A note on “A novel correlation coefficient of intuitionistic fuzzy sets based on the connection number of set pair analysis and its application”

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Abstract

Garg and Kumar (Scientia Iranica, 2017, https://doi.org/ 10.24200/SCI.2017.4454) proposed some new correlation coefficient between intuitionistic fuzzy sets (IFSs). To point out the advantages of their proposed correlation coefficient over the existing correlation coefficient, Garg and Kumar applied their proposed correlation coefficient as well as the existing correlation coefficient to identify a suitable classifier for an unknown pattern, represented by an intuitionistic fuzzy set (IFS), from the known patterns, each represented by IFS. Garg and Kumar suggested that the existing correlation coefficient fails to identify a suitable classifier, whereas, the correlation coefficient, proposed by them, does not fail to identify a suitable classifier. So, it is appropriate to use the correlation coefficient, proposed by them, instead of the existing correlation coefficient. In this note, it is shown that the correlation coefficient, proposed by Garg and Kumar, also fails to identify a suitable classifier. Furthermore, it is shown that more computational efforts are required to apply the correlation coefficient, proposed by Garg and Kumar, as compared to the existing correlation coefficient. In the actual case, it is inappropriate to apply the correlation coefficient for identifying a suitable classifier.

A note on “A novel correlation coefficient of intuitionistic fuzzy sets based on the connection number of set pair analysis and its application”

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Abstract: Garg and Kumar (Scientia Iranica, 2017, https://doi.org/ 10.24200/SCI.2017.4454) proposed some new correlation coefficient between intuitionistic fuzzy sets (IFSs). To point out the advantages of their proposed correlation coefficient over the existing correlation coefficient, Garg and Kumar applied their proposed correlation coefficient as well as the existing correlation coefficient to identify a suitable classifier for an unknown pattern, represented by an intuitionistic fuzzy set (IFS), from the known patterns, each represented by IFS. Garg and Kumar suggested that the existing correlation coefficient fails to identify a suitable classifier, whereas, the correlation coefficient, proposed by them, does not fail to identify a suitable classifier. So, it is appropriate to use the correlation coefficient, proposed by them, instead of the existing

correlation coefficient. In this note, it is shown that the correlation coefficient, proposed by Garg and Kumar, also fails to identify a suitable classifier. Furthermore, it is shown that more computational efforts are required to apply the correlation coefficient, proposed by Garg and Kumar, as compared to the existing correlation coefficient. In the actual case, it is inappropriate to apply the correlation coefficient for identifying a suitable classifier.

Keywords: Set pair analysis; Connection number (CN); IFS; Pattern recognition; Medical diagnosis; Decision making.

1. Introduction

Garg and Kumar [1] discussed a brief review of the existing correlation coefficient [2-11]. Furthermore, Garg and Kumar [1, Section 4, Eqn. 13, pp. 8] used the existing expression(1) [12] to evaluate the correlation coefficient of three known patterns,$A_1 = \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.1\rangle, \langle x_3, 0.3, 0.3\rangle\}$,$A_2 = \{\langle x_1, 0.5, 0.4\rangle, \langle x_2, 0.7, 0.2\rangle, \langle x_3, 0.4, 0.3\rangle\}andA_3 = \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.1\rangle, \langle x_3, 0.4, 0.3\rangle\}$with the unknown pattern,$B = \{\langle x_1, 0.1, 0.1\rangle, \langle x_2, 1.0, 0.0\rangle, \langle x_3, 0.0, 1.0\rangle\}$and shown that the obtained correlation coefficient between $A_1$and B, the obtained correlation coefficient between A_2 and$B$ as well as the obtained correlation coefficient between $A_3$and B are equal. Therefore, expression (1) [12] cannot be used to identify a suitable pattern, for the unknown pattern$B$, from the known patterns A_1, A_2, and A_3.

$$\mathrm{ZL}(A,B) = \frac{\sum_{t=1}^{n}(u_A(x_t)u_B(x_t)+v_A(x_t)v_B(x_t)+\pi_A(x_t)\pi_B(x_t))}{\sqrt{\sum_{t=1}^{n}\left(u_A^2(x_t)+v_A^2(x_t)+\pi_A^2(x_t)\right).\sum_{t=1}^{n}\left(u_B^2(x_t)+v_B^2(x_t)+\pi_B^2(x_t)\right)}} (1)$$

Garg and Kumar [1, Section 4, Eqn. 14, pp. 9] used the existing expression (2) [13] to evaluate the correlation coefficient between the IFSs,$A = \{\langle x_1, 0.1, 0.2\rangle, \langle x_2, 0.2, 0.1\rangle, \langle x_3, 0.29, 0.0\rangle\}andB = \{\langle x_1, 0.1, 0.3\rangle, \langle x_2, 0.2, 0.2\rangle, \langle x_3, 0.29, 0.1\rangle\}$and shown that the obtained correlation coefficient between A and$B$ is 1, which is mathematically incorrect as it indicates that the IFSs A and B are equal. Whereas, it is obvious that both the IFSs are not equal. Therefore, the existing expression(2) [13] cannot be used to obtain correlation coefficient between A and B.

$$r(A,B) = \frac{r_1(A,B)+r_2(A,B)+r_3(A,B)}{3} (2)$$

where,

$$r_1(A,B) = \frac{\sum_{t=1}^{n}(u_A(x_t)-\overline{u_A})(u_B(x_t)-\overline{u_B})}{\sqrt{\sum_{t=1}^{n}(u_A(x_t)-\overline{u_A})^2.\sum_{t=1}^{n}(u_B(x_t)-\overline{u_B})^2}},$$

$$r_2(A,B) = \frac{\sum_{t=1}^{n}(v_A(x_t)-\overline{v_A})(v_B(x_t)-\overline{v_B})}{\sqrt{\sum_{t=1}^{n}(v_A(x_t)-\overline{v_A})^2.\sum_{t=1}^{n}(v_B(x_t)-\overline{v_B})^2}},$$

$$r_3(A,B) = \frac{\sum_{t=1}^{n}(\pi_A(x_t)-\overline{\pi_A})(\pi_B(x_t)-\overline{\pi_B})}{\sqrt{\sum_{t=1}^{n}(\pi_A(x_t)-\overline{\pi_A})^2.\sum_{t=1}^{n}(\pi_B(x_t)-\overline{\pi_B})^2}},$$

$$\overline{u_A} = \frac{1}{n}\sum_{t=1}^{n}u_A(x_t), \overline{u_B} = \frac{1}{n}\sum_{t=1}^{n}u_B(x_t),$$

$$\overline{v_A} = \frac{1}{n}\sum_{t=1}^{n}v_A(x_t), \overline{v_B} = \frac{1}{n}\sum_{t=1}^{n}v_B(x_t),$$

$$\overline{\pi_A} = \frac{1}{n}\sum_{t=1}^{n}\pi_A(x_t), \overline{\pi_B} = \frac{1}{n}\sum_{t=1}^{n}\pi_B(x_t).$$

Garg and Kumar [1, Section 4, Eqn. 15, pp. 10] used the existing expression (3) [14] to evaluate the correlation coefficient between the three known patterns,$A_1 = \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.1\rangle, \langle x_3, 0.3, 0.3\rangle\}$,$A_2 = \{\langle x_1, 0.5, 0.4\rangle, \langle x_2, 0.7, 0.2\rangle, \langle x_3, 0.4, 0.3\rangle\}andA_3 = \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.7, 0.1\rangle, \langle x_3, 0.4, 0.3\rangle\}$with the unknown pattern,$B = \{\langle x_1, 0.1, 0.1\rangle, \langle x_2, 1.0, 0.0\rangle, \langle x_3, 0.0, 1.0\rangle\}$and shown that the obtained correlation coefficient between $A_1$and B, the obtained correlation coefficient between A_2 and$B$ as well as the obtained correlation coefficient between $A_3$and B are equal. Therefore, expression (3) [14] cannot be used to identify a suitable pattern, for the unknown pattern$B$, from the known patterns A_1, A_2 and A_3.

$$Xu_1(A,B) = \frac{\sum_{t=1}^{n}(u_A(x_t)u_B(x_t)+v_A(x_t)v_B(x_t)+\pi_A(x_t)\pi_B(x_t))}{\max\left\{\sum_{t=1}^{n}\left(u_A^2(x_t)+v_A^2(x_t)+\pi_A^2(x_t)\right).\ \sum_{t=1}^{n}\left(u_B^2(x_t)+v_B^2(x_t)+\pi_B^2(x_t)\right)\right\}} (3)$$

To overcome this limitation of the existing expressions$(1)-(3)$, Garg and Kumar [1], firstly, proposed the expression (4) [1, Section 3, Eqn. 6, pp. 4] to transform an intuitionistic fuzzy number (IFN)$\langle\mu_p(x_t),\nu_p(x_t)\rangle$into a CN$a_p(x_t)+b_p(x_t)\,i+c_p(x_t)\,j$[15]. Then, using the proposed expression (4), Garg and Kumar [1] proposed expression (5) [1, Section 3, Eqn. 9, pp. 5], expression (6) [1, Section 3, Eqn. 10, pp. 7] for evaluating the correlation coefficient and expression (7) [1, Section 3, Eqn. 11, pp. 7], expression (8) [1, Section 3, Eqn. 12, pp. 7], for evaluating the weighted correlation coefficient between two IFSs$A=\{\langle\mu_1(x_t),\nu_1(x_t)\rangle\ \}andB=\{\langle\mu_2(x_t),\nu_2(x_t)\rangle\,,\ \}\quad t=1,2,\ldots,n$.

$a_p(x_t)=\mu_p(x_t)(1-\nu_p(x_t))$,$b_p(x_t)=1-\mu_p(x_t)(1-\nu_p(x_t))-\nu_p(x_t)(1-\mu_p(x_t))andc_p(x_t)=\nu_p(x_t)(1-\mu_p(x_t))$(4)

$$K_1(A,B)=\frac{\sum_{t=1}^{n}(a_1(x_t)a_2(x_t)+b_1(x_t)b_2(x_t)+c_1(x_t)c_2(x_t))}{\sqrt{\sum_{t=1}^{n}\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right).\ \sum_{t=1}^{n}\left(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\right)}}(5)$$

$$K_2(A,B)=\frac{\sum_{t=1}^{n}(a_1(x_t)a_2(x_t)+b_1(x_t)b_2(x_t)+c_1(x_t)c_2(x_t))}{\max\left\{\sum_{t=1}^{n}\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right),\ \sum_{t=1}^{n}\left(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\right)\right\}}(6)$$

$$K_3(A,B)=\frac{\sum_{t=1}^{n}w_t(a_1(x_t)a_2(x_t)+b_1(x_t)b_2(x_t)+c_1(x_t)c_2(x_t))}{\sqrt{\sum_{t=1}^{n}w_t\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right).\ \sum_{t=1}^{n}w_t\left(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\right)}}(7)$$

$$K_4(A,B)=\frac{\sum_{t=1}^{n}w_t(a_1(x_t)a_2(x_t)+b_1(x_t)b_2(x_t)+c_1(x_t)c_2(x_t))}{\max\left\{\sum_{t=1}^{n}w_t\left(a_1^2(x_t)+b_1^2(x_t)+c_1^2(x_t)\right),\ \sum_{t=1}^{n}w_t\left(a_2^2(x_t)+b_2^2(x_t)+c_2^2(x_t)\right)\right\}}(8)$$

In this note, it is shown that the correlation coefficients$(5)-(8)$, proposed by Garg and Kumar [1], also fails to identify a suitable classifier. Furthermore, it is shown that more computational efforts are required to apply the correlation coefficients $(5)-(8)$, proposed by Garg and Kumar [1], as compared to the existing correlation coefficients $(1)-(3)$. In the actual case, it is inappropriate to apply the correlation coefficient for identifying a suitable classifier.

2. Counter examples for Garg and Kumar's the correlation coefficient

As discussed in Section 1, Garg and Kumar [1] have shown that the existing correlation coefficients$(1)-(3)$ fails to identify a suitable classifier for the unknown pattern$B=\{\langle x_1,0.1,0.1\rangle\,,\ \langle x_2,1.0,0.0\rangle\,,\ \langle x_3,0.0,1.0\rangle\}$from the three known patterns,$A_1=\{\langle x_1,0.4,0.5\rangle\,,\ \langle x_2,0.7,0.1\rangle\,,\ \langle x_3,0.3,0.3\rangle\}$,$A_2=\{\langle x_1,0.5,0.4\rangle\,,\ \langle x_2,0.7,0.2\rangle\,,\ \langle x_3,0.4,0.3\rangle\}andA_3=\{\langle x_1,0.4,0.5\rangle\,,\ \langle x_2,0.7,0.1\rangle\,,\ \langle x_3,0.4,0.3\rangle\}$. Therefore, it is inappropriate to use the existing correlation coefficients $(1)-(3)$.

On the same direction, in this section two known patterns$A_1=\{\langle x_1,0.1,0.4\rangle\,,\ \langle x_2,0.4,0.3\rangle\,,\ \langle x_3,0.25,0.35\rangle\}$,$A_2=\{\langle x_1,0.4,0.1\rangle\,,\ \langle x_2,0.3,0.4\rangle\,,\ \langle x_3,0.35,0.25\rangle\}$and an unknown pattern$B=\{\langle x_1,0.3,0.3\rangle\,,\ \langle x_2,0.2,0.2\rangle\,,\ \langle x_3,0.1,0.1\rangle\}$, represented by IFSs, are considered. Also, the weights of a relative importance are considered as $(0.40,\ 0.45,0.15)$, and shown that the correlation coefficients$(5)-(8)$, proposed by Garg and Kumar [1], also fails to identify that either A_1 or A_2 is a suitable classifier for the unknown pattern B.

To apply the correlation coefficients$(5)-(8)$ [1], proposed by Garg and Kumar [1], firstly, there is a need to transform each element ofA_1,A_2, and B into a CN.

Using the expression (4), proposed by Garg and Kumar [1] for transforming an IFN into a CN, the elements$\langle 0.1,0.4\rangle\,,\langle 0.4,0.3\rangle,\langle 0.25,0.35\rangle,\langle 0.4,0.1\rangle\,,\langle 0.3,0.4\rangle\,,\langle 0.35,0.25\rangle\,,\langle 0.3,0.3\rangle,\langle 0.2,0.2\rangle$ and$\langle 0.1,0.1\rangle$ can be transformed into its equivalent CNs $\langle 0.06,0.58,0.36\rangle,\langle 0.28,0.54,0.18\rangle,\langle 0.1625,0.575,0.2625\rangle,\langle 0.36,0.58,0.06\rangle,\langle 0.18,0.54,0.2$
and$\langle 0.09,0.82,0.09\rangle$ respectively. Therefore,A_1, A_2 and B in terms of CNs can be rewritten as

$$A_1=\{\langle x_1,0.06,0.58,0.36\rangle\,,\ \langle x_2,0.28,0.54,0.18\rangle\,,\ \langle x_3,0.1625,0.575,0.2625\rangle\},$$

$$A_2=\{\langle x_1,0.36,0.58,0.06\rangle\,,\ \langle x_2,0.18,0.54,0.28\rangle\,,\ \langle x_3,0.2625,0.575,0.1625\rangle\},$$

and,

$$B=\{\langle x_1,0.21,0.58,0.21\rangle\,,\ \langle x_2,0.16,0.68,0.16\rangle\,,\ \langle x_3,0.09,0.82,0.09\rangle\}.$$

Now,

1. Using the existing expression (5), proposed by Garg and Kumar [1] for evaluating the correlation coefficient between IFSs,$K_1(A_1, B) = 0.946359402$ and$K_1(A_2, B) = 0.946359402$. Since $K_1(A_1, B) = K_1(A_2,\ B)$so it is not possible to identify the suitable classifier for the unknown pattern B from the known patterns A_1 and A_2. Hence, the limitation pointed out by Garg and Kumar [1] in the existing correlation coefficients$(1) - (3)$, is also occurring in Garg and Kumar's expression (5) [1].
2. Using the existing expression (6), proposed by Garg and Kumar [1] for evaluating the correlation coefficient between the IFS,

$K_2(A_1, B) = 0.84530981$ and$K_2(A_2, B) = 0.84530981$.Since $K_2(A_1, B) = K_2(A_2, B)$ so it is not possible to identify the suitable classifier for the unknown pattern B from the known patterns A_1 and A_2. Hence, the limitation pointed out by Garg and Kumar [1] in the existing correlation coefficients$(1) - (3)$, is also occurring in Garg and Kumar's expression (6) [1].

Using the existing expression (7), proposed by Garg and Kumar [1] for evaluating the correlation coefficient between the IFSs,

$K_3(A_1, B) = 0.951828261$and $K_3(A_2, B) = 0.951828261$. Since $K_3(A_1, B) = K_3(A_2, B)$ so it is not possible to identify the suitable classifier for the unknown pattern B from the known patterns A_1 and A_2. Hence, the limitation pointed out by Garg and Kumar [1] in the existing correlation coefficients$(1) - (3)$, is also occurring in Garg and Kumar's expression (7) [1].

Using the existing expression (8), proposed by Garg and Kumar [1] for evaluating the correlation coefficient between the IFSs,

$K_4(A_1, B) = 0.881829449$ and$K_4(A_2, B) = 0.881829449$.

Since $K_4(A_1, B) = K_4(A_2, B)$ so it is not possible to identify the suitable classifier for the unknown pattern B from the known patterns A_1 and A_2.

Hence, the limitation pointed out by Garg and Kumar [1] in the existing correlation coefficients$(1) - (3)$, is also occurring in Garg and Kumar's expression (8) [1].

3. Advantages of existing correlation coefficients over Garg and Kumar's correlation coefficients

It is obvious from Section 1 and Section 2 that although neither the existing correlation coefficients$(1) - (3)$ nor the correlation coefficients$(5) - (8)$, proposed by Garg and Kumar [1], can be used for identifying a suitable classifier. But, to apply the correlation coefficients$(5) - (8)$, proposed by Garg and Kumar [1], there is a need to transform each element of the known patterns, represented by an intuitionistic fuzzy number, into a CN. While applying the existing correlation coefficients$(1) - (3)$, no such transformation is required i.e., more computational efforts are required for applying the correlation coefficients $(5) - (8)$, proposed by Garg and Kumar [1], as compared to the existing correlation coefficients $(1) - (3)$. Therefore, it is better to use the existing correlation coefficients$(1) - (3)$ as compared to Garg and Kumar's correlation coefficients $(5) - (8)$[1].

4. Conclusions

It is shown that the limitation, pointed out by Garg and Kumar [1] in the existing correlation coefficients$(1) - (3)$, is also occurring in the correlation coefficients $(5) - (8)$. Also, it is pointed out that more computational efforts are required to apply in the existing correlation coefficients$(5) - (8)$ as compared to the correlation coefficients $(1) - (3)$.

COMPLIANCE WITH ETHICAL STANDARDS

CONFLICT OF INTEREST.

The authors declare that they do not have any financial or associative interest indicating a conflict of interest in about submitted work.

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