Discontinuous Sturm-Liouville Problem with Eigenparameter-Dependent Boundary conditions and Herglotzs transmission

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Abstract

This paper is to study a Sturm-Liouville equation Ly:=-p(x)y''+q(x)y=\lambda y with discontinuities in the case that eigenparameter appears not only in the differential equation but also appears in both the boundary conditions \lambda(\alpha'_{1}&y(-a)-\alpha'_{2}y'(-a))-(\alpha_{1}y(-a)-\alpha_{2}y'(-a))=0, \lambda(\beta'_{1}y(b)-\beta'_{2}y'(b))+(\beta_{1}y(b)-\beta_{2}y'(b))=0 and transmission conditions as -y(0^{+})&\bigg(\lambda\eta-\xi-\sum_{i=1}^{N}\frac{b_{i}^{2}}{\lambda-c_{i}}\bigg)=y'(0^{+})-y'(0^{-}), y'(0^{-})&\bigg(\lambda\kappa+\zeta-\sum_{j=1}^{M}\frac{a_{j}^{2}}{\lambda-d_{j}}\bigg)=y(0^{+})-y(0^{-}). In particular, in the space L^{2}([-a,b])\oplus\mathbb{C}\oplus\mathbb{C}\oplus \mathbb{C}^{N'}\oplus \mathbb{C}^{M'}, the considered problem can be interpreted as the eigenvalue problem of self-adjoint operator A. Moreover, we construct the Green's function of the considered problem and resolvent operator of A.

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