A new study of Lassa hemorrhagic fever model via Caputo-Fabrizio derivative

Abdullahi Yusuf [1,2], **Pushpendra Kumar** [3], **Tukur Abdulkadir Sulaiman** [1,2,⋆]

[1]Department of Computer Engineering, Biruni University Istanbul, Turkey; yusufabdullahi@fud.edu.ng

[2]Federal University Dutse, Science Faculty, Department of Mathematics, Jigawa 7156, Nigeria

[3]Department of Mathematics and Statistics, School of Basic and Applied Sciences, Central University of Punjab, Bathinda, Punjab-151001, India; kumarsaraswatpk@gmail.com

⋆ Correspondence: sulaiman.tukur@fud.edu.ng

Abstract

In the last three decades, we can see that an enthralling research topic which attracted the consideration of many researchers is mathematical modelling of biological systems. This paper is organised with the aim of getting some new simulations of lassa hemmorhagic fever; a deathly diseases in pregnant women via Caputo-Fabrizio fractional order derivative with the help of Euler method. Lassa hemorrhagic fever is biocidal and epidemical disease, whose outbreaks were first confirmed in African countries. As compare to the Ebola virus this virus kills pregnant women's more. On 8 January, Lassa virus was detached in Vero cell cultures from a blood sample, which was ejective to be 12 days after the invasion of the illness. In this manuscript, important lemma and theorems are considered to exhibit the existence and uniqueness analysis, stability of proposed fractional approximation method. Results are provided to confirm the effectiveness of used approximation method by graphical simulations for different values of β.

Keywords: Lassa hemorrhagic fever, epidemic mathematical model, Caputo-Fabrizio derivative, fractional Euler method.

AMS Subject Classification(2010): 37N25, 92D30.

1 Introduction

A well known infectious disease called Lassa hemorrhagic fever [4] [5] is classified under the family of arenaviridae virus. This disease was first recognised in the town of Lassa, in Borno State, Nigeria in 1969. A multimammate mouse (named Mastomys natalensis), an animal found in sub-Saharan Africa is the main host of this virus. Annually, this virus has affected 2-3 million people and spread out by the direct interplay with the infected persons blood, faeces or urine and by manifestation to sewage of animals via respiratory tracts. The uncover of infective material by dint of cracked skin or the lesser particles mixture in the air [6] is one of the main reason. It becomes more risky for those who are belongs to rural areas with tired abstersion conditions. It is recorded that 2 to 21 days is the incubation term of this fever. This lassa disease invasion with fever and weakness, successively reaching to headache, abdominal pain, muscle pain, sore throat, nausea, chest pain, vomiting, cough, and diarrhoea. In the observations of hospitalized patients of this disease it is seen that 20% of them goes die and in the duration of epidemic this percentage can reach to 50%. The death rate is near about 80% in the case of spreading in pregnant women which looks like more dangerous [2] [3]. Recently, most of the maternal deaths are cause of several infections, eclampsia, haemorrhage, virus inspection, unsafe abortion and others. Especially, in this list arenaviridae viruses has been an swelling virus which catch many pregnant women's into death. Lassa and Ebola hemorrhagic fevers are the two main viruses of Arenaviridae virus family. Reverse transcriptase polymerase chain reaction test, antigen detection tests, and virus isolation by cell culture observations can be used to diagnosed this virus infection. This lassa virus has a moderate treatment convenience and there is no licensed vaccine is avowed till now to control this infection [7]. For an initial stage detection only ribavirin is establish to be an dominant antiviral drug. 4 new confirmed cases were reported from two states of Nigeria in June 10- 16, 2019 with a death and then it is reached to a total of 2763 suspected cases from 1 January to 16 June, 2019 in 22 states of Nigeria. In these 2763 cases there were 591 confirmed positive cases and 132 confirmed deaths. In these confirmed cases the case fatality is 22.3% while the ratio of male-female is 1.2:1. It is noticed that the age group of 21- 40 years founded affected mainly. In 2019, there are 22 states in Nigeria have founded at least one confirmed case [8].

In epidemiology, so many non-linear models have been successfully studied with the help of fractional calculus with various approaches such as Riemann, Liouville, Caputo, Atangana- baleanu and Fabrizio etc. However, these fractional approaches have their own issues or limitations. In Caputo-Fabrizio, non-singular kernel and non-local property have been pointed out. The given model is studied through Caputo-Fabrizio fractional operator. A numerical method in the fractional derivative sense is also used to solve the system of equations. The important class of the new achievements gained within this manuscript is, only a simple recursive algebraic formula is needed to solve for given CF fractional operator. The stability and existence and uniqueness analysis of the proposed technique is described in this paper which make the suggested scheme adept and ingenious to use as compared to the other current methods. It may also be commented that the Lassa hemorrhagic fever study has been done in [2] using beta differential operator and then studied by Goyal in [9]. A study via q- homotopy analysis transform method has been also done in [15] and via Atangana-Baleanu fractional derivative in [19].
In the organization of this paper sect. 2 is for preliminary description of the Caputo-Fabrizio derivative and proposed numerical method. In sect. 3, fractional model description, in sect. 4, the proposed fractional approximation method with existence and uniqueness analysis, stability analysis will be shown. Results will be discussed in sect. 5.

2 Preliminaries

Definition 1. [20] For $T \in H^1(c,d)$ and $0 < \beta < 1$, the Caputo- Fabrizio (CF) fractional derivative(FD) of order β is defined by

$$ {}_{c}^{CF}D_t^{\beta}T(t) = \frac{1}{1-\beta}\int_c^t \frac{dT(\eta)}{d\eta} exp[-\alpha(t-\eta)]d\eta $$

where $\alpha = \frac{\beta}{1-\beta}$
The CF non- integer order integral is defined as

$$ {}_{c}^{CF}I_t^{\beta}T(t) = (1-\beta)T(t) + \beta\int_c^t T(\eta)d\eta. $$

Lemma 1. [10] If $0 < \beta < 1$ and m is an integer (nonnegative), then $\exists$ the $+ve$

constants $C_{\beta,1}$ and $C_{\beta,2}$ only dependent on β, s.t

$$(m+1)^{\beta}-m^{\beta}\leq C_{\beta,1}(m+1)^{\beta-1},$$

and

$$(m+2)^{\beta+1}-2(m+1)^{\beta+1}+m^{\beta+1}\leq C_{\beta,2}(m+1)^{\beta-1}.$$

Lemma 2. [10] Let us assume $d_{p,n}=(n-p)^{\beta-1}(p=1,2,...,n-1)$ & $d_{p,n}=0$ for $p\geq n,\beta,M,h,T>0,mh\leq T$ & m is a $+ve$ integer. Let $\sum_{p=m}^{p=n}d_{p,n}|e_p|=0$ for $k>n\geq 1$. If

$$|e_n|\leq Mh^{\beta}\sum_{p=1}^{n-1}d_{p,n}|e_p|+|\eta_0|,\quad n=1,2,...,m,$$

then

$$|e_m|\leq C|\eta_0|,\quad m=1,2,...$$

where C is a $+ve$ constant independent of m & h.

Lemma 3. [10] Let the fractional ordinary differential equation in the Caputo sense

$$\begin{cases}{}_0^CD_t^{\beta}\xi(t)=f(t,\xi(t)), & t\in(0,K),\ K>0,\\ \xi^{(k)}(0)=\xi_0^{(k)}, & k=0,1,...,r-1,\end{cases}\tag{1}$$

Where the Caputo derivative defined as

$${}_0^CD_t^{\beta}\xi(t)=\frac{1}{\Gamma(r-\beta)}\int_0^t(t-s)^{r-\beta-1}\xi^{(r)}(s)ds,\quad r-1<\xi<r\in\mathbb{Z}^+.$$

The corresponding Volterra integral equation can be written as

$$\xi(t)=\sum_{k=0}^{r-1}\frac{t^k}{k!}y_0^{(k)}+\frac{1}{\Gamma(\beta)}\int_0^t(t-s)^{\beta-1}f(s,\xi(s))ds.\tag{2}$$

Let $\xi(t)$ is considered as the solution to (2), $f(t,\xi)$ is bounded where h is sufficiently small. Then, the given inequality holds

$$|\xi(t+h)-\xi(t)|\leq C_1h^{\sigma(\beta)},\quad t\in[0,K-h],$$

where constant C_1 is not depend on h, and

$$\sigma(\beta)=\begin{cases}\beta, & 0<\beta<1;\\ 1, & \beta\geq 1.\end{cases}$$

Remark: We will use the given lemma for the system of equation in the Caputo-Fabrizio derivative sense.

Lemma 4. [10] Let when $\xi(t)$ is considered as the solution of (2), and $f(t,\xi)$ is continuous and satisfies the Lipschitz condition with respect to ξ and t with a Lipschitz constant L for sufficiently small h, then we have

$$|\int_0^{t_{m+1}}(t_{m+1}-t)^{\beta-1}f(t,\xi(t))dt-\frac{h^{\beta}}{\beta}\sum_{j=0}^{m}d_{j,m+1}f(t_j,\xi(t_j))|\leq C_2h^{\sigma(\beta)}.$$

Lemma 5. (Gronwall Inequality [21]) Let $c,d>0$, and $\{\gamma_i\}$ satisfy

$$|\gamma_n|\leq d+ch\sum_{i=0}^{r-1}|\gamma_i|,\quad r=k,k+1,...,rh\leq T,$$

then

$$|\gamma_r|\leq e^{aT}(d+ckhM_0),\quad r\geq k,\quad rh\leq T,$$

where $M_0=max(|\gamma_0|,|\gamma_1|,...,|\gamma_{k-1}|)$.

3 Model description

Here, we introduce a Lassa hemorrhagic fever time fractional model studied by Atangana et al. [2]. In [2], the author presented and derived the projected model in the sense of beta derivative. The formulation of the system for susceptible, infected and recovery class of lassa haemorrhagic fever in the sense of Caputo-Febrizio fractional derivative presented as:

$$\begin{cases} {}^{CF}_{0}D^{\beta}_{t}S(t) = -gS(t)I(t) + pN - uN + dR(t) + rS(t) - uS(t) \\ {}^{CF}_{0}D^{\beta}_{t}I(t) = gS(t)I(t) - (d+r)I(t) - rS(t) \\ {}^{CF}_{0}D^{\beta}_{t}R(t) = rI(t) - dR(t) \end{cases} \tag{3}$$

with initial conditions

$$S(0) = S_0, \quad I(0) = I_0, \quad and \quad R(0) = R_0. \tag{4}$$

Here, $N = S + I + R$ is the adult women population in a given country, S, I, R are the susceptible, infected and recovery population of pregnant women respectively. Let p is the pregnancy rate of women, s is the susceptible rate, infected at a rate g, infected women are dying at a rate d, women are recover at a rate r, u is the death rate cause of other disease or natural death.

To calculate the endemic equilibrium points, let us assume

$$\begin{cases} 0 = -g\bar{S}\bar{I} + pN - uN + d\bar{R} + r\bar{S} - u\bar{S} \\ 0 = g\bar{S}\bar{I} - (d+r)\bar{I} - r\bar{S} \\ 0 = r\bar{I} - d\bar{R} \end{cases} \tag{5}$$

put the values of $\bar{R}$ and $\bar{S}$ from last two equations into the first one of the system, we obtain

$$E\bar{I}^2 + F\bar{I} + G = 0, \quad E = -dg, \quad F = (r-u)d + gN(p-u) - ur, \quad G = -r(p-u)N,$$

Thus the given $\bar{I}$ is the solution of the above equation which is presented as

$$\bar{I}_{\mp} = \frac{-F \mp \sqrt{F^2 + 4Gdg}}{2E}$$

We obtain the reproductive number by considering the positive solution of the equation.

$$\bar{I} = \frac{\sqrt{F^2 + 4Cdg}}{2E} - \frac{F}{2E} = \frac{F}{2E}\left(\frac{\sqrt{F^2 + 4Gdg}}{F} - 1\right) = \frac{F}{2E}(R_0 - 1)$$

The equilibrium points for the disease-free system are $\left(\frac{p-u}{u}, 0, 0\right)$
where

$$R_0 = \frac{\sqrt{F^2 + 4Gdg}}{F}$$

when modelling an infectious disease it would be a most important concerns to find its efficiency to assail a population. R_0 (basic reproductive number) is a very familiar term in the study of disease spread in a target population and a precious idea in epidemic theory. The measurement of R_0 represents the secondary cases average born by an individual who is infected to the virus and moved into a susceptible population with no impregnability to the disease in the privation of interference to infection control.

In our case, if F is negative, then $R_0 < 1$ which means the stability of disease- free equilibrium and un stability of the endemic equilibrium points. If F is $+ve$ and Gdg is also $+ve$, then $R_0 > 1$ which means that the endemic equilibrium is stable and disease-free equilibrium is unstable.

4 The Euler fractional approximation method

Here, we are discussing an efficient numerical method for the proposed model (3) and (4) in the sense of CF fractional operator. In the way of simplify the relations, here we represent Eqs. (3) and (4) in a compact form

$$\begin{cases} {}_0^{CF}D_t^{\beta}\zeta(t) = g(\zeta(t)), & 0 < t < d < \infty, \\ \zeta(0) = \zeta_0. \end{cases} \tag{6}$$

where $\zeta = (S, I, R) \in \mathbb{R}_+^3$, g is a continuous real- valued vector function agree the Lipschitz condition

$$\| g(\zeta_1(t)) - g(\zeta_2(t)) \| \leq L \| \zeta_1(t) - \zeta_2(t) \|, L > 0, \tag{7}$$

and $\zeta_0 = (S_0, I_0, R_0)$ is the vector for initial conditions named as initial state vector. Now applying CF non- integer order integral operator to eq. (6), we derive

$$\zeta(t) = \zeta_0 +_0^{CF} I_t^{\beta} g(\zeta(t)), \ 0 < t < d < \infty. \tag{8}$$

where ${}_0^{CF}I_t^{\beta}$ represents the fractional integral operator respect to the CF fractional derivative. For proposed numerical method, we consider a interval length $[0, d]$ with time step size $h = \frac{d-0}{N}$, where $N \in \mathbb{N}$. Let ζ_k be the numerical approximation of $\zeta(t)$ at $t = t_k$, where $t_k = 0 + kh$ & $k = 0, 1, ..., N$. Describing Eq. (8) by applying the Euler method [10], we infer the following formula for the CF operator

$$\zeta_{k+1} = \zeta_0 + (1-\beta)g(\zeta_{k+1}) + \beta h \sum_{i=0}^{k} g(\zeta_i), \ k = 0, ..., N-1, \tag{9}$$

The stability analysis of the proposed numerical method is given by the following theorem.

Theorem 1. The numerical scheme (9) is conditionally stable.
Proof. Let us approximate ζ_0 and ζ_i $(i = 0, ..., k+1)$ by $\bar{\zeta_0}$ and $\bar{\zeta_i}$, respectively. From eq. (9), this method yields

$$\zeta_{k+1} + \bar{\zeta}_{k+1} = \zeta_0 + \bar{\zeta_0} + (1-\beta)g(\zeta_{k+1} + \bar{\zeta}_{k+1}) + \beta h \sum_{i=0}^{k} g(\zeta_i + \bar{\zeta_i}). \tag{10}$$

Using Eq. (9) in (10), we derive

$$|\bar{\zeta}_{k+1}| = |\bar{\zeta_0} + (1-\beta)[g(\zeta_{k+1} + \bar{\zeta}_{k+1}) - g(\zeta_{k+1})] + \beta h \sum_{i=0}^{k} [g(\zeta_i + \bar{\zeta_i}) - g(\zeta_i)]|. \tag{11}$$

Applying the Lipschitz condition and triangle inequality, we obtain

$$|\bar{\zeta}_{k+1}| \leq (1 + hL)|\bar{\zeta_0}| + (1-\beta)L|\bar{\zeta}_{k+1}| + \beta hL \sum_{i=1}^{k} |\bar{\zeta_i}|. \tag{12}$$

Eq.(12) can be simplified as

$$|\bar{\zeta}_{k+1}| \leq \frac{1 + hL}{(\beta-1)L + 1}|\bar{\zeta_0}| + \frac{\beta hL}{(\beta-1)L + 1} \sum_{i=1}^{k} |\bar{\zeta_i}|. \tag{13}$$

Finally, from Lemmas (1) and (2), we conclude $|\bar{\zeta}_{k+1}| \leq C\frac{1+hL}{(\beta-1)L+1}|\bar{\zeta}_0|$, where C is a $+ve$ constant not dependent on h and k.

Theorem 2. For the numerical method (9) we have

$$|\zeta(t_{k+1}) - \zeta_{k+1}| \leq Ch^{\beta}, \; k = 0, ..., N-1, \tag{14}$$

where C is a $+ve$ constant not depends on h and k.
Proof. For the projected numerical scheme (9), we express the following relation by assuming $\zeta(t_0) = \zeta_0$, using Eq. (8) in the CF sense and employing Eq. (9)

$$\zeta(t_{k+1}) - \zeta_{k+1} = (1-\beta)[g(\zeta(t_{k+1})) - g(\zeta_{k+1})] + \beta\Big[\int_0^{t_{k+1}} g(\zeta(\eta))d\eta - h\sum_{i=0}^{k} g(\zeta_i)\Big]$$

$$= (1-\beta)[g(\zeta(t_{k+1})) - g(\zeta_{k+1})] + \beta\Big[\int_0^{t_{k+1}} g(\zeta(\eta))d\eta - h\sum_{i=0}^{k} g(\zeta(t_i))\Big]$$

$$+\beta h\sum_{i=1}^{k}[g(\zeta(t_i)) - g(\zeta_i)].$$

Now using the Lipschitz condition (7) and lemma (3) and (4) in the above expression, it becomes

$$|\zeta(t_{k+1}) - \zeta_{k+1}| \leq L(1-\beta)|\zeta(t_{k+1}) - \zeta_{k+1}| + \beta C_2 h + \beta hL\sum_{i=1}^{k}|\zeta(t_i) - \zeta_i|. \tag{15}$$

where C_2 is a constant free from h. Also we can written the Eq. (15) as follows

$$|\zeta(t_{k+1}) - \zeta_{k+1}| \leq \frac{\beta C_2 h}{(\beta-1)L+1} + \frac{\beta hL}{(\beta-1)L+1}\sum_{i=1}^{k}|\zeta(t_i) - \zeta_i|. \tag{16}$$

Lastly, we derive Eq.(14) by employing the Gronwall's inequality from Lemma 5.

Theorem 3. (Existence and uniqueness) Let $g(\zeta(\xi))$ be a continuous real-valued function, defined under the domain H of a plane (ξ, ζ), and agree with the Lipschitz condition in H with respect to ζ, i.e.,

$$|g(\zeta_1(\xi)) - g(\zeta_2(\xi))| \leq L|\zeta_1(\xi) - \zeta_2(\xi)|.$$

Then, $\exists$ a unique solution $\zeta(\xi)$ of the fractional IVP (6), when the following condition is satisfied

$$(1-\beta)L + \beta L d^{\beta} < 1. \tag{17}$$

Proof: Let us take the volterra integral equation (8)

$$\zeta(\xi) = \zeta_0 + (1-\beta)g(\zeta(\xi)) + \beta\int_0^{\xi} g(\zeta(\eta))d\eta \tag{18}$$

If $\zeta(\xi)$ is a solution of (6), then it satisfies (18). Applying the CF FD operator on, we derive for $\zeta(\xi)$ the fractional differential equation (6). Let $V = (0, d)$ & $Z : C(V, \mathbb{R}) \to C(V, \mathbb{R})$ be the following operator

$$Z[\zeta(\xi)] = \zeta_0 + (1-\beta)g(\zeta(\xi)) + \beta\int_0^{\xi} g(\zeta(\eta))d\eta \tag{19}$$

Then, Eq. (18) can be written in the form

$$\zeta(\xi) = Z[\zeta(\xi)]. \tag{20}$$

Let $\| \,.\, \|_V$ is the defined supremum norm on V; such that

$$\| \zeta(\xi) \|_V = \sup |\zeta(\xi)|, \quad \zeta(\xi) \in C(V, \mathbb{R}). \tag{21}$$

Then, $C(V, \mathbb{R})$ with $\| \,.\, \|_V$ is a Banach space. Moreover, it is easily shown that

$$\| \int_0^{\xi} K(\xi,\eta)\zeta(\eta)d\eta \|_V \leq d\| K(\xi,\eta) \|_V \| \zeta(\xi) \|_V, \tag{22}$$

where $\zeta(\xi) \in C(V, \mathbb{R}), K(\xi,\eta) \in C(V^2, \mathbb{R})$ and

$$\| K(\xi,\eta) \|_V = \sup_{\xi,\eta \in J} |K(\xi,\eta)|, \ K(\xi,\eta) \in C(V^2, \mathbb{R}). \tag{23}$$

Now we enumerate the following

$$\| Z[\zeta_1(\xi)] - Z[\zeta_2(\xi)] \|_V = \sup_{\xi \in V} |Z[\zeta_1(\xi)] - Z[\zeta_2(\xi)]|. \tag{24}$$

By the defi- of operator Z in Eq. (19), and with the help of Eq. (22) and Lipschitz condition, we have

$$\begin{gathered} \| Z[\zeta_1(\xi)] - Z[\zeta_2(\xi)] \|_V \\ \leq (1-\beta)\| g(\zeta_1(\xi)) - g(\zeta_2(\xi)) \|_V + \beta d^{\beta} \| g(\zeta_1(\eta)) - g(\zeta_2(\eta)) \|_V \\ \leq \Big((1-\beta)L + \beta L d^{\beta} \Big) \| \zeta_1 - \zeta_2 \|_V. \end{gathered}$$

Therefore, we obtain

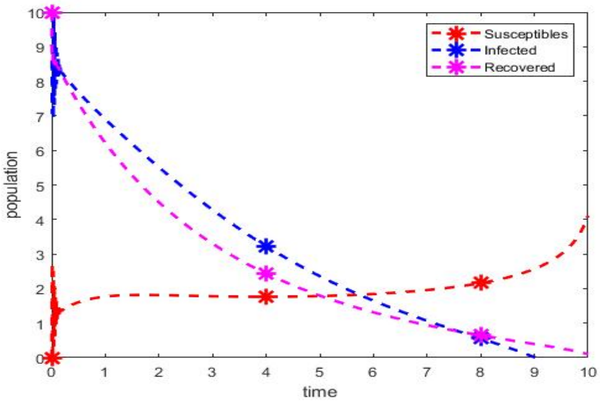
$$\| Z[\zeta_1(\xi)] - Z[\zeta_2(\xi)] \|_V \leq M \| \zeta_1(\xi) - \zeta_2(\xi) \|_V, \tag{25}$$

where $M = (1-\beta)L + \beta L d^{\beta}$. If the given condition of eq. (17) is satisfied, then the operator Z will be a contraction on $C(V, \mathbb{R})$. Thus, as a Banach fixed point theorem consequence, the proposed system (6) has a unique solution.
The inequality (17) also shows that this sufficient condition depends on three parameters including the Lipschitz constant L, terminal time d and fractional order β.

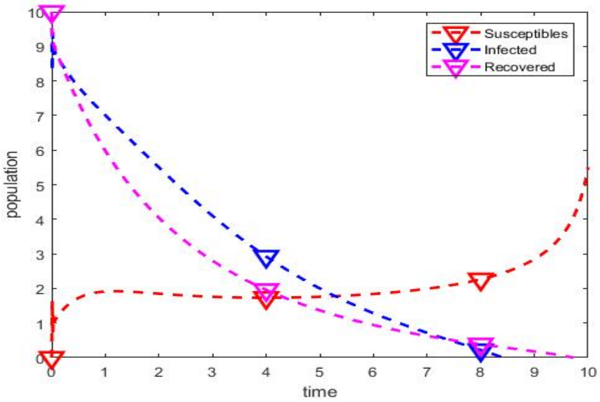
5 Simulation results

In this paper, all numerical results are done by the help MATLAB software for S, I and R at $\beta = 0.75, 0.85, 0.95$ and 1 respectively. The numerical solution of the proposed system is attained by using fractional Eulers method with parameters values mentioned in Table. In the group of Figure 1, fig. (i) shows the approximation nature of the no. of susceptible, infected and recovered patients (pregnant women) with time for $u = 0.2$ and $\beta = 0.75$. In the same way fig. (ii) is for $\beta = 0.85$, fig. (iii) is for $\beta = 0.95$, and fig. (iv) shows the all three classes for $\beta = 1$. Similarly in the class of Fig. 2, all separate figures are for particular values of β when the death rate by other diseases $u = 0.5$.

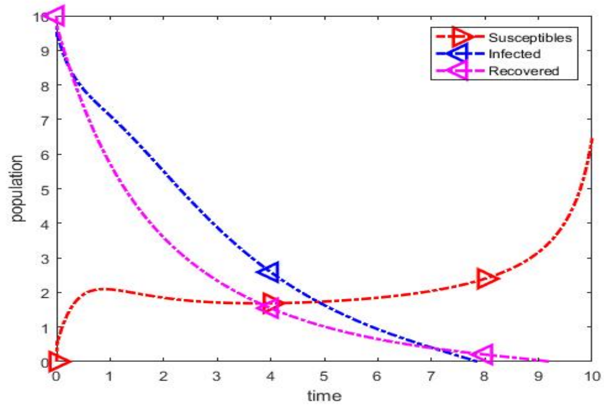
Parameter	**values**
N	S+I+R
u	0.2/ 0.5
p	0.3
d	0.8
r	0.2
g	0.4
$S(0)$	30
$I(0)$	10
$R(0)$	0
$D(0)$	0



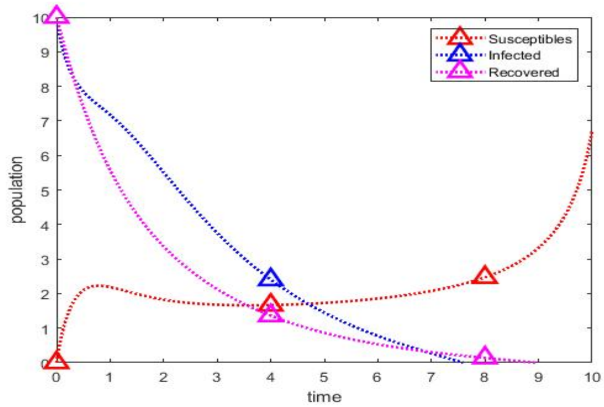
$(\boldsymbol{i})$



$(\boldsymbol{ii})$

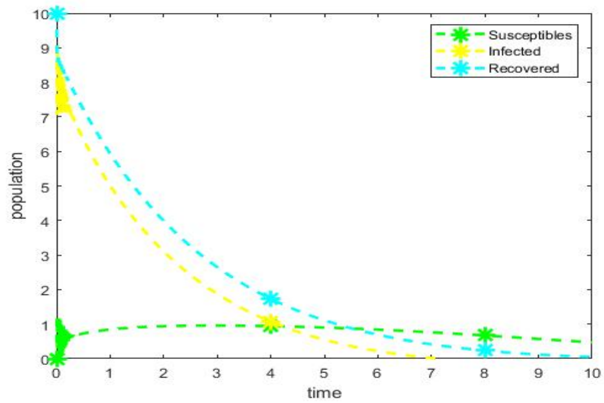


$(\boldsymbol{iii})$

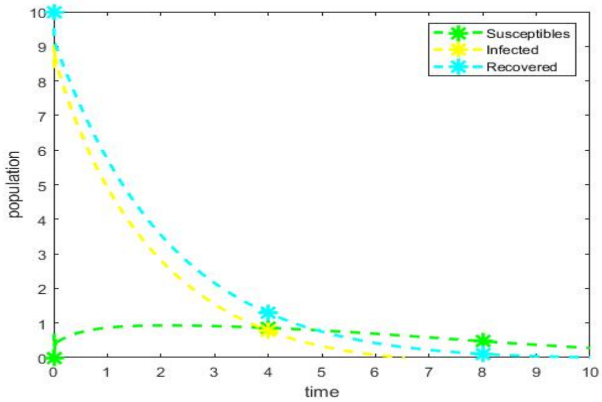


$(\boldsymbol{iv})$

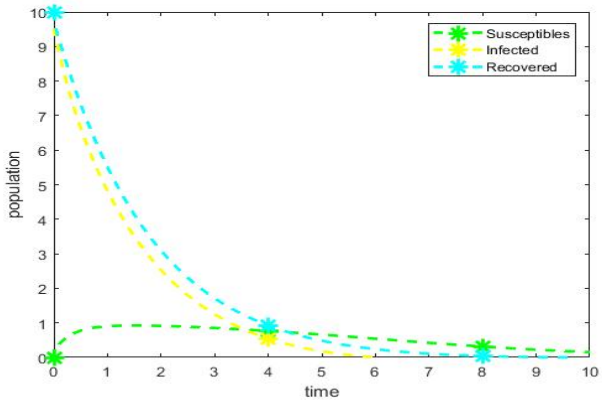
Figure 1 Nature of the achieved results at $u = 0.2$ for $(i)\, \beta = 0.75,\ (ii)\, \beta = 0.85,\ (iii)\, \beta = 0.95,\ (iv)\, \beta = 1.$



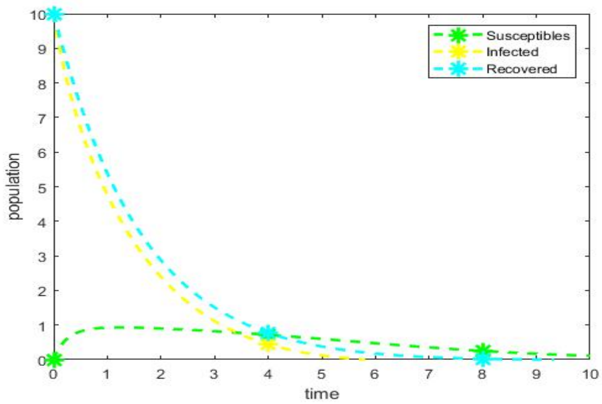
$(\boldsymbol{i})$



*(**ii**)*



*(**iii**)*



*(**iv**)*

Figure 2 Nature of the achieved results at $u = 0.5$ for
$(i)\,\beta = 0.75,\ (ii)\,\beta = 0.85,\ (iii)\,\beta = 0.95,\ (iv)\,\beta = 1.$

6 Conclusions

In the last three decades, so many fatal diseases have manifest their entity in different countries all over the world. In this way, lassa and ebola hemorrhagic fever in Africa are one of the fatal disease. Ebola virus first founded in 1975 in Zaire, and then in Sierra Leone, Guinea, Nigeria, and Liberia in 2014. Lassa was classified first time in town of Lassa in 1969, in Borno state Nigeria and the cases of this epidemic have been founded in Sierra Leone, Guinea, Liberia, Nigeria, and the Central Africa Republic. This particular disease is more dangerous for those women who are in third trimester of pregnancy in which the death rate of about 80%. In this paper we observed the results with the help of fractional Euler approximation method via Caputo-Fabrizio fractional derivative. In this work we give the prediction for susceptible, infected and recovered population of pregnant woman for different values of β. We presented the conditional stability of the proposed numerical scheme by the help of some important lemmas. Existence and uniqueness analysis also presented by the help of Lipschitz condition and Banach fixed point theorem. All figures are clearly described which give the idea of effectiveness of the proposed numerical scheme.

References

[1] Okuonghae, D., & Okuonghae, R. (2006). A mathematical model for Lassa fever. Journal of the Nigerian Association of Mathematical Physics, 10(1).

[2] Atangana, A. (2015). A novel model for the lassa hemorrhagic fever: deathly disease for pregnant women. Neural Computing and Applications, 26(8), 1895-1903.

[3] McCormick, J. B., Webb, P. A., Krebs, J. W., Johnson, K. M., & Smith, E. S. (1987). A prospective study of the epidemiology and ecology of Lassa fever. Journal of Infectious Diseases, 155(3), 437-444.

[4] Ogbu, O., Ajuluchukwu, E., & Uneke, C. J. (2007). Lassa fever in West African sub-region: an overview. Journal of vector borne diseases, 44(1), 1.

[5] Frame, J. D., Baldwin Jr, J. M., Gocke, D. J., & Troup, J. M. (1970). Lassa fever, a new virus disease of man from West Africa. The American journal of tropical medicine and hygiene, 19(4), 670-676.

[6] Emond, R. T., Bannister, B., Lloyd, G., Southee, T. J., & Bowen, E. T. (1982). A case of Lassa fever: clinical and virological findings. Br Med J (Clin Res Ed), 285(6347), 1001-1002.

[7] Warner, B. M., Safronetz, D., & Stein, D. R. (2018). Current research for a vaccine against Lassa hemorrhagic fever virus. Drug design, development and therapy, 12, 2519.

[8] Ekechi, H. U., Ibeneme, C., Ogunniyi, B., Awosanya, E., Gbadebo, B., Usman, A., & Ihekweazu, C. (2020). Factors associated with a confirmed Lassa fever outbreak in Eguare community of Esan West, Edo State, Nigeria: January-March, 2019. Journal of Interventional Epidemiology and Public Health, 3(1).

[9] Goyal, M., Baskonus, H. M., & Prakash, A. (2019). An efficient technique for a time fractional model of lassa hemorrhagic fever spreading in pregnant women. The European Physical Journal Plus, 134(10), 482.

[10] Li, C., & Zeng, F. (2013). The finite difference methods for fractional ordinary differential equations. Numerical Functional Analysis and Optimization, 34(2), 149-179.

[11] Jajarmi, A., & Baleanu, D. (2018). A new fractional analysis on the interaction of HIV with CD4+ T-cells. Chaos, Solitons & Fractals, 113, 221-229.

[12] Diethelm, K., Ford, N. J., Freed, A. D., & Luchko, Y. (2005). Algorithms for the fractional calculus: a selection of numerical methods. Computer methods in applied mechanics and engineering, 194(6-8), 743-773.

[13] Li, C., Chen, A., & Ye, J. (2011). Numerical approaches to fractional calculus and fractional ordinary differential equation. Journal of Computational Physics, 230(9), 3352-3368.

[14] Sheng, Q., & Tang, T. (1995). Optimal convergence of an Euler and finite difference method for nonlinear partial integro-differential equations. Mathematical and computer modelling, 21(10), 1-11.

[15] Gao, W., Veeresha, P., Prakasha, D. G., Baskonus, H. M., & Yel, G. (2020). New approach for the model describing the deathly disease in pregnant women using Mittag-Leffler function. Chaos, Solitons & Fractals, 134, 109696.

[16] Baleanu, D., Jajarmi, A., & Hajipour, M. (2018). On the nonlinear dynamical systems within the generalized fractional derivatives with MittagLeffler kernel. Nonlinear dynamics, 94(1), 397-414.

[17] Podlubny, I. (1999). Fractional differential equations, vol. 198 of Mathematics in Science and Engineering.

[18] Li, C., & Zeng, F. (2015). Numerical methods for fractional calculus (Vol. 24). CRC Press.

[19] Jain, S., & Atangana, A. (2018). Analysis of lassa hemorrhagic fever model with non-local and non-singular fractional derivatives. International Journal of Biomathematics, 11(08), 1850100.

[20] Caputo, M., & Fabrizio, M. (2015). A new definition of fractional derivative without singular kernel. Progr. Fract. Differ. Appl, 1(2), 1-13.

[21] Lin, R., & Liu, F. (2007). Fractional high order methods for the nonlinear fractional ordinary differential equation. Nonlinear Analysis: Theory, Methods & Applications, 66(4), 856-869.