Optical solitons of the (2 + 1)-dimensional Kundu-Mukherjee-Naskar equation with Lokal M-derivative

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Abstract

This paper aims to uncover fairly interesting optical soliton solutions in (2 + 1)-dimensions. The fractional temporal Kundu-Mukherjee-Naskar (KMN) equation is reviewed as a governing model. Local M-derivative along with the unified approach is used to acquire these soliton solutions. The predicted solutions are yielded with the constraint conditions and highlighted by their graphical portrayal. Lastly, the influence of a local fractional parameter upon predicted solutions are depicted through 2D and 3D graphs.

Keywords: Kundu-Mukherjee-Naskar equation; Local M-derivative; Unified method; Optical solitons in (2 + 1)-dimensions.

1 Introduction

Over the previous couple of decades, various different techniques are discovered to deal with nonlinear partial differential equations (NPDEs) due to symbolic programming innovations and new computational methodologies. Classical computing packages like Maple and Mathematica have also been

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modified several times to enhance performance. While on another side, the mathematical features of solution strategies have been developed and different new approaches have been implemented to address various issues based on NPDEs [1–4]. The most familiar techniques in the literature can be classified as expansion based approaches such as the technique of Kudryashov and its varied expansions and the direct substitution of some specific expected solution-based techniques.

To date, numerous mathematical approaches to study the nonlinear aspects of analytical solutions have been applied across several disciplines like fluid dynamics,mechanics, plasma physics, biology, economics, and many other scientific and engineering fields [5–11]. Such approaches include the Darboux transformation technique [12], the $\tan(\phi(\eta)/2)$-expansion technique [13], the Exp-function technique [14, 15], the sine-Gordon expansion technique [16, 17], the homogeneous balance technique [18], the modified auxiliary equation technique [19], the first integral technique [20, 21], the exponential rational function technique [22, 23], the extended trial function technique [24], the homotopy perturbation technique [25], the bilinear transformation technique [26] and so on.

During the last few years, fractional calculus is one in every of the foremost prolific fields of mathematical study alongside fractional operators, for example, Caputo, Grunwald-Letnikov, and Riemann-Liouville [27]. The integer order integro-differential calculus presented by Leibniz and Newton was an enormous disclosure in mathematics with different implementations in such a significant number of fields of science and engineering: today its applications within the economy, signal processing, as well as image processing, are among the most interesting. In 1695, l'Hospital asked Leibniz in a letter concerning the chance of enlarging the sense of an integer order derivative to the case of a fraction of the order. This idea initiated the development of a modern calculus that was termed the arbitrary order calculus and is currently widely known as the fractional calculus. Up to date, different kinds of fractional derivatives have been developed such as Riemann-Liouville, Caputo, Hadamard, Caputo-Hadamard, and Riesz [27, 28]. Practically such derivatives are described within the Riemann-Liouville sense on the basis of the corresponding fractional integral.

In 2014, Katugampola presented a new fractional derivative generalizing the so-called alternative fractional derivative [29]. This new differential operator is identified by $D_M^{\gamma,\mu}$, where γ is the order, such that $\gamma \in (0, 1), \mu > 0$ and M indicates the derived function involves a function known as Mittag-Leffler along side one parameter. This new kind of derivative is familiar as local M-derivative, it assures certain characteristics of integer order calculus, for example, linearity, product rule, quotient rule, chain rule, and composition of functions. Additionally, the local M-derivative of a constant function is zero. Since the Mittag-Leffler function is a generalized form of the exponential function, a number of the classical outcomes of integer order calculus can be enlarged, such as the mean value theorem, Rolle's theorem, and its enlargement. In particular, if we assume that the order of fractional derivative $\lambda = 1$ and the Mittag-Leffler function parameter is also unitary, then our perception is identical to ordinary derivative of order one.

It is important to point out that a number of authors have discussed soliton dynamics in the past using a diverse range of effective mathematical schemes due to an abundance of physical systems. They are used to illustrate the particle-like features of nonlinear pulses. The significance of solitons is because of its presence in a context of nonlinear differential equations describing numerous complex nonlinear dynamics as well as water waves, nonlinear optics, telecommunication industry, plasma physics, solid-state physics, and engineering [30–34]. Particularly, many authors discussed solitons in (2+1)-dimensions, but significantly between 2005 [35] and 2009 [36]. Kundu-Mukherjee-Naskar (KMN) equation is one of the models which highlights optical soliton dynamics in (2 + 1)-dimensions. It is also explored in the aspect of Fluid Dynamics by which rogue wave solutions were developed. This became first presented in 2013/ 2014 and later it has received attention and a number of researchers are streaming outcomes from this model [37–41]. The model was considered, previously, to encounter the bright and singular solitons by using an extended trial function method [37]. Furthermore, higher-order rational solutions were additionally reported [38]. During 2016, the rogue wave solutions for KMN equation were also introduced [39]. Afterward, a diversity of soliton solutions were developed utilizing the versatile approaches, namely; the trial equation technique and modified simple equation technique [40, 41].

This study aims to construct specific fractional temporal optical solitons of the KMN equation by employing a versatile approach, namely the unified method. The rest of this paper is conducted as follows: Section 2 carried out with the basic definition of local M-derivative and its properties. A brief overview of the used methodology is encountered in Section 3. The interpretation of the considered model is presented in Section 4 and Section 5 providing the extraction of temporal fractional solitons of the governing model. The conclusion is eventually illustrated in Section 6.

2 Local M-derivative

The basic definition and fundamental properties of Local M-derivative, which generalizes the classical definition of a derivative and overcome the drawbacks of the existing properties, are given by:

Definition : If $h : [0, \infty) \to \mathbb{R}$ and $t > 0$, then the local M-derivative of order $\gamma \in (0, 1)$ of function

h, is defined as

$$D_M^{\gamma;\mu}\{h(t)\} = \lim_{\epsilon\to 0} \frac{h\big(tE_\mu(\epsilon t^{-\gamma})\big) - h(t)}{\epsilon}, \quad \forall\ t > 0, \tag{1}$$

where M shows the derived function includes a function called Mittag-Leffler $(E_\mu(\cdot), \forall \mu > 0)$ alongside one parameter. Also, if $h(t)$ is r-differentiable within a given range $(0, r)$, $r > 0$ and $\lim_{t\to 0^+} D_M^{\gamma;\mu}\{h(t)\}$ exists, then we have

$$D_M^{\gamma;\mu}\{h(0)\} = \lim_{t\to 0^+} D_M^{\gamma;\mu}\{h(t)\}, \tag{2}$$

certain local M-derivative characteristics are

$$D_M^{\gamma;\mu}\{h(t)\} = \frac{t^{1-\gamma}}{\Gamma(\mu+1)} \frac{d}{dt}\{h(t)\}, \tag{3}$$

therefore,

$$D_M^{\gamma;\mu}\Big(\frac{\Gamma(\mu+1)t^\gamma}{\gamma}\Big) = 1. \tag{4}$$

Such a derivative of fractional order also satisfy the property mentioned below:

$$D_M^{\gamma;\mu}(g \cdot h)(r) = g'(h(r)) D_M^{\gamma;\mu} h(r), \tag{5}$$

therefore, from Eq.(4) and Eq.(5), the corresponding relationship can be developed:

$$D_M^{\gamma;\mu} F\left[\frac{\Gamma(\mu+1)t^\gamma}{\gamma}\right] = F'\left[\frac{\Gamma(\mu+1)t^\gamma}{\gamma}\right] D_M^{\gamma;\mu}\left[\frac{\Gamma(\mu+1)t^\gamma}{\gamma}\right] = F'\left[\frac{\Gamma(\mu+1)t^\gamma}{\gamma}\right], \tag{6}$$

with

$$\eta = \frac{b}{\gamma}\Gamma\big(\mu+1\big)t^\gamma, \tag{7}$$

where b is a constant and eventually we get the relation given by

$$D_M^{\gamma;\mu}\{F(\eta)\} = bF'(\eta). \tag{8}$$

3 Description of the proposed method

Consider the general structure of temporal fractional evolution as follows

$$H\Big(x, y, t, \frac{\partial^\gamma u}{\partial t^\gamma}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^{2\gamma} u}{\partial t^{2\gamma}}, \frac{\partial^2 u}{\partial x^2}, ...\Big) = 0, \quad t \geq 0,\ 0 < \gamma \leq 1. \tag{9}$$

where H is a polynomial function of $u(x,y,t)$ with its arguments. By using the fractional traveling wave variable of the type $s = x + y - \nu\frac{t^\gamma}{\gamma}$, Eq.(9) can be reduced to

$$G\Big(u, \frac{du}{ds}, \frac{d^2u}{ds^2}, ...\Big) = 0, \tag{10}$$

where ν is velocity of soliton, G is a function of $u(s)$ as well as corresponding derivatives. To hunt the solutions for Eq.(10), by UM method are to be grouped into two classes such as a polynomial function or rational function solutions [42].

3.1 Polynomial function solution

The basic principle of this approach is to assume that Eq.(10) has polynomial solution as

$$u(\xi) = \sum_{j=0}^{n} a_j \phi^j(\xi), \tag{11}$$

with satisfying

$$(\phi'(\xi))^\sigma = \sum_{j=0}^{\sigma k} m_j \phi^j(\xi), \quad \xi = x + y - \nu\frac{t^\gamma}{\gamma}, \quad \sigma = 1, 2. \tag{12}$$

Here, in Eq.(11) and Eq.(12), a_j and m_j are the unknown coefficients, such that solution given in Eq.(11) satisfies the Eq.(10).

It is necessary to note that the value of n and k is assessed by the balancing of the highest order of linear and nonlinear terms in Eq.(10) [43]. In addition, a condition that is usually referred to as a consistency condition that claims the unknown coefficients of Eq.(11) could be reliably decided is used. Also, the UM method solves Eq.(11) for elementary or elliptic solutions when $\sigma = 1$ or $\sigma = 2$, respectively.

3.2 Rational function solution

To acquire the rational solution of Eq.(10), we suppose the formal solution is as follows:

$$u(\xi) = \frac{\sum_{j=0}^{n} r_j \phi^j(\xi)}{\sum_{j=0}^{r} s_j \phi^j(\xi)}, \quad n \geq r, \tag{13}$$

with satisfying

$$(\phi'(\xi))^\sigma = \sum_{j=0}^{\sigma k} h_j \phi^j(\xi), \quad \xi = x + y - \nu\frac{t^\gamma}{\gamma}, \quad \sigma = 1, 2. \tag{14}$$

Here, in Eq.(13) and Eq.(14), r_j, s_j and h_j are the unknown coefficients to be determined, such that solution given in Eq.(13) satisfies the Eq.(10).

It is important to keep in mind that the value of n and k are assessed due to balancing the highest order of linear and the nonlinear terms in Eq.(10) [43]. In addition, a condition that is usually referred to as a consistency condition that claims the unknown coefficients of Eq.(13) could be reliably decided is used. Also, the UM method solves Eq.(13) for elementary or elliptic solutions when $\sigma = 1$ or $\sigma = 2$, respectively.

To obtain the solutions for Eq.(10) in polynomial function or rational function solutions, the following steps are involved:

i. Solve the algebraic system of equations.

ii. Solve the auxiliary equation.

iii.Find the exact formal solutions given in Eq.(11) (or Eq.(13)).

4 Governing model

Consider the temporal fractional KMN equation as follows:

$$i\frac{\partial^\gamma u}{\partial t^\gamma} + \alpha\frac{\partial^2 u}{\partial x \partial y} + i\beta u\Big(u\frac{\partial u^*}{\partial x} - u^*\frac{\partial u}{\partial x}\Big) = 0, \tag{15}$$

where $u(x, y, t)$ is wave portrait signifying the complex nonlinear wave envelope, γ is a local fractional parameter and $*$ denotes the complex conjugation. The first term in Eq.(15) indicates the fractional temporal wave evolution followed by means of the dispersion term that is identified by the coefficient of α. Ultimately, the nonlinear term which appears as a coefficient of β is different from the conventional Kerr law nonlinearity or any known non-Kerr law media.

Now, applying the definition of local M-derivative and its properties the time fractional KMN equation is presented as follows:

$$iD^{\gamma;\mu}_{M,t}u + \alpha\frac{\partial^2 u}{\partial x \partial y} + i\beta u\Big(u\frac{\partial u^*}{\partial x} - u^*\frac{\partial u}{\partial x}\Big) = 0, \tag{16}$$

where $D^{\gamma;\mu}_{M,t}u$ indicates the local M-derivative of order γ of function $u(x, y, t)$, with respect to t. It is worthwhile to note that when $\gamma = \mu = 1$, Eq.(16) can be transformed into original KMN equation. Assume that the complex transformation of fractional traveling wave variable is as follows

$$u(x, y, t) = q(\xi)e^{i\Phi}, \xi = P_1 x + P_2 y - \frac{\nu}{\gamma}\Gamma(\mu + 1)t^\gamma, \tag{17}$$

where $q(\xi)$ and ν are the amplitude and velocity of the soliton respectively, and

$$\Phi = Q_1 x + Q_2 y + \frac{\omega}{\gamma}\Gamma(\mu+1)t^{\gamma}, \tag{18}$$

here, Q_1 and Q_2 are respectively the soliton frequencies in $x-$ and $y-$directions while ω indicates the soliton wave number.
Substituting Eq.(17) into Eq.(16), yields

$$\alpha P_1 P_2 q'' - (\omega + \alpha Q_1 Q_2)q + 2Q_1\beta q^3 = 0, \tag{19}$$

from real part and

$$\nu = \alpha\big(P_1 Q_2 + P_2 Q_1\big), \tag{20}$$

from imaginary part.

5 Extraction of fractional optical solitons

Here, we implement the UM approach to obtain the fractional optical solitons of Eq.(16).

5.1 Polynomial function solution

To search for polynomial solution of Eq.(19), assume that

$$\begin{aligned} q(\xi) &= \sum_{j=0}^{n} \kappa_j \phi^j(\xi), \\ (\phi'(\xi))^{\sigma} &= \sum_{j=0}^{\sigma k} m_j \phi^j(\xi), \quad \xi = P_1 x + P_2 y - \frac{\nu}{\gamma}\Gamma(\mu+1)t^{\gamma}, \quad \sigma = 1, 2. \end{aligned} \tag{21}$$

Where κ_j and m_j are arbitrary coefficients to be determined. Apply the balancing condition between the terms q'' and q^3 in Eq.(19), yields the relation $n = k - 1$, where $k = 2, 3, ...$
Here, we restrict to hunt those solutions for $k = 2$ and $\sigma = 1$ or $\sigma = 2$. Hence, the Eq.(21) can be converted into

$$\begin{aligned} q(\xi) &= \kappa_0 + \kappa_1 \phi(\xi), \\ (\phi'(\xi))^{\varrho} &= \sum_{i=0}^{2\sigma} m_i \phi^i(\xi), \quad \sigma = 1, 2. \end{aligned} \tag{22}$$

5.1.1 Solitary wave solution

To acquire the solitary wave solution of Eq.(19), put $\sigma = 1$ in Eq.(22) and we have

$$q(\xi) = \kappa_0 + \kappa_1 \phi(\xi), \qquad (23)$$
$$\phi'(\xi) = m_0 + m_1\phi(\xi) + m_2\phi^2(\xi).$$

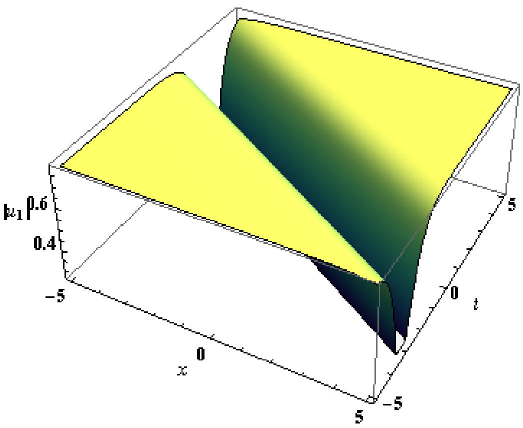
Now, applying Eq.(23) into Eq.(19) yields an algebraic system of equations. By adopting any symbolic computing package to solve this system for $\kappa_0, \kappa_1, m_0, m_1, m_2, \omega$ and we obtained the following results:

$$\omega = \frac{1}{2}\alpha\Big((4\kappa_1 m_0 R_1 - m_1^2)P_1P_2 - 2Q_1Q_2\Big), \quad \kappa_0 = -\frac{1}{2}\frac{\alpha m_1 P_1 P_2 R_1}{\beta Q_1}, \quad m_2 = \kappa_1 R_1, \qquad (24)$$

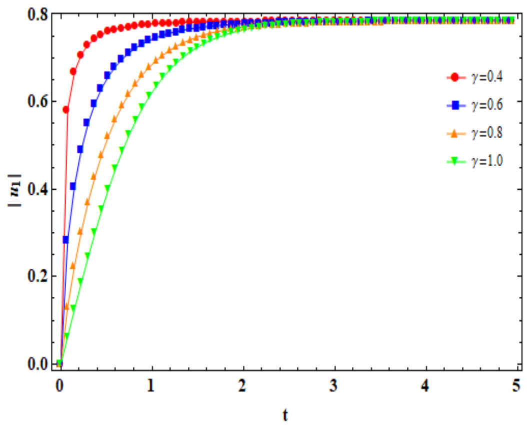
where $R_1 = \sqrt{-\frac{\beta Q_1}{\alpha P_1 P_2}}$. On solving the auxiliary equation Eq.$(23)_2$ and putting together with Eq.(24) into Eq.(23), we get the following solution for Eq.(16) for $\mu = 1$, namely

$$u_1(x,y,t) = -\frac{1}{2}\frac{R_2 \tanh(\frac{1}{2}\xi R_2) e^{i\Big(Q_1x + Q_2y + \frac{1}{2}\alpha\big((4\kappa_1 m_0 R_1 - m_1^2)P_1P_2 - 2Q_1Q_2\big)\frac{t^\gamma}{\gamma}\Big)}}{R_1}, \qquad (25)$$

where $\xi = P_1x + P_2y - \alpha\big(P_1Q_2 + P_2Q_1\big)\frac{t^\gamma}{\gamma}$, $R_2 = \sqrt{m_1^2 - 4m_0\kappa_1 R_1}$ and $R_1 = \sqrt{-\frac{\beta Q_1}{\alpha P_1 P_2}}$ provided that $m_1^2 > 4m_0\kappa_1 R_1$ and $(\beta Q_1)(\alpha P_1 P_2) < 0$.



(a) y=0



(b) x=0, y=0

Figure 1. Depicts the pictorial representation of the modulus of wave solution specified by Eq.(25) through 3D and 2D-plots at $\gamma = 1$ and $\gamma = 0.4, 0.6, 0.8, 1$ respectively, and with with a series of given choice of parameters $m_0 = 0.4, m_1 = 0.2, P_1 = 0.5, P_2 = 0.3, Q_1 = 1.2, Q_2 = 0.4, \alpha = -1.2, \beta = 0.5$ and $\kappa_1 = -2.5$.

5.1.2 Soliton wave solution

To get the soliton wave solution for Eq.(19), put $\sigma = 2$ in Eq.(22) and assume solution of the following form

$$\begin{aligned} q(\xi) &= \kappa_0 + \kappa_1 \phi(\xi), \\ \phi'(\xi) &= \phi(\xi)\sqrt{m_0 + m_1\phi(\xi) + m_2\phi^2(\xi)}. \end{aligned} \tag{26}$$

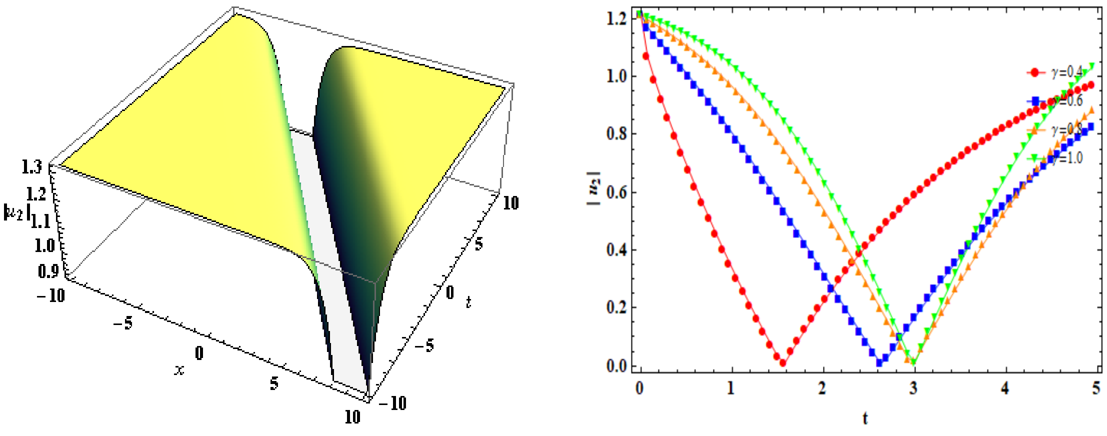
Substituting Eq.(26) into Eq.(19) in a similar fashion as we have done in the last case gives:

$$\kappa_0 = \sqrt{\frac{\alpha Q_1 Q_2 + \omega}{2\beta Q_2}}, m_0 = -\frac{2(\alpha Q_1 Q_2 + \omega)}{\alpha P_1 P_2}, m_1 = -\frac{4\beta Q_1 \kappa_1 \sqrt{\frac{\alpha Q_1 Q_2 + \omega}{2\beta Q_2}}}{\alpha P_1 P_2}, m_2 = -\frac{\beta Q_1 \kappa_1^2}{\alpha P_1 P_2}. \tag{27}$$

On solving the auxiliary equation Eq.$(26)_2$ and putting together with Eq.(27) into Eq.(23), we get the following solution for Eq.(16) for $\mu = 1$, namely

$$u_2(x,y,t) = \frac{\left(\sqrt{\frac{2(\alpha Q_1 Q_2+\omega)}{\beta Q_2}}\alpha P_1 P_2 - 8\kappa_1 e^{\xi R_3}(\alpha Q_1 Q_2 + \omega)\right) e^{i(Q_1 x + Q_2 y + \omega \frac{t^\gamma}{\gamma},)}}{\alpha P_1 P_2 + 4\beta Q_1 \kappa_1 \left(\frac{2(\alpha Q_1 Q_2+\omega)}{\beta Q_2}\right) e^{\xi R_3}}, \tag{28}$$

where $\xi = P_1 x + P_2 y - \alpha\big(P_1 Q_2 + P_2 Q_1\big)\frac{t^\gamma}{\gamma}, R_3 = \sqrt{-\frac{2(\alpha Q_1 Q_2+\omega)}{\alpha P_1 P_2}}$ provided that $(\alpha Q_1 Q_2 + \omega)(\alpha P_1 P_2) < 0$ and $(\alpha Q_1 Q_2 + \omega)(\beta Q_2) > 0$.



(c) y=0 (d) x=0, y=0

Figure 2. Depicts the pictorial representation of the modulus of wave solution specified by Eq.(28) through 3D and 2D-plots at $\gamma = 1$ and $\gamma = 0.4, 0.6, 0.8, 1$ respectively, and with a series of given choice of parameters $P_1 = -0.6, P_2 = 0.3, Q_1 = 1.2, Q_2 = 0.4, \alpha = 3.5, \beta = 0.5, \omega = 0.4$ and $\kappa_1 = -2.5$.

5.1.3 Elliptic wave solution

To acquire the elliptic wave solution of Eq.(19), assume the initial solution of the following form:

$$
\begin{aligned}
q(\xi) &= \kappa_0 + \kappa_1 \phi(\xi), \\
\phi'(\xi) &= \sqrt{m_0 + m_2\phi^2(\xi) + m_4\phi^4(\xi)}.
\end{aligned} \tag{29}
$$

Substituting Eq.(29) into Eq.(19) and by adopting a similar fashion as we did in previous sections gives:

$$
\omega = -\alpha(Q_1Q_2 - P_1P_2m_2), \kappa_0 = 0, \kappa_1 = \sqrt{-\frac{\alpha P_1P_2m_4}{\beta Q_1}}, \tag{30}
$$

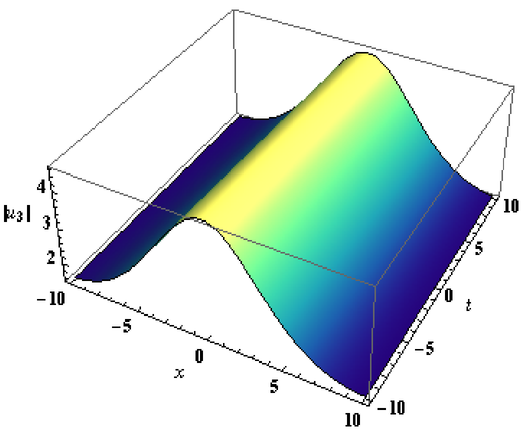
where m_j $(j = 0, 2, 4)$ are constants. For specific values of m_j, we obtain various solutions in the context of Jacobi elliptic functions. Following the categorization defined in [44], for

$$
m_4 = -\frac{1}{4}, m_2 = \frac{1+\lambda^2}{2}, m_0 = -\frac{(1-\lambda^2)^2}{2}, \quad 0 < \lambda < 1, \tag{31}
$$

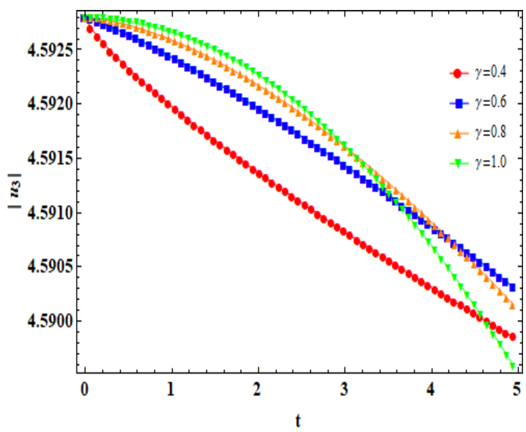
we take $\phi(\xi) = \lambda\ \mathrm{cn}(\xi, \lambda)+\mathrm{dn}(\xi, \lambda)$ and yields the following solution for Eq.(16) for $\mu = 1$, namely

$$
u_3(x, y, t) = \sqrt{\frac{\alpha P_1P_2}{4\beta Q_1}}\Big(\lambda cn(\xi, \lambda) + dn(\xi, \lambda)\Big)e^{i\left(Q_1x+Q_2y-\frac{\alpha}{2}\left(2Q_1Q_2-P_1P_2(1+\lambda^2)\right)\frac{t^\gamma}{\gamma}\right)}, \tag{32}
$$

where $\xi = P_1x + P_2y - \alpha\big(P_1Q_2 + P_2Q_1\big)\frac{t^\gamma}{\gamma}$ and $\lambda \in (0, 1)$ is known as the modulus of jacobi elliptic functions. It is important to bear in mind that when λ tends to zero, $sn(\xi)$, $cn(\xi)$ and $dn(\xi)$ approaches to $\sin(\xi), \cos(\xi)$ and 1, respectively. While, when λ tends to one, $sn(\xi)$, $cn(\xi)$ and $dn(\xi)$ degenerate to $\tanh(\xi)$, $\mathrm{sech}(\xi)$ and $\mathrm{sech}(\xi)$, respectively.



(e) y=0



(f) x=0, y=0

Figure 3. Depicts the pictorial representation of the modulus of wave solution specified by Eq.(32) through 3D and 2D-plots at $\gamma = 1$ and $\gamma = 0.4, 0.6, 0.8, 1$ respectively, and with a series of given choice of parameters $P_1 = 0.3, P_2 = 0.25, Q_1 = 0.01, Q_2 = 0.3, \alpha = 0.1, \beta = 0.02$ and $\lambda = 0.5$.

5.2 Rational function solution

In order to get the Eq.(19) rational solution, we assume that the solution has form as

$$
\begin{aligned}
q(\xi) &= \frac{\sum_{j=0}^{n} a_j \phi^j(\xi)}{\sum_{j=0}^{r} b_j \phi(\xi)}, \quad n \geq r, \\
(\phi'(\xi))^\sigma &= \sum_{j=0}^{\sigma k} m_j \phi^j(\xi), \quad \xi = P_1 x + P_2 y - \frac{\nu}{\gamma}\Gamma(\mu+1)t^\gamma, \quad \sigma = 2.
\end{aligned} \tag{33}
$$

Where a_j, b_j and m_j are arbitrary coefficients to be determined. Using the balance condition given by Lemma 2.3 [43], we get $k = 1$ and n is free by taking $n = r$. Here, we find those optical wave solutions when $n = r = 1, 2$ and $\sigma = 2$.

Case 1. Here, we take $n = r = 1$ and the solution results in the following form

$$
\begin{aligned}
q(\xi) &= \frac{a_0 + a_1\phi(\xi)}{b_0 + b_1\phi(\xi)}, \\
\phi'(\xi) &= \sqrt{m_0 + m_1\phi(\xi) + m_2\phi^2(\xi)}.
\end{aligned} \tag{34}
$$

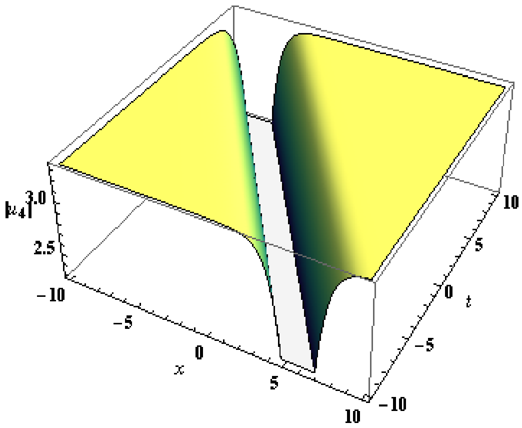
Now, applying Eq.(34) into Eq.(19) yields a system of algebraic equations. By using any symbolic computing package to solve this system for $a_0, a_1, b_0, b_1, m_0, m_1, m_2, \alpha, \beta$ and we obtained the following results:

$$
\begin{aligned}
\beta &= \frac{1}{2}\frac{b_1^2(\alpha Q_1 Q_2 + \omega)}{Q_1 a_1^2}, m_0 = -\frac{1}{2}\frac{(\alpha Q_1 Q_2 + \omega)(b_0 a_1 + a_0 b_1)^2}{\alpha P_1 P_2 b_1^2 a_1^2}, \\
m_1 &= -\frac{2\big(\alpha Q_1 Q_2 + \omega\big)(b_0 a_1 + a_0 b_1)}{\alpha P_1 P_2 b_1 a_1}, m_2 = -\frac{2(\alpha Q_1 Q_2 + \omega)}{\alpha P_1 P_2}.
\end{aligned} \tag{35}
$$

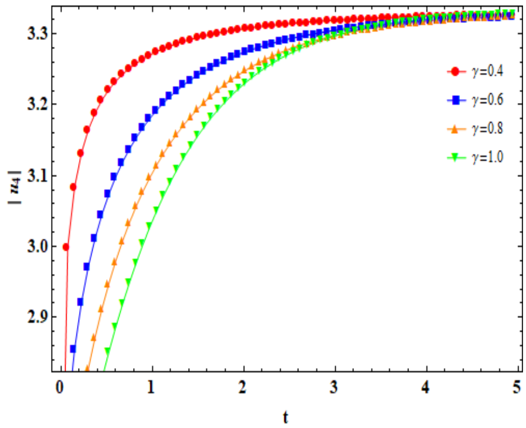
On solving the auxiliary equation Eq.$(34)_2$ and putting together with Eq.(35) into Eq.(34), we get the following solution for Eq.(16) for $\mu = 1$, namely

$$
u_4(x, y, t) = -\frac{a_1\Big((a_0 b_1 - b_0 a_1)R_3 + e^{\xi R_3} b_1 a_1\Big)e^{i(Q_1 x + Q_2 y + \omega \frac{t^\gamma}{\gamma})}}{b_1\Big((a_0 b_1 - b_0 a_1)R_3 - e^{\xi R_3} b_1 a_1\Big)}, \tag{36}
$$

where $\xi = P_1 x + P_2 y - \alpha\big(P_1 Q_2 + P_2 Q_1\big)\frac{t^\gamma}{\gamma}, R_3 = \sqrt{-\frac{2(\alpha Q_1 Q_2 + \omega)}{\alpha P_1 P_2}}$ provided that $(\alpha Q_1 Q_2 + \omega)(\alpha P_1 P_2) < 0$.



(g) y=0



(h) x=0, y=0

Figure 4. Depicts the pictorial representation of the modulus of wave solution specified by Eq.(36) through 3D and 2D-plots at $\gamma = 1$ and $\gamma = 0.4, 0.6, 0.8, 1$ respectively, and with a series of given choice of parameters $P_1 = -0.6, P_2 = 0.3, Q_1 = 1.2, Q_2 = 0.4, \alpha = 3.5, \omega = 0.4, a_0 = 0.5, a_1 = 1, b_0 = 1$ and $b_1 = 0.3$.

Case 2. Here, we take $n = r = 2$ and the solution results in the following form

$$q(\xi) = \frac{a_0 + a_1\phi(\xi) + a_2\phi^2(\xi)}{b_0 + b_1\phi(\xi) + b_2\phi^2(\xi)},$$
$$\phi'(\xi) = \sqrt{m_0 + m_1\phi(\xi) + m_2\phi^2(\xi)}. \tag{37}$$

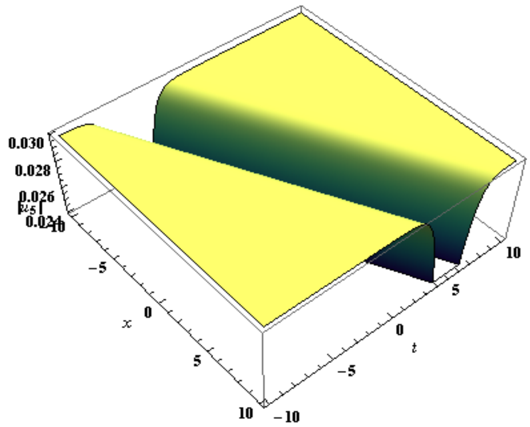
Substituting Eq.(37) for Eq.(19) in a similar fashion as we have done in the last case gives:

$$\beta = -\frac{1}{4}\frac{b_2^2\alpha P_1 P_2 m_2}{Q_1 a_2^2}, m_0 = \frac{m_1^2}{4m_2}, a_0 = \frac{1}{4}\frac{m_2^2(b_2^2a_1^2 - b_1^2a_2^2) - m_1^2a_2^2b_2^2 + 2m_1m_2a_2^2b_1b_2}{m_2^2b_2^2a_2},$$
$$b_0 = -\frac{1}{4}\frac{m_2^2(b_2^2a_1^2 - b_1^2a_2^2) + m_1^2a_2^2b_2^2 - 2m_1m_2b_2^2a_1a_2}{m_2^2a_2^2b_2}, \omega = -\frac{\alpha}{2}\big(P_1P_2m_2 + 2Q_1Q_2\big). \tag{38}$$

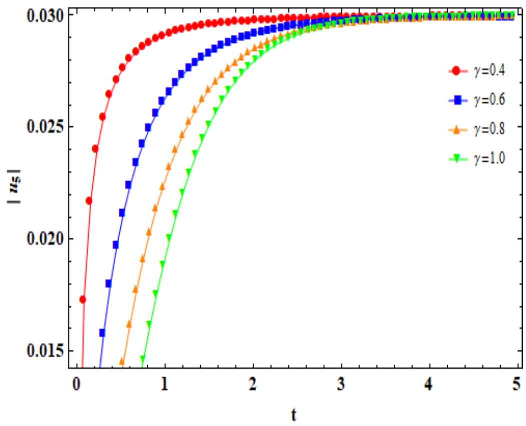
On solving the auxiliary equation Eq.$(37)_2$ and putting together with Eq.(38) into Eq.(37), we get the following solution for Eq.(16) for $\mu = 1$, namely

$$u_5(x,y,t) = \frac{a_2\Big(m_2^{\frac{3}{2}}(b_2^2a_1^2 - b_1^2a_2^2) + 2a_2b_2m_1\sqrt{m_2}(a_2b_1 - a_1b_2) + 2b_2^2a_2(a_1m_2 - a_2m_1)e^{\xi\sqrt{m_2}} + a_2^2b_2^2\sqrt{m_2}e^{2\xi\sqrt{m_2}}\Big)}{b_2\Big(m_2^{\frac{3}{2}}(b_1^2a_2^2 - b_2^2a_1^2) + 2a_2b_2m_1\sqrt{m_2}(a_1b_2 - a_2b_1) + 2a_2^2b_2(b_1m_2 - b_2m_1)e^{\xi\sqrt{m_2}} + a_2^2b_2^2\sqrt{m_2}e^{2\xi\sqrt{m_2}}\Big)}$$
$$\times e^{i\left(Q_1x + Q_2y - \frac{\alpha}{2}\left(P_1P_2m_2 + 2Q_1Q_2\right)\frac{t^\gamma}{\gamma}\right)}, \tag{39}$$

where $\xi = P_1x + P_2y - \alpha\big(P_1Q_2 + P_2Q_1\big)\frac{t^\gamma}{\gamma}$ provided that $m_2 > 0$ and $a_2, b_2 \neq 0$.



(i) y=0



(j) x=0, y=0

Figure 5. Depicts the pictorial representation of the modulus of wave solution specified by Eq.(39) through 3D and 2D-plots at $\gamma = 1$ and $\gamma = 0.4, 0.6, 0.8, 1$ respectively, and with a series of given choice of parameters $P_1 = 2, P_2 = 0.3, Q_1 = 1.2, Q_2 = 0.4, \alpha = 3.5, \omega = 0.4, a_1 = 0.5, a_2 = 0.1, b_1 = 2, b_2 = 0.3, m_1 = 0.4$ and $m_2 = 0.2$.

6 Conclusion

This work addresses the unified method together with local M-derivative to uncover fairly interesting optical solitons of the governing equation. The imaginary and real part of the equation led to the soliton solutions. We reach to a constraint relation and the velocity of the soliton by equating the soliton parameters such the amplitude and the width. The predicted results are yielded with constraint conditions and highlighted by their 3D graphical portrayal. The impact of a local fractional parameter on dispersion is also presented via 2D graphs. The obtained results revealed that the suggested approach with the local M-derivative is useful and effective for addressing other nonlinear evolution equations in mathematical physics. The reportable solutions are novel and gift a valuable addition to the literature in soliton wave theory.

Being a recently developed model, there is plenty of flexibility to extend the horizon in this regard. The UM technique being a nontrivial scheme, further integration approaches are to be implemented, later on, to tie down additional new solutions for KMN model.

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