Adaptive NN Control for Nominal Backstepping Form with Periodically Time-varying and Nonlinearly Parameterized Switching Functions

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Abstract

In this paper, the prescribed tracking performance control problem is addressed for uncertain nonlinear systems with unknown periodically time-varying parameters and arbitrary switching signal. By utilizing radial basis function neural network and fourier series expansion, an approximator is developed to overcome the difficulty of identifying unknown periodically time-varying and nonlinearly parameterized functions. To achieve the ideal tracking control performance and eliminate the influence of filtering error, a performance function is constructed in advance, and then, a novel command filter-based adaptive neural network controller and a new compensating signal are designed. Differently from the traditional Backstepping technique, the proposed control scheme eliminates the "explosion of complexity" problem and relaxes the constraint condition on the reference signal. And then, it is warranted that the closed-loop system is semi-globally ultimately uniformly bounded and the tracking error is always limited to the specified region bounded by the performance functions. Two simulation examples are used to demonstrate the feasibility of the developed technique in this paper.

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Xiaoli Yang, Jing Li*, Shuzhi Sam Ge, *Fellow, IEEE* and Xiaobo Li

***Abstract*—In this paper, the prescribed tracking performance control problem is addressed for uncertain nonlinear systems with unknown periodically time-varying parameters and arbitrary switching signal. By utilizing radial basis function neural network and fourier series expansion, an approximator is developed to overcome the difficulty of identifying unknown periodically time-varying and nonlinearly parameterized functions. To achieve the ideal tracking control performance and eliminate the influence of filtering error, a performance function is constructed in advance, and then, a novel command filter-based adaptive neural network controller and a new compensating signal are designed. Differently from the traditional Backstepping technique, the proposed control scheme eliminates the "explosion of complexity" problem and relaxes the constraint condition on the reference signal. And then, it is warranted that the closed-loop system is semi-globally ultimately uniformly bounded and the tracking error is always limited to the specified region bounded by the performance functions. Two simulation examples are used to demonstrate the feasibility of the developed technique in this paper.**

***Index Terms*—Periodically Time-Varying Parameters, Tracking Control, Prescribed Performance, Command Filter, Neural Networks**

I. Introduction

As a distinct type of hybrid systems, switched systems are composed of several subsystems and a logical switching law that governs how to switch between subsystems. Switched systems have captured many scholars' attention because of their wide applications, where quintessential examples include networked systems, robotic systems [1], traffic surveillance control systems [2] and switched RLC circuits [3]. It is common knowledge that the asymptotically stability of the switched systems with arbitrary switching can be guaranteed if a common Lyapunov function (CLF) exists for all subsystems satisfy [4]. However, constructing the CLF for an arbitrary switching systems is not an effortless task. By constructing proper CLFs, some important results on arbitrary switching systems have been deduced in [5]–[8]. For instance, by introducing two switching functions in [5] and [6], the tracking control problems with prior tracking accuracy have been examined for the arbitrarily switched nonlinear systems. Noted that, these control strategies reported in [5]–[8] are designed in the framework of the traditional Backstepping method. Nevertheless, when order n of the controlled systems rises, the repeated derivatives of the virtual control functions will become very complicated, which is called as the "explosion of complexity" (EOC) problem. The EOC issue results in the implementation difficulty of the designed controllers. Thus, there naturally arises a problem that how to deal with the EOC issue.

In order to tackle the EOC problem, Swaroop D et al., in [9], propose a dynamic surface control (DSC) strategy for the first time. In [10] and [11], by combining the DSC strategy with the Backstepping method, the EOC problem is solved successfully. However, in these studies, the filtering errors caused by the filtering process are ignored, which may affect the system control performance. To avoid the influence of the filtering error, afterwards, a mechanism with error compensation is introduced to compensate the filtering error, and then, a command filtered Backstepping (CFB) control method is proposed in [12]. Recently, quiet a few remarkable advances have been made by using the command filter and adaptive Backstepping (CFAB) method, see [13]–[20]. For instance, a CFB control scheme is reported in [13] to deal with the tracking control problem for the strict-feedback nonlinear systems. For a kind of the nonlinear systems with unknown constant parameter, a new implementation technique is proposed [14] by combining the CFAB control method. In addition, by combining CFB design with some approximation techniques, e.g., fuzzy logic systems (FLSs) and neural networks (NNs), some adaptive control strategies are developed for uncertain switched nonlinear system [18]–[20]. Noted that there is considerable amount of literatures on the systems with constant parameters. However, to date, far too little attention has been paid to control the systems with periodically time-varying parameters (PTVPs).

The periodically time-varying disturbances/parameters play a significant role in practical applications, such as numerical control machines [21], [22] and van der Pol oscillator [23]. Obviously, the PTVPs are more complex than constants parameters, so thus its existence increases the difficulty of system control design undoubtedly. Recently, a new adaptive learning control technique is constructed for the nonlinear systems [24], where an fourier series expansion (FSE) method

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is leveraged to overcome the challenges that arise from the periodically time-varying reference signals. Coincidentally, some appropriate techniques have been carried out by combining FSE [22], [25]–[30]. For instance, in [25] and [26], FSE and radial basis function neural network (RBFNN) are combined together to handle the nonlinearly parameterised and periodically disturbed functions, and two adaptive neural DSC schemes are constructed for strict-feedback nonlinear systems with unknown control gain functions and uncertain multiple input multiple output (MIMO) strict-feedback nonlinear systems, respectively. In addition, the tracking control problems are addressed in [27], [28] by introducing the DSC into the backstepping method, and the unknown periodically disturbed system functions are approximated by using FLS and FSE. Moreover, Chen et al. in [29], [30] propose a novel approximation method by utilizing multilayer neural network and FSE to model the periodically time-varying and nonlinearly parameterized (PTVNP) system functions. However, the previously published studies are limited to nonlinear systems without switching function, and the PTVNP switching systems have rarely been studied up to now.

Another, the tracking control, as one of the important issues in the control field, has received much attention recently. In practical applications, high tracking control performances, such as convergence domain, convergence speed and overshoot, are required. However, the most existing results only assure that the tracking error falls into to a small but unknown neighborhood of zero [28]–[31]. To obtain better performance, predefined performance control (PPC) is proposed in [32], [33], where the tracking error converges to a predetermined small set of residuals, the speed of convergence is great than or equal a pre-specified value and the maximum permissible overshoot is below a sufficiently small pre-specified constant. Obviously, this method takes into account the transient and steady-state performance of the system simultaneously, and thus, in recent years, a large number of meaningful results are reported [34]–[38] by using the PPC technique. For instance, by introducing a novel error transformation function, an event-triggered PPC scheme is developed for the nonlinear system with unknown control direction and actuator fault [34]. Moreover, the tracking control with prescribed performance is addressed for the uncertain strict-feedback nonlinear systems in [35], [36]. Furthermore, some control strategies by combining with the PPC method are proposed for the switching systems in [37], [38], such that tracking performance is ensured. However, it is worth noting that few scholars pay attention to the prescribed tracking control performance for the nonlinear systems with unknown PTVPs and switching functions.

As a result of the foregoing observation, we first attempt to address the predefined tracking control performance issue for a category of uncertain nonlinear systems subject to unknown nonlinear parameterized switching functions with PTVPs. The approximator by combining FSE with RBFNN is constructed to approximate the unknown PTVNP functions. By using an error transformation method and the CFAB technique, a novel command filter-based adaptive NN control scheme is developed and the new compensating signals are constructed, such that the predefined performance of the tracking error is warranted. In comparison to existing works, the following contributions are worth to be emphasized:

1) This paper focuses on a class of nonlinear systems with unknown PTVNP switching functions. The considered systems are more realistic and more complex than those with unknown functions of constant parameters published in [18]–[20]. Moreover, in comparison with the results in [25], [26], where the approximation errors are ignored, the estimates of these approximation errors's upper bounds can be acquired by the designed adaptive laws in our paper, and the performance of the approximation is presented in a more concise way.

2) The prescribed tracking performance control problem of the considered system is first investigated. Compared with the tracking control performance obtained in [28]–[31] where the tracking error converges to a small but unknown residual set, in this context, the tracking error is always restricted to the given region defined by the performance functions and converges with the performance function. Although the PPC problem is solved [10], [34]–[38], they do not consider nonlinear systems with unknown PTVNP switching functions.

3) A new command filter-based adaptive prescribed tracking performance control scheme is developed. Differently from the traditional Backstepping technique in [5]–[8], [29], [30], the method proposed in our paper eliminates the EOC problem, reduces the computational burden, and relaxes the assumption of the reference signal whose nth-order derivatives are continuous. Additionally, in contrast to the control strategies published [10] based on DSC technique, to achieve better system performance, the novel compensating signals are designed to compensate for the filtering errors.

This paper is structured as follows: Section 2 contains the description of the problem and some preliminaries for the reference signal, performance function and unknown system functions. In Section 3, a command-filter-based adaptive NN control is provided. And the stability of the closed-loop system is analyzed in Section 4. Two simulation studies are performed in the next section to indicate the validity of the created control approach. Finally, Section V provides the conclusions of our paper.

Notations

R^n	n-dimensional Euclidean space
$\|X\|$	the Euclidean norm of vector X
$\mathbf{c}_{[m]}$	m-dimensional column vector with elements c
$\mathrm{diag}\{\mathbf{c}_{[m]}\}$	the $m \times m$ diagonal matrix with elements c
$\Lambda > 0$	positive definite matrix Λ
Λ^T	the transposition matrix of matrix Λ
$\|\Lambda\|_F$	the Frobenius norm of matrix Λ
$tr(\Lambda^T\Lambda)$	the trace of matrix $\Lambda^T\Lambda$ (i.e., sum of diagonal elements of matrix $\Lambda^T\Lambda$)

II. Problem Formulation and Preliminaries

A. Problem Formulation

Consider the following class of nonlinear systems:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{i,\varrho(t)}(\bar{x}_i, \theta_i(t)), \ i = 1, 2, \ldots, n-1, \\ \dot{x}_n = u + f_{n,\varrho(t)}(\bar{x}_n, \theta_n(t)), \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i, i = 1, \ldots, n$ is the system state vector; u and y denote the system control input and output, respectively; $\varrho(t) : [0, +\infty) \to \mathcal{M} = \{1, 2, \ldots, m\}$ is a switching signal; for any $i = 1, 2, \ldots, n$, $f_{i,\varrho(t)}(\bar{x}_i, \theta_i(t))$ is an unknown smooth nonlinear function with unknown periodic parameter $\theta_i(t)$, and satisfies $f_{i,\varrho(t)}(0, 0) = 0$.

Remark 1. *Parameter $\theta_i(t)$ in (1) is bounded and represents any periodic signal or any compound periodical function, such as sine function, tangent function, and so forth.*

The objective of this paper is to design a command filter-based adaptive NN control strategy, such that all the signals of the closed-loop system is semi-globally ultimately uniformly bounded (SGUUB) [39] and it can be guaranteed that the tracking error signal keeps within the prescribed bounds of the performance function.

To achieve this purpose, some preliminaries are provided for the next controller design.

Assumption 1. *[14]. Reference trajectory $y_d(t)$ and its first derivative are continuous.*

Remark 2. *Compared with the constraint condition on the reference signal which is nth-order differentiable in [5]–[8], [29], [30], in our paper, the continuity of the reference signal and its first derivative is requested. Obviously, the condition is more relaxed.*

B. Performance Function

In this section, performance function $\rho(t)$ originally proposed by Bechlioulis et in [32], [33] is introduced to confine that tracking error $e_1(t) = y_1(t) - y_d(t)$ satisfies that

$$-\underline{g}\rho(t) < e_1(t) < \bar{g}\rho(t), \tag{2}$$

where $\underline{g}, \bar{g}$ are positive constants, and smooth function $\rho(t)$ has the following properties [32]: 1) $\rho(t) > 0$; 2) $\dot{\rho}(t) \leq 0$; 3) $\lim_{t\to\infty} \rho(t) = \rho_\infty > 0$. Apparently, from (2), for initial error $e_1(0)$, one has the following inequality

$$-\underline{g}\rho(0) < e_1(0) < \bar{g}\rho(0). \tag{3}$$

The relationship between tracking error $e_1(t)$ and performance function $\rho(t)$ stated above is clearly shown in Fig. 1.

Throughout this paper, $\rho(t) = (\rho_0 - \rho_\infty) e^{-ct} + \rho_\infty$, ($\rho_0$, ρ_∞ and c are positive constants) is used as a performance function, which satisfies all aforementioned properties of $\rho(t)$ obviously.

Remark 3. *Constant ρ_∞ indicates the maximum permissible dimension of tracking error $e_1(t)$ in steady state. Moreover, c is the decreasing pace of $\rho(t)$, which controls the speed of convergence of $e_1(t)$. And the maximum overshoot of $e_1(t)$*

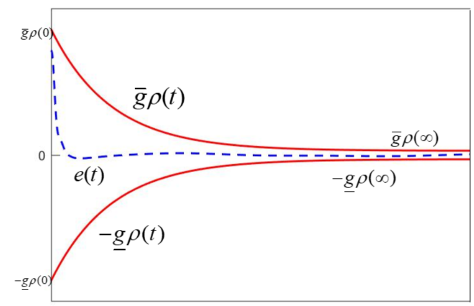

Fig. 1. performance function

is less than $\bar{g}\rho(0)$ and is more than $\underline{g}\rho(0)$, and $\bar{g}$ and $\underline{g}$ are arbitrary positive constant values and may not be equal. Thus, different tracking performance can be achieved by selecting appropriate performance functions $\rho(t)$ and constants $\bar{g}$ and $\underline{g}$. In other word, when parameters ρ_0, ρ_∞ and c are given, the performance of the tracking error signal is predetermined.

To achieve the prescribed performance, an error transformation related to the performance function and tracking error is introduced to transform constraint condition (2) into the unconstrained one [10].

$$e_1(t) = \rho(t) R\left(\underline{\zeta}_1\right), \tag{4}$$

where smooth function

$$R\left(\underline{\zeta}_1\right) = \frac{\bar{g}e^{\underline{\zeta}_1} - \underline{g}e^{-\underline{\zeta}_1}}{e^{\underline{\zeta}_1} + e^{-\underline{\zeta}_1}} \tag{5}$$

is strictly increasing and satisfies that $\lim_{\underline{\zeta}_1\to-\infty} R\left(\underline{\zeta}_1\right) = -\underline{g}$ and $\lim_{\underline{\zeta}_1\to+\infty} R\left(\underline{\zeta}_1\right) = \bar{g}$. And thus, the inverse function of (4) is

$$\underline{\zeta}_1 = R^{-1}\left(\frac{e_1}{\rho}\right) = \frac{1}{2}\ln\left(\frac{\frac{e_1}{\rho} + \underline{g}}{\bar{g} - \frac{e_1}{\rho}}\right). \tag{6}$$

Afterwards, to solve the zero equilibrium point inconformity problem in the state transformation, transformed error $\underline{\zeta}_1$ can be rewritten as

$$\zeta_1 = \underline{\zeta}_1 - \frac{1}{2}\ln\left(\frac{\underline{g}}{\bar{g}}\right), \tag{7}$$

And then, the derivation of ζ_1 is

$$\dot{\zeta}_1 = \chi_1\left(\dot{e}_1 - \frac{\dot{\rho}}{\rho}e_1\right), \tag{8}$$

where

$$\chi_1 = \frac{1}{2\rho}\left(\frac{1}{\frac{e_1}{\rho} + \underline{g}} - \frac{1}{\frac{e_1}{\rho} - \bar{g}}\right) > 0, \tag{9}$$

$$\dot{\rho} = -c\left(\rho_0 - \rho_\infty\right)e^{-ct}. \tag{10}$$

C. FSE-RBFNN based approximator

To successfully control system (1), the unknown system dynamics should first be addressed. In recent years, for unknown system functions, several general intelligent approximators are proposed, where the popular design methods mainly include

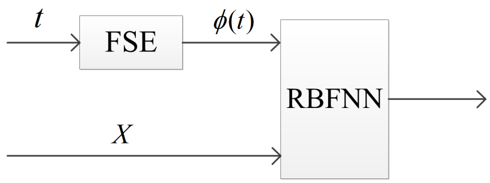

Fig. 2. FSE-RBFNN Approximator

FLS and NNs. By combining FLS or NNs with the adaptive method, several control problems for the uncertain systems have been solved in [39]–[46].

In our article, the NN approximation method is applied to identify unknown continuous function $f(x, \theta(t))$. As cited in [47], $f(x, \theta(t))$ with constant parameter ($\theta(t) = \theta$) or no parameter ($\theta(t) = 0$) can be identified by using RBFNN over a compact set Ω. However, when $\theta(t)$ is an unknown periodically time-varying parameter (PTVP), it is not an easy task to cope with unknown system functions. Inspired by [24] and [47], the discussion on a new approximation method is presented in what follows.

First of all, for unknown PTVP $\theta(t)$, a common linearly parameterized approximation method, FSE reported in [24] can be represented as

$$\theta(t) = a^T \phi(t) + \varepsilon(t), \tag{11}$$

where $a = [a_1, \ldots, a_q]^T \in R^q$ represents the weight vector; and node number q is an odd integer; $\varepsilon(t)$ is the truncation error; $\phi(t) = [\phi_1(t), \ldots, \phi_q(t)]^T$ with $\phi_1(t) = 1$, $\phi_{2j}(t) = \sqrt{2}\sin(2\pi jt/\mathrm{T})$ and $\phi_{2j+1}(t) = \sqrt{2}\cos(2\pi jt/\mathrm{T})$, $j = 1, \ldots, (q-1)/2$, where T means the period of parameter $\theta(t)$, which is a known constant.

Combining the RBFNN reported in [47], a new approximation method, FSE-RBFNN, is constructed over compact set Ω. And it is described as (see Fig. 2)

$$f(x, \theta(t)) = W^{*T} S(A^{*T} Z) + \mu(Z), \ |\mu(Z)| \le \bar{\mu} \tag{12}$$

where $Z = (x, \phi(t)) \in R^{q+1}$ is the NN input vector with number of FSE nodes q; μ denotes the inherent NN approximation error and $\bar{\mu}$ is the upper bound of μ; $A^* = [A_1, A_2, \ldots, A_p]^T \in R^{(q+1)\times p}$ with $A_\iota = [1, a_{\iota,1}, a_{\iota,2}, \ldots, a_{\iota,q}]^T \in R^{q+1}$ ($\iota = 1, 2, \ldots, p$) represents the first-to-second layer interconnection weight; and $W^* = [w_1, w_2, \ldots, w_p]^T \in R^p$ is the second-to-third layer interconnection vector with number of NN nodes $p > 1$. And the ideal weight is

$$(W^*, A^*) = \arg \min_{(W,V)} \left\{ \sup_{Z\in\Omega} \mid f(x, \theta(t)) - \hat{W}^T S(\hat{A}^T Z) \mid \right\}, \tag{13}$$

where $\hat{A}$ and $\hat{W}$ are the estimates of ideal weight vectors A^* and W^*. And $S(Z) = [s_1(Z), s_2(Z), \cdots, s_p(Z)]^T \in R^p$ denotes the basis function vector. Throughout this paper, $s_\iota(Z)$ are chosen as the commonly used Gaussian functions $s_\iota(Z) = e^{-\frac{\|Z-c_\iota\|^2}{2\sigma_\iota^2}}$ with $c_\iota = [c_{\iota 1}, c_{\iota 2}, \ldots, c_{\iota i}]^T$ ($\iota = 1, 2, \ldots, p, i = 1, 2, \ldots, q+1$); Z, $c_{\iota i}$ and σ_ι are input vector, the center and width of Gaussian function $s_\iota(Z)$, respectively.

Lemma 1. *[47]. The estimation error of approximator (12) can be described as*

$$\begin{aligned} &\hat{W}^T S\left(\hat{A}^T Z\right) - W^{*T} S\left(A^{*T} Z\right) \\ &= \tilde{W}^T \left(\hat{S} - \hat{S}'\hat{A}^T Z\right) + \hat{W}^T \hat{S}' \tilde{A}^T Z + d_u, \end{aligned} \tag{14}$$

where $\hat{S} = S\left(\hat{A}^T Z\right)$, $\hat{S}' = \mathrm{diag}\left\{\hat{s}'_1, \hat{s}'_2, \ldots, \hat{s}'_p\right\}$ *with* $\hat{s}'_l = \frac{\mathrm{d}[s(Z_a)]}{\mathrm{d}Z_a}|_{Z_a=\hat{A}_\iota^T Z}$, $(\iota = 1, 2, \ldots, p)$*; and error* d_u *satisfies that*

$$\|d_u\| \le \|A\|_F \left\|Z\hat{W}^T\hat{S}'\right\|_F + \|W^*\| \left\|\hat{S}'\hat{A}^T Z\right\| + |W^*|_1, \tag{15}$$

and $\|d_u\| \le d$ *with upper bound* $d > 0$.

III. Command Filter-Based Adaptive NN Control Design

The command filter-based adaptive NN controllers are presented for the considered nonlinear system in this section. Before displaying the designed controllers, we first define the tracking errors and their compensating signals.

Define error variables

$$\begin{cases} z_1 = \zeta_1, \\ z_i = x_i - x_{i,c}, \quad i = 2, \ldots, n, \end{cases} \tag{16}$$

where $x_{i,c}$ $(i = 2, 3, \ldots, n)$ are the outputs of the command filter. The command filter reported in [12] is defined as

$$\begin{cases} \dot{\eta}_{i,1} = \omega_n \eta_{i,2}, \quad i = 1, 2, \ldots, n-1, \\ \dot{\eta}_{i,2} = -2\zeta\omega_n\eta_{i,2} - \omega_n\left(\eta_{i,1} - \alpha_i\right), \end{cases} \tag{17}$$

with $x_{i+1,c}(t) = \eta_{i,1}$ and $\dot{x}_{i+1,c}(t) = \omega_n\eta_{i,2}$ as the filter outputs. The initial values of filter (17) are $\eta_{i,1}(0) = 0$ and $\eta_{i,2}(0) = 0$. The design parameters of filter (17) satisfy $\omega_n > 0$ and $\zeta \in (0, 1]$.

Next, we define compensated error signal v_i

$$\begin{cases} v_1 = z_1 - \xi_1, \\ v_i = z_i - \xi_i. \quad i = 2, \ldots, n, \end{cases} \tag{18}$$

where ξ_i is a compensating signal to be devised.

Remark 4. *It should be pointed out that a filtering error will be generated when virtual function* α_i *passes through the filter, which will add the difficulty to obtain better tracking performance. Thus, to eliminate the effect of errors* $x_{i+1,c} - \alpha_i$ $(i = 1, 2, \ldots, n)$*, the compensating signals are introduced.*

To achieve the tracking control performance, when $i = 1$, the virtual controller is designed as

$$\alpha_1 = -c_1 v_1 \chi_1 \hat{W}_1^T S(\hat{A}_1^T Z_1) - c_1 v_1 \chi_1 \hat{\delta}_1 + \dot{y}_d + \frac{\dot{\rho}}{\rho} e_1 - l_1 z_1. \tag{19}$$

And the dynamic of compensating signal ξ_1 is constructed as

$$\dot{\xi}_1 = \chi_1[-l_1\xi_1 + (x_{2,c} - \alpha_1) + \xi_2], \tag{20}$$

with $\xi_1(0) = 0$. The adaptive laws are

$$\begin{cases} \dot{\hat{W}}_1 = \Gamma_1(\hat{S}_1 - \hat{S}'_1\hat{A}_1^T Z_1)c_1 v_1^2 \chi_1^2, \\ \dot{\hat{A}}_1 = \Upsilon_1 Z_1 \hat{W}_1^T \hat{S}'_1 c_1 v_1^2 \chi_1^2, \\ \dot{\hat{\delta}}_1 = \lambda_1 c_1 v_1^2 \chi_1^2. \end{cases} \tag{21}$$

In what follows, the i-th $(i = 2, \ldots, n)$ virtual controllers are constructed as

$$\alpha_i = -k_i z_i + \dot{x}_{i,c} - \chi_1(v_{i-1} - \xi_{i-1}) - c_i v_i \hat{\delta}_i - c_i v_i \hat{W}_i^T S(\hat{A}_i^T Z_i). \tag{22}$$

Controller α_n represents system (1) control input u. And then, the dynamics of compensating signal ξ_i are designed as

$$\begin{cases} \dot{\xi}_i = -k_i \xi_i + (x_{i+1,c} - \alpha_i) + \xi_{i+1} - \chi_1 \xi_{i-1}, \\ \dot{\xi}_n = -k_n \xi_n - \xi_{n-1}, \end{cases} \tag{23}$$

with $\xi_i(0) = 0$. The adaptive laws are designed as

$$\begin{cases} \dot{\hat{W}}_i = \Gamma_i(\hat{S}_i - \hat{S}_i' \hat{A}_i^T Z_i) c_i v_i^2, \\ \dot{\hat{A}}_i = \Upsilon_i Z_i \hat{W}_i^T \hat{S}_i' c_i v_i^2, \ i = 2, \ldots, n, \\ \dot{\hat{\delta}}_i = \lambda_i c_i v_i^2. \end{cases} \tag{24}$$

In Eqs. (19)–(24), for $i = 1, 2, \ldots, n$, parameters l_1, c_i and k_i are positive constants; $\hat{W}_i$, $\hat{A}_i$ and $\hat{\delta}_i$ are the estimations of W_i^*, A_i^* and δ_i, $\tilde{W}_i = \hat{W}_i - W_i^*$, $\tilde{A}_i = \hat{A}_i - A_i$ and $\tilde{\delta}_i = \hat{\delta}_i - \delta_i$; λ_i are positive constants; $\Gamma_i = \Gamma_i^T > 0$ and $\Upsilon_i = \Upsilon_i^T > 0$ are both the adaptive gain matrixes.

IV. Stability Analysis

The stability of the closed-loop system will be analyzed and proved in this section. In what follows, the system stability is summarized as Theorem 1.

Theorem 1. *Under Assumption 1, consider the plant consisting of system (1), controllers (19) and (22), the dynamics of compensating signal (20) and (23), and parameters adaptive laws (21) and (24), such that*

a) all the signals of the closed-loop system keep uniformly ultimately bounded, and

b) the prescribed tracking control performance is assured.

Proof. Based on the Lyapunov function theory, a new command filter-based prescribed tracking performance control strategy is developed to prove Theorem 1 by using Backstepping method and the PPC technique. For clarity, the design process of the proposed control algorithm is shown as follows.

Step *1***.** According to the error definitions in (7), (16) and (18), the dynamic of compensated error signal v_1 is

$$\begin{aligned} \dot{v}_1 &= \chi_1(\dot{x}_1 - \dot{y}_d - \frac{\dot{\rho}}{\rho} e_1) - \dot{\xi}_1 \\ &= \chi_1(x_2 + f_{1,\varrho(t)}(\bar{x}_1, \theta_1(t)) - \dot{y}_d - \frac{\dot{\rho}}{\rho} e_1) - \dot{\xi}_1. \end{aligned} \tag{25}$$

Consider Lyapunov function candidate

$$V_1 = \frac{1}{2} v_1^2,$$

and its time derivative is obtained as

$$\begin{aligned} \frac{\mathrm{d}V_1}{\mathrm{d}t} =& v_1 \chi_1 [(x_2 - x_{2,c}) + (x_{2,c} - \alpha_1) + \alpha_1 \\ &+ f_{1,\varrho(t)}(\bar{x}_1, \theta_1(t)) - \dot{y}_d - \frac{\dot{\rho}}{\rho} e_1 - \chi_1^{-1} \dot{\xi}_1], \end{aligned} \tag{26}$$

where α_1 is a virtual controller.

By using the Young's inequality, we obtain

$$\nu_1 \chi_1 f_{1,\varrho(t)}(\bar{x}_1, \theta_1(t)) \le c_1 \nu_1^2 \chi_1^2 \Lambda_1 + \frac{1}{4c_1}, \tag{27}$$

where $\Lambda_1 = \sum_{l=1}^m f_{1,l}^2(\bar{x}_1, \theta_1(t))$ is an unknown smooth function with unknown PTVP $\theta_1(t)$, $l = \varrho(t) \in \mathcal{M} = \{1, 2, \ldots, m\}$ represents l-th subsystem is active, c_1 is an arbitrarily positive constant.

By using (12), we have

$$\Lambda_1 = W_1^{*T} S(A_1^{*T} Z_1) + \mu_1(Z_1), \quad |\mu_1(Z_1)| \le \bar{\mu}_1, \tag{28}$$

where the input of basis function $S(A_1^{*T} Z_1)$ is vector $Z_1 = [\bar{x}_1, \phi(t)]^T \in \Omega$, and $\bar{\mu}_1 > 0$ is an unknown constant.

Substituting (19), (20) and (28) into (26) and based on Lemma 1, we get

$$\begin{aligned} \frac{\mathrm{d}V_1}{\mathrm{d}t} \le& v_1 \chi_1 \Big[(x_2 - x_{2,c}) + (x_{2,c} - \alpha_1) - l_1 v_1\Big] \\ &+ \frac{1}{4c_1} + c_1 v_1^2 \chi_1^2 \Big[W_1^{*T} S(A_1^{*T} Z_1) \\ &- \hat{W}_1^T S(\hat{A}_1^T Z_1) + \bar{\mu}_1 - \hat{\delta}_1\Big] \\ \le& - l_1 \chi_1 v_1^2 + \chi_1 v_1 v_2 - c_1 v_1^2 \chi_1^2 [\hat{W}_1^T \hat{S}_1' \tilde{A}_1^T Z_1 \\ &+ \tilde{W}_1^T (\hat{S}_1 - \hat{S}_1' \hat{A}_1^T Z_1) + \tilde{\delta}_1] + \frac{1}{4c_1}, \end{aligned} \tag{29}$$

where $\delta_1 = \bar{\mu}_1 - d_1$.

Further, consider

$$\bar{V}_1 = V_1 + \frac{1}{2} \tilde{W}_1^T \Gamma_1^{-1} \tilde{W}_1 + \frac{1}{2} tr\{\tilde{A}_1^T \Upsilon_1^{-1} \tilde{A}_1\} + \frac{1}{2\lambda_1} \tilde{\delta}_1^2. \tag{30}$$

From (29), the first derivative of $\bar{V}_1$ is obtained

$$\begin{aligned} \frac{\mathrm{d}\bar{V}_1}{\mathrm{d}t} \le& - l_1 \chi_1 v_1^2 + \chi_1 v_1 v_2 - c_1 v_1^2 \chi_1^2 [\hat{W}_1^T \hat{S}_1' \tilde{A}_1^T Z_1 \\ &+ \tilde{W}_1^T (\hat{S}_1 - \hat{S}_1' \hat{A}_1^T Z_1) + \tilde{\delta}_1] + \frac{1}{\lambda_1} \tilde{\delta}_1 \dot{\hat{\delta}}_1 + \frac{1}{4c_1} \\ &+ \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 + tr\left\{\tilde{A}_1^T \Upsilon_1^{-1} \dot{\hat{A}}_1\right\}. \end{aligned} \tag{31}$$

It is known that

$$\hat{W}_1^T \hat{S}_1' \tilde{A}_1^T Z_1 = \mathrm{tr}\left\{\tilde{A}_1^T Z_1 \hat{W}_1^T \hat{S}_1'\right\}. \tag{32}$$

By substituting (32) into (31), we obtain

$$\begin{aligned} \frac{\mathrm{d}\bar{V}_1}{\mathrm{d}t} \le& - l_1 \chi_1 v_1^2 + \chi_1 v_1 v_2 + \frac{1}{\lambda_1} \tilde{\delta}_1 (\dot{\hat{\delta}}_1 - \lambda_1 c_1 v_1^2 \chi_1^2) \\ &+ \tilde{W}_1^T \Gamma_1^{-1} \Big(\dot{\hat{W}}_1 - \Gamma_1 (\hat{S}_1 - \hat{S}_1' \hat{A}_1^T Z_1) c_1 v_1^2 \chi_1^2\Big) \\ &+ tr\left\{\tilde{A}_1^T \Upsilon_1^{-1} (\dot{\hat{A}}_1 - \Upsilon_1 Z_1 \hat{W}_1^T \hat{S}_1' c_1 v_1^2 \chi_1^2)\right\}. \end{aligned} \tag{33}$$

From (21), Eq. (33) can be further written as

$$\dot{\bar{V}}_1 \le -k_1 \chi_1 v_1^2 + \chi_1 v_1 v_2 + \frac{1}{4c_1}, \tag{34}$$

where $k_1 := l_1 \chi_1 > 0$ since $l_1 > 0$ and $\chi_1 > 0$.

Step i $(2 \le i \le n-1)$**.** Combining (1) with (16), the dynamic of compensating error v_i is expressed as

$$\dot{v}_i = x_{i+1} + f_{i,\varrho(t)}(\bar{x}_i, \theta_i(t)) - \dot{x}_{i,c} - \dot{\xi}_i. \tag{35}$$

Define

$$V_i = \bar{V}_{i-1} + \frac{1}{2} v_i^2. \tag{36}$$

Subsequently, we achieve

$$\begin{aligned}\frac{\mathrm{d}V_i}{\mathrm{d}t} =&\frac{\mathrm{d}\bar{V}_{i-1}}{\mathrm{d}t} + v_i\Big[(x_{i+1} - x_{i+1,c}) + (x_{i+1,c} - \alpha_i) \\ &+ \alpha_i + f_{i,\varrho(t)}(\bar{x}_i, \theta_i(t)) - \dot{x}_{i,c} - \dot{\xi}_i\Big],\end{aligned} \tag{37}$$

where α_i is a virtual controller.

By using $a \leq ca^2 + 1/(4c)$, similar to (27), Eq. (37) can be rewritten as

$$\begin{aligned}\frac{\mathrm{d}V_i}{\mathrm{d}t} \leq& -\sum_{j=1}^{i-1} k_j v_j^2 + \sum_{j=1}^{i} \frac{1}{4c_j} + v_i\Big[v_{i-1} + (x_{i+1} - x_{i+1,c}) \\ &+ (x_{i+1,c} - \alpha_i) + \alpha_i + c_i v_i \Lambda_i - \dot{x}_{i,c} - \dot{\xi}_i\Big],\end{aligned} \tag{38}$$

where $\Lambda_i = \sum_{l=1}^{m} f_{i,l}^2(\bar{x}_i, \theta_i(t))$ is an smooth unknown function needing to be identified.

From (12), we get

$$\Lambda_i = W_i^{*T} S(A_i^{*T} Z_i) + \mu_i(Z_i), \quad |\mu_i(Z_i)| \leq \bar{\mu}_i, \tag{39}$$

where $\bar{\mu}_i > 0$ has the same meaning as $\bar{\mu}_1$ in (28).

Substituting (22), (23) and (39) into (38) results in

$$\begin{aligned}\frac{\mathrm{d}V_i}{\mathrm{d}t} \leq& -\sum_{j=1}^{i} k_j v_j^2 + v_i v_{i+1} + c_i v_i^2\Big[W_i^{*T} S(A_i^{*T} Z_i) \\ &- \hat{W}_i^T S(\hat{A}_i^T Z_i) + \bar{\mu}_i - \hat{\delta}_i\Big] + \sum_{j=1}^{i} \frac{1}{4c_j}.\end{aligned} \tag{40}$$

Based on Lemma 1, it is easy to acquire

$$\begin{aligned}\frac{\mathrm{d}V_i}{\mathrm{d}t} \leq& -\sum_{j=1}^{i} k_j v_j^2 + v_i v_{i+1} + c_i v_i^2\Big[\hat{W}_i^T \hat{S}_i' \tilde{A}_i^T Z_i \\ &+ \tilde{W}_i^T(\hat{S}_i - \hat{S}_i' \hat{A}_i^T Z_i) + \tilde{\delta}_i\Big] + \sum_{j=1}^{i} \frac{1}{4c_j},\end{aligned} \tag{41}$$

where $\delta_i = \bar{\mu}_i - d_i$.

Consider

$$\bar{V}_i = V_i + \frac{1}{2}\tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i + \frac{1}{2} tr\{\tilde{A}_i^T \Upsilon_i^{-1} \tilde{A}_i\} + \frac{1}{2\lambda_i}\tilde{\delta}_i^2. \tag{42}$$

Then, we have

$$\begin{aligned}\frac{\mathrm{d}\bar{V}_i}{\mathrm{d}t} =& \frac{\mathrm{d}V_i}{\mathrm{d}t} + \tilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i + tr\{\tilde{A}_i^T \Upsilon_i^{-1} \dot{\hat{A}}_i\} + \frac{1}{\lambda_i}\tilde{\delta}_i \dot{\hat{\delta}}_i \\ \leq& -\sum_{j=1}^{i} k_j v_j^2 + \sum_{j=1}^{i} \frac{1}{4c_j} + v_i v_{i+1} + \frac{1}{\lambda_i}\tilde{\delta}_i(\dot{\hat{\delta}}_i - \lambda_i c_i v_i^2) \\ &+ \tilde{W}_i^T \Gamma_i^{-1}\Big(\dot{\hat{W}}_i - \Gamma_i(\hat{S}_1 - \hat{S}_1' \hat{A}_i^T \phi(t)) c_i v_i^2\Big) \\ &+ tr\left\{\tilde{A}_i^T \Upsilon_i^{-1}(\dot{\hat{A}}_i - \Upsilon_i Z_i \hat{W}_i^T \hat{S}_i' c_i v_i^2)\right\}.\end{aligned} \tag{43}$$

According to (24), we obtain

$$\frac{\mathrm{d}\bar{V}_i}{\mathrm{d}t} \leq -\sum_{j=1}^{i} k_j v_j^2 + \sum_{j=1}^{i} \frac{1}{4c_j} + v_i v_{i+1}. \tag{44}$$

Remark 5. *From (44) with $i = 2$, it is obtained that when $i > 2$, Eqs (22)–(23) will be independent of χ_1. That is, $\chi_1(v_{i-1} - \xi_{i-1})$ of controller (22) and $\chi_1\xi_{i-1}$ of (23) becomes $v_{i-1} - \xi_{i-1}$ and ξ_{i-1}, respectively.*

Step *n*. Consider

$$\bar{V}_n = V_n + \frac{1}{2}\tilde{W}_n^T \Gamma_n^{-1} \tilde{W}_n + \frac{1}{2} tr\{\tilde{A}_n^T \Upsilon_n^{-1} \tilde{A}_n\} + \frac{1}{2\lambda_n}\tilde{\delta}_n^2, \tag{45}$$

and $V_n = \bar{V}_{n-1} + \frac{1}{2}v_n^2$.

Using the same computational process as done in **Step *i***, noticing (44) with $i = n - 1$ and combining (22) and (24) with $i = n$, it is easy to obtain that

$$\frac{\mathrm{d}\bar{V}_n}{\mathrm{d}t} \leq -\sum_{i=1}^{n} k_i v_i^2 + \sum_{i=1}^{n} \frac{1}{4c_i}. \tag{46}$$

From (46), it is clearly evident that $\frac{\mathrm{d}\bar{V}_n}{\mathrm{d}t} \leq 0$. According to the Boundedness Theorem (e.g., Theorem 4.18 in [48]), we can conclude directly that for $1 \leq i \leq n$, error signals v_i, $\tilde{W}_i$, $\tilde{A}_i$ and $\tilde{\delta}_i$ are uniformly ultimately bounded. And then, $\hat{W}_i$, $\hat{A}_i$ and $\hat{\delta}_i$ are bounded owing to $\tilde{W}_i = \hat{W}_i - W_i^*$, $\tilde{A}_i = \hat{A}_i - A_i^*$, and $\tilde{\delta}_i = \hat{\delta}_i - \delta_i$. Furthermore, the stability of the compensated error signal is provided. Choose the Lyapunov function of the compensating system as follows:

$$V_\xi = \frac{1}{2}\sum_{i}^{n} \xi_i^2. \tag{47}$$

Substituting (20) and (23) into the time derivative of function V_ξ, we have

$$\begin{aligned}\frac{\mathrm{d}V_\xi}{dt} =& \xi_1\dot{\xi}_1 + \xi_2\dot{\xi}_2 + \cdots + \xi_n\dot{\xi}_n \\ =& -l_1\chi_1\xi_1^2 + \chi_1\xi_1(x_{2,c} - \alpha_1) + \chi_1\xi_1\xi_2 \\ &- k_2\xi_2^2 + \xi_2(x_{i+1,c} - \alpha_i) + \xi_2\xi_3 - \chi_1\xi_1\xi_2 \\ &- k_i\xi_i^2 + \xi_i(x_{i+1,c} - \alpha_i) + \xi_i\xi_{i+1} - \xi_i\xi_{i-1} \\ &+ \cdots - k_n\xi_n^2 - \xi_n\xi_{n-1},\end{aligned} \tag{48}$$

As cited in [12], $|x_{i+1,c} - \alpha_i| \leq \varpi_i$ can be ensured with arbitrarily small positive constants ϖ_i $(i = 1, 2, \ldots, n)$. Therefore, with the help of the inequality that $ab \leq 1/2a^2 + 1/2b^2$, Eq. (48) can be rewritten as

$$\frac{\mathrm{V_{n+1}}}{dt} \leq -\mathcal{K}V_{n+1} + 1/2\sum_{i=1}^{n-1} \varpi_i^2 \tag{49}$$

where $\mathcal{K} = 2\min\{l_1\chi_1 - 1/2, k_2 - 1/2, \ldots, k_n - 1/2\}$. Thus, according to the Boundedness Theorem [48], the compensating signals are bounded. And then, based on the boundedness of v_i and ξ_i and from (18), we can obtain that error signals z_i are bounded. Furthermore, when z_1 is bounded, the prescribed performance of tracking error e_1 is warranted, which is proved in [33].

In conclusion, from (46), it can be readily available that all the signals are SGUUB, and the prescribed performance of tracking error e_1 is guaranteed. That is, Eq. (2) holds.

Consequently, the proof of Theorem 1 is completed. □

Remark 6. *From (5) and the boundedness of transformation error ξ_1, it is concluded that the value of $e(t)$ will not be infinitely close to the value of $\bar{g}\rho(t)$ or $-\underline{g}\rho(t)$. That is, the tracking error is always limited in the bound $(-\underline{g}\rho(t), \bar{g}\rho(t))$, which means that inequality (2) holds.*

TABLE I: Control Parameters

Parameters in (53)			Other parameters		
Γ_1	diag$\{\mathbf{10}_{[5]}\}$		Node numbers	p	5
Γ_2	diag$\{\mathbf{10}_{[5]}\}$			q	5
Υ_1	diag$\{\mathbf{2}_{[5]}\}$		Control gains	l_1	5
Υ_2	diag$\{\mathbf{10}_{[5]}\}$			k_2	10
λ_1	2		Command filter (17)	ζ	0.5
λ_2	5			ω_n	100
$\underline{g}$	1		performance function	ρ_0	2
$\bar{g}$	1			ρ_∞	0.02
				c	2

V. Simulation Studies

In this section, two simulations are performed to the validity of the constructed control strategy.

Consider the nonlinear system described below

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\varrho(t)}(\bar{x}_1, \theta_1(t)), \\ \dot{x}_2 = u + f_{2,\varrho(t)}(\bar{x}_2, \theta_2(t)), \quad \varrho(t) = 1, 2, \\ y = x_1, \end{cases} \tag{50}$$

where u, y and $\bar{x}_2 = [x_1, x_2]^T$ denote the system control input, output and state vector, respectively; $\theta_1(t)$ and $\theta_2(t)$ are unknown PTVPs; reference signal $y_d(t)$ is chosen as $y_d(t) = \sin(2t) + 2\cos(t)$; the performance function is $\rho(t) = (2 - 0.1)e^{2t} + 0.1$; the system functions are $f_{1,1} = x_1 \cos(t), f_{1,2} = \cos(x_1 \sin(2t)), f_{2,1} = \sin^2(t)\cos(x_1 x_2)$, and $f_{2,2} = \sin(x_1 x_2 \cos(2t))$. Given a set of initial values $x_1(0) = 1$, $x_2(0) = 1.5$, $\xi_1(0) = 0$, $\hat{W}_1(0) = \mathbf{0}_{[5]}$, $\hat{W}_2(0) = \mathbf{0}_{[5]}$, $\hat{A}_1(0) = \mathbf{0}_{[5]}$, $\hat{A}_2(0) = \mathbf{0}_{[5]}$, $\hat{\delta}_1(0) = 0$ and $\hat{\delta}_2(0) = 0$.

The control strategy developed in Section III are employed to control system (50). Similar to the design process of controllers (19) and (22), the controllers for system (50) are formulated as

$$u = -k_2 z_2 + \dot{x}_{2,c} - \chi_1(v_1 - \xi_1) - c_2 v_2 \hat{W}_2^T S(\hat{A}_2^T Z_2) - c_2 v_2 \hat{\delta}_2, \tag{51}$$

$$\alpha_1 = -c_1 \chi_1 v_1 \hat{W}_1^T S(\hat{A}_1^T Z_1) - c_1 \chi_1 v_1 \hat{\delta}_1 + \dot{y}_d + \frac{\dot{\rho}}{\rho} e_1 - l_1 z_1. \tag{52}$$

At the same time, the parameter learning laws are

$$\begin{cases} \dot{\hat{W}}_1 = \Gamma_1(\hat{S}_1 - \hat{S}_1' \hat{A}_1^T Z_1) c_1 v_1^2 \chi_1^2, \\ \dot{\hat{A}}_1 = \Upsilon_1 Z_1 \hat{W}_1^T \hat{S}_1' c_1 v_1^2 \chi_1^2, \\ \dot{\hat{\delta}}_1 = \lambda_1 c_1 v_1^2 \chi_1^2, \\ \dot{\hat{W}}_2 = \Gamma_2(\hat{S}_2 - \hat{S}_2' \hat{A}_2^T Z_2) c_2 v_2^2, \\ \dot{\hat{A}}_2 = \Upsilon_2 Z_2 \hat{W}_2^T \hat{S}_2' c_2 v_2^2, \\ \dot{\hat{\delta}}_2 = \lambda_2 c_2 v_2^2, \end{cases} \tag{53}$$

and the adaptive law of the compensating signal is

$$\begin{cases} \dot{\xi}_1 = \chi_1[-l_1 \xi_1 + (x_{2,c} - \alpha_1) + \xi_2], \\ \dot{\xi}_2 = -k_2 \xi_2 - \chi_1 \xi_1. \end{cases} \tag{54}$$

Moreover, the corresponding control parameters in (51)–(54) are displayed in Table I.

The simulation results for system (50) are shown in Figs. 3–9. The output signal follows the reference signal, as seen in Fig. 3. To further display clearly, Fig. 4 is provided, where the dotted line represents error signal $e_1(t)$ and the solid lines indicates performance function $\rho(t)$. From Fig. 4, we can see that the trajectory of the tracking error fluctuates between the bounds predetermined by performance function $\rho(t)$. The tracking performance is unquestionably assured. And the trajectories of the designed compensated signal and the control input is shown in Fig. 5 and Fig. 6. Moreover, Fig. 7 depicts the arbitrarily given switching signal's trajectory. The norms of weight estimations $\hat{W}_1, \hat{W}_2$, $\hat{A}_1, \hat{A}_2$ and the upper bound estimations of approximation errors δ_1, δ_2 are shown in Figs. 8–10, respectively. Seeing from Figs. 8–10, apparently, these estimated trajectories are settled within bounded regions.

Example 2: To further illustrate the vitality of the proposed algorithm, the designed adaptive NN controller is applied to the following well-known van der Pol oscillator (van der Pol,

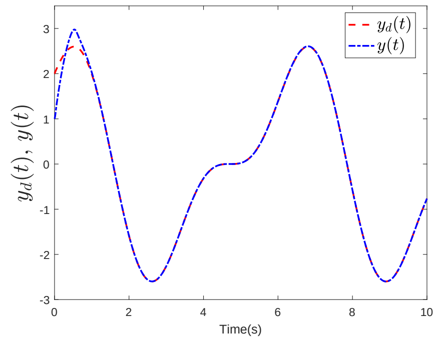

Fig. 3. Reference signal $y_d(t)$ and output signal $y(t)$

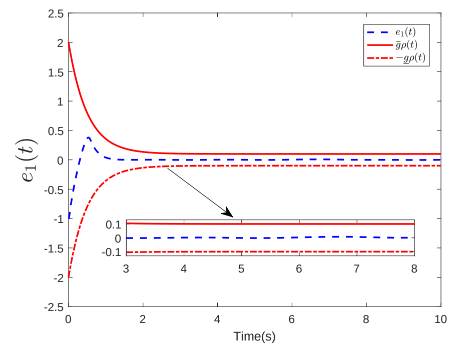

Fig. 4. Error signal $e_1(t)$ and performance function $\rho(t)$

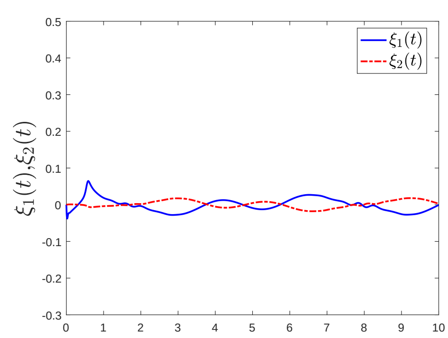

Fig. 5. Compensating signal $\xi_1(t)$

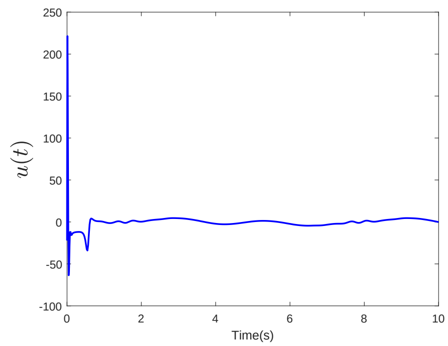

Fig. 6. Control input $u(t)$

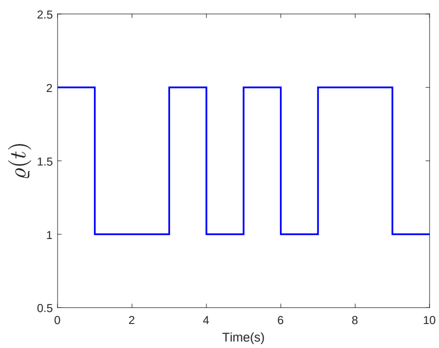

Fig. 7. Switching signal $\varrho(t)$

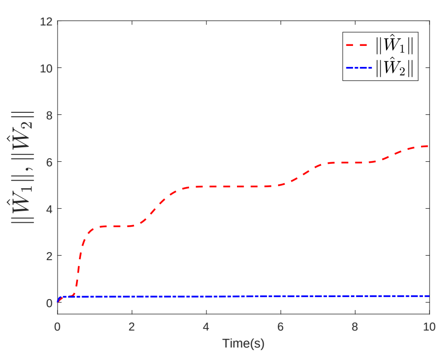

Fig. 8. Norms of the weight estimations $\|\hat{W}_1\|$ and $\|\hat{W}_2\|$

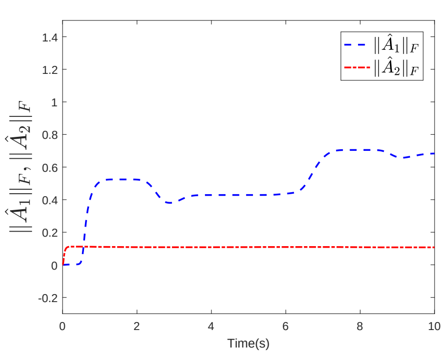

Fig. 9. F-Norms of the weight estimations $\|\hat{A}_1\|_F$ and $\|\hat{A}_2\|_F$

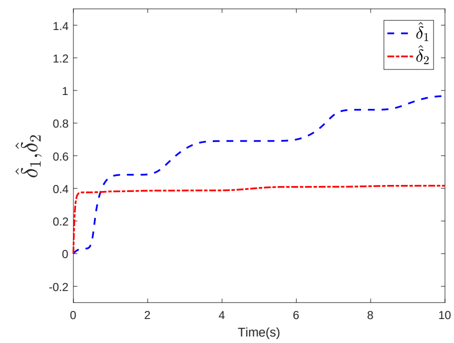

Fig. 10. Estimations of the approximation errors $\hat{\delta}_1$ and $\hat{\delta}_2$

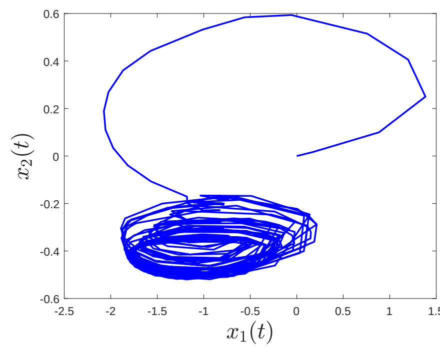

Fig. 11. Van der Pol chaotic attractor

1927):

$$\left\{\begin{array}{l} \dot{x}_1 = x_1 - \frac{1}{3}x_1^3 - x_2 + p + F(t), \\ \dot{x}_2 = 0.1\left(x_1 + a - bx_2\right) + u, \\ y = x_1, \end{array}\right. \tag{55}$$

where $F(t) = q\cos(\omega t)$ is a periodically function. In [23], the chaotic behavior of system (55) without control input (i.e., $u = 0$) is shown in Fig. 11 with $a = 0.7, b = 0.8, p = 0, q = 0.74$ and $\omega = 1$.

In this study, it is assumed that the system exhibits the switching phenomena and $F(t) = q\cos(\omega t)$ is an unknown function. The system with switching signal can be expressed as

$$\left\{\begin{array}{l} \dot{x}_1 = -x_2 + f_{1,\varrho(t)}(x_1, F_1(t)), \\ \dot{x}_2 = u + f_{2,\varrho(t)}(x_1, x_2), \\ y = x_1, \end{array}\right. \tag{56}$$

where $F_1(t) = F(t)$ means the unknown periodic signal with known period $T = 2\pi$; $\varrho(t) = \{1, 2\}$ and the system functions are $f_{1,2} = x_1 - \frac{1}{3}x_1^3 - 0.74\cos(0.5t), f_{1,2} = x_1 - \frac{1}{3}x_1^3 + 0.74\cos(t)$ and $f_{2,1} = f_{2,2} = 0.1(x_1 + 0.7 + 0.8x_2)$. Moreover, the reference signal is $y_d(t) = \cos(t) + \sin(2t)$.

The goal of the control is to design an adaptive controller for system (56) such that i) all the signals of the closed-loop system are SGUUB; ii) tracking error $e_1(t)$ is stable within the prescribed bounds of the performance function, i.e., $-\underline{g}\rho(t) < e_1(t) < \bar{g}\rho(t)$.

TABLE II: Simulation Parameters

Parameters	Values	Parameters	Values
$x_1(0)$	0.2	p	5
$x_2(0)$	1	q	5
$\hat{W}_1(0)$	$\mathbf{0}_{[5]}$	l_1	0.01
$\hat{W}_2(0)$	$\mathbf{0}_{[5]}$	k_2	10
$\hat{A}_1(0)$	$\mathbf{0}_{[5]}$	λ_1	2
$\hat{\delta}_1(0)$	0	λ_2	5
$\hat{\delta}_2(0)$	0	Γ_1	diag$\{\mathbf{1}_{[5]}\}$
ζ	0.5	Γ_2	diag$\{\mathbf{2}_{[5]}\}$
ω_n	100	Υ_1	diag$\{\mathbf{1}_{[5]}\}$
ρ_0	2	$\underline{g}$	1
ρ_∞	0.1	$\bar{g}$	1
c	3	c_1	1

Apparently, the gain of system (56) state x_2 is not positive constant. Hence, the virtual controller is revised as

$$\alpha_1 = -[-c_1 v_1 \chi_1 \hat{W}_1^T S(\hat{A}_1^T Z_1) - c_1 v_1 \chi_1 \hat{\delta}_1 + \dot{y}_d + \frac{\dot{\rho}}{\rho} e_1 - l_1 z_1], \tag{57}$$

and the dynamics of compensating signal ξ_1 is developed as

$$\dot{\xi}_1 = -\chi_1[-l_1\xi_1 + (x_{2,c} - \alpha_1) + \xi_2]. \tag{58}$$

Moreover, the adaptive learning laws are

$$\begin{cases} \dot{\hat{W}}_1 = \Gamma_1(\hat{S}_1 - \hat{S}_1' \hat{A}_1^T Z_1) c_1 v_1^2 \chi_1^2, \\ \dot{\hat{A}}_1 = \Upsilon_1 Z_1 \hat{W}_1^T \hat{S}_1' c_1 v_1^2 \chi_1^2, \\ \dot{\hat{\delta}}_1 = \lambda_1 c_1 v_1^2 \chi_1^2. \end{cases} \tag{59}$$

In addition, function $f_{2,\varrho(t)}(x_1, x_2)$ is not related to the unknown PTVPs, hence, for simplicity, RBFNN instead of FSE-RBFNN is utilized to model the unknown system function. And the controller is

$$u = -k_2 z_2 + \dot{x}_{2,c} + \chi_1(v_1 - \xi_1) - c_2 v_2 \hat{W}_2^T S(\bar{x}_2) - c_2 v_2 \hat{\delta}_2, \tag{60}$$

and the dynamics of compensating signal ξ_2 is developed as

$$\dot{\xi}_2 = -k_2\xi_2 - \chi_1\xi_1. \tag{61}$$

and the adaptive control laws are constructed as

$$\begin{cases} \dot{\hat{W}}_2 = \Gamma_2 S(\bar{x}_2) c_2 v_2^2, \\ \dot{\hat{\delta}}_2 = \lambda_2 c_2 v_2^2. \end{cases} \tag{62}$$

The simulation parameters are provided in Table II, which contains the initial conditions, the number of the FSE-RBFNN nodes, and the suggested parameters in controllers and parameter adaptive laws.

The simulation results are shown in Figs. 12–18. As shown in Fig. 12, the output signal tracks the reference signal. Further analysis showed in Fig. 13 that the tracking error trajectory never exceeds the bounds defined by the performance function, which suggests that the designed algorithm has achieved good tracking performance. Furthermore, Fig. 14 exhibits the trajectory of the arbitrarily chosen switching signal. In addition, the trajectory of the compensating signal is presented in Fig.15. Moreover, the norms of weight estimations $\hat{W}_1, \hat{W}_2$, $\hat{A}_1$ and the upper bound estimations of approximation errors δ_1, δ_2 are shown in Figs. 16–18, respectively. From Fig. 15–18, we can see that these trajectories are stable in a bounded zones.

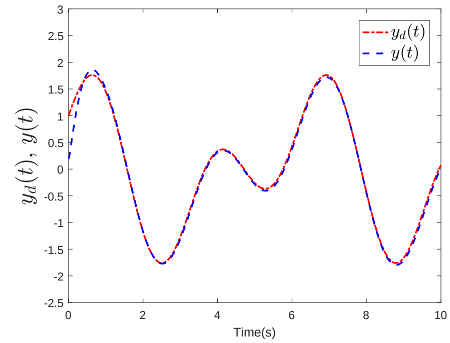

Fig. 12. Reference signal $y_d(t)$ and output signal $y(t)$

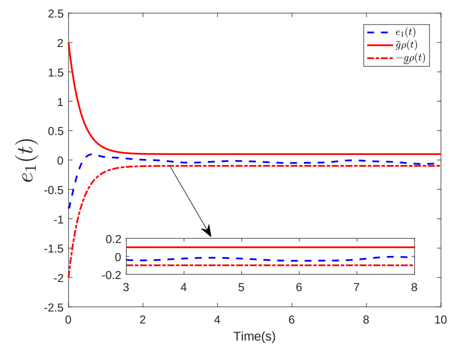

Fig. 13. Error signal $e_1(t)$ and performance function $\rho(t)$

In summary, according to Examples 1 and 2, these simulation results suggest that all the signals of the closed-loop system are SGUUB and the performance of the tracking error is achieve, which accord to Theorem 1. Furthermore, the achieved control performance validates the proposed control method's efficacy.

VI. Conclusion

This paper is to address the prescribed tracking control performance problem for PTVNP uncertain systems. The unknown switching nonlinear functions with PTVPs are identified by exploiting the approximation method which combines RBFNN with FSE. In addition, to avoid the EOC problem, a new adaptive NN tracking control strategy is constructed by

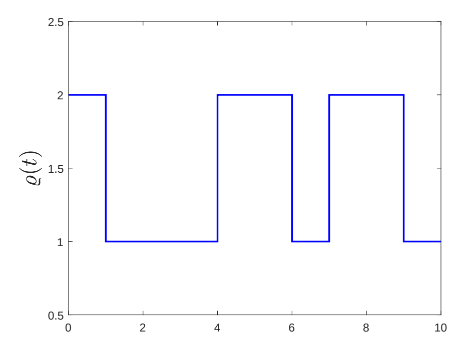

Fig. 14. Switching signal $\varrho(t)$

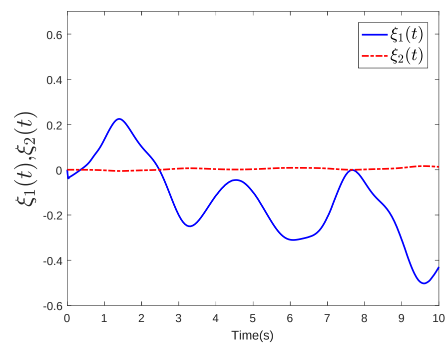

Fig. 15. Compensating signals $\xi_1(t)$ and $\xi_2(t)$

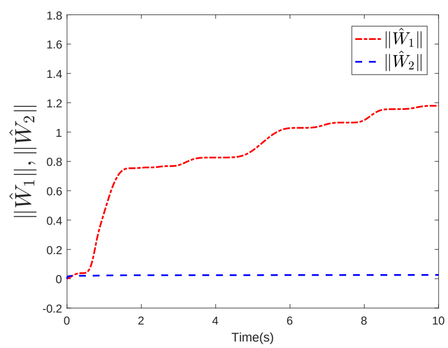

Fig. 16. Norms of the weight estimations $\|\hat{W}_1\|$ and $\|\hat{W}_2\|$

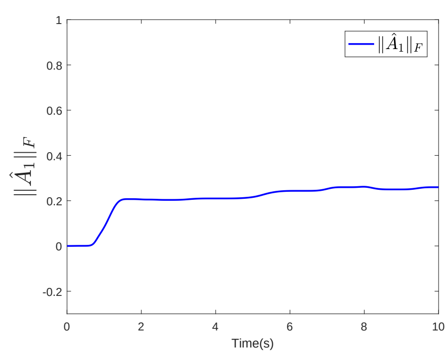

Fig. 17. F-Norm of the weight estimation $\|\hat{A}_1\|_F$

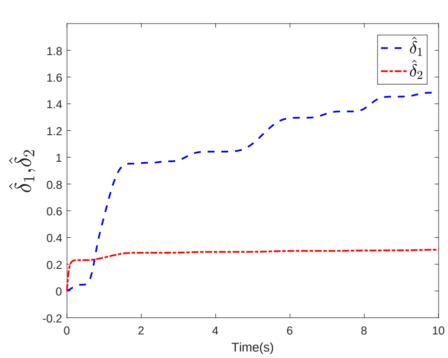

Fig. 18. Estimations of the approximation errors $\hat{\delta}_1$ and $\hat{\delta}_2$

using CFAB technique. At the same time, the adaptive laws including the approximation weights, the approximation errors' upper bounds and the compensating signals, are proposed. Furthermore, it can be proven that the closed-loop system is SGUUB, and the tracking performance is warranted, i.e., the tracking error signal is always kept within the prescribed bounds by the performance functions. Finally, the simulation studies substantiated these properties.

In our context, the tracking control scheme is proposed for a class of the strict-feedback nonlinear systems with switching function and PTVPs. In the future work, we will consider extending the results of this article to a class of the nonstrict-feedback systems.

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Conflict of interest

The authors declare no potential conflict of interests.

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