3.7}{4}$.

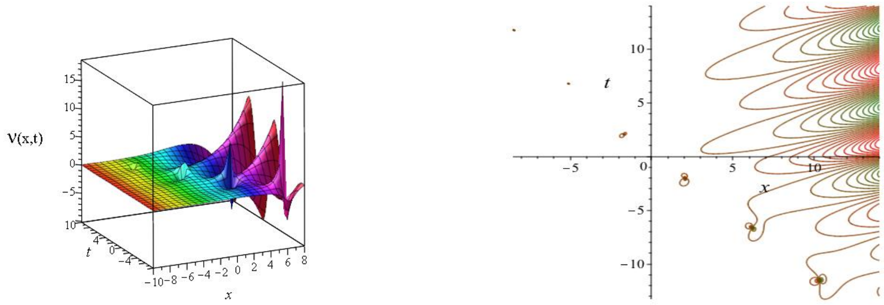


Figure 2: The 3D and contour plot periodic solitons and rogue waves of $\mathcal{V}_{1,13}(x,t)$, from the complex part with $\alpha=\beta=w=1$, $\varepsilon=1+2i$, $\lambda=0, \mu=0, \nu=2$, $\gamma=\delta=\rho=\vartheta=-1$, $\varsigma=2$, $q=\frac{3.7}{4}$.

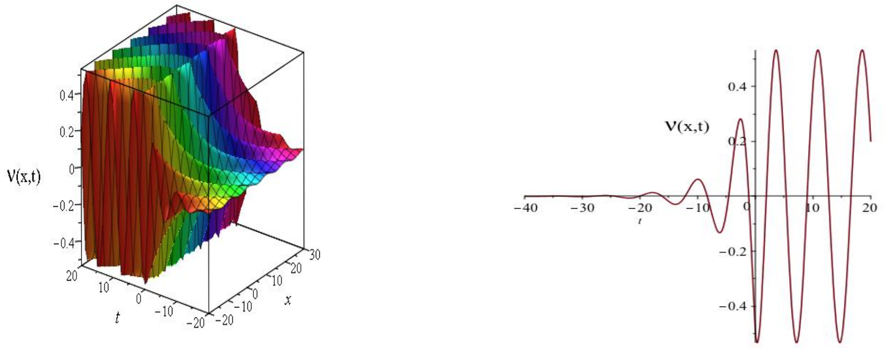


Figure 3: The 3D and 2D periodic solitons and shock waves of $\mathcal{V}_{1,2}(x,t)$, from the real part with $\alpha = \beta = \rho = w = 1$, $\varepsilon = 2$, $\lambda = 3, \mu = 2, \nu = 1$, $\gamma = \delta = \vartheta = -1$, $\varsigma = 2$, $q = \frac{3.7}{4}$.

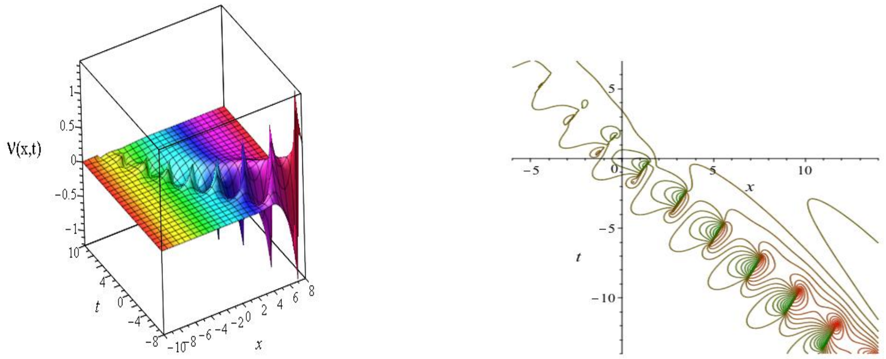


Figure 4: The 3D and contour plots combo periodic solitons and rogue waves of $\mathcal{V}_{1,6}(x,t)$, from the real part with $\alpha = \beta = w = 1$, $\varepsilon = 1 + 2i$, $\lambda = 0, \mu = 2, \nu = 0$, $\gamma = \delta = \rho = \vartheta = -1$, $\varsigma = 2$, $q = \frac{3.7}{4}$.

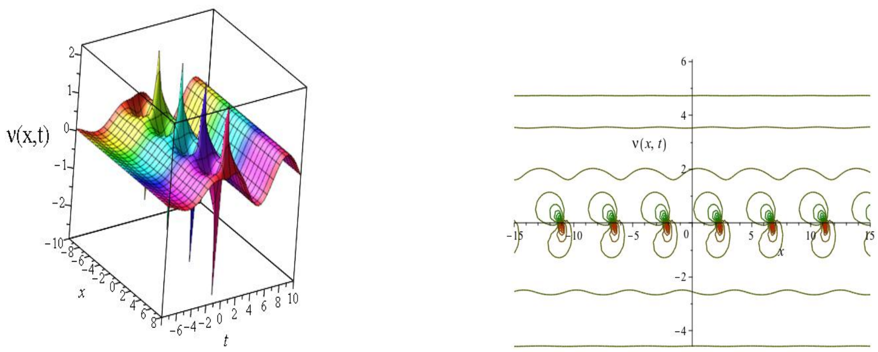


Figure 5: The 3D and contour plots combo periodic solitons and rogue waves of $\mathcal{V}_{1,6}(x,t)$, from the real part with $\alpha = \beta = \mu = w = 1$, $\gamma = -1 + i$, $\varepsilon = 1 + 2i$, $\lambda = \nu = 2$, $\delta = \rho = \vartheta = -1$, $\varsigma = 2$, $q = \frac{3.7}{4}$.

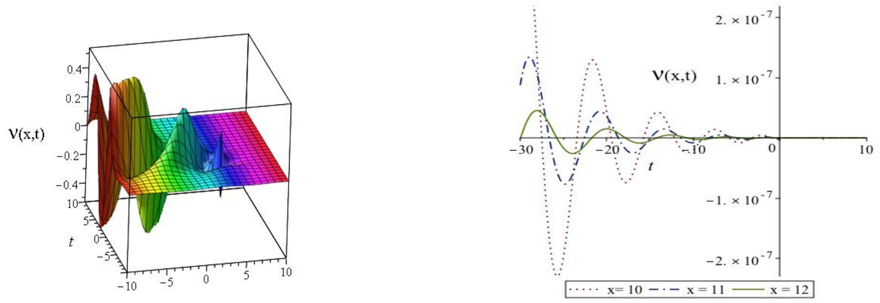


Figure 6: The 3D and 2D combo rogue waves and periodic solitons of $\mathcal{V}_{3,1}(x,t)$, from the real part with $\alpha = \beta = \nu = \gamma = 1$, $\delta = 1$ $\varepsilon = w = \rho = 1$, $\varsigma = 0$, $A = 0$ $C = h = 1$, $B = -1$, $q = \frac{3}{4}$.

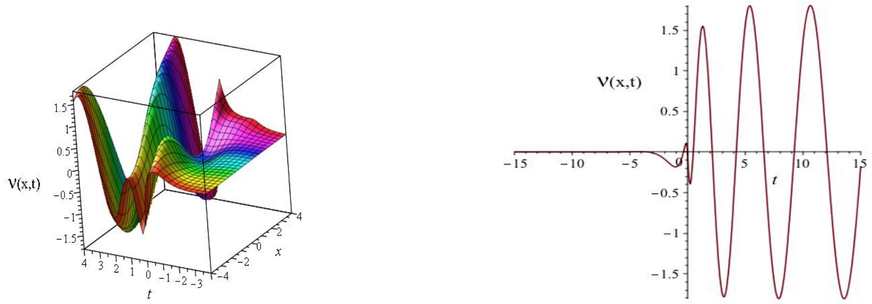
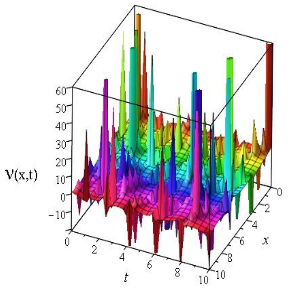


Figure 7: The 3D and 2D double periodic solitons of $\mathcal{V}_{3,2}(x,t)$, for the real part of $\mathcal{V}_{3,2}(x,t)$ with $\alpha = \beta = \nu = \gamma = 1$, $\delta = -1$ $\varepsilon = w = \rho = 2$, $\varsigma = 0$, $A = -1$ $C = h = 1$, $B = 0$, $q = \frac{3}{4}$.



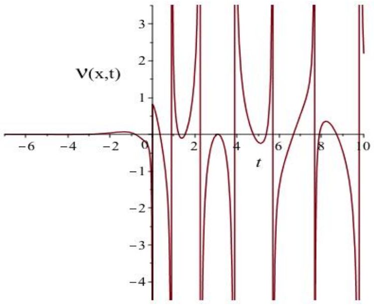


Figure 8: The 3D and 2D double periodic solitons of $\mathcal{V}_{3,5}(x,t)$, for the real part of $\mathcal{V}_{3,5}(x,t)$ with $\rho = \alpha = \beta = \nu = \delta = \varepsilon = 1$, $\gamma = w = 2$, $\varsigma = 0$, $A = C = h = 1$, $B = 0$, $q = \frac{3}{4}$.