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How frequent are Antarctic sudden stratospheric warmings in present and future climate?

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Key Points:

- Antarctic sudden stratospheric warmings occur once every 22 years in present-day (1990) climate conditions.
- The warmings will become much rarer under future climate change, irrespective of their exact definition.
- The future decrease in frequency is linked to a strengthening of the Antarctic polar vortex.

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Abstract

Southern Hemisphere (SH) Stratospheric Sudden Warmings (SSWs) result in smaller Antarctic ozone holes and are linked to extreme midlatitude weather on subseasonal to seasonal timescales. Therefore, it is of interest how often such events occur and whether we should expect more events in the future. Here, we use a pair of novel multi-millennial simulations with a stratosphere-resolving coupled ocean-atmosphere climate model to show that the frequency of SSWs, such as observed 2002 and 2019, is about one in 22 years for 1990 conditions. In addition, we show that we should expect the frequency of SSWs—and that of more moderate vortex weakening events—to strongly decrease by the end of this century.

Plain Language Summary

The stratosphere at 10-50 km height can influence surface weather for several months. In 2002 and 2019, the stratosphere warmed over Antarctica within a few days to weeks. This caused dry and hot summers in Australia and South America. And it reduced the size of the ozone hole. Since these warming events are rare, it is difficult to say how often they occur. We therefore use long computer simulations to answer that question. We find that without climate change, warming events occur about every 22 years. But with climate change, the warming events will happen only once every 300 years. From this, we believe that the quick succession of two events in 2002 and 2019 will remain special in history.

1 Introduction

The stratospheric polar vortex forms in the winter hemisphere due to the lack of solar heating at high latitudes and the resulting strong equator-to-pole temperature gradient. In the Northern Hemisphere (NH), strong and planetary scale waves originating in the troposphere from orographic forcing and land-sea contrast periodically propagate upward into the stratosphere and perturb the polar vortex via momentum deposition when the waves break (Eliassen & Palm, 1960; Charney & Drazin, 1961; Matsuno, 1971). In extreme cases, this disruption of the polar vortex leads to a rapid warming and reversal of wind directions in the polar stratosphere, a so-called (major) Sudden Stratospheric Warming (SSW) (Butler et al., 2015). These SSWs occur around every other winter in the NH.

However, over the six decades that we have station records (and later satellite observations) of the Southern Hemisphere (SH) polar vortex, only one such wind reversal has been recorded in 2002 (Roscoe et al., 2005). This event substantially decreased the size of the ozone hole thanks to higher than usual stratospheric polar temperatures and transport of ozone-rich air from lower latitudes into the polar regions (Fig. S1a) (Stolarski et al., 2005). There was also a dynamical effect of the 2002 SSW at the surface, as an extreme negative polarity of the Southern Annular Mode (SAM) was recorded at the surface for the 10-90 day period following the event (Thompson et al., 2005). Even though no wind reversal at 60° S and 10 hPa was registered in 2019, the polar vortex in this more recent event weakened dramatically and also led to a smaller ozone hole (Fig. S1b) with almost 30% higher total column ozone values compared to the previous decade (Safieddine et al., 2020). The event has also been linked to the severe bushfire season in South Eastern Australia the following spring and summer (SPARC, 2020).

Due to the impacts on stratospheric ozone and surface weather on the subseasonal to seasonal timescale, it is important to determine how rare SSWs are in the SH, and whether we should expect more or less frequent SSWs under future climate change. However, given the shortness of the observational record it is impossible to get an observational estimate of how often SSWs do occur on average. Recently, Wang et al. (2020)

65 analyzed hindcasts of a seasonal forecasting system and found an average Antarctic SSW
66 frequency of one every 25 years. However, the underlying model of this study had a strong
67 mean westerly wind bias, raising some doubts on the validity of their results. Here, we
68 revisit the question of how frequent Antarctic SSWs are in present climate and also ad-
69 dress possible changes under future climate change. This is accomplished by investigat-
70 ing two nearly 10,000-year long simulations with a well-performing stratosphere-resolving
71 coupled ocean-atmosphere model based on present-day (1990) and future (increased CO₂)
72 conditions and by considering integrations from the sixth Climate Model Intercompar-
73 ison Project (CMIP6).

74 **2 Model data and SSW definitions**

75 **2.1 Multi-millennial coupled GCM simulations**

76 We use a set of two 9,900-year long simulations with the stratosphere-resolving ver-
77 sion of the the Geophysical Fluid Dynamics Laboratory’s CM2.1 atmosphere-ocean cou-
78 pled climate model (Delworth et al., 2006; Horan & Reichler, 2017), which has been used
79 in particular for studies of stratosphere-troposphere coupling in the past (Horan & Re-
80 ichler, 2017; Jucker & Reichler, 2018). The model has 48 vertical levels with approxi-
81 mately half of the levels situated in the stratosphere and a model top at 0.002 hPa. The
82 horizontal resolution is approximately 2° in latitude and 2.5° in longitude. The bound-
83 ary conditions are set to perpetual 1990 conditions. More specifically, ozone in the year
84 1990 is comparable to both 2002 and the 2010s (Newman & Nash, 2019). The two sim-
85 ulations differ in their greenhouse gas forcing; CO₂ is set to 353 ppm in the ‘present-day’
86 and 1120 ppm in the ‘future’ simulation, which is a quadrupling relative to pre-industrial
87 CO₂ concentration (and 3.2 times present-day concentration). This is the only difference
88 between the two simulations. Atmospheric variables are stored on a daily frequency to
89 allow for detailed dynamical analysis, including Eliassen-Palm fluxes.

90 In agreement with Reichler and Kim (2008) and Horan and Reichler (2017) who
91 have shown that this model compares well to reanalysis in the troposphere and north-
92 ern hemisphere stratosphere, both the southern hemisphere stratospheric zonal mean zonal
93 wind and vertical component of the Eliassen-Palm flux from our present-day simulation
94 show excellent agreement with those from ERA5 reanalysis (1979-2019) (Hersbach et al.,
95 2020), for both mean and standard deviation (Fig. 1a,c). Besides its performance in the
96 stratosphere which is of particular relevance here, the oceanic component has been val-
97 idated extensively and also found to have a good representation of tropical (including
98 ENSO, Wittenberg et al., 2006) as well as extratropical southern hemisphere ocean dy-
99 namics (Gnanadesikan et al., 2006).

100 Having multi-millennial simulations with a model showing such small bias will al-
101 low us to robustly estimate SSW frequencies. In addition, having future projections will
102 make it possible to address the question of whether or not we should expect another SSW
103 to occur, which we will show to be impossible without information on the impact of in-
104 creased greenhouse gas concentrations.

105 **2.2 SSW definitions**

106 We follow the most common definition of Sudden Stratospheric Warmings as the
107 reversal of u_{1060} , the zonal mean zonal wind at 60°S and 10 hPa (‘SSW-reversal’, Charl-
108 ton & Polvani, 2007). However in observations, only the September 2002 event is a SSW-
109 reversal event, while the 2019 event is widely considered a SSW but did not show wind
110 reversal at 60°S and 10 hPa. Therefore, we have performed our analysis with an addi-
111 tional definition, allowing for a more general determination of SSW frequency and fu-
112 ture change.

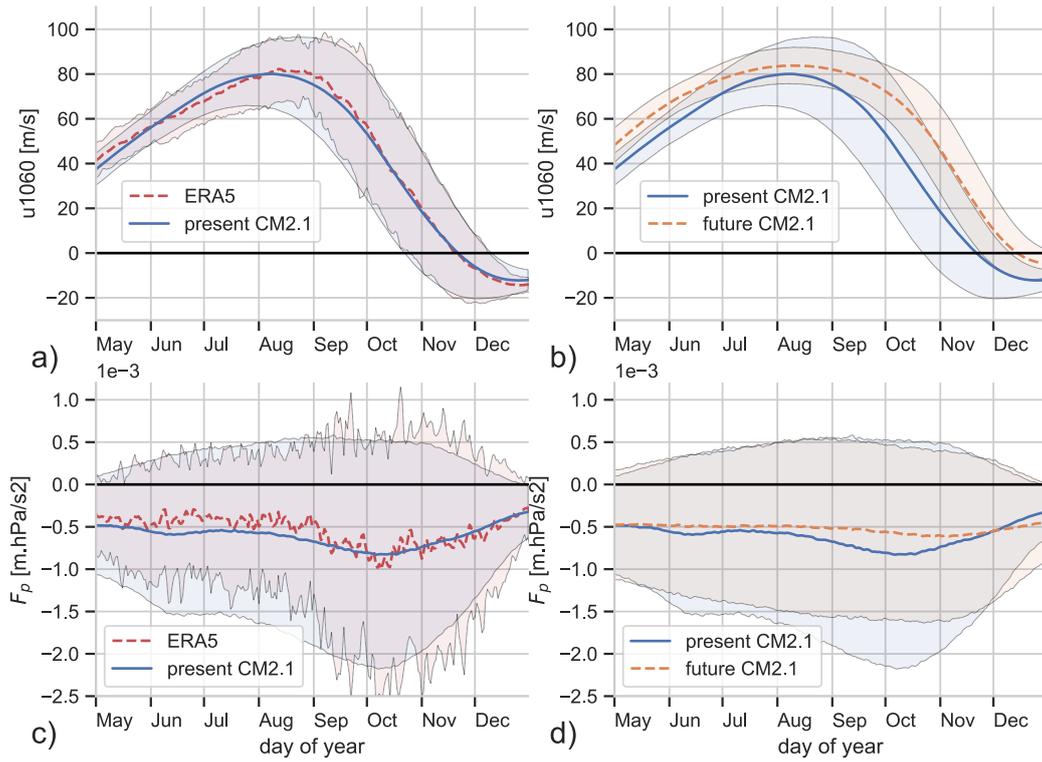


Figure 1. (top) Climatological mean (solid) and 2σ variability (shaded) of zonal mean zonal wind at 60°S and 10 hPa (u_{1060}) for (a) present-day CM2.1 and ERA5 and (b) present-day and future CM2.1. (bottom) same but for vertical EP flux. The present-day simulation (blue, solid) reproduces both mean and variability of the ERA5 reanalysis (1979-2019; red, dashed) in both u_{1060} (a) and vertical EP flux (c). The future simulation (orange, dashed) shows a clear strengthening of the polar vortex throughout the year (b) and a weakening of the vertical EP flux (d), in particular during the spring.

113 We found that the simplest method to define SSWs in the SH which detects both
 114 2002 and 2019 as the only events during the satellite era is that the zonal mean zonal
 115 wind anomaly with respect to the day of the year at 60°S and 10 hPa passes below -40 m/s.
 116 The onset date is then defined as the day when the zonal mean zonal wind anomaly crosses
 117 -20 m/s for the last time before crossing -40 m/s. These ‘SSW-weak’ events follow the
 118 common features of stratosphere-troposphere coupling in the SH in their significant sur-
 119 face impact on monthly timescales (Fig. S2).

120 For both definitions, two events have to be separated by at least 20 days, and the
 121 onset date has to be at least 20 days before the vortex breakdown, which is defined as
 122 the last day of the year when u_{1060} becomes negative.

123 Finally, we follow Lim et al. (2018) who showed that weaker events can also have
 124 an impact at the surface, and we will also report results from their detection method based
 125 on the yearly timeseries of the first Principal Component of de-seasonalized monthly mean
 126 zonal mean zonal wind between 55 and 65°S. The corresponding Empirical Orthogonal
 127 Function is two-dimensional but in month of the year–pressure space (instead of the con-
 128 ventional longitude–latitude space) and is centered around the vortex breakdown in spring
 129 (the ‘L18’ method). This method does not provide onset dates, as there is only one value
 130 per year, and L18 is closely related to variations in the date of the vortex breakdown (pos-
 131 itive for earlier breakdown; the correlation coefficient between the first Principal Com-
 132 ponent and the vortex breakdown date is $r = 0.79$ in ERA5 data, not shown). Follow-
 133 ing Lim et al. (2019), we apply a threshold of 0.8 standard deviations, which detects many
 134 more events than the other two definitions.

135 3 Occurrence of SSWs in the Southern Hemisphere

136 The present-day 9,900-year simulation produces 458 SSW-weak and 159 SSW-reversal
 137 events, corresponding to an average frequency of about one SSW-weak every 22 years and
 138 one SSW-reversal every 59 years. This compares well with the single SSW-reversal and
 139 only two SSW-weak events in the 42-year long satellite observation record (and the 63-
 140 year long non-satellite observational record since 1957 (Roscoe et al., 2005; Naujokat &
 141 Roscoe, 2005)), as well as Wang et al. (2020). In addition to yearly occurrence, we also
 142 analyze the seasonal occurrence of SSWs and find that the SSW-weak criterion detects
 143 events during the entire winter, with a peak occurrence in late August to September (Fig. 2d)
 144 and a mean occurrence of 27 August (note that early events in June and July have a sim-
 145 ilar impact as later events, not shown). The 2002 SSW occurred in late September, a
 146 time of the year when we estimate the mean return time of SSW-weak events to be 113 years,
 147 and the 2019 SSW occurred in early September, when the mean return time is estimated
 148 to be 102 years (Fig. 2a). Irrespective of time of the year, our present-day simulations
 149 indicate that we should expect between 0 and 6 SSW-reversals and between 0 and 12
 150 SSW-weak events per century, with most likely numbers of 0-2 SSW-reversal and 3-6 SSW-
 151 weak events per century (25th and 75th percentiles, Figs. 2b,e). As indicated before, L18
 152 events are much more abundant, with an occurrence of 7-36 events per century and a
 153 mean return time of one in 5 years (Fig. 2h).

154 To get an estimate of when the next SSW might occur, we perform a return time
 155 analysis where we produce a histogram of the number of SSWs which occur within a given
 156 time interval (Fig. 2c,f,i). If SSWs are independent and random events, we can compare
 157 the observed return time distribution to a theoretical distribution (S.5). The return time
 158 histogram follows closely the theoretical distribution for all methods, suggesting that in
 159 the SH, SSWs are independent and random, with a mean return time of about 59 years
 160 for SSW-reversal and 22 years for SSW-weak, or an annual probability of occurrence of
 161 1.6% for SSW-reversal and 4.6% for SSW-weak. Using the theoretical survival function,
 162 we can then compute the probabilities of various scenarios (reported in Table 1). All of
 163 these probabilities are consistent with the observational record of one SSW-reversal and

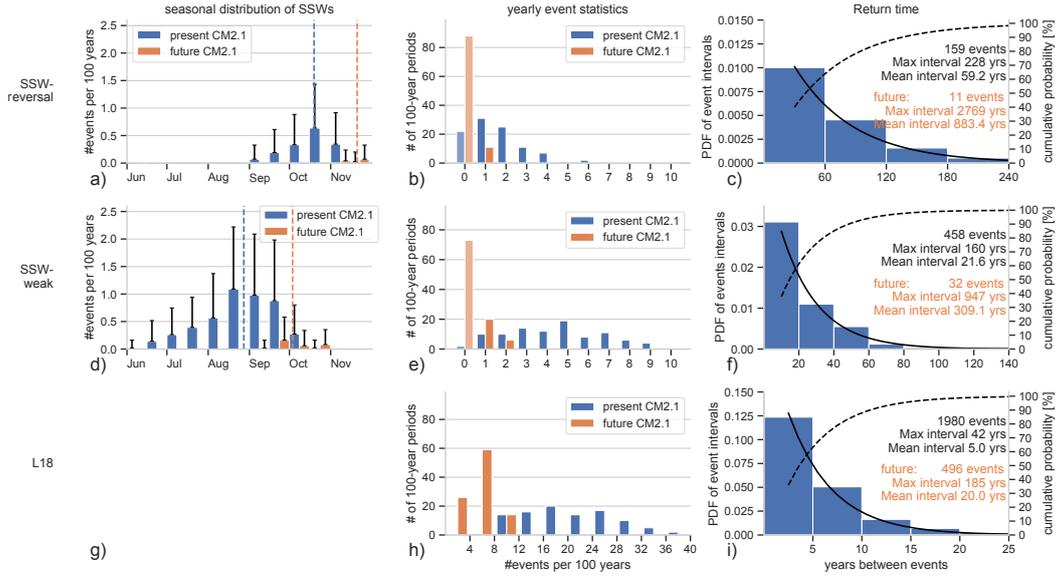


Figure 2. Event statistics: (left) Seasonal distribution, (middle) histogram of number of events per century, and (right) return time distribution histograms (bars) and theoretical distribution (black lines) for probability (solid) and cumulative distribution functions (dashed). Statistics are shown for (top) SSW-reversal, (middle) SSW-weak and (bottom) L18. For all plots, the present-day simulation is in blue and increased CO₂ (‘future’) in orange. On the left panels, statistics are shown for half-monthly intervals, the black whiskers show the standard deviation, and the vertical dashed lines indicate the mean date of occurrence. Panel (g) is empty as there is no seasonal information for L18. Note the differences in scales between rows. In panels (b) and (e), bars are drawn for each year, whereas in panel (h), the bars are drawn within intervals designated by the tick marks. Bars showing the number of centuries without event are pale.

		SSW-weak	SSW-reversal
present	Yearly probability	4.6%	1.6%
	Probability of less than observation	43%	52%
	Probability of exact observation	28%	35%
	Probability of more than observation	30%	15%
	Probability > 50% after	15 years	41 years
future	Yearly probability	0.3%	0.1%
	Probability > 50% after	214 years	612 years
	Probability of at least one SSW in 80 years	23%	8.7%
	Probability of at least two SSWs in 80 years	2.8%	0.4%

Table 1. Results from the theoretical fitting of the return times (Figs. 2c and f). Yearly probability is the probability of an event occurring during any given year (1/mean return time), probability of exact observation is computed for 2 SSW-weak and 1 SSW-reversal in 41 years. Time periods give the interval after which an SSW is more probable than not (probability of one or more events > 50%). The labels ‘present’ and ‘future’ refer to the relevant CM2.1 simulations, and we use an 80-year period to represent the time span 2021-2100 in the future simulation. Note that the observation percentages in the present simulation add to 101 instead of 100 due to rounding errors.

164 two SSW-weak events during the satellite era. Finally, neglecting any changes in climate
 165 from further greenhouse gas forcing since 1990, we estimate from the present-day sim-
 166 ulation that the probability of at least one SSW by the end of the century (next 80 years)
 167 would be 74% for SSW-reversals and 98% for SSW-weak events. Of course, this is only
 168 hypothetical as greenhouse gas concentrations have already risen since 1990 and are pro-
 169 jected to further increase in the future.

170 4 Enhanced greenhouse gas forcing

171 To estimate the impact of enhanced greenhouse gas forcing on the occurrence of
 172 SSWs in the SH, we conducted a second 9,900 year long simulation using increased CO₂
 173 corresponding to the end of the century (1120 ppm instead of 353 ppm, henceforth called
 174 ‘future’). The occurrence of SSWs in this simulation decreases drastically. The number
 175 of SSW-reversals reduces from 159 SSWs for present-day to only 11 in the future sim-
 176 ulation, while SSW-weak events decrease from 458 to only 32 (Fig. 2). This translates
 177 into a return time of one SSW-reversal every 883 and one SSW-weak every 309 years,
 178 and a maximum of 1 SSW-reversal and 2 SSW-weak events per century. Note how the
 179 most probable outcome by far for any given 100-year period is zero SSWs (median is zero
 180 for both SSW-reversal and SSW-weak; Fig. 2b and e, orange). From the theoretical fit,
 181 the probability of occurrence of at least one SSW-weak event in 80 years is now about
 182 23% (2.8% for at least two SSWs; Table 1). The analysis also suggests that SSW-reversals
 183 become very rare (probability of 8.7% within 80 years). SSWs not only become much rarer,
 184 but are also occurring later in the year, with a mean date of 3 October for SSW-weak,
 185 i.e. more than one month later than in the present-day simulation. For all definitions,
 186 there is a strong tendency for fewer SSWs in the future—including L18, which reduce to
 187 0-11 events per century. Thus, while the 2019 event is consistent with the occurrence rate
 188 in our present-day simulation, it is inconsistent with the rate seen in our future simu-
 189 lation. Given the trend in SSW frequency, and that we are already one-third of the way
 190 towards the year 2080, we conclude that this latest event should not be attributed to in-
 191 creased CO₂ forcing, and might indeed be the last observed event this century.

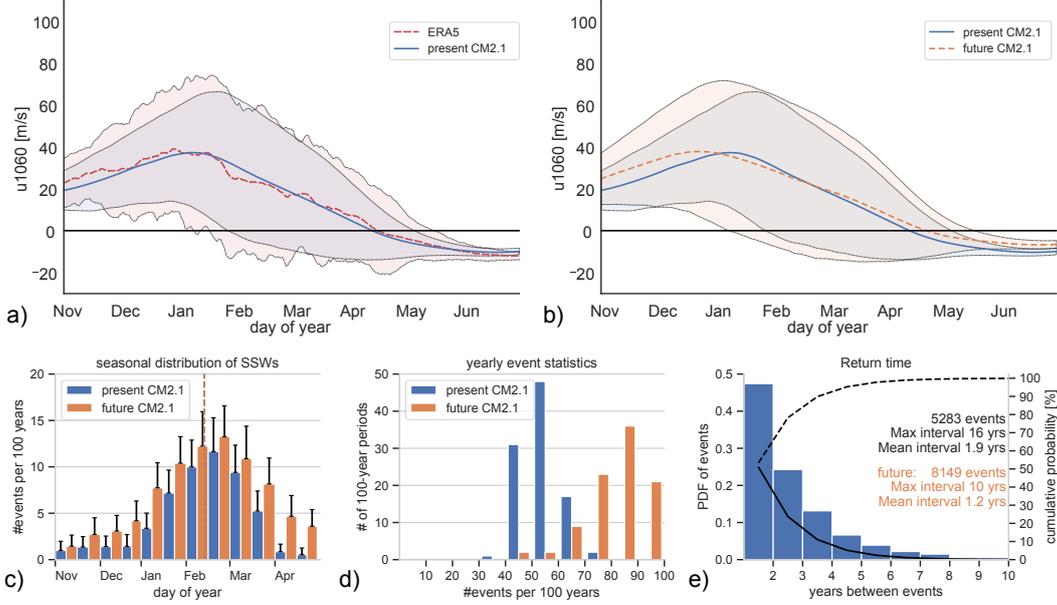


Figure 3. (top) u_{1060} for the Northern Hemisphere (NH) for (a) present-day and (b) increased CO_2 ('future'), similar to Fig. 1. (bottom) NH SSW-reversal statistics for (c) seasonal distribution, (d) number of events per century and (e) return time, similar to Fig. 2. Note the differences in scale of the bottom row compared to Fig. 2, which is a result of the higher occurrence rate for the NH.

192 The decrease in SSW frequency in the future is accompanied by a strengthening
 193 of the SH polar vortex (Fig. 1b), which can be linked to stronger radiative cooling un-
 194 der increased greenhouse gas concentrations (Thompson et al., 2012; Santer et al., 2013).
 195 In addition, our simulations suggest a decrease in wave forcing, more so during spring
 196 than other times of the year (Fig. 1d). Together with an earlier study, which found a di-
 197 rect link between the SSW-reversal frequency and polar vortex strength (Jucker et al.,
 198 2014), our results suggest that the projected strengthening of the polar vortex along with
 199 a decrease in wave forcing are responsible for a substantial decrease in the probability
 200 of occurrence of SSWs.

201 5 Comparison to NH

202 The occurrence of SSWs in the NH is very different from the SH, not just because
 203 of the much higher SSW frequency at present, but also in terms of future projections of
 204 both polar vortex strength and SSW frequency. As discussed in detail by Horan and Re-
 205 ichler (2017), our model climatology and variability in the NH compares well to reanal-
 206 ysis products (Fig. 3), and it produces about five SSWs per decade in the NH, in accor-
 207 dance with observations (Jucker & Reichler, 2018). Therefore, we perform the same anal-
 208 ysis for the NH and briefly report our findings here.

209 The return time distribution shows that at intervals shorter than four years, NH
 210 SSWs are not independent and random (Fig. 3e), probably reflecting the influence of slowly
 211 evolving large scale climate modes, such as the El Niño Southern Oscillation or the Quasi-
 212 Biennial Oscillation, on the occurrence of SSWs (Holton & Tan, 1980; Taguchi & Hart-
 213 mann, 2006; Anstey & Shepherd, 2014). The NH polar vortex is also weaker and more
 214 influenced by upward propagating planetary waves from the troposphere, resulting in a

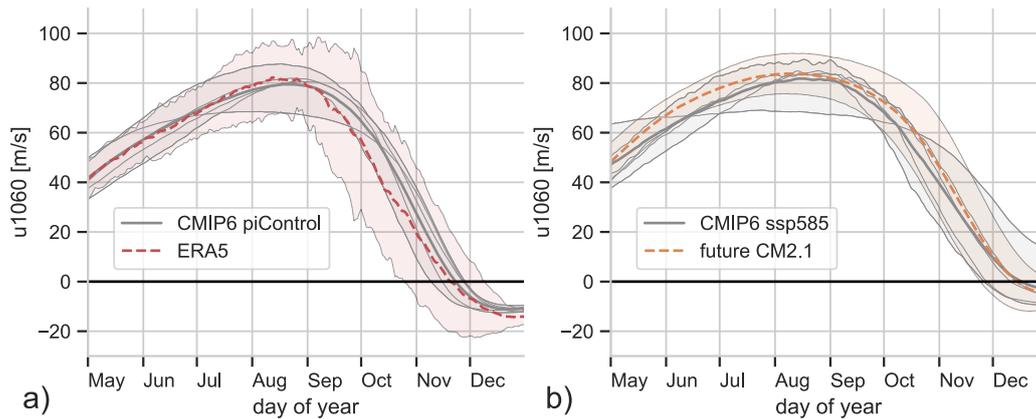


Figure 4. Analysis similar to Fig. 1, but using CMIP6 data. SSP585 data represents the climatology over 2080-2100. Shading corresponds to the range of model means (min to max), whereas the thin solid gray lines show each model and the thick gray and purple lines the multi model means.

215 more variable polar vortex than in the SH (Fig. 3, top). Our simulations suggest a slightly
 216 weaker polar vortex and more SSWs in the future NH (Fig. 3, bottom; SSW-reversal only).
 217 However, we have less confidence in this result because strong dynamical coupling be-
 218 tween the troposphere and the stratosphere in the NH complicates future projections,
 219 and also because several past studies were unable to reach a consensus on possible fu-
 220 ture changes of SSW occurrence rates over the NH (Manzini et al., 2014; Ayarzagüena
 221 et al., 2018; Wu et al., 2019; Ayarzagüena et al., 2020). Coincidentally, there is also no
 222 consensus about the future strength of the polar vortex (Simpson et al., 2018), which
 223 is in agreement with our conclusion that the polar vortex strength is important for the
 224 frequency of SSWs.

225 6 CMIP6

226 To check the robustness of our single model simulations, we repeat our analysis with
 227 CMIP6 data (see Sec. S.3 for details). We find that these models show a positive polar
 228 vortex strength bias (Fig. 4) and generally struggle to produce the observed frequency
 229 of SSWs, with a range of 0.3-2.4 SSW-weak events on average in 80 years for piControl
 230 (Table S1). The low SSW frequency in CMIP6 was also briefly noted in recent work (Ayarzagüena
 231 et al., 2020). However, the statistical analysis again suggests a decrease in SSWs in the
 232 future, with three models producing one single and two models producing no SSW-weak
 233 event in SSP585 between 2021 and 2100 (Table S1b). The CMIP6 models consistently
 234 project a strengthening of the SH polar vortex of similar amplitude as our simulations
 235 with CM2.1 (Fig. 4), suggesting that our main conclusion that SSWs will become much
 236 rarer in the future is robust.

237 Our enhanced CO₂ CM2.1 simulation only considers future increases in CO₂. Changes
 238 in other radiatively active gases, in particular the expected recovery of the ozone hole
 239 by 2080 (Dhomse et al., 2018), are not included. However, our 1120 ppm CO₂ concen-
 240 tration is equal to the CO₂ concentration at the end of the century following the SSP585
 241 scenario (which in addition to CO₂ also increases other greenhouse gases such as methane
 242 and nitrous oxide (O’Neill et al., 2016; Meinshausen & Nicholls, 2020)). Consequently,
 243 u₁₀₆₀ of our future simulation compares well to the end of the 21st century in CMIP6
 244 SSP585 model data (Fig. 4b). This is consistent with previous findings that over the long

245 term, the greenhouse effect from increasing CO₂ concentrations dominates the effect of
 246 the ozone hole recovery (Barnes & Polvani, 2013). The similarities in u₁₀₆₀ and CO₂ con-
 247 centrations between our CM2.1 simulations and CMIP6 models gives us confidence that
 248 our enhanced CO₂ simulation is relevant for end-of-century projections.

249 7 Conclusions

250 The 2002 and 2019 SSWs both resulted in exceptionally small ozone holes as have
 251 not been observed since the 1980s. They were also followed by extended periods of neg-
 252 ative Southern Annular Mode at the surface, and 2019 in particular was linked to the
 253 catastrophic fire season in South Eastern Australia. While possibly predictable on the
 254 seasonal time scale, it has been difficult to determine how often SSWs should be expected
 255 in the southern hemisphere, due to a relatively short observational record on one hand
 256 and large model biases in the southern hemisphere stratosphere in most comprehensive
 257 climate models on the other hand. Using a pair of exceptionally long and low bias cli-
 258 mate model runs, we found that while SSWs in the SH have significant impacts on strato-
 259 spheric ozone and surface weather, such events are rare and will become even rarer as
 260 CO₂ concentrations increase. In our simulation based on 1990 conditions, the mean re-
 261 turn time for events similar to the 2002 and 2019 SSWs is about 22 years, with a 57%
 262 chance of at least two and a 30% chance of three or more SSW-weak events happening
 263 within the time period spanned by the satellite record. Thus, it is no surprise that two
 264 events have been observed, and there would be a fair chance of another SSW (of either
 265 flavor) in the near future if CO₂ levels were kept constant. However, we show that one
 266 should not make predictions of future occurrence from past data; given that the world
 267 follows a high emissions pathway, our projections suggest that events similar to 2002 and
 268 2019 will become extremely rare, with a mean return time of one in 309 years (or 0.3%
 269 each year) by the end of the century.

270 Acknowledgments

271 M.J. was supported by the Australian Research Council grant ARC grant FL150100035
 272 and the ARC Centre of Excellence for Climate Extremes which is supported by the Aus-
 273 tralian Research Council via grant CE170100023. T. R. acknowledges support from NSF
 274 grant 1446292. We also acknowledge the Center for High Performance Computing at the
 275 University of Utah and the National Computational Infrastructure in Canberra for pro-
 276 viding compute infrastructure and computing time. This work used the xarray (Hoyer
 277 & Hamman, 2017) and aostools (Jucker, 0) packages. The zonal mean zonal wind time-
 278 series from our simulations are available at [https://data.mendeley.com/datasets/
 279 hknv82tz7v/draft?a=84a23625-6306-440f-ae69-19011c620c7a](https://data.mendeley.com/datasets/hknv82tz7v/draft?a=84a23625-6306-440f-ae69-19011c620c7a). All authors contributed
 280 to conceptualization, methodology and writing of the original draft. T.R. provided the
 281 GCM simulations and M.J. performed the formal analysis. Authors declare no compet-
 282 ing interests.

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Supplementary material

S.1 The 2002 and 2019 events

Fig. S1 shows the evolution of u_{1060} and polar cap stratospheric ozone during the springs of 2002 and 2019 from ERA5.

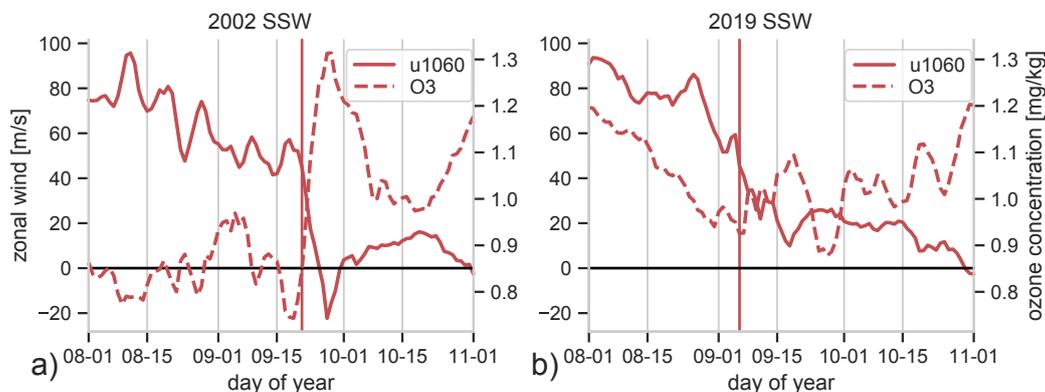


Figure S1. u_{1060} (solid), and polar cap (60–90°S) averaged ozone mass mixing ratio at 50 hPa (dashed) for the springs of (a) 2002 and (b) 2019 from ERA5 reanalysis. The solid vertical lines denote the onset date based on the SSW-weak definition.

S.2 Surface impacts

Just as for the observed SSWs, in our present-day simulation SSWs are followed by a negative phase of the SAM on a monthly to seasonal timescale (Fig. S2a; only composites for SSW-weak are shown) (Thompson et al., 2005), accompanied by colder and wetter conditions over New Zealand and South America as well as warmer and drier conditions over Eastern Australia (Figs. S2b and S2c). These surface impacts agree well with previous work (Gillett et al., 2006; Lewis, 2019; Lim et al., 2019) and the reanalysis data from the 2002 and 2019 events, confirming that our model reproduces the dynamical evolution of SSWs well and that our definition based on anomalous u_{1060} does indeed capture events with considerable surface impact. We note that the surface impact of early SSW-weak events (e.g. those occurring in June and July) is similar to the impact of later events (not shown).

S.3 CMIP6 model selection

We consider pre-industrial control (piControl) and Shared Socioeconomic Pathway 585 (SSP585) (O'Neill et al., 2014) simulations (which include e.g. ozone hole recovery, Fig. 4 and Table S1). The models from the CMIP6 archive were chosen based on the availability of daily data for both piControl and SSP585 scenarios, and given the lack of stratospheric variability in low top models (Charlton-Perez et al., 2013), we require a well resolved stratosphere with at least 30 vertical levels and a model top at or above 1 hPa. For piControl we required at least 100 years of data for sufficient statistics. The five models that fulfil all these conditions are CESM2-WACCM, CanESM5, GFDL-CM4, INM-CM5-0, MIROC6, and the data used comprise a total of 3,341 years of piControl as well as 5x80 years of SSP585 (from 2021 to 2100). One ensemble member (r1i1p1f) for each model was considered.

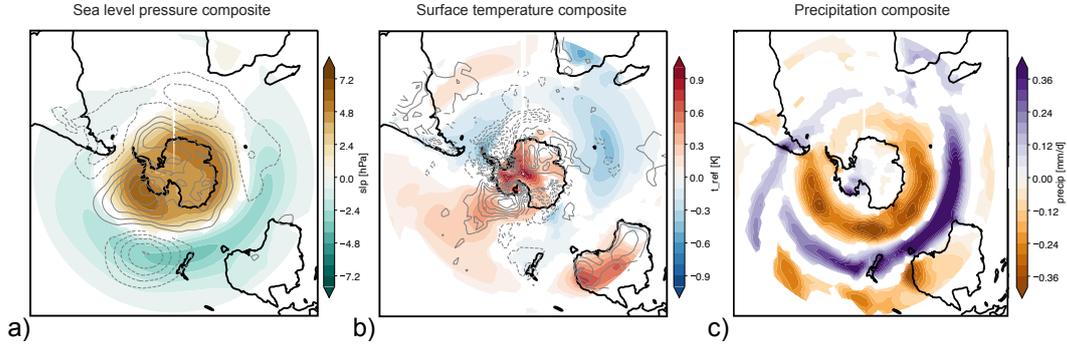


Figure S2. Composites surface anomalies averaged 0-60 days after the onset day for (a) surface pressure, (b) surface temperature and (c) precipitation for present-day CM2.1 SSW-weak events. The composites of the 2002 and 2019 events from ERA5 are added in gray contours for direct comparison except for precipitation which is too noisy in ERA5. Anomalies are relative to daily climatology, and only values which are statistically significant at the 5% level (t -test) are plotted.

Model	# years	# SSW-weak	mean return time [years]	# SSW-weak per 80 years	# SSW-weak 2021-2100
CESM2-WACCM	500	15	33.3	2.40	0
CanESM5	1000	6	166.7	0.48	1
GFDL-CM4	140	1	140.0	0.57	1
INM-CM5-0	1201	5	240.2	0.33	0
MIROC6	500	5	100.0	0.80	1

Table S1. Statistical information for CMIP6 data. All columns except the last refer to pi-Control simulations, whereas the last reports results from the SSP585 simulations. The second last column normalizes the number of SSW-weak events in piControl to a 80-year period for direct comparison to 2021-2100. All models except CESM2-WACCM strongly underestimate the number of SSWs, and no model produces more than one single event between 2021 and 2100.

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S.4 Uncertainty estimates

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For the two CM2.1 simulations, frequency uncertainties in Fig. 2 are computed by splitting the 9,900 years (after 90 years spinup) into 99 century-long non-overlapping segments, and computing the mean and standard deviation from this ensemble. For CMIP6 there are not enough events for similar statistical calculations, and the raw results are reported in Table S1.

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S.5 Return time

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If SSWs are random and independent, we should be able to model them as a Poisson process. For such a process, the return or waiting time can be computed using an exponential distribution with an expectation value equal to the mean occurrence frequency: PDF = $\lambda \exp(-\lambda x)$, where λ is the average frequency (e.g. 1/21.6 years for present-day SH SSW-weak events) and x is the waiting time in years. This is an approximation to a binomial distribution assuming large sample size and low probability. Since in our case we do not always have large sample size, we compute the return time using the binomial distribution. Then, the return time distribution is determined by the probability of zero events during a given time period ($k = 0$, $n =$ number of years, $p = 1/\text{mean return time to be fitted}$). This has the advantage of being able to compute the probabilities for an arbitrary number of events, while still being able to check the validity of randomness and independence. The cumulative distribution function of the exponential distribution is an approximation (again large sample size and low probability) for the survival function of a binomial distribution for zero events. Therefore, we use the latter to compute the probability of one or more events within a given time period, provided the events are independent and random.

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Explicitly, the return time PDF of a random and independent process follows a binomial distribution of zero events, as the return time corresponds to the probability of no event happening within a given time interval:

$$P(y) = \left[\sum_{k=0}^{k=N} \binom{y}{k} \left(\frac{1}{\tau}\right)^k \left(1 - \frac{1}{\tau}\right)^{y-k} \right]_{N=0} = \left(1 - \frac{1}{\tau}\right)^y, \quad (1)$$

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where y is the time interval in years, $N(=0)$ is the number of events, and τ is the mean time interval between two SSWs (e.g. 21.6 years for present-day SSW-weak events). This is what is shown as solid black line in the return time plots of Fig. 2.

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The probability of one or more events within a given time interval is simply $1 - P(y)$, which is shown as dashed black line in the return time plots. This so-called 'survival function' is used along with the cumulative probability function (as shown above but not setting $N = 0$) and the mass probability function (without the summation) to compute the various probabilities reported in the text and Table 1. For instance, the probability of at least one SSW-reversal in 80 years is $1 - (1 - 1/883)^{80} \approx 8.7\%$, and the probability of exactly two SSW-weak events in 41 years of present-day conditions is

$$\binom{41}{2} \left(\frac{1}{21.6}\right)^2 \left(1 - \frac{1}{21.6}\right)^{41-2} \approx 28\%, \quad (2)$$

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as reported in Table 1.