Stability and Numerical solutions of Second Wave Mathematical Modeling on COVID-19 outbreak strategy in India pandemic: Analytical and Error analysis of Approximate series solutions by using HPM

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Abstract

This paper deals the mathematical modeling of second wave COVID19 pandemic in India, also we discussed such as uniformly bounded of the system, Equilibrium analysis and basic reproduction number R0. We calculated the analytic solutions by HPM (Homotopy Perturbation Method) and used Mathematica 12 software for numerical analysis up to 8th order approximation. It checked the error values of the approximation while the system has residual error, absolute error and h curve initial derivation of square error at up to 8th order approximation. The basic reproduction number ranges between 0.8454 and 2.0317 form numerical simulation, it helps to identify the whole system fluctuations. Finally, our proposed model validated from real life data for highly affected 5 states

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Abstract This paper deals the mathematical modeling of second wave COVID-19 pandemic in India, also we discussed such as uniformly bounded of the system, Equilibrium analysis and basic reproduction number R_0. We calculated the analytic solutions by HPM (Homotopy Perturbation Method) and used Mathematica 12 software for numerical analysis up to 8th order approximation. It checked the error values of the approximation while the system has residual error, absolute error and h curve initial derivation of square error at up to 8th order approximation. The basic reproduction number ranges between 0.8454 and 2.0317 form numerical simulation, it helps to identify the whole system fluctuations. Finally, our proposed model validated from real life data for highly affected 5 states.

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1 Introduction

Now COVID-19 active cases were decreased in India and it is very soon end for this pandemic. Covid-19 India cases are as on 29 October 2021, we have collected data from WHO (World Health Organization) such live status of passenger screened on Airport (1524266), Active Cases (161334), Cured or Discharged (33627632), Deaths (457191), Total Active Cases (160989), Last Total Cured (33614434),Last Total death (456386) and total Samples Tested (605885769). The total vaccination doses as on current dated is 1061440335. Even though there are some vaccinations and medicines for treatment, the recent pandemic caused by COVID-19 gives a big challenge to the people. Particularly in India, Covid-19 has drawn attention to the strategies of quarantine and there are so many governmental measures, like lockdown, social isolation, speed up of treatment and improvement of public hygiene, etc to control the disease. The present paper describes a mathematical modeling and dynamics of a COVID-19 in India. We have presented a thorough dynamical behavior of the model in terms of the basic reproduction number. Moreover, we perform the equilibrium analysis of COVID-19 to lessen the infected individuals in India. We have given the convergent, comparable and most appropriate solution of each and every compartment involved in the model by using the most powerful and elegant method viz Homotopy Perturbaton Method.

Finally, we concluded that the five highly affected states Maharashtra, kerala, Karnataka, Tamilnadu and Andra Pradesh need more attention to reduce the contact of susceptible humans. The number of people infected with the corona virus was still high in many areas, and transmission of the virus was easily regenerated once people increased their activities and contact with each other. The current pandemic situation is to reduce the infection of COVID-19 cases in India. Scientists are currently working to find opt vaccine for corona virus disease from various countries. In this regard, we calculated the active cases from the mathematical modeling and then create a new model in second wave. We consider the available infection cases for the period August 2021 to October 2021 and parameter estimation of the model. We compute the basic reproduction number for the model. The described model is then solved numerically as well as approximated analytically using Homotopy perturbation method by presenting many graphical results, which can be very helpful for the infection controlling.

The same strength continues now itself. The equilibrium state is endemic. Because it is the end of second wave of Indian epidemic spread. Endemic means weekly reports in steady state with minor fluctuations. Last seven weeks of this same cases uniform. The four states (Kerala, Sikkim, Mizoram and Meghalaya) are exception to the endemic state (they are not yet to endemic). It will soon change and become endemic. Almost the majority of population is infected state. The affected population had 68 percentage (nearly 1000 million) antibodies from 4th ICMR survey by end of July 2021. The cumulative COVID-19 cases had 30,410,577 (3.2 percentage out of affected population).

Chakraborty T, Ghosh I.[1], it discusses the real time forecasts and risk assessment of novel coronavirus by using data-driven analysis. The analysis and forecast of COVID-19 spreads are in China, Italy and France in [2]. The isolation of cases and contacts are to control COVID-19 outbreaks [3]. We collected the Indian data separately in Indian council of medical research (ICMR) [4]. Stability and bifurcation analysis are of an epidemic model in [5]. Kucharski AJ et al. [6], it gives a mathematical modelling study on transmission and control of COVID-19. The basic reproductive number of COVID-19 is calculated and higher compared to other coronavirus [7]-[8]. Ndariou F, et.al [9], the Mathematical modeling of COVID-19 transmission with study of Wuhan is considered and control strategies in [10] with similar to Brazil [11]. The Mathematical modelling of improved SIR model with real life government control strategies [12] with SARSCoV-2 in India [13]. We collected the tracker data from crowd sourced India [14]. The good model is Modified SEIR model to predict COVID-19 outbreak in all countries with control scenarios and multi scale epidemics for source data [15]-[18]. M. A. Khan, A. Atangana [19], A Mathematical Modeling of the dynamics of novel Corona-virus (2019-nCov) is studied with numerical simulation and asymptomatic carrier transmission [20]. Its applications to compartmental models in [21] with phase based [22]. The Estimating the reproductive number and the outbreak size of COVID-19 are in all countries, we used this procedure the calculations [23]-[25]. It helps to all the analysis such as outbreak in Wuhan, China with individual reaction and governmental action [26]-[27]. The Indian dynamics are of transmission and control strategy derived from the mathematical modeling [28] with New dynamical behavior in [29]. In generally we collected all data's from WHO [30] with Optimal control theories [31]-[33]. The supporting data collected from other government recongnised websites [34]-[37].

This paper organized as below: In section 2, we have given the detailed mathematical modeling of second wave Indian COVID-19 pandemic. In section 3, Stability analysis of the model like uniformly bounded of the system, equilibrium analysis such as disease free equilibrium and endemic free equilibrium and basic reproduction number is studied. In Section 4 and 5, the approximate analytical expressions of each and every compartments appeared in the given model are derived using HPM. Also we briefly discuss the numerical analysis and error analysis for the data versus model fitting for the given period will be shown in Section 6 and 7. The concluding remarks provided in Section 7.

2 Mathematical modelling of second wave COVID-19

In Indian perspective, the analysis of different strategies on COVID-19 transmission dynamics in the presence of different intervention schemes becomes significant. Considering the significant role of intervention strategies, there are many researchers have obtained a new epidemic model with different intervention strategies of COVID-19 in a homogeneous host population to control the spread of COVID-19. The appearance and recurrence of coronavirus epidemics sparked renewed concerns from global epidemiology researchers, public health administrators and Mathematical Modeling researchers to model this. In the present investigation, we consider the compartmental mathematical model (epidemic model) has been developed by Kham and Atangana [19] for understanding the transmission of virus and presented and derived some interesting results for the projected model by comparison with some practical values (see also [9,20,25,30]). In this epidemic model a total number of populations N at a time t, is divided into the following six compartments: $S(t)$ the susceptible people; $E(t)$ the exposed people; $I(t)$ the infected strength; $I_a(t)$ the asymptotically infected people; $R(t)$ the recovered people; $M(t)$ the reservoir. The system of nonlinear ordinary differential equations representing this epidemic model as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= \alpha_0 - \alpha_1 S - \frac{\alpha_2 S(I + \alpha_3 I_a)}{N} - \alpha_4 SM \\
\frac{dE}{dt} &= \frac{\alpha_2 S(I + \alpha_3 I_a)}{N} + \alpha_4 SM - (1 - \alpha_5)\alpha_6 E - \alpha_5 \alpha_7 E - \alpha_1 E \\
\frac{dI}{dt} &= (1 - \alpha_5)\alpha_6 E - (\alpha_8 + \alpha_1) I \\
\frac{dI_a}{dt} &= \alpha_5 \alpha_7 E - (\alpha_9 + \alpha_1) I_a \\
\frac{dR}{dt} &= \alpha_8 I + \alpha_9 I_a - \alpha_1 R \\
\frac{dM}{dt} &= \alpha_{10} I + \alpha_{11} I_a - \alpha_1 M
\end{aligned}
\tag{1}
$$

To understand the above system of eqn. (1) more clearly, we rewrite the same system in the following way by substituting some more constants as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \alpha_0 - \alpha_1 S - \alpha_{13} S(I + \alpha_3 I_a) - \alpha_4 SM \\
\frac{dE}{dt} &= \alpha_{13} S(I + \alpha_3 I_a) + \alpha_4 SM - \alpha_{14} E - \alpha_{15} E - \alpha_1 E \\
\frac{dI}{dt} &= \alpha_{14} E - \alpha_{16} I \\
\frac{dI_a}{dt} &= \alpha_{15} E - \alpha_{17} I_a \\
\frac{dR}{dt} &= \alpha_8 I + \alpha_9 I_a - \alpha_1 R \\
\frac{dM}{dt} &= \alpha_{10} I + \alpha_{11} I_a - \alpha_1 M
\end{aligned} \tag{2}$$

where

$$\alpha_{13} = \frac{\alpha_2}{N}, \alpha_{14} = (1-\alpha_5)\alpha_6, \alpha_{15} = \alpha_5\alpha_7, \alpha_{16} = \alpha_8+\alpha_1 \text{ and } \alpha_{17} = \alpha_9+\alpha_1 \tag{3}$$

with the initial conditions for finding the solution of eqn.(2) are

$$S(0) = S_0, E(0) = E_0, I(0) = I_0, I_a(0) = I_{a_0}, R(0) = R_0 \text{ and } M(0) = M_0 \tag{4}$$

3 Stability analysis of second wave COVID-19

Uniformly bounded of the system In this section, we produced uniformly boundedness of the system. Let

$$\begin{aligned}
&X = S + E + I + I_a + R + M \\
&\frac{dX}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dI_a}{dt} + \frac{dR}{dt} + \frac{dM}{dt} \\
&\frac{dX}{dt} = \alpha_0 - \alpha_1 X + \alpha_{10} I + \alpha_{11} I_a \\
&\frac{dX}{dt} + \alpha_1 X \leq \alpha_0,\ t \to \infty. \\
&\text{Region} = \left\{ X \in \mathbb{R}_+^6 : 0 \leq X(S, E, I, I_a, R, M) < \frac{\alpha_0}{\alpha_1} + \varepsilon \right\}, \varepsilon > 0.
\end{aligned}$$

Equilibrium analysis of COVID-19
The study on equilibrium of COVID-19 deals with several things such as the equilibrium on the regional economies of a country, the equilibrium at the

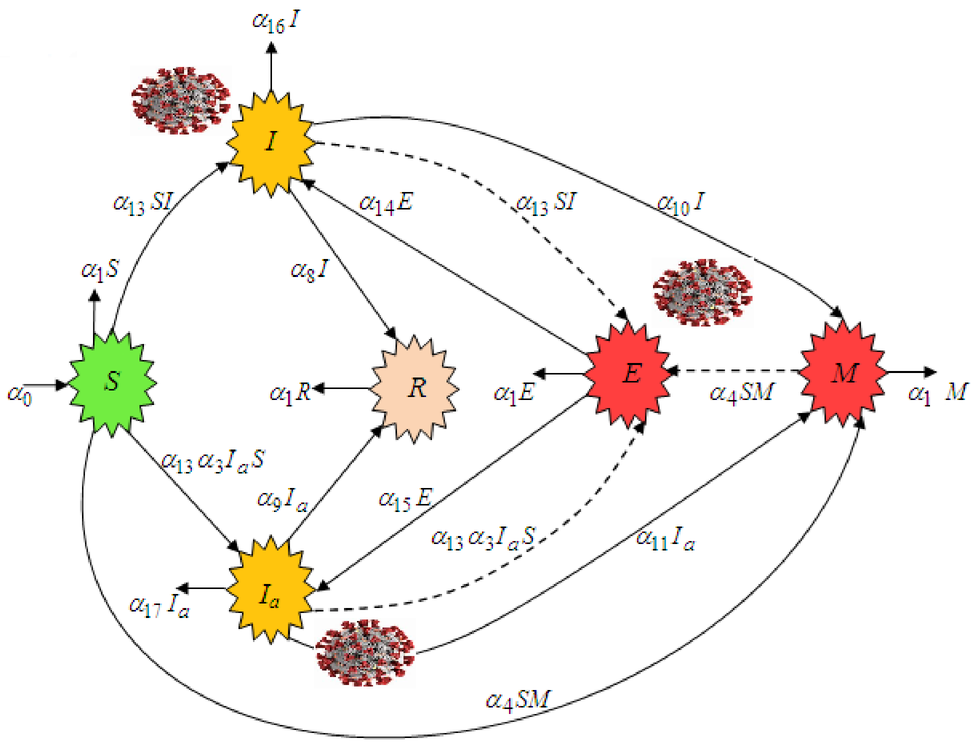

Fig. 1 The compartmental diagram for COVID-19 epidemic model

population level. There are basically two types of equilibrium in disease epidemics, one is disease free equilibrium and the other is endemic equilibrium. The disease free equilibrium is the point at which no disease is present in the population. Here the same is considered for COVID-19. The disease free equilibrium point results to be locally asymptotically stable if the reproduction number is less than unity while the endemic equilibrium point is locally asymptotically stable if such a number exceeds unity. The present section explore the stability for the model ( ) by considering first the disease free equilibrium and the endemic equilibrium second the basic reproduction number denoted by R_0 . It is calculated to derive the disease free and endemic equilibrium points. These two cases the derivative is equal to zero.

$$\begin{aligned}
&\alpha_0 - \alpha_1 S - \alpha_{13} S(I + \alpha_3 I_a) - \alpha_4 SM = 0\\
&\alpha_{13} S(I + \alpha_3 I_a) + \alpha_4 SM - \alpha_{14} E - \alpha_{15} E - \alpha_1 E = 0\\
&\alpha_{14} E - \alpha_{16} I = 0\\
&\alpha_{15} E - \alpha_{17} I_a = 0\\
&\alpha_8 I + \alpha_9 I_a - \alpha_1 R = 0\\
&\alpha_{10} I + \alpha_{11} I_a - \alpha_1 M = 0
\end{aligned}$$

Then we solved the eqauilibrium points of S, E, I, I_a, R and M.

Disease free equilibrium for COVID-19
This case there is no infection of COVID-19. We put E = I = I_a = 0.

The disease free equilibrium points are:

$$S = \frac{\alpha_0}{\alpha_1}, E = 0, I = 0, I_a = 0, R = 0, M = 0$$

Endemic free equilibrium for COVID-19
It is used to find the spread of COVID-19 infection. This case all compartments are not equal to zero. The endemic equilibrium points are:

$$S = \frac{\alpha_1\alpha_{16}\alpha_{17}\left(\alpha_{14}+\alpha_{15}+\alpha_1\right)}{\alpha_{14}\alpha_{17}(\alpha_4\alpha_{10}+\alpha_{13}\alpha_1)+\alpha_{15}\alpha_{16}(\alpha_4\alpha_{11}+\alpha_{13}\alpha_3\alpha_1)}$$

$$E = \frac{\alpha_0(\alpha_4\alpha_{10}+\alpha_{13}\alpha_1)\alpha_{14}\alpha_{17}+\alpha_{15}\alpha_{16}(\alpha_4\alpha_{11}+\alpha_{13}\alpha_1\alpha_3)-\alpha_1\alpha_{16}\alpha_{17}\alpha_1(\alpha_{14}+\alpha_{15}+\alpha_1)}{(\alpha_{14}+\alpha_{15}+\alpha_1)[\alpha_4(\alpha_{10}\alpha_{14}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})+\alpha_{13}(\alpha_{14}\alpha_{17}\alpha_1+\alpha_{15}\alpha_3\alpha_{16}\alpha_1)]}$$

$$I = \frac{-\alpha_{14}\left(\alpha_1\alpha_{16}\alpha_{17}\alpha_1(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_0\alpha_4(\alpha_{14}\alpha_{10}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})+\alpha_0\alpha_{13}(\alpha_{14}\alpha_{17}\alpha_1-\alpha_{15}\alpha_3\alpha_{16}\alpha_1\right)}{\alpha_{16}[(\alpha_{14}+\alpha_{15}+\alpha_1)(\alpha_4(\alpha_{10}\alpha_{14}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})+\alpha_{13}(\alpha_{14}\alpha_{17}\alpha_1+\alpha_{15}\alpha_3\alpha_{16}\alpha_1)]}$$

$$I_a = \frac{-\alpha_{15}[\alpha_1\alpha_{16}\alpha_{17}\alpha_1(\alpha_{14}+\alpha_{15}+\alpha_1)-\alpha_0\alpha_4(\alpha_{10}\alpha_{14}\alpha_{17}-\alpha_{11}\alpha_{15}\alpha_{16})-\alpha_0\alpha_{13}\alpha_1(\alpha_{14}\alpha_{17}-\alpha_{15}\alpha_3\alpha_{16})]}{\alpha_{17}(\alpha_{14}+\alpha_{15}+\alpha_1)\left((\alpha_4\alpha_{14}(\alpha_{10}\alpha_{17}+1)+\alpha_1\alpha_{13}(\alpha_{14}\alpha_{17}+\alpha_{15}\alpha_3\alpha_{16})\right)}$$

$$R = \frac{\begin{array}{c}-((\alpha_1\alpha_{16}\alpha_{17}\alpha_1(\alpha_{14}\alpha_{15}+\alpha_1)-\alpha_0\alpha_4(\alpha_{10}\alpha_{14}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})\\ -\alpha_0\alpha_1\alpha_{13}(\alpha_{14}\alpha_{17}-\alpha_{15}\alpha_3))(\alpha_8\alpha_{14}\alpha_{17}+\alpha_9\alpha_{15}\alpha_{16}))\end{array}}{(\alpha_1\alpha_{16}\alpha_{17})\left(\alpha_{14}+\alpha_{15}+\alpha_1\right)[\alpha_4(\alpha_{10}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})+\alpha_1\alpha_{13}(\alpha_{14}\alpha_{17}+\alpha_{15}\alpha_3\alpha_{16})]}$$

$$M = \frac{\begin{array}{c}-((\alpha_1\alpha_{16}\alpha_{17}\alpha_1(\alpha_{14}+\alpha_{15}+\alpha_1)-\alpha_0\alpha_4(\alpha_{10}\alpha_{14}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16}\\ -\alpha_0\alpha_1\alpha_{13}(\alpha_{14}\alpha_{17}+\alpha_{15}\alpha_3\alpha_{16}))(\alpha_{10}\alpha_{14}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})))\end{array}}{((\alpha_{14}+\alpha_{15}+\alpha_1)(\alpha_4(\alpha_{10}\alpha_{14}\alpha_{17}+\alpha_{11}\alpha_{15}\alpha_{16})+\alpha_1\alpha_{13}(\alpha_{14}\alpha_{17}\alpha_1+\alpha_{15}\alpha_3\alpha_{16}\alpha_1)(\alpha_1\alpha_{16}\alpha_{17})))}$$

we calculate the Jacobian matix

$$J = \begin{vmatrix} -\alpha_1 & 0 & -\alpha_{13}s & \alpha_3 s & 0 & -\alpha_4 s \\ 0 & -(\alpha_{14}+\alpha_{15}+\alpha_1) & \alpha_{13}s & \alpha_3 s & 0 & \alpha_4 s \\ 0 & \alpha_{14} & -\alpha_{16} & 0 & 0 & 0 \\ 0 & \alpha_{15} & 0 & -\alpha_{17} & 0 & 0 \\ 0 & 0 & \alpha_8 & \alpha_9 & -\alpha_1 & 0 \\ 0 & 0 & \alpha_{10} & \alpha_{11} & 0 & -\alpha_1 \end{vmatrix}$$

Then to find the eigen values of the above matrix

$$|\lambda I - J| = \begin{vmatrix} \lambda+\alpha_1 & 0 & -\alpha_{13}s & \alpha_3 s & 0 & -\alpha_4 s \\ 0 & \lambda+\alpha_{14}+\alpha_{15}+\alpha_1 & \alpha_{13}s & \alpha_3 s & 0 & \alpha_4 s \\ 0 & \alpha_{14} & \lambda+\alpha_{16} & 0 & 0 & 0 \\ 0 & \alpha_{15} & 0 & \lambda+\alpha_{17} & 0 & 0 \\ 0 & 0 & \alpha_8 & \alpha_9 & \lambda+\alpha_1 & 0 \\ 0 & 0 & \alpha_{10} & \alpha_{11} & 0 & \lambda+\alpha_1 \end{vmatrix} = 0$$

$$a_0\lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6 = 0$$

$$
\begin{aligned}
&a_0 = 1,\\
&a_1 = (4\alpha_1 + \alpha_{14} + \alpha_{15} + \alpha_{17} + \alpha_{16}),\\
&a_2 = (\alpha_{14}\alpha_{17} + 2\alpha_{16}\alpha_1 + \alpha_{16}(\alpha_{15} + \alpha_{14} + \alpha_{17}) + \alpha_{14}\alpha_1 + \alpha_{17}\alpha_1\\
&+\alpha_{16}\alpha_1 + 2\alpha_{17}\alpha_1 + \alpha_1^2 + 2\alpha_1\alpha_1 - \alpha_{14}\alpha_{13}s + \alpha_{15}\alpha_1 + \alpha_{15}\alpha_1 + \alpha_{15}\alpha_{17} + \alpha_{14}\alpha_1 - \alpha_{15}\alpha_{13}\alpha_3 s - \alpha_1(-\alpha_{14}\\
&-2\alpha_1 - \alpha_{15} - \alpha_1 - \alpha_{17} - \alpha_{16})),\\
&a_3 = (\alpha_1^2(\alpha_{16} + \alpha_{17} + \alpha_1) - \alpha_{15}\alpha_{13}\alpha_3 s(\alpha_1 - \alpha_1) + \alpha_{15}\alpha_{16}\alpha_1 - \alpha_{14}\alpha_{13}s\alpha_1\\
&+\alpha_{15}\alpha_{11}\alpha_4 s - \alpha_{14}\alpha_{13}s\alpha_1 + \alpha_{14}\alpha_{10}\alpha_4 s - \alpha_{14}\alpha_{13}s\alpha_{17} - \alpha_1(-\alpha_{14}\alpha_{17} - \alpha_{16}(2\alpha_1 + \alpha_{15} + \alpha_{14} + \alpha_{17} + \alpha_1)\\
&-\alpha_1(\alpha_{14} + 2\alpha_{17} + \alpha_1 + 2\alpha_1) + \alpha_{14}\alpha_{13}s - \alpha_{15}(\alpha_1 + \alpha_1 + \alpha_{17} - \alpha_{13}\alpha_3 s) - \alpha_1(\alpha_{14} + \alpha_{17})) + \alpha_{14}\alpha_1\alpha_1\\
&+\alpha_{15}\alpha_{17}\alpha_1 + \alpha_{15}\alpha_{17}\alpha_1 + \alpha_{14}\alpha_{17}\alpha_1 + 2\alpha_{17}\alpha_1^2 + \alpha_{15}\alpha_{16}\alpha_1 + \alpha_{14}\alpha_{16}\alpha_{17} + \alpha_{14}\alpha_{17}\alpha_1 + \alpha_{16}\alpha_{17}\alpha_1\\
&+2\alpha_{16}\alpha_1^2 + \alpha_{15}\alpha_1^2 + \alpha_{14}\alpha_{16}\alpha_1 + \alpha_{14}\alpha_{16}\alpha_1 - \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s + \alpha_{15}\alpha_{16}\alpha_{17} + 2\alpha_{16}\alpha_{17}\alpha_1),\\
&a_4 = (\alpha_{14}\alpha_{16}\alpha_1^2 + \alpha_{14}\alpha_{16}\alpha_{17}\alpha_1 + \alpha_{15}\alpha_{16}\alpha_{17}\alpha_1 + \alpha_{14}\alpha_{10}\alpha_4 s\alpha_1 - \alpha_{14}\alpha_{13}s\alpha_1^2 + \alpha_{14}\alpha_{10}\alpha_4 s\alpha_{17}\\
&-\alpha_{14}\alpha_{13}s\alpha_{17}\alpha_1 + \alpha_{17}\alpha_1^2\alpha_{12} + \alpha_{17}\alpha_{15}\alpha_1^2 + \alpha_{15}\alpha_{16}\alpha_1^2 + \alpha_{15}\alpha_{16}\alpha_{11}\alpha_4 s + \alpha_{16}\alpha_1^2\alpha_{17} + \alpha_{14}\alpha_{17}\alpha_1^2\\
&+\alpha_{15}\alpha_{16}\alpha_{17}\alpha_1 + \alpha_{16}\alpha_1^3 + \alpha_{15}\alpha_{11}\alpha_4 s\alpha_1 - \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s\alpha_1 - \alpha_{14}\alpha_{13}\alpha_{17}s\alpha_1 + \alpha_{14}\alpha_{16}\alpha_{17}\alpha_1\\
&-\alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s\alpha_1 + 2\alpha_{16}\alpha_{17}\alpha_1^2 - \alpha_{15}\alpha_{13}\alpha_3 s\alpha_1^2 - \alpha_1(-\alpha_1^2\alpha_{16} - \alpha_{17}\alpha_1^2 - \alpha_1^3 + \alpha_{15}\alpha_{13}\alpha_3 s\alpha_1\\
&+\alpha_{15}\alpha_{13}\alpha_3 s\alpha_1 - \alpha_{15}\alpha_{16}\alpha_1 + \alpha_{14}\alpha_{13}s\alpha_1 - \alpha_{15}\alpha_{11}\alpha_4 s + \alpha_{14}\alpha_{13}s\alpha_1 - \alpha_{14}\alpha_{10}\alpha_4 s + \alpha_{14}\alpha_{13}s\alpha_{17} - \alpha_{14}\alpha_1^2\\
&-\alpha_{15}\alpha_{17}\alpha_1 - \alpha_{15}\alpha_{17}\alpha_1 - \alpha_{14}\alpha_{12}\alpha_{17} - 2\alpha_{17}\alpha_1^2 - \alpha_{15}\alpha_{16}\alpha_1 - \alpha_{14}\alpha_{16}\alpha_{17} - \alpha_{14}\alpha_{17}\alpha_1 - -\alpha_{17}\alpha_{16}\alpha_1\\
&-2\alpha_{16}\alpha_1^2 - \alpha_{15}\alpha_1 - \alpha_{14}\alpha_{16}\alpha_1 - \alpha_{14}\alpha_{16}\alpha_1 + \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s - \alpha_{15}\alpha_{16}\alpha_{17} - 2\alpha_{16}\alpha_{17}\alpha_1)),\\
&a_5 = (-\alpha_1(-\alpha_{14}\alpha_{16}\alpha_1^2 - \alpha_{14}\alpha_{16}\alpha_{17}\alpha_1 - \alpha_{15}\alpha_{16}\alpha_{17}\alpha_1 - \alpha_{14}\alpha_{10}\alpha_4 s\alpha_1 + \alpha_{14}\alpha_{13}s\alpha_1^2 - \alpha_{14}\alpha_{10}\alpha_4 s\alpha_{17}\\
&+\alpha_{14}\alpha_{13}s\alpha_{17}\alpha_1 - \alpha_{17}\alpha_1^3 - \alpha_{15}\alpha_{17}\alpha_1^2 - \alpha_{15}\alpha_{16}\alpha_1^2 - \alpha_{15}\alpha_{16}\alpha_{11}\alpha_4 s - \alpha_{17}\alpha_1^2\alpha_{16} - \alpha_{14}\alpha_{17}\alpha_1^2\\
&-\alpha_{15}\alpha_{16}\alpha_{17}\alpha_1 - \alpha_{16}\alpha_1^3 - \alpha_{15}\alpha_{11}\alpha_4 s\alpha_1 + \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s\alpha_1 + \alpha_{14}\alpha_{13}s\alpha_{17}\alpha_1 - \alpha_{14}\alpha_{16}\alpha_{17}\alpha_1 + \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s\alpha_1\\
&-2\alpha_{16}\alpha_{17}\alpha_1^2 + \alpha_{15}\alpha_{13}\alpha_3 s\alpha_1^2) + \alpha_{15}\alpha_{16}\alpha_{11}\alpha_4 s\alpha_1 + \alpha_{14}\alpha_{10}\alpha_4 s\alpha_{17}\alpha_1 + \alpha_{15}\alpha_{16}\alpha_{17}\alpha_1^2 + \alpha_{14}\alpha_{16}\alpha_{17}\alpha_1^2\\
&-\alpha_{14}\alpha_{13}s\alpha_{17}\alpha_1^2 + \alpha_1^2\alpha_{16}\alpha_{17}\alpha_1 - \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s\alpha_1^2),\\
&a_6 = -\alpha_1(-\alpha_{15}\alpha_{16}\alpha_{11}\alpha_4 s\alpha_1 - \alpha_{14}\alpha_{10}\alpha_4 s\alpha_{17}\alpha_1 - \alpha_{15}\alpha_{16}\alpha_{17}\alpha_1^2 - \alpha_{14}\alpha_{16}\alpha_{17}\alpha_1^2\\
&+\alpha_{14}\alpha_{13}s\alpha_{17}\alpha_1^2 - \alpha_1^3\alpha_{16}\alpha_{17} + \alpha_{15}\alpha_{16}\alpha_{13}\alpha_3 s\alpha_1^2).
\end{aligned}
$$

We changed the above characteristic equation by using Descartes' rule of sign as follows:

$$a_0\lambda^6 - a_1\lambda^5 + a_2\lambda^4 - a_3\lambda^3 + a_4\lambda^2 - a_5\lambda + a_6 = 0,$$

with conditions $a_0, a_1, a_2, a_3, a_4, a_5, a_6 > 0\,\&\,\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 < 0$.

The eigen values are negative, the equilibrium point is globally asymptotic stable.

The basic reproduction number

The basic reproduction number is the most important epidemiological parameter for determining the nature of a disease. There are various techniques available to evaluate for an epidemic spread. In this present article, we use the method—-.

$$F = \begin{vmatrix} 0 & \alpha_{13}s & \alpha_{13}\alpha_3 s & \alpha_4 s \\ \alpha_{14} & 0 & 0 & 0 \\ \alpha_{15} & 0 & 0 & 0 \\ 0 & \alpha_{10} & \alpha_{11} & 0 \end{vmatrix}$$

$$V^{-1} = \begin{vmatrix} \frac{1}{\alpha_{14}+\alpha_{15}+\alpha_1} & -\frac{\alpha_{14}}{\alpha_{16}(\alpha_{14}+\alpha_{15}+\alpha_1)} & -\frac{\alpha_{14}}{\alpha_{17}(\alpha_{14}+\alpha_{15}+\alpha_1)} & 0 \\ 0 & \frac{1}{\alpha_{16}} & 0 & 0 \\ 0 & 0 & \frac{1}{\alpha_{17}} & 0 \\ 0 & 0 & 0 & \frac{1}{\alpha_1} \end{vmatrix}$$

$$P = FV^{-1} = \begin{vmatrix} 0 & \frac{\alpha_{13}s}{\alpha_{16}} & \frac{\alpha_{13}\alpha_3 s}{\alpha_{17}} & \frac{\alpha_4 s}{\alpha_1} \\ \frac{\alpha_{14}}{\alpha_{14}+\alpha_{15}+\alpha_1} & \frac{\alpha_{14}^2}{\alpha_{16}(\alpha_{14}+\alpha_{15}+\alpha_1)} & \frac{\alpha_{14}\alpha_{15}}{\alpha_{17}(\alpha_{14}+\alpha_{15}+\alpha_1)} & 0 \\ \frac{\alpha_{15}}{\alpha_{14}+\alpha_{15}+\alpha_1} & \frac{-\alpha_{14}\alpha_{15}}{\alpha_{16}(\alpha_{14}+\alpha_{15}+\alpha_1)} & \frac{-\alpha_{15}^2}{\alpha_{17}(\alpha_{14}+\alpha_{15}+\alpha_1)} & 0 \\ 0 & \frac{\alpha_{10}}{\alpha_{16}} & \frac{\alpha_{11}}{\alpha_{17}} & 0 \end{vmatrix}$$

$$R_0 = -\frac{2\alpha_4\alpha_{15}^2\alpha_{10}\alpha_{14}\alpha_0}{\alpha_1^2\alpha_{16}\alpha_{17}(\alpha_{14}+\alpha_{15}+\alpha_1)^2}$$

4 Homotopy perturbation method (HPM) procedure for COVID-19 model

Let us consider the given equation is converted to below form:

(1-p) (linear terms of given differential equations) + p (linear and nonlinear all terms of given differential equations) = 0.

Let us organised the given model all variables as follows:

$$S(t) = S_0 + pS_1 + p^2 S_2 + ... + \infty.$$

$$E(t) = E_0 + pE_1 + p^2 E_2 + ... + \infty.$$

$$I(t) = I_0 + pI_1 + p^2 I_2 + ... + \infty.$$

$$I_a(t) = I_{a_0} + pI_{a_1} + p^2 I_{a_2} + ... + \infty.$$

$$R(t) = R_0 + pR_1 + p^2 R_2 + ... + \infty.$$

$$M(t) = M_0 + pM_1 + p^2 M_2 + ... + \infty.$$

Approximate solutions of COVID-19 are:

$$S(t) = \lim_{p\to 1} S(t) = S_0 + S_1 + S_2 + ... + \infty.$$

$$E(t) = \lim_{p\to 1} E(t) = E_0 + E_1 + E_2 + ... + \infty.$$

$$I(t) = \lim_{p\to 1} I(t) = I_0 + I_1 + I_2 + ... + \infty.$$

$$I_a(t) = \lim_{p\to 1} I_a(t) = I_{a_0} + I_{a_1} + I_{a_2} + ... + \infty.$$

$$R(t) = \lim_{p\to 1} R(t) = R_0 + R_1 + R_2 + ... + \infty.$$

$$M(t) = \lim_{p\to 1} M(t) = M_0 + M_1 + M_2 + ... + \infty.$$

Here the first two terms are enough to get the approximate analytic asolutions of converges of numerical simulations. This method is very helpful for solving nonlinear ordinary differential equations.

5 Application of HPM in COVID-19 model

The solution of the system of eqns.(2) can be obtained by using HPM as follows:

$$\frac{dS}{dt} = \alpha_0 - \alpha_1 S - \alpha_{13} S(I + \alpha_3 I_a) - \alpha_4 SM \tag{5}$$

$$\frac{dE}{dt} = \alpha_{13} S(I + \alpha_3 I_a) + \alpha_4 SM - \alpha_{14} E - \alpha_{15} E - \alpha_1 E \tag{6}$$

$$\frac{dI}{dt} = \alpha_{14} E - \alpha_{16} I \tag{7}$$

$$\frac{dI_a}{dt} = \alpha_{15} E - \alpha_{17} I_a \tag{8}$$

$$\frac{dR}{dt} = \alpha_8 I + \alpha_9 I_a - \alpha_1 R \tag{9}$$

$$\frac{dM}{dt} = \alpha_{10} I + \alpha_{11} I_a - \alpha_1 M \tag{10}$$

To obtain the analytical solution, we construct the homotopy as follows:

$$(1-p)\left(\frac{dS}{dt} - \alpha_0 + \alpha_1 S\right) + p\left(\frac{dS}{dt} - \alpha_0 + \alpha_1 S + \alpha_{13} S(I + \alpha_3 I_a) + \alpha_4 SM\right) = 0 \tag{11}$$

$$(1-p)\left(\frac{dE}{dt} + (\alpha_{14} + \alpha_{15} + \alpha_1)E\right)$$

$$+\,p\left(\frac{dE}{dt}+(\alpha_{14}+\alpha_{15}+\alpha_1)E-\alpha_{13}S(I+\alpha_3 I_a)+\alpha_4 SM\right)=0 \tag{12}$$

$$(1-p)\left(\frac{dI}{dt}+\alpha_{16}I\right)+p\left(\frac{dI}{dt}+\alpha_{16}I-\alpha_{14}E\right)=0 \tag{13}$$

$$(1-p)\left(\frac{dI_a}{dt}+\alpha_{17}I_a\right)+p\left(\frac{dI_a}{dt}+\alpha_{17}I_a+\alpha_{15}E\right)=0 \tag{14}$$

$$(1-p)\left(\frac{dR}{dt}+\alpha_1 R\right)+p\left(\frac{dR}{dt}+\alpha_1 R-\alpha_8 I-\alpha_9 I_a\right)=0 \tag{15}$$

$$(1-p)\left(\frac{dM}{dt}+\alpha_1 M\right)+p\left(\frac{dM}{dt}+\alpha_{12}M-\alpha_{10}I-\alpha_{11}I_a\right)=0 \tag{16}$$

Equating *p0* terms on both sides of the above system of eqns. (11)–(16), we get

Constructing homotopy, we get

$$p^0:\frac{dS_0}{dt}=\alpha_0-\alpha_1 S_0 \tag{17}$$

$$p^0:\frac{dE_0}{dt}=\alpha_{14}E_0-\alpha_{15}E_0-\alpha_1 E_0 \tag{18}$$

$$p^0:\frac{dI_0}{dt}=-\alpha_{16}I_0 \tag{19}$$

$$p^0:\frac{dI_{a_0}}{dt}=-\alpha_{17}I_{a_0} \tag{20}$$

$$p^0:\frac{dR_0}{dt}=-\alpha_1 R_0 \tag{21}$$

$$p^0:\frac{dM_0}{dt}=-\alpha_1 M_0 \tag{22}$$

The solution for these equations are given as follows

$$S_0=\frac{\alpha_0}{\alpha_1}+c_1 e^{-\alpha_1 t} \tag{23}$$

$$E_0=c_2 e^{-(\alpha_{14}+\alpha_{15}+\alpha_1)t} \tag{24}$$

$$I_0=c_3 e^{-\alpha_{16}t} \tag{25}$$

$$I_{a_0}=c_4 e^{-\alpha_{17}t} \tag{26}$$

$$R_0=c_5 e^{-\alpha_1 t} \tag{27}$$

$$M_0=c_6 e^{-\alpha_1 t} \tag{28}$$

Applying initial conditions,

$$S(0)=\beta_0;E(0)=\beta_1;I(0)=\beta_2;I_a(0)=\beta_3;R(0)=\beta_4;M(0)=\beta_5, \tag{29}$$

for all $\beta_i > 0, i = 0, 1, 2, 3, 4, 5$
and initial approximations,

$$S(i) = 0; E(i) = 0; I(i) = 0; I_a(i) = 0; R(i) = 0; M(i) = 0 \text{ for all } i = 1, 2, 3... \tag{30}$$

By applying eqn. (29) into eqn. (23), we get

$$c_1 = \beta_0 - \frac{\alpha_0}{\alpha_1} \tag{31}$$

Therefore

$$S_0 = \frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right) e^{-\alpha_1 t} \tag{32}$$

Similarly by applying eqn. (29) into eqns. (24)–(28), we get

$$c_2 = \beta_1 \tag{33}$$

$$E_0 = \beta_1 e^{-E_0(\alpha_{14}+\alpha_{15}+\alpha_1)t} \tag{34}$$

$$c_3 = \beta_2 \tag{35}$$

$$I_0 = \beta_2 e^{-\alpha_{16} t} \tag{36}$$

$$c_4 = \beta_3 \tag{37}$$

$$I_{a_0} = \beta_3 e^{-\alpha_{17} t} \tag{38}$$

$$c_5 = \beta_4 \tag{39}$$

$$R_0 = \beta_4 e^{-\alpha_1 t} \tag{40}$$

$$c_6 = \beta_5 \tag{41}$$

$$M_0 = \beta_5 e^{-\alpha_1 t} \tag{42}$$

Again equating p^1 terms, we get

$$p^1 : \frac{dS_1}{dt} = \alpha_0 - \alpha_1 S_1 - \alpha_{13} S_0 I_0 - \alpha_{13}\alpha_3 S_0 I_{a0} - \alpha_4 S_0 M_0 \tag{43}$$

$$p^1 : \frac{dE_1}{dt} = \alpha_{13} S_0 I_0 + \alpha_{13}\alpha_3 S_0 I_{a0} + \alpha_4 S_0 M_0 - \alpha_{14} E_1 - \alpha_{15} E_1 - \alpha_1 E_1 \tag{44}$$

$$p^1 : \frac{dI_1}{dt} = \alpha_{14} E_0 - \alpha_{16} I_1 \tag{45}$$

$$p^1 : \frac{dI_{a1}}{dt} = \alpha_{15} E_0 - \alpha_{17} I_{a1} \tag{46}$$

$$p^1 : \frac{dR_1}{dt} = \alpha_8 I_0 + \alpha_9 I_{a0} - \alpha_1 R_1 \tag{47}$$

$$p^1 : \frac{dM_1}{dt} = \alpha_{10} I_0 + \alpha_{11} I_{a0} - \alpha_1 M_1 \tag{48}$$

From eqn.(43) $\Rightarrow$

$$\frac{dS_1}{dt} = \alpha_0 - \alpha_1 S_1 - \alpha_{13}\left(\frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\right)\beta_2\exp(-\alpha_{16}t)$$
$$- \alpha_{13}\alpha_3\left(\frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\right)\beta_3\exp(-\alpha_{17}t)$$
$$- \alpha_4\left(\frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\right)\beta_5\exp(-\alpha_1 t)$$

$$S_1 = \begin{bmatrix} -\frac{\alpha_0}{\alpha_1} + \frac{\alpha_{13}\beta_2}{-\alpha_{16}+\alpha_1}\frac{\alpha_0}{\alpha_1} - \frac{1}{\alpha_{16}}\left[\alpha_{13}\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\right] + \frac{\alpha_{13}\alpha_3\beta_3}{-\alpha_{17}+\alpha_1}\frac{\alpha_0}{\alpha_1} \\ -\frac{1}{\alpha_{17}}\left[\alpha_{13}\alpha_3\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\right] + \frac{\alpha_4\beta_5}{-\alpha_2+\alpha_1}\frac{\alpha_0}{\alpha_1} - \frac{1}{\alpha_1}\left[\alpha_4\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\right] \end{bmatrix} e^{-\alpha_1 t}$$
$$+ \frac{\alpha_0}{\alpha_1} - \frac{\alpha_{13}\beta_2}{-\alpha_{16}+\alpha_1}\frac{\alpha_0}{\alpha_1}e^{-\alpha_{16}t} + \frac{\alpha_{13}\beta_2}{\alpha_{16}}\left[\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)e^{-(\alpha_1+\alpha_{16})t}\right] - \frac{\alpha_{13}\alpha_3\beta_3}{\alpha_1 - \alpha_{17}}\frac{\alpha_0}{\alpha_1}e^{-\alpha_{17}t}$$
$$+ \frac{\alpha_{13}\alpha_3\beta_3}{\alpha_{17}}\left[\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)e^{-(\alpha_1+\alpha_{17})t}\right] - \frac{\alpha_4\beta_5}{\alpha_1 - \alpha_2}\frac{\alpha_0}{\alpha_1}e^{-\alpha_{12}t} + \frac{\beta_3}{\alpha_1}\left[\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)e^{-(2\alpha_1)t}\right] \tag{49}$$

$$\frac{dE_1}{dt} = \alpha_{13}\left(\frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\right)\beta_2\exp(-\alpha_{16}t) \tag{50}$$
$$+ \alpha_{13}\alpha_3\left(\frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\right)\beta_3\exp(-\alpha_{17}t)$$
$$+ \alpha_4\left(\frac{\alpha_0}{\alpha_1} + \left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\right)\beta_5\exp(-\alpha_1 t) - \alpha_{14}E_1 - \alpha_{15}E_1 - \alpha_1 E$$

$$E_1 = \begin{bmatrix} -\frac{1}{-\alpha_{16}+(\alpha_{14}+\alpha_{15}+\alpha_1)}\alpha_{13}\frac{\alpha_0}{\alpha_1}\beta_2 \\ -\frac{1}{-(\alpha_1+\alpha_{16})+(\alpha_{14}+\alpha_{15}+\alpha_1)}\left[\alpha_{13}\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\right] \\ -\frac{1}{-\alpha_{17}+(\alpha_{14}+\alpha_{15}+\alpha_1)}\alpha_{13}\alpha_3\frac{\alpha_0}{\alpha_1}\beta_3 \\ -\frac{1}{-(\alpha_1+\alpha_{17})+(\alpha_{14}+\alpha_{15}+\alpha_1)}\left[\alpha_{13}\alpha_3\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\right] \\ -\frac{1}{-\alpha_1+(\alpha_{14}+\alpha_{15}+\alpha_1)}\alpha_4\frac{\alpha_0}{\alpha_1}\beta_5 \\ -\frac{1}{-(2\alpha_1)+(\alpha_{14}+\alpha_{15}+\alpha_1)}\left[\alpha_4\left(\beta_0 - \frac{\alpha_0}{\alpha_1}\right)\right] \end{bmatrix} \exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)$$
$$+ \frac{1}{-\alpha_{16}+(\alpha_{14}+\alpha_{15}+\alpha_1)}\alpha_{13}\frac{\alpha_0}{\alpha_1}\beta_2\exp(-\alpha_{16}t)$$

$$+\frac{1}{-(\alpha_1+\alpha_{16})+(\alpha_{14}+\alpha_{15}+\alpha_1)}\left[\alpha_{13}\left(\beta_0-\frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\beta_2\exp(-\alpha_{16}t)\right]$$
$$+\frac{1}{-\alpha_{17}+(\alpha_{14}+\alpha_{15}+\alpha_1)}\alpha_{13}\alpha_3\frac{\alpha_0}{\alpha_1}\beta_3\exp(-\alpha_{17}t)$$
$$+\frac{1}{-(\alpha_1+\alpha_{17})+(\alpha_{14}+\alpha_{15}+\alpha_1)}\left[\alpha_{13}\alpha_3\left(\beta_0-\frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\beta_3\exp(-\alpha_{17}t)\right]$$
$$+\frac{1}{-\alpha_1+(\alpha_{14}+\alpha_{15}+\alpha_1)}\alpha_4\frac{\alpha_0}{\alpha_1}\beta_5\exp(-\alpha_1 t)$$
$$+\frac{1}{-(2\alpha_1)+(\alpha_{14}+\alpha_{15}+\alpha_1)}\left[\alpha_4\left(\beta_0-\frac{\alpha_0}{\alpha_1}\right)\exp(-\alpha_1 t)\beta_3\exp(-\alpha_1 t)\right] \tag{51}$$

$$I_1=c_9\exp(-\alpha_{16}t)+\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{16}}\alpha_{14}\beta_1\exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)$$

Applying initial condition $I(0)=0$

$$c_9+\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{16}}\alpha_{14}\beta_1=0$$

$$I_1=\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{16}}\alpha_{14}\beta_1\exp(-\alpha_{16}t) \tag{52}$$
$$+\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{16}}\alpha_{14}\beta_1\exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)$$

$$\frac{dI_1}{dt}=\alpha_{14}\beta_1\exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)-\alpha_{16}I_1$$

$$\frac{dI_{a1}}{dt}=\alpha_{15}\beta_1\exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)-\alpha_{17}I_{a1}$$

$$I_{a1}=c_{10}\exp(-\alpha_{17}t)+\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{17}}\alpha_{15}\beta_1\exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)$$

Applying initial condition $I_a(0)=0$

$$c_{10}+\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{17}}\alpha_{15}\beta_1=0$$

$$I_{a1}=\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{17}}\alpha_{15}\beta_1\exp(-\alpha_{17}t) \tag{53}$$
$$+\frac{1}{-(\alpha_{14}+\alpha_{15}+\alpha_1)+\alpha_{17}}\alpha_{15}\beta_1\exp(-(\alpha_{14}+\alpha_{15}+\alpha_1)t)$$

$$\frac{dR_1}{dt}=\alpha_8\beta_2\exp(-\alpha_{16}t)+\alpha_9\beta_3\exp(-\alpha_{17}t)-\alpha_1R_1$$

$$R_1=c_{11}\exp(-\alpha_1 t)+\frac{1}{-\alpha_{16}+\alpha_1}\alpha_8\beta_2\exp(-\alpha_{16}t)+\frac{1}{-\alpha_{17}+\alpha_1}\alpha_9\beta_3\exp(-\alpha_{17}t)$$

Applying initial condition

$$R(0) = 0$$

$$c_{11} + \frac{1}{-\alpha_{16} + \alpha_1}\alpha_8\beta_2 + \frac{1}{-\alpha_{17} + \alpha_1}\alpha_9\beta_3 = 0$$

$$R_1 = \left[-\frac{1}{-\alpha_{16} + \alpha_1}\alpha_8\beta_2 - \frac{1}{-\alpha_{17} + \alpha_1}\alpha_9\beta_3\right]\exp(-\alpha_1 t) \tag{54}$$

$$+ \frac{1}{-\alpha_{16} + \alpha_1}\alpha_8\beta_2\exp(-\alpha_{16}t) + \frac{1}{-\alpha_{17} + \alpha_1}\alpha_9\beta_3\exp(-\alpha_{17}t)$$

$$\frac{dM_1}{dt} = \alpha_{10}\beta_2\exp(-\alpha_{16}t) + \alpha_{11}\beta_3\exp(-\alpha_{17}t) - \alpha_1 M_1$$

$$M_1 = c_{12}\exp(-\alpha_1 t) + \frac{1}{-\alpha_{16} + \alpha_1}\alpha_{10}\beta_2\exp(-\alpha_{16}t) + \frac{1}{-\alpha_{17} + \alpha_1}\alpha_{11}\beta_3\exp(-\alpha_{17}t) \tag{55}$$

Applying initial condition

$$M(0) = 0$$

$$c_{12} + \frac{1}{-\alpha_{16} + \alpha_1}\alpha_{10}\beta_2 + \frac{1}{-\alpha_{17} + \alpha_1}\alpha_{11}\beta_3 = 0$$

$$M_1 = \left[-\frac{1}{-\alpha_{16} + \alpha_1}\alpha_{10}\beta_2 - \frac{1}{-\alpha_{17} + \alpha_1}\alpha_{11}\beta_3\right]\exp(-\alpha_1 t) \tag{56}$$

$$+ \frac{1}{-\alpha_{16} + \alpha_1}\alpha_{10}\beta_2\exp(-\alpha_{16}t) + \frac{1}{-\alpha_{17} + \alpha_1}\alpha_{11}\beta_3\exp(-\alpha_{17}t)$$

6 Numerical Analysis

We consider the parameter values as follows [34]-[37]:

$$N = 6,757,131 S_0 = 6,757,131, E_0 = 20000, I_0 = 104,591, I_{a0} = 200,$$

$$R_0 = 5,744,693, M_0 = 907883\alpha_0 = 50, \alpha_1 = \frac{1}{76.79 \times 365} = 0.0000356, \alpha_2 = 0.05,$$

$$\alpha_3 = 0.02, \alpha_4 = 0.000001231, \alpha_5 = 0.1243, \alpha_6 = 0.00047876, \alpha_7 = 0.005,$$

$$\alpha_8 = 0.09871, \alpha_9 = 0.854302, \alpha_{10} = 0.000398, \alpha_{11} = 0.001,$$

$$\alpha_{12} = 0.01, \alpha_{14} = (1 - \alpha_5)\,\alpha_6 = 0.000419, \alpha_{15} = \alpha_5$$

$$\alpha_7 = 0.000622\alpha_{16} = \alpha_8 + \alpha_1 = 0.098745678, \alpha_{17} = \alpha_9 + \alpha_1 = 0.85433768$$

Let us use Mathematica 12 software to obtain 8^{th} order approximation for $S(t)$, $E(t)$,......$M(t)$.

$$\mathrm{S(t)} = 9065518 + 6151989\mathrm{ht} + 2648092\mathrm{h}^2\mathrm{t} + 451983\mathrm{h}^3\mathrm{t} + 66467\mathrm{h}^4\mathrm{t} + 6258\mathrm{h}^5\mathrm{t} + 878\,\mathrm{h}^6\mathrm{t} + 71\mathrm{h}^7\mathrm{t} + 65\mathrm{h}^8\mathrm{t} + 55\mathrm{h}^2\mathrm{t}^2 + 53\mathrm{h}^3\mathrm{t}^2 + 43\mathrm{h}^4\mathrm{t}^2 + 39\mathrm{h}^5\mathrm{t}^2 + 27\mathrm{h}^6\mathrm{t}^2 + 22\mathrm{h}^7\mathrm{t}^2 + 17\mathrm{h}^8\mathrm{t}^2 + ...,$$

$$\mathrm{E(t)} = 300000 + 480795\mathrm{ht} + 371138\,\mathrm{h}^2\mathrm{t} + 77144\,\mathrm{h}^3\mathrm{t} + 18273\mathrm{h}^4\mathrm{t} + 2853\mathrm{h}^5\mathrm{t} + 355\mathrm{h}^6\mathrm{t} + 36\mathrm{h}^7\mathrm{t} + 33\mathrm{h}^8\mathrm{t} + 30\mathrm{h}^2\mathrm{t}^2 + 26\mathrm{h}^3\mathrm{t}^2 + 22\mathrm{h}^4\mathrm{t}^2 + 19\mathrm{h}^5\mathrm{t}^2 + 17\mathrm{h}^6\mathrm{t}^2 + 15\mathrm{h}^7\mathrm{t}^2 + 9\mathrm{h}^8\mathrm{t}^2 + ...,$$

$$\mathrm{I(t)} = 280 + 115\mathrm{ht} + 110\mathrm{h}^2\mathrm{t} + 99\mathrm{h}^3\mathrm{t} + 83\mathrm{h}^4\mathrm{t} + 79\mathrm{h}^5\mathrm{t} + 76\mathrm{h}^6\mathrm{t} + 75\mathrm{h}^7\mathrm{t} + 71\mathrm{h}^8\mathrm{t} + 67\mathrm{h}^2\mathrm{t}^2 + 63\,\mathrm{h}^3\mathrm{t}^2 + 59\mathrm{h}^4\mathrm{t}^2 + 44\mathrm{h}^5\mathrm{t}^2 + 39\mathrm{h}^6\mathrm{t}^2 + 33\mathrm{h}^7\mathrm{t}^2 + 21\mathrm{h}^8\mathrm{t}^2 + ...,$$

$$\mathrm{I_a(t)} = 199 + 190\mathrm{ht} + 173\mathrm{h}^2\mathrm{t} + 151\mathrm{h}^3\mathrm{t} + 143\mathrm{h}^4\mathrm{t} + 137\mathrm{h}^5\mathrm{t} + 129\mathrm{h}^6\mathrm{t} + 119\mathrm{h}^7\mathrm{t} + 115\mathrm{h}^8\mathrm{t} + 101\mathrm{h}^2\mathrm{t}^2 + 91\mathrm{h}^3\mathrm{t}^2 + 83\mathrm{h}^4\mathrm{t}^2 + 77\mathrm{h}^5\mathrm{t}^2 + 65\mathrm{h}^6\mathrm{t}^2 + 52\mathrm{h}^7\mathrm{t}^2 + 41\mathrm{h}^8\mathrm{t}^2 + ...,$$

$$\mathrm{R(t)} = 197 + 190\mathrm{ht} + 167\mathrm{h}^2\mathrm{t} + 150\mathrm{h}^3\mathrm{t} + 143\mathrm{h}^4\mathrm{t} + 133\mathrm{h}^5\mathrm{t} + 123\mathrm{h}^6\mathrm{t} + 111\mathrm{h}^7\mathrm{t} + 109\mathrm{h}^8\mathrm{t} + 99\mathrm{h}^2\mathrm{t}^2 + 87\mathrm{h}^3\mathrm{t}^2 + 85\mathrm{h}^4\mathrm{t}^2 + 77\mathrm{h}^5\mathrm{t}^2 + 64\mathrm{h}^6\mathrm{t}^2 + 53\mathrm{h}^7\mathrm{t}^2 + 41\mathrm{h}^8\mathrm{t}^2 + ...,$$

$$\mathrm{M(t)} = 60000 + 190\mathrm{ht} + 183\mathrm{h}^2\mathrm{t} + 177\mathrm{h}^3\mathrm{t} + 165\,\mathrm{h}^4\mathrm{t} + 157\mathrm{h}^5\mathrm{t} + 147\mathrm{h}^6\mathrm{t} + 133\mathrm{h}^7\mathrm{t} + 129\mathrm{h}^8\mathrm{t} + 119\mathrm{h}^2\mathrm{t}^2 + 107\mathrm{h}^3\mathrm{t}^2 + 96\mathrm{h}^4\mathrm{t}^2 + 88\mathrm{h}^5\mathrm{t}^2 + 71\mathrm{h}^6\mathrm{t}^2 + 67\mathrm{h}^7\mathrm{t}^2 + 55\mathrm{h}^8\mathrm{t}^2 + ...,$$

7 Error Analysis

In this section, an error analysis is produced to obtain the optimal values of parameters.

$$ER_1(S;h_1) = \frac{d\varphi_S(t,h_1)}{dt} = \alpha_0 - \alpha_1 s(t,h_1) - \alpha_B^s(t,h_1)\left(I(t,h_1) + I_a(t,h_1)\alpha_3\right)$$

$$-\alpha_4 S(t,h_1) M(t,h_1)$$

$$ER_2(E;h_2) = \frac{d\phi_E(t,h_2)}{dt} = \alpha_{13} S(t,h_2)\left(I(t,h_2) + I_a(t,h_2)\alpha_3\right)$$

$$+\alpha_4 S(t,h_2) M(t,h_2) - \alpha_{14} E(t,h_2) - \alpha_{13} E(t,h_2) - \alpha_1(t,h_2)$$

$$ER_3(I;h_3) = \frac{d\phi_I(t,h_3)}{dt} = \alpha_{14} E(t,h_3) - \alpha_{16} I(t,h_3)$$

$$ER_4(I_a;h_4) = \frac{d\phi_{I_a}(t,h_4)}{dt} = \alpha_{15} E(t,h_4) - \alpha_{17} I_a(t,h_4)$$

$$ER_5(R;h_5) = \frac{d\phi_R(t,h_5)}{dt} = \alpha_8 I(t,h_5) + \alpha_9 I_a(t,h_5) - \alpha_1 R(t,h_5)$$

$$ER_6(M;h_6) = \frac{d\phi_M(t,h_6)}{dt} = \alpha_{10} I(t,h_6) + \alpha_{11} I_a(t,h_6) - \alpha_{12} M(t,h_6)$$

Table 1 The h values range of Compartments

S(t)	$-1.1 \leq$ h ≤ -0.4
E(t)	$-1.3 \leq$ h ≤ -0.8
I(t)	$-1.4 \leq$ h ≤ -0.7
I_a(t)	$-1.5 \leq$ h ≤ -0.4
R(t)	$-1.7 \leq$ h ≤ -0.2
M(t)	$-1.8 \leq$ h ≤ -0.1

Table 2 The optimal solutions of $S(h_1*)$, $E(h_2*)$,$I(h_3*)$, $I_a(h_4*)$,$R(h_5*)$,$M(h_6*)$

	h*	Optimum solution of compartment
S(h_1)	-1.1	2×10^{-4}
E(h_2)	-1.2	3×10^{-6}
I(h_3)	-1.3	4×10^{-8}
Ia(h_4)	-1.4	5×10^{-10}
R(h_5)	-1.5	6×10^{-12}
M(h_6)	-1.6	7×10^{-14}

Table 3 The residual errors for ER_1,ER_2,$ER_3 ER_4$,$ER_5$and $ER_6$for $t \in (0,1)$

t	ER$_1$	ER$_2$	ER$_3$	ER$_4$	ER_5	ER_6
0.0	3.4×10^{-1}	2.3×10^{-1}	1.1×10^{-1}	1.8×10^{-1}	3.1×10^{-1}	1.9×10^{-1}
0.1	1.2×10^{-2}	4.7×10^{-2}	6.8×10^{-2}	2.5×10^{-2}	4.5×10^{-2}	3.6×10^{-2}
0.2	4.5×10^{-3}	9.9×10^{-3}	4.2×10^{-3}	9.2×10^{-3}	9.2×10^{-3}	4.9×10^{-3}
0.3	1.1×10^{-4}	6.7×10^{-4}	3.3×10^{-4}	8.3×10^{-4}	6.3×10^{-4}	9.2×10^{-4}
0.4	6.1×10^{-5}	3.5×10^{-5}	2.1×10^{-5}	7.5×10^{-5}	5.5×10^{-5}	8.7×10^{-5}
0.5	7.3×10^{-6}	1.9×10^{-6}	5.9×10^{-6}	1.6×10^{-6}	4.9×10^{-6}	5.4×10^{-6}
0.6	5.6×10^{-7}	2.7×10^{-7}	6.3×10^{-7}	1.5×10^{-7}	3.7×10^{-7}	9.1×10^{-7}
0.7	2.8×10^{-8}	4.4×10^{-8}	7.5×10^{-8}	3.8×10^{-8}	2.9×10^{-8}	2.6×10^{-8}
0.8	3.7×10^{-9}	6.1×10^{-9}	2.2×10^{-9}	4.9×10^{-9}	1.6×10^{-9}	3.9×10^{-9}
0.9	4.9×10^{-10}	7.8×10^{-10}	3.9×10^{-10}	5.6×10^{-10}	2.9×10^{-10}	4.1×10^{-10}
1	5.1×10^{-11}	9.1×10^{-11}	4.9×10^{-11}	9.2×10^{-11}	8.4×10^{-11}	8.8×10^{-11}

Let us consider the square residual error for 8^{th} order approximation:

$$S(h_1) = \int_0^1 \left(ER_1(S,E,I,I_a,R,M;h_1)\right)^2 dt,$$

$$E(h_2) = \int_0^1 \left(ER_2(S,E,I,I_a,R,M;h_2)\right)^2 dt,$$

$$I(h_3) = \int_0^1 (ER_3 (S, E, I, I_a, R, M; h_3))^2 \, dt,$$

$$I_a(h_1) = \int_0^1 (ER_4 (S, E, I, I_a, R, M; h_4))^2 \, dt,$$

$$R(h_5) = \int_0^1 (ER_5 (S, E, I, I_a, R, M; h_5))^2 \, dt,$$

$$M(h_6) = \int_0^1 (ER_6 (S, E, I, I_a, R, M; h_6))^2 \, dt,$$

The minimal values of $RX(h_1), RY(h_2), RV(h_3)$ and $RZ(h_4)$ are shown below.

$$\frac{dS(h_1*)}{dh_1} = 0, \frac{dE(h_2*)}{dh_2} = 0, \frac{dI(h_3*)}{dh_3} = 0, \frac{dI_a(h_4*)}{dh_4} = 0. \frac{dR(h_5*)}{dh_5} = 0. \frac{dM(h_6*)}{dh_6} = 0.$$

We consider the optimal values of $h_1*, h_2*, h_3*, h_4*, h_5*$ and h_6* for all of the cases are

$$h_1 * \; = -1.1, h_2 * \; = -1.2, h_3 * \; = -1.3, h_4 * \; = -1.4, h_5 * \; = -1.5, h_6 * \; = -1.6.$$

There are 3 types of errors are calculated from the numerical experiment. It is very useful for accuracy of exact solutions and numerical simulations. The residual error of 8^{th} order approximation is defined for $ER_1, ER_2, ER_3, ER_4, ER_5$ and ER_6 in fig 2. The Absolute error of 8^{th} order approximation is defined for $ER_1, ER_2, ER_3, ER_4, ER_5$ and ER_6 in fig 3. The h curves initial derivatives of 7^{th} and 8^{th} order approximation is calculated from HPM in fig 4. The Square residual error of 8^{th} order approximation is derived in fig 5. Numerical simulation of ranges of Reproduction numbers are $R_0 = 2.0317$; 1.2922; 1.4809; 1.5972; 0.9844; 0.8454. in fig. 6. It gives the fluctuations of the overall model validation.

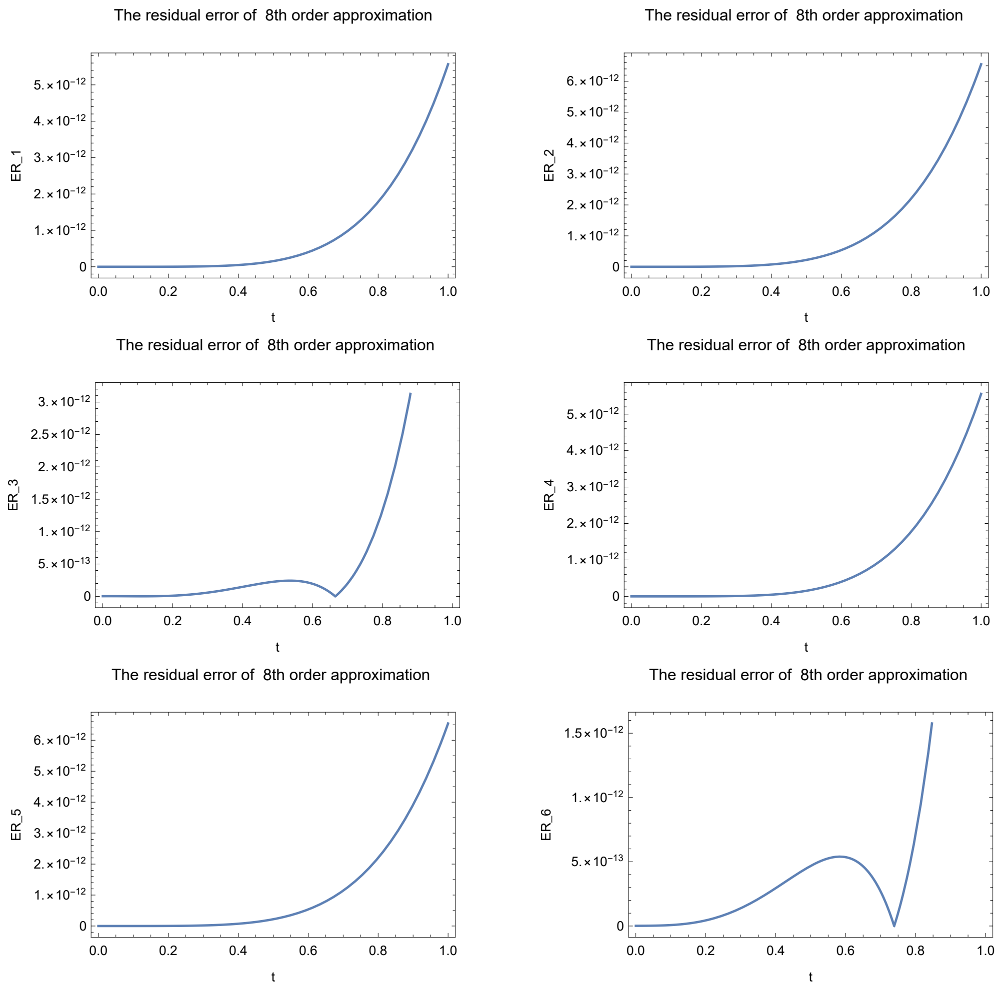

Fig. 2 The residual error of 8^{th} order approximation for $ER_1, ER_2, ER_3, ER_4, ER_5$ and ER_6

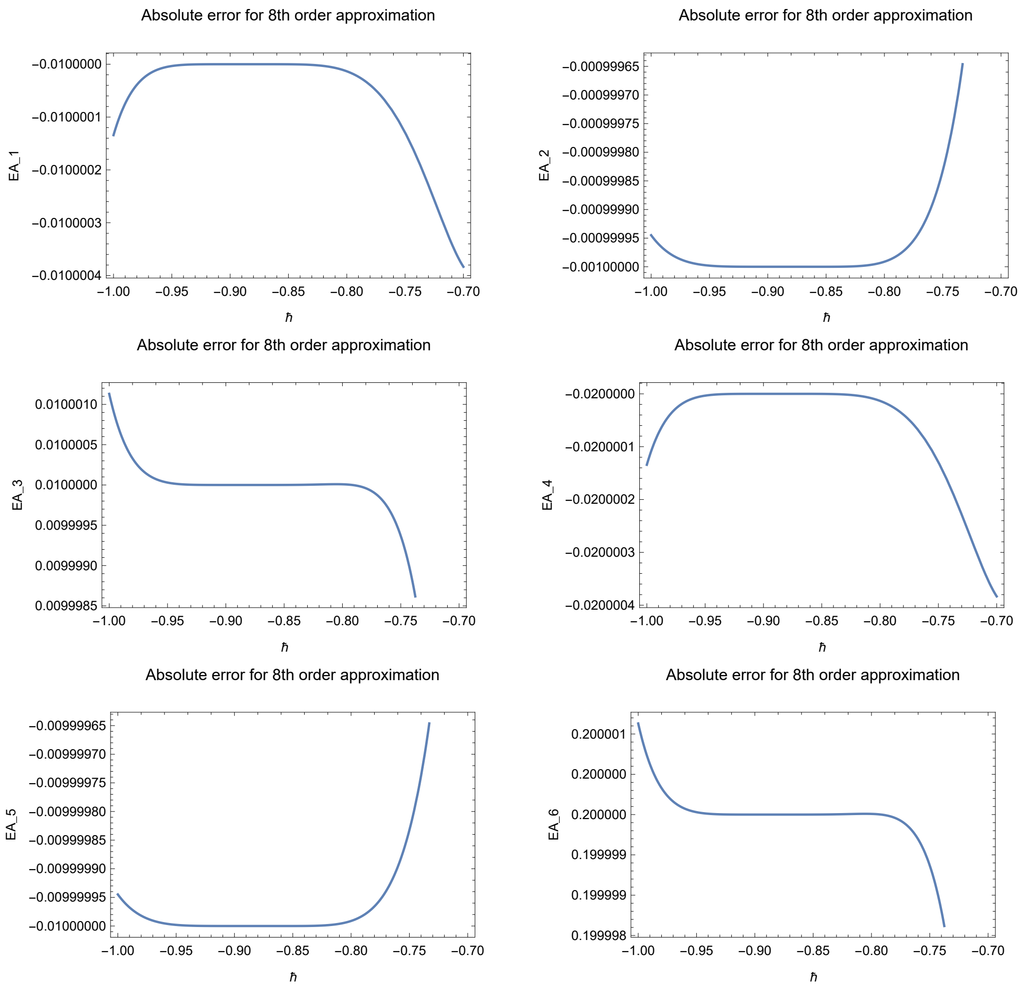

Fig. 3 The Absolute error of 8^{th} order approximation for $ER_1, ER_2, ER_3, ER_4, ER_5$ and ER_6

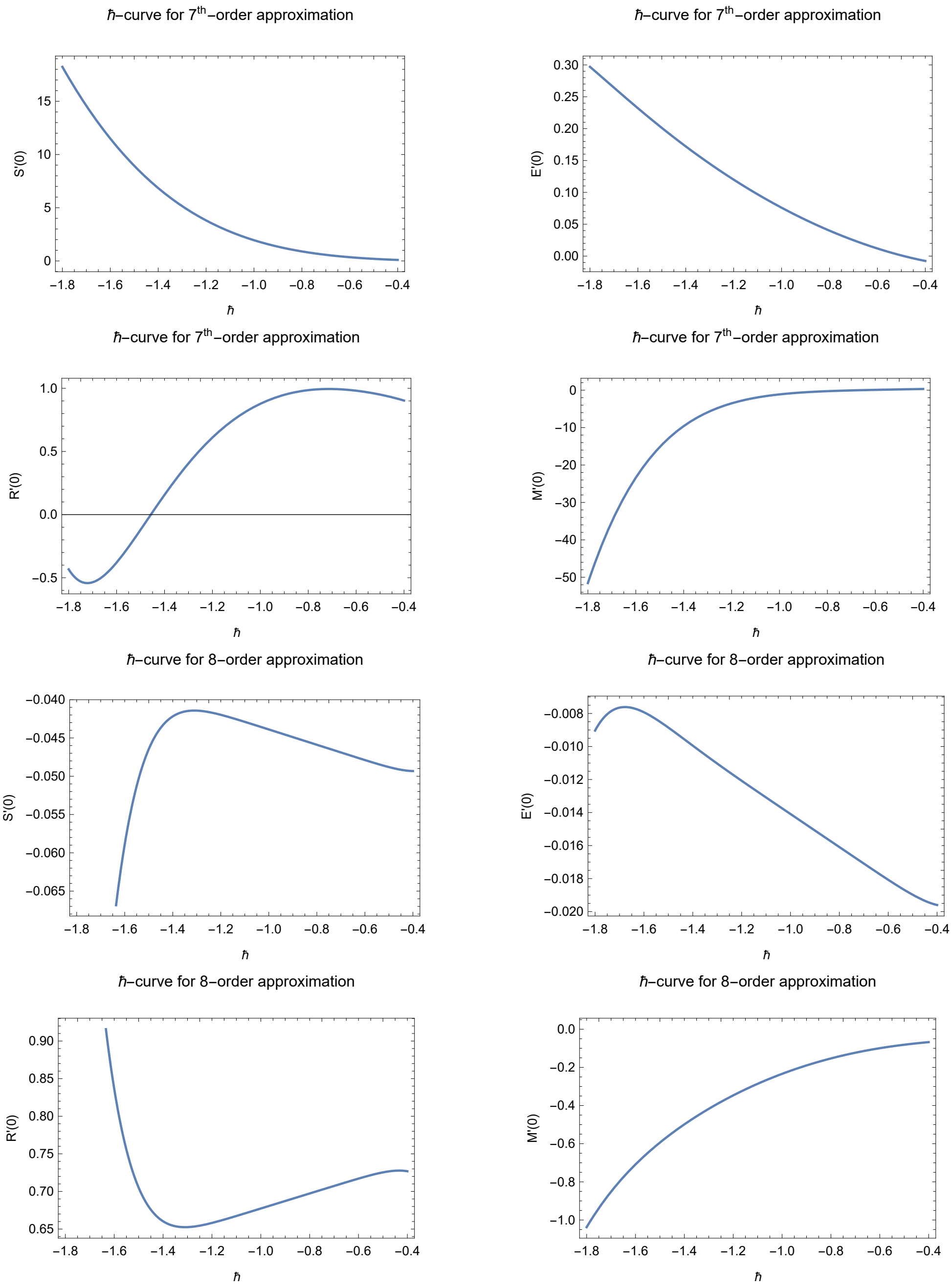

Fig. 4 The h curves initial derivatives of 7^{th} and 8^{th} order approximation from HPM

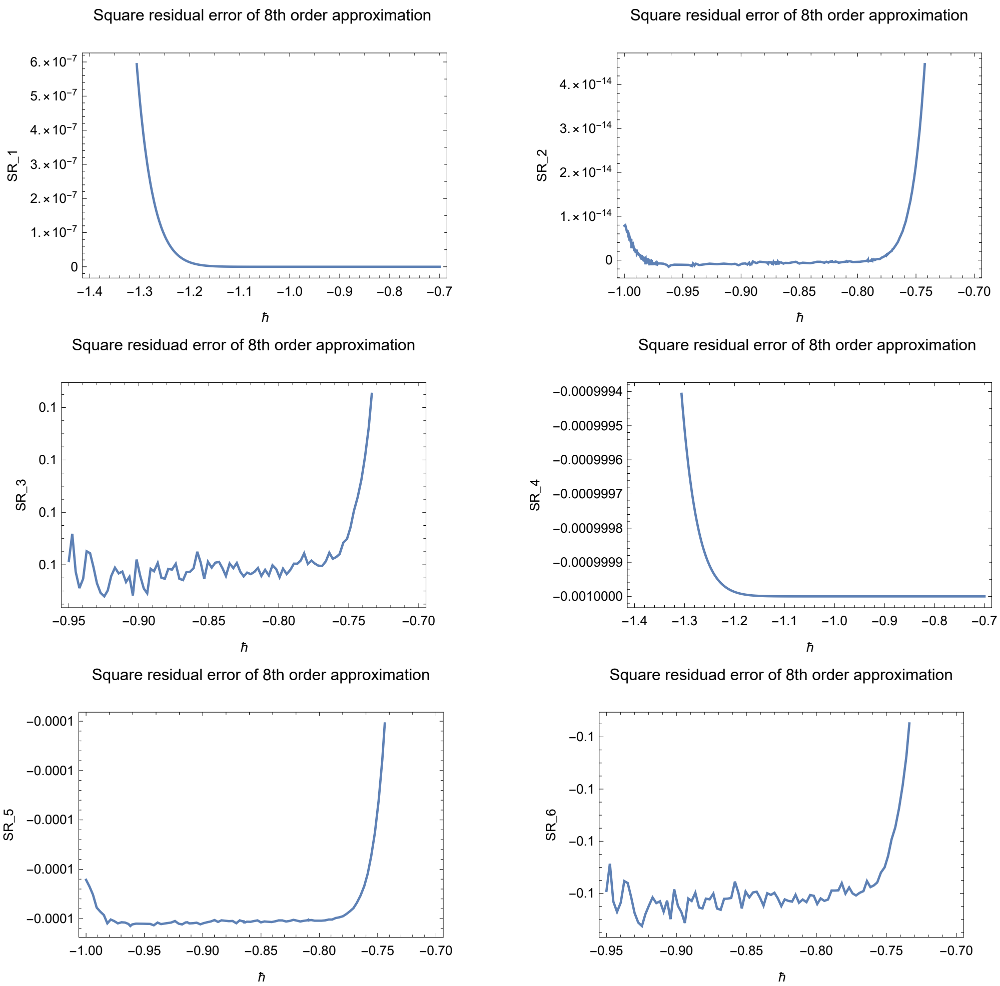

Fig. 5 The Square residual error of 8^{th} order approximation

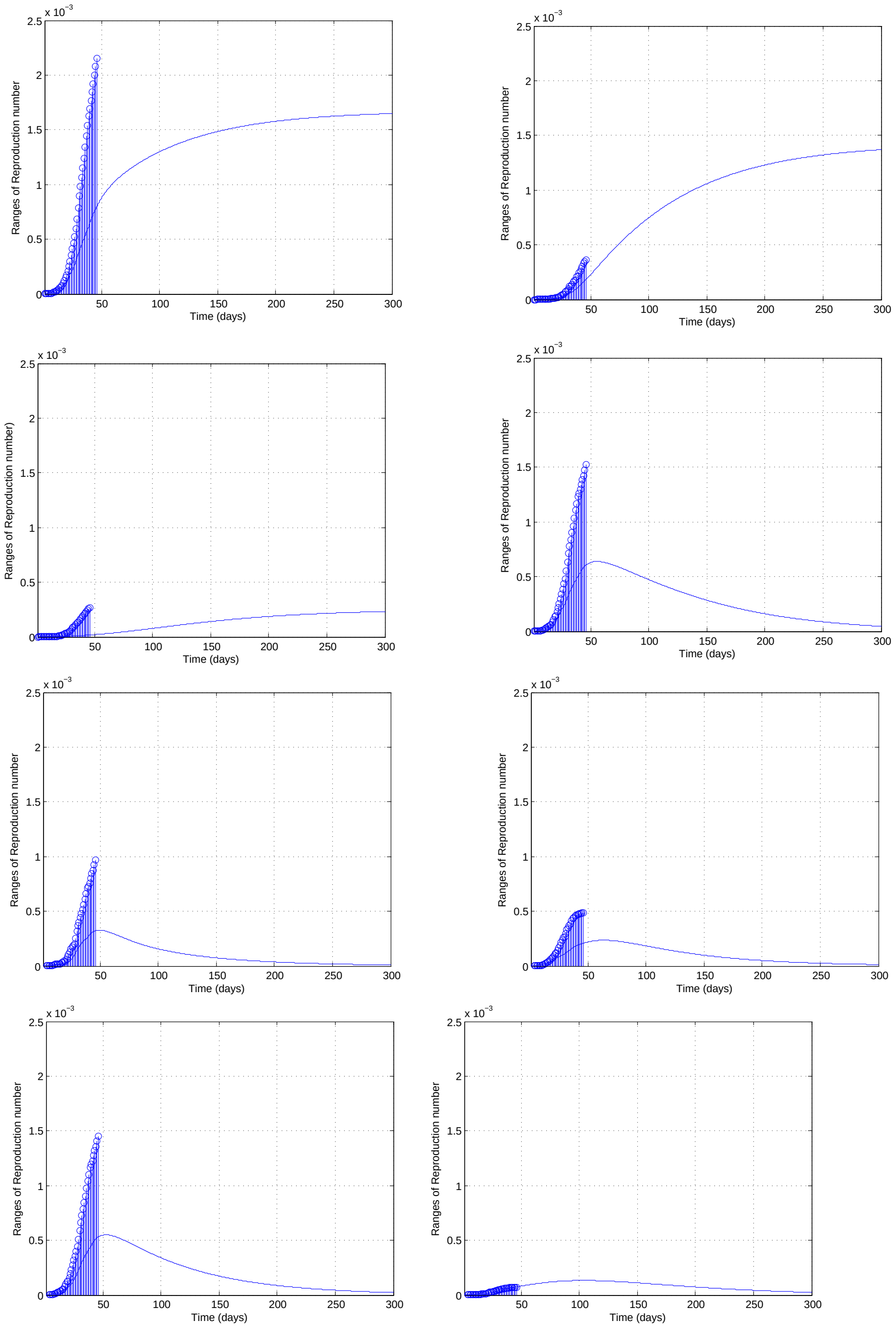

Fig. 6 Numerical simulation of ranges of Reproduction number

8 End of Second wave validity checking

In this section, we discussed five affected states (Maharashtra, Kerala, Karnataka, Tamil Nadu, Andra Pradesh) in India. The 4 important parameter values (confirmed, active, recovered, deceased) are given in Table 4. Another Table 5 shows, an Initial Values of parameters in the states of Maharashtra,Kerala, Karnataka, Tamil Nadu and Andhra Pradesh. We mainly discussed one parameter for Active cases in Maharashtra,Kerala, Karnataka, Tamil Nadu and Andhra Pradesh. Also we drawn the diagram for all states at active cases (see Fig 7 to Fig 11). It used for proposed model validation from real life data and this case approximately equal to the proposed mathematical model. So this model helps for our future prediction from current data.

Table 4 Number of COVID-19 cases across Indian states and union territories as of October 25, 2021

Parameters	Maharashtra	Kerala	Karnataka	Tamil Nadu	Andra Pradesh	References
confirmed	6602961	4915331	2985986	2695216	2063577	[**34–37**]
Active	27506	77964	8740	13034	5102	[**34–37**]
Recovered	6435439	4808775	2939239	2646163	2044132	[**34–37**]
Deceased	140016	28592	38007	36019	14343	[**34–37**]

Table 5 Initial Values of parameters in the states of Maharashtra,Kerala, Karnataka, Tamil Nadu and Andhra Pradesh

Initial values	Maharashtra	Kerala	Karnataka	Tamil Nadu	Andra Pradesh	References
S(0)	3301480	2515300	1590900	1390300	1070600	[**34–37**]
E(0)	13500	35800	4700	680150	5060300	**Calculated**
I(0)	70016	17599	2800	6040	5000	[**34–37**]
Ia(0)	3500	9000	1400	3200	2500	[**34–37**]
R(0)	3234400	2408700	1540300	1340170	1040100	[**34–37**]
M(0)	1630500	14600	19008	730169	7400	**Calculated**

9 Conclusion

We presented the mathematical modeling and the dynamics of second wave COVID-19 which is emerged recently in India. Homotopy perturbation method has been successfully applied to solve the analytical solutions of the dynamics model of COVID-19 with the given initial conditions is effectively analyzed. This method is simple, easy to apply and it provides most approximate analytical expressions. HPM provides an explicit solution which is very useful to analyze the epidemic model based COVID -19 by understanding the parameters. In numerical simulation part, we used Mathematica 12 software for up to

8th order approximation with error analysis which calculated from residual error, absolute error and square error respectively. The growth of the dangerous corona virus and deadly disease in the current pandemic yields the death of millions of people still date. The basic reproduction number R_0 ranges between 0.8454 and 2.0317, derived from numerical simulations, it helps to identify the spread of the disease. Finally, our proposed model is verified from the real life data and it obtained the validity of the system of equations, the same model is defined for all future data.

Competing Interests

The authors declare there are no competing interests.

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Authors' Contributions

For the writing of this paper all authors are equally contributed and also read and agreed the final copy of the manuscript.

Availability of data and material

- Indian council of medical research (ICMR), government of India. 2020. https: //icmr.nic.in
- Ministry of health and welfare, government of India. 2020. https://www.mohfw.gov.in
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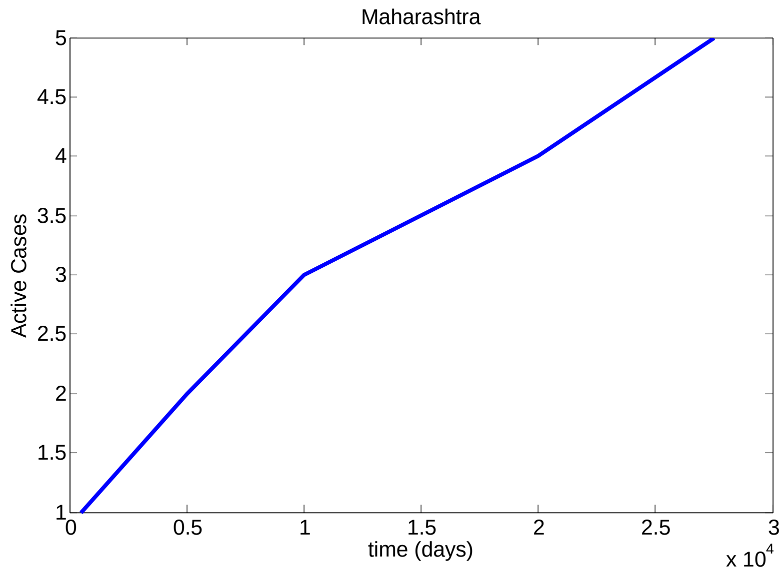

Fig. 7 Active cases of Maharshtra from real life data

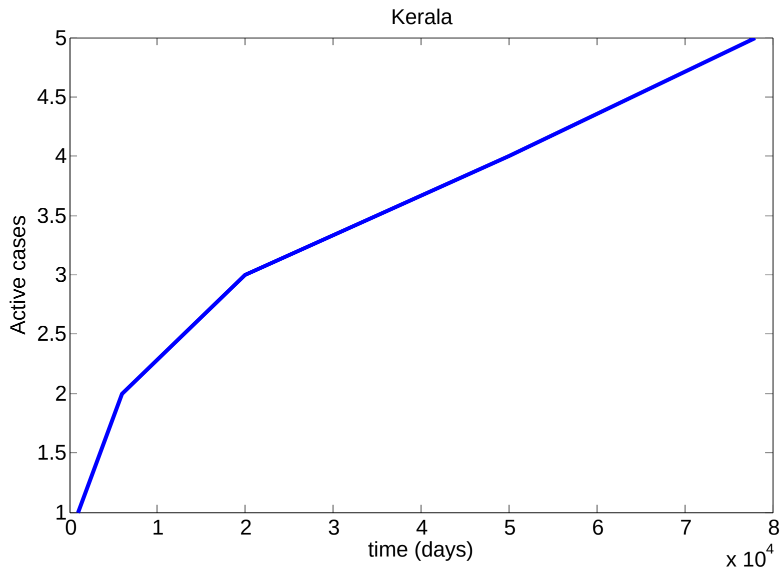

Fig. 8 Active cases of Kerala from real life data

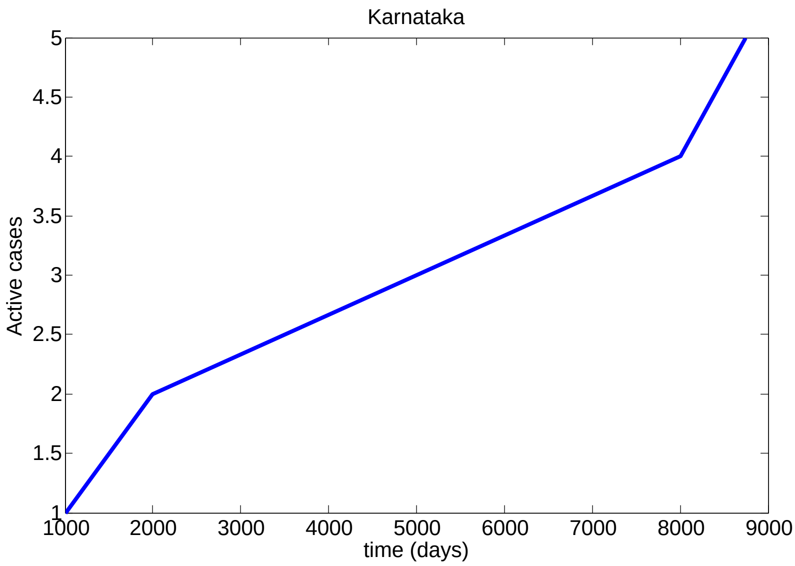

Fig. 9 Active cases of Karnataka from real life data

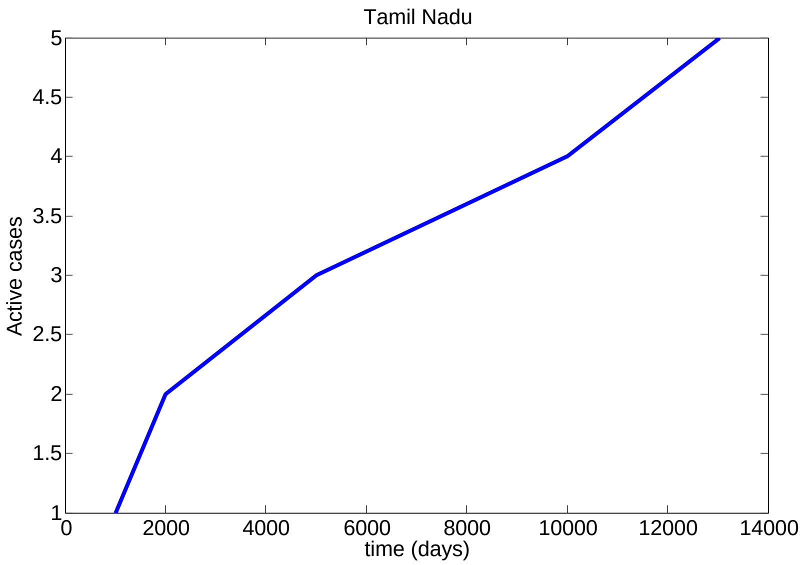

Fig. 10 Active cases of Tamil Nadu from real life data

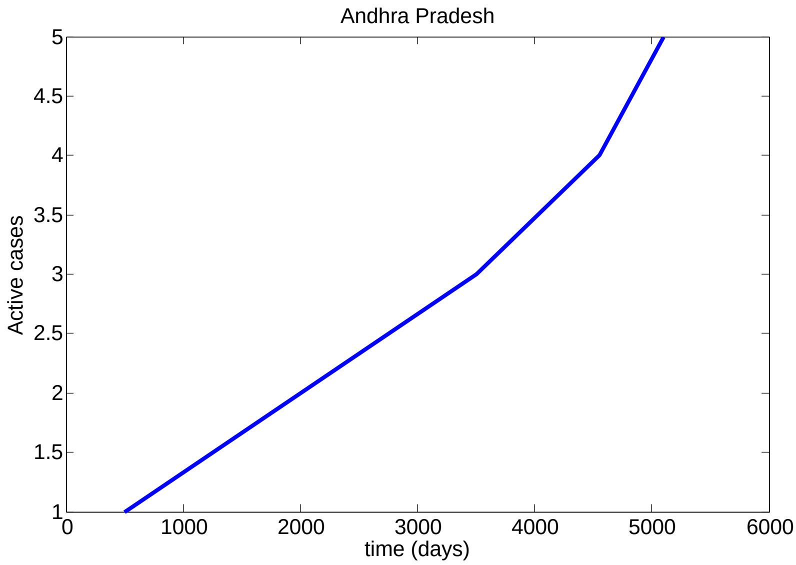

Fig. 11 Active cases of Andhra Pradesh from real life data